JOEEP

e-ISSN: 2651-5318
Homepage: http://dergipark.org.tr/joeep

Araştırma Makalesi ● Research Article

Forewarned is forearmed: Forecasting expansions and contractions of the Saudi Economy

Tedbiri Elden Bırakma: Suudi Ekonomisine İlişkin Genişleme Ve Daralma Tahminleri

Pierre Rostan [a, *], Alexandra Rostan [b] & Mohammad Nurunnabi [c]

[a] American University of Iraq Baghdad, College of Business, Airport Road, Baghdad / Iraq
ORCID: 0000-0003-1046-0214
[b] American University of Iraq Baghdad, College of Business, Airport Road, Baghdad / Iraq
ORCID: 0000-0002-8204-1361
[c] Prince Sultan University: Salahuddin, Riyadh 12435, Riyadh / Saudi Arabia
ORCID: 0000-0003-0848-3556

MAKALE BİLGİSİ

Makale Geçmişi:
Başvuru tarihi: 1 Nisan 2023
Düzeltme tarihi: 28 Nisan 2023
Kabul tarihi: 3 Mayıs 2023

Anahtar Kelimeler:
Dalgacık Analizi
Çok Ölçekli Temel Bileşen Analizi
Suudi Arabistan

ÖZ

Bu çalışmada, Suudi ekonomisinin 2021 ile 2060 yılları arasındaki genişlemelerini ve daralmalarını belirlemeye çalışılmaktadır. Suudi ekonomisi petrole dayalı olduğundan, petrol fiyatı ile Suudi Reel GSYH'sı arasındaki özel ilişki, dalgacık analiziyle birleştirilmiş Çok Ölçekli Temel Bileşen Analizi (MPCA) ile ortaya konulmaya çalışılmıştır. MPCA analizini içeren ve içermeyen iki senaryo varsayıldığında, daha olası olan orta nokta senaryosu, 2021-2060 ortalama Reel GSYH yıllık büyüme oranını - %0,37 olarak göstermektedir. Suudi Arabistan, İran ile kıyaslandığında 2021-2060 petrol fiyatı tahminleri, 2020'den sonra Suudi Arabistan'ın GSYH'sını yukarı çekecek bir toparlanma öngörülmektedir. Ancak 2031'de Suudi Arabistan ve İran'ın GSYH büyüme oranları farklılaşmaktadır. İran'ın büyüme oranı 2044'e kadar pozitif bölgede kalırken, Suudi ekonomisinin 2048'e kadar uzun bir durgunluk yaşayacağı görülmektedir. 2048'den sonra iki ekonomi durgunluktan çıkacak ama 2060 öncesinde geri döneceklerdir.

ARTICLE INFO

Article history:
Received: April 1, 2022
Received in revised form: April 28, 2023
Accepted: May 3, 2023

Keywords:
Wavelet Analysis
Multiscale Principal Component Analysis
Saudi Arabia

ABSTRACT

This study attempts to determine the expansions and contractions of the Saudi economy between 2021 and 2060. The Saudi economy being oil-driven, the special relationship between oil price and Saudi Real GDP is unfolded with Multiscale Principal Component Analysis (MPCA) combined with wavelet analysis. Assuming two scenarios with and without MPCA, the more likely midpoint scenario returns a 2021-2060 average of the Real GDP annual growth rate of - 0.37%. Saudi Arabia is benchmarked to Iran. 2021-2060 estimates of oil price forecast a rebound after 2020 that will pull up Saudi Arabia's GDP. However, in 2031, Saudi Arabia's and Iran's GDP growth rates will diverge, Iran's growth rate remaining in positive territory until 2044, whereas Saudi economy enduring a lengthy recession until 2048. After 2048, the two economies will emerge from recession but will eventually return to it before 2060.

1. Introduction

This work is an application of Surah Yusuf, verse 43 (Holy Quran, 2023) which relates to the Famine episode which took place in Egypt around 1700s BC during the sixteenth Dynasty of King Ibbabi I (ElKholy, 2017). The Famine lasted seven years and no grain grew anywhere in the Near East. 'The King said, "I see seven fat cows being eaten by seven lean ones, and seven green spikes, and others dried up. O elders, explain to me my vision, if you are able to interpret visions."' Thanks to Prophet Yusuf's (Alayhi al-salām) interpretation of the King's dream and its wise inventory management, although there was famine in every country of the Near East for seven years, there was food throughout the

* *Sorumlu yazar/Corresponding author*.
e-posta: *rostan.pierre@gmail.com*

Atıf/Cite as: Rostan, P., Rostan A. & Nurunnabı, M. (2023). Forewarned is forearmed: Forecasting expansions and contractions of the Saudi Economy. *Journal of Emerging Economies and Policy*, 8(1), 178-190.

land of Egypt all this time. Amid the COVID-19 pandemic where the global demand for oil collapsed in 2020 and rebounded in 2021, Saudi Arabia's economy must cope nowadays with a roller coaster oil market, the country being an oil economy whose proven oil reserves represent about 16% of the world's reserves. It is the world's largest oil exporter. The relationship between oil price and the Saudi economy is blatant, explained by the oil sector accounting for nearly 87% of its government revenues, 90% of its export earnings and 42% of its GDP (Forbes, 2018). Saudi Arabia needs oil to be priced at around $85 per barrel to fund its budget, but prices have been well short of that mark for years (Riedel, 2020). In 2016, the Saudi government unveiled a plan called Saudi Vision 2030 which has triggered structural economic reforms leading to an unprecedented strategy of transition from an oil-driven economy to a modern market economy. The accompanying National Transformation Plan to Saudi Vision 2030 would enable the Kingdom to diversify its heavily oil-dependent revenue base, reduce its growing budget deficits, balance its budgets, and promote long-term economic growth (Moshashai et al., 2018). This paper attempts to forecast expansions and contractions of the Saudi Economy until 2060, estimating the Real GDP annual growth rate at constant national prices, not seasonally adjusted with wavelet analysis. Economic expansions (contractions) are increases (decreases) in the level of economic activity measured by a rise (decline) in real GDP. The special relationship between oil price and the Saudi GDP is unfolded with Multiscale Principal Component Analysis (MPCA). The transformed Real GDP annual growth rate time series after MPCA is decomposed in simpler signal called approximations and details in the framework of the one-dimensional discrete wavelet analysis. The simplified signals are recomposed after Burg extension. Vision 2030 should ultimately diversify the Saudi economy away from oil dependency in the medium term. The economic forecasts of this paper represent therefore a worst-case scenario that should make government policy makers aware of future economic contractions and expansions of the Saudi economy. Patterns in oil price having a direct impact on Saudi economy are formally identified with MPCA. Signal processing achieves forecasting Real GDP as demonstrated in the methodology section. Next section will review the literature regarding the influence of oil price shocks on economies and presents some applications of signal processing in the literature, especially as a forecasting tool.

2. Literature Review

The literature covering the influence of oil price shocks on financial markets and therefore on the economy using stock markets as proxies is abundant. Among seminal papers, we may cite Huang et al. (1996), Faff and Brailsford (1999), Ciner (2001), El-Sharif et al. (2005), Boyer and Filion (2009), Filis (2010), Arouri and Nguyen (2010), Elyasiani et al. (2011), Broadstock et al. (2012). More recently, the impact of oil price shocks has been reviewed on large emerging countries' stock prices such as China, India, and Russia (Fang and You, 2014). Kang et al. (2015) showed that the structural oil shocks account for a quarter of the long-run variation in real stock returns overall, with substantial change in levels and sources of contribution over time. Gupta (2016) added that oil price shocks positively impact firm-level returns. Soyemi et al. (2017) analyzes the destabilizing direct and indirect effects of oil price shocks on economies. They showed that oil shocks have a direct positive effect and indirect effect on company stock returns. The reasons for using stock market prices in the literature is the convenience of having large datasets of daily stocks and oil prices. Investigating the effect of oil shocks on economic indicators is less tractable since the frequency of macroeconomic data is scarcely monthly, more often quarterly if not annual. It is more challenging to make the data speaking with low-frequency data. A few articles have covered the effect of oil price shocks on Saudi Arabia economy. Al-Nakib (2015) showed that the recent oil price slump is likely to slow growth and widen fiscal deficit in the Saudi economy. Mseddi and Benlagha (2017) showed a bidirectional causality between oil consumption and economic growth. Hemrit and Benlagha (2018) argued that the volatility of oil prices results in large fluctuations which increases the economy's reliance upon the petroleum sector. Abdel-Latif et al. (2018) investigated the effect of oil price shocks on government expenditures on the health and education sectors in Saudi Arabia. Al Rasasi et al. (2019) showed no discernable causality relationship between oil prices and money supply in Saudi Arabia, may be due to prudent and stable fiscal and monetary policy on the part of the central bank, the Saudi Arabian Monetary Authority.

Rostan and Rostan (2022a) forecasted GDPs of Persian Gulf Economies with wavelet analysis until 2050. Persian Gulf Economies being oil-driven, the special relationship between oil price and Persian Gulf Economies was unfolded with Multiscale Principal Component Analysis and integrated in the forecasts. In this paper, the forecasting model similarly aggregates algorithms from wavelet analysis and principal component analysis to consider the special relationship between oil price and the Saudi economy. The added value of this paper is to track the latest impacts of the drastic economic reforms of the Saudi Vision 2030 plan which has focused on diversifying the Saudi economy and transforming the country into a modern market economy. It also estimates periods of expansions and contractions of the Saudi economy between 2021 and 2060. Traditional economic forecasting methods include causal methods (regression analysis, logit, probit), time series methods (moving average, exponential smoothing, trend and seasonal decomposition, Box-Jenkins ARIMA, 1976, 1994) and qualitative methods (Delphi Method, Jury of Executive Opinion, Sales Force Composite, Consumer Market Survey) (FHI, 2021). Signal processing used in this paper to forecast Saudi Real GDP belongs to time series methods. Signal processing is borrowed from Physics and focuses on the analysis, synthesis, and modification of signals. The basic assumption of this paper is that oil price and GDP time series behave like signals propagating through time instead of propagating through space like physics phenomena such as audio, video, speech, geophysical, sonar, radar, medical or musical signals (IEEE, 2019). Wavelet analysis is a tool of signal processing. In Physics, wavelets have practical applications to model physical phenomena such as electrical, audio, or seismic signals which propagate through space in waveforms. Wavelets mimic signals with specific properties that make them useful for signal processing. Signal processing focuses on the analysis, synthesis, and modification of signals. Spectral (or spectrum) analysis focuses on data analysis of signals. More specifically (Stoica and Moses, 2005), from a finite record of a stationary data sequence, spectral analysis estimates how the total power is distributed over frequency. In meteorology, astronomy and other fields, spectral analysis may reveal 'hidden periodicities' in data, which are to be associated with cyclic behavior or recurring processes. Regarding wavelet analysis, forecasters have focused on the Discrete Wavelet Transform (DWT) due to several not tractable properties of Continuous Wavelet Transform (CWT) such as highly redundant wavelet coefficients (Valens, 1999), infinite number of wavelets in the wavelet transform and no analytical solutions found for most functions of the wavelet transforms. A wavelet-based forecasting method using redundant "à trous" wavelet transform and multiple resolution signal decomposition was presented in Renaud et al. (2002). Forecasting day-ahead electricity prices based on the wavelet transform and ARIMA models was a challenge detailed in Conejo et al. (2005). Schlüter and Deuschle (2010) were able to capture seasonalities with time-varying period and intensity, incorporated the wavelet transform to improve forecasting methods. Tan et al. (2010) proposed a price forecasting method based on wavelet transform combined with ARIMA and GARCH models. Kao et al. (2013) integrated wavelet transform, multivariate adaptive regression splines (MARS), and support vector regression (SVR called Wavelet-MARSSVR) to address the problem of wavelet sub-series selection and to improve forecast accuracy.

Ortega and Khashanah (2013) proposed a wavelet neural network model for the short-term forecast of stock returns from high-frequency financial data. Kriechbaumer et al. (2014) showed the cyclical behavior of metal prices. With wavelet analysis, they were able to capture the cyclicality by decomposing a time series into its frequency and time domain. They presented a wavelet-autoregressive integrated moving average (ARIMA) approach for forecasting monthly prices of aluminum, copper, lead and zinc. He et al. (2014) proposed an entropy optimized wavelet-based forecasting algorithm to forecast the exchange rate movement. Berger (2016) transformed financial return series into its frequency and time domain via wavelet decomposition to separate short-run noise from long-run trends and assess the relevance of each frequency to value-at-risk (VaR) forecast. Rostan and Rostan (2018a) illustrated with market data the versatility of wavelet analysis to the forecast of financial times series with distinctive properties. Rostan et al. (2017) forecasted the yield curve with a wavelet analysis model. Rostan et al. (2015) appraised the financial sustainability of the Spanish pension system and Rostan et Rostan (2018b) of the Saudi pension system using wavelet analysis. With a refined methodology using multiscale principal component analysis to consider the co-dynamics of age groups, Rostan and Rostan (2017) forecasted European and Asian populations and the European Muslim population (2019) with signal processing. Rostan and Rostan applied wavelet analysis to the forecasts of Spanish (2018c), Greek (2018d), Austrian (2020), Saudi (2021a), Persian Gulf (2022a), Turkish (2022b), UK (2022c), Korean (2023a) and Australian (2023b) economies. Rostan and Rostan (2021b) applied wavelet analysis to the projections of fossil fuels prices. This paper is an extension of their work. Section 3 presents the methodology. Section 4 gathers the results and section 5 concludes.

3. Methodology

The objective of the paper is to identify future contractions and expansions of the Saudi economy by forecasting until 2060 real GDP annual growth rate at constant national prices, not seasonally adjusted. Figure 1 illustrates historical Real GDP at Constant National Prices for Saudi Arabia and Annual Average nominal oil price of Illinois Crude to demonstrate the strong and positive relationship between these two variables explained by the fact that the oil sector in Saudi Arabia accounts for nearly 87% of its government revenues, 90% of its export earnings and 42% of its GDP (Forbes, 2018). Illinois Crude oil price is a few dollars cheaper per barrel than West Texas Intermediate (WTI) because Illinois Crude requires a bit more refining.

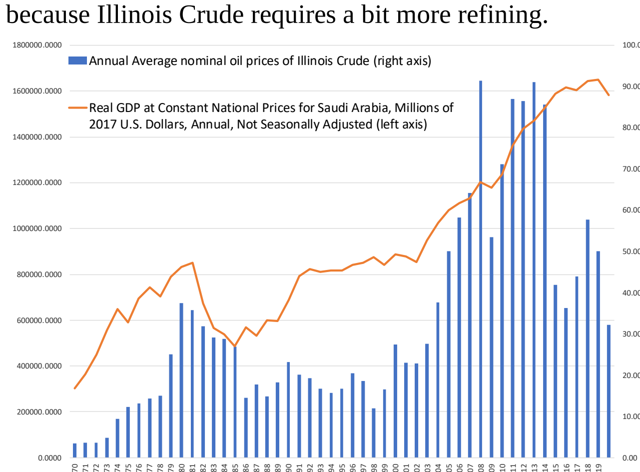


Figure 1: 1970-2020 Real GDP of Saudi Arabia, Annual, and 1970-2020 Annual Average Nominal oil prices of Illinois Crude, Annual (51 annual data for each series) Sources: World Bank, https://data.worldbank.org/country/SA and Inflationdata.com, https://inflationdata.com/articles/inflation-adjusted-prices/historical-crude-oil-prices-table/

From Figure 1, a strong and positive relationship is observed between the two series. The correlation coefficient between the Saudi Real GDP and the Annual Average Nominal oil prices of Illinois Crude is equal to 77%. Signal processing brings together sophisticated tools able to capture and predict the evolving behavior, frequency, rate of change, amplitude, shape and form of oil price and Real GDP through time. Observing physical phenomena such as electrical, audio, or seismic signals, they propagate through space in waveforms. The basic idea of this paper is to apply a model that captures waveforms in physics to the waveform of the oil price and GDP time series, considering the relationship between the two variables. This relationship is captured using Multiscale Principal Component Analysis (MPCA). The forecast of the Real GDP time series after MPCA is realized with wavelet analysis which expand functions in terms of wavelets generated in the form of translations and dilations of a fixed function called the mother wavelet. The resulting wavelets have special scaling properties, localized in time and frequency, permitting a closer connection between the represented function and their coefficients. Wavelet analysis brings greater numerical stability in reconstruction and manipulation of signals (Lee and Yamamoto, 1994).

The forecasting accuracy of the model used in this paper is illustrated with historical data of Saudi Real GDP growth rates by dividing an initial sample of 47 annual observed data between 1971 and 2017 into two samples whose sizes reflect the size of the forecasted window of GDP annual growth rates between 2021 and 2060 (40 forecasted annual rates). The forecasting model is fed with the first 24 in-sample growth rates and forecasts the next 23 in-sample growth rates.

3.1. Step 1: Adjusting the co-dynamics of Real GDP and Annual Average Nominal Oil Price with Multiscale Principal Component Analysis Multiscale Principal Component Analysis (MPCA) is applied to the 24 first in-sample annual growth rates. MPCA and its application to Real GDP and Oil price is explained by an analogy to the processing of parallel time series obtained from several sensors with electroencephalogram (EEG) sensors (Prochazka et al., 2010) or electrocardiogram (ECG) sensors (Castells et al., 2007). MPCA helps interpreting EEG and ECG signals by denoising the signals to have a more simplistic representation of them. Since the signals represent the electrical activity of the brain (EEG) or of the cardiac muscle (ECG) monitored from different angles depending on electrode placements, the information captured by each electrode repeats itself from one signal to another. MPCA aims at reconstructing from the multivariate electrical signal, using a simple representation at each resolution level, a simplified multivariate signal. The analogy between EEG and ECG signals and Saudi Real GDP and the Annual Average Nominal Oil Price (Rostan and Rostan, 2017) is explained by the fact that Saudi Arabia is an oil economy. In EEG and ECG, signals convey similar information at almost the same time, depending on electrode location. MPCA will capture the influence of oil price on the Saudi economy represented by the Real GDP growth rate. The information coming from oil price lags on the Saudi economy. MPCA aims at reconstructing from the two original signals of oil price and Real GDP two new signals, considering the information that repeats itself from one variable to the other. As an illustration of step 1, Figure 2 illustrates the denoising after MPCA of the two original growth rates time series of Saudi Real GDP and oil price.

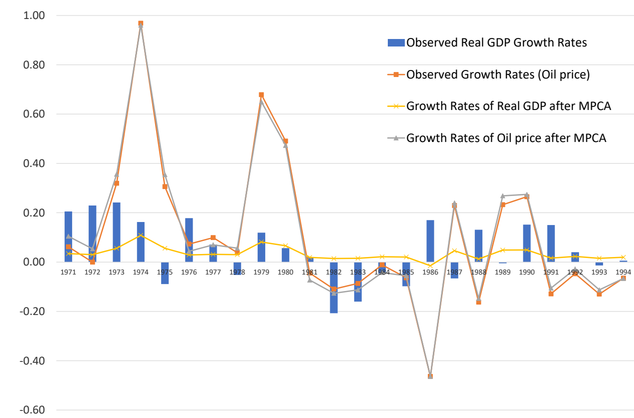


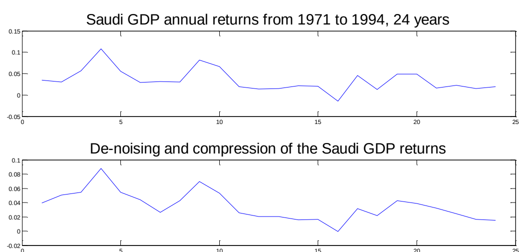
Figure 2: Annual growth rates before and after Multiscale Principal Component Analysis of Real GDP at Constant National Prices for Saudi Arabia, Millions of 2011 U.S. Dollars, Annual, Not Seasonally Adjusted and Annual Average Nominal oil prices of Illinois Crude, Frequency: Annual from 1971 to 1994 (24 annual growth rates) Sources: Federal Reserve Bank of St. Louis and University of Groningen https://fred.stlouisfed.org/series/RGDPNASAA666NRUG and Inflationdata.com https://inflationdata.com/articles/inflation-adjusted-prices/historical-crude-oil-prices-table/

3.2. Step 2: De-noising and Compression of the annual growth rate of the Saudi Real GDP after MPCA The annual growth rate of the Saudi Real GDP after MPCA is de-noised using a onedimensional de-noising and compression-oriented function using wavelets. The function is called 'wdencmp' in Matlab (Misiti et al., 2015).

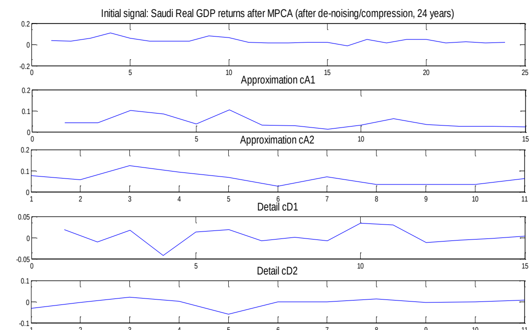
The underlying model for the noisy signal is of the form:

$$s(n) = f(n) + \sigma e(n) \tag{1}$$

where time n is equally spaced, e(n) is a Gaussian white noise N(0,1) and the noise level σ is supposed to be equal to 1. The de-noising objective is to suppress the noise part of the signal s and to recover f. The de-noising procedure proceeds in three steps: 1) Decomposition. We choose the wavelet sym4 and choose the level 2-decomposition. Sym4 is a Symlets wavelet of order 4 used as the mother wavelet for decomposition and reconstruction. It is a nearly symmetrical wavelet belonging to the family of Symlets proposed by Daubechies (1994). We compute the wavelet decomposition of the signal s at level 2. 2) Detail coefficients thresholding. For each level from 1 to 2, we select a threshold and apply soft thresholding to the detail coefficients. 3) Reconstruction. We compute wavelet reconstruction based on the original approximation coefficients of level 2 and the modified detail coefficients of levels from 1 to 2. Like de-noising, the compression procedure contains three steps: 1) Decomposition. 2) Detail coefficient thresholding. For each level from 1 to 2, a threshold is selected, and hard thresholding is applied to the detail coefficients. 3) Reconstruction. The difference with the de-noising procedure is found in step 2. The notion behind compression is based on the concept that the regular signal component can be accurately approximated using a small number of approximation coefficients (at a suitably selected level) and some of the detail coefficients. Figure 3 illustrates the Saudi Real GDP annual growth rate time series after MPCA (24 years, top) and the time series after de-noising and compression (bottom). Figure 3: Saudi Real GDP annual growth rate after MPCA from 1971 to 1994, 24 years (top), De-noising and Compression of Saudi Real GDP annual growth rate (bottom).



3.3. Step 3: Wavelet Decomposition The signal is decomposed after being differentiated, de-noised and compressed. The signal, i.e., the 24-year time series of Saudi Real GDP annual growth rate after MPCA transformed at step 2, is decomposed into decomposed signals cAs named approximations and cDs named details. The Discrete Wavelet Transform is a kind of decomposition scheme evaluated by passing the signal through lowpass and highpass filters (Corinthios, 2009), dividing it into a lower frequency band and an upper band. Each band is subsequently divided into a second level lower and upper bands. The process is repeated, taking the form of a binary, or “dyadic” tree. The lower band is referred to as the approximation cA and the upper band as the detail cD. The two sequences cA and cD are downsampled. The downsampling is costly in terms of data: with multilevel decomposition, at each one-level of decomposition the sample size is reduced by half (in fact, slightly more than half the length of the original signal, since the filtering process is implemented by convolving the signal with a filter. The convolution “smears” the signal, introducing several extra samples into the result). Therefore, the decomposition can proceed only until the individual details consist of a single sample. Thus, the number of levels of decomposition will be limited by the initial number of data of the signal. Figure 3 illustrates the 2nd-level decomposition of Saudi Real GDP annual growth rate after MPCA (after de-noising/compression, 24 years). We observe in Figure 4 that details cDs are small and look like high-frequency noise, whereas the approximation cA2 contains much less noise than does the initial signal. In addition, the higher the level of decomposition, the lower the noise generated by details. For a better understanding of signal decomposition using discrete wavelet transform, refer to the methodology section of Rostan and Rostan (2018a). Figure 4: 2nd-level decomposition of Saudi Real GDP annual growth rate after MPCA (after de-noising/compression, 24 years) using one-dimensional discrete wavelet analysis.

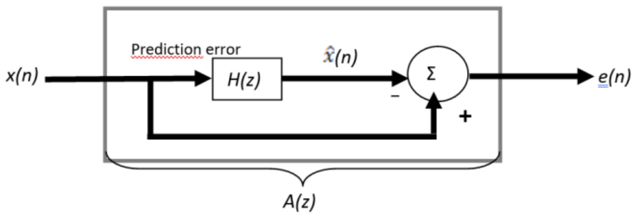


3.4. Step 4: Burg extension of approximations and details We apply Burg extension to cA and cD as presented in Figure 6. To run the Burg extension, we apply an autoregressive p th order from historical data, in this paper we choose a p th order equal to the longest available order when forecasting. For instance in 2017, when forecasting Saudi Real GDP growth rates for the subsequent 43 years, the longest p th order available is 45 out of 47 available data. Given x the decomposed signal (which is cA or cD), we generate a vector a of all-pole filter coefficients that model an input data sequence using the Levinson-Durbin algorithm (Levinson, 1946; Durbin, 1960). We use the Burg (1975) model to fit a p th order autoregressive (AR) model to the input signal, x, by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy Levinson-Durbin recursion. x is assumed to be the output of an AR system driven by white noise. Vector a contains the normalized estimate of the AR system parameters, A(z), in descending powers of z:

$$H(z) = \frac{\sqrt{e}}{A(z)} = \frac{\sqrt{e}}{1+a_2 z^{-1}+\ldots+a_{(p+1)} z^{-p}} \tag{2}$$

Since the method characterizes the input data using an all-pole model, the correct choice of the model order *p* is important. In Figure 5, the prediction error, *e(n)*, can be viewed as the output of the prediction error filter *A(z)*, where *H(z)* is the optimal linear predictor, *x(n)* is the input signal, and *(n)* is the predicted signal.

Figure 5: Prediction error filter to run the Burg extension



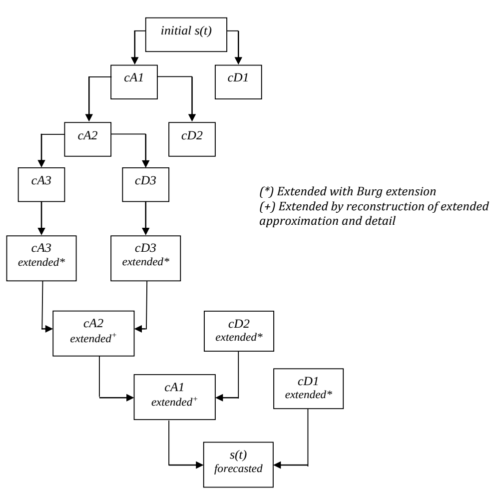
Source: Matlab.

In a last step, the Infinite Impulse Response (*IIR*) filter extrapolates the index values for each forecast horizon. *IIR* filters are digital filters with infinite impulse response. Unlike finite impulse response (*FIR*) filter, *IIR* filter has the feedback (a recursive part of a filter) and is also known as recursive digital filter.

Step 5: Wavelet Reconstruction

We recompose the forecasted signals after Burg extension using the methodology illustrated in Figure 6. We present the 3rd-level decomposition/reconstruction diagram.

Figure 6: Diagram of a 3rd-level wavelet decomposition/reconstruction tree to forecast the initial signal *s(t)*. Source: The Authors.



Assessing the Forecasting Ability of Wavelet Analysis combined with Multiscale Principal Component Analysis (MPCA)

An additional exercise is to assess the forecasting ability of wavelet analysis combined with Multiscale Principal Component Analysis. We divide the 47 Saudi Real GDP annual growth rate time series after MPCA into two subsamples whose sizes reflect the forecast of the GDP between 2021 and 2060 (40 forecasted annual growth rates). The methodology from steps 1 to 5 is applied to the first 24 in-sample growth rates and forecast the next 23 in-sample growth rates. To acknowledge the superior ability of MPCA to provide more accurate forecasts than a plain spectral analysis forecasting method, the GDP annual growth rate forecasts are compared and without MPCA (i.e. with or without step 1).

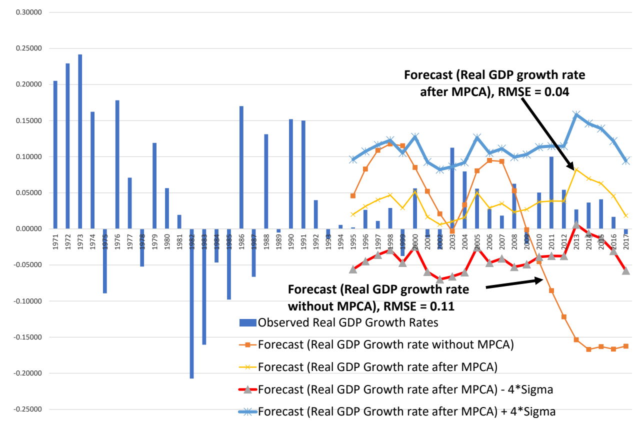


Figure 7: In-sample Saudi Real GDP annual growth rate forecasts from 1995 to 2017 with and without MPCA (23 years), using Spectrum Analysis (level-2 decomposition/reconstruction, pth order = 22).

The method of Multiscale Principal Component Analysis (MPCA) applied to the original time series of annual growth rate of Real GDP and oil price improved the forecasting accuracy of spectrum analysis by almost 3 times based on the Root Mean Square Error (RMSE) criteria applied to the 23 forecasted annual growth rates from 1995 to 2017: the RMSE of forecasted annual growth rates decreases from 0.11 without MPCA to 0.04 with MPCA.

Based on the sample standard deviation of the twenty-three forecasted annual growth rates of the Saudi Real GDP from 1995 to 2017, equal in this example to 1.90%, a plus or minus 4 times the standard deviation interval is built where 95.7% of the observed data are included. The interval is drawn to help improving the interpretation of the forecasts in the Results section since 95% of the forecasted data should be within the interval.

In a second example, steps 1 to 5 are applied to in-sample quarterly growth rates from Q1 2010 to Q2 2014 (18 quarters) of the Saudi GDP quarterly growth rate, seasonally adjusted. We obtained Figure 8 that again illustrates how MPCA improves the forecasting ability of spectrum analysis based on RMSE that decreases from 0.01424 without MPCA to 0.01138 with MPCA. In this case, MPCA is applied to quarterly GDP growth rates and to WTI oil price quarterly average growth rates.

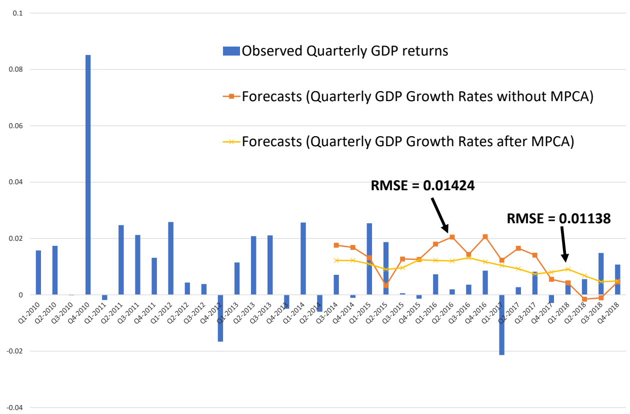


Figure 8: In-sample Saudi GDP quarterly growth rate forecasts from Q3 2014 to Q4 2018 (18 quarters) with and without MPCA, using Spectrum Analysis (level-2 decomposition/reconstruction, pth order = 16). Sources: OECD, https://stats.oecd.org/index.aspx?queryid=33940 and Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org/series/DCOILWTICO

Identifying the optimal level of decomposition/reconstruction

In this section, the optimal level of decomposition/reconstruction of spectral forecasting model combined with MPCA is identified. The historical data are divided into two samples of 24 years versus 23 as explained in the previous section, then the forecasting error is measured, expressed in RMSE over the last 23 in-sample years of real GDP annual growth rates from 1995 to 2017. To generate the forecasts, steps 1 to 5 are applied to the first half of the sample (24 years from 1971 to 1994) making the level of decomposition/reconstruction varying from 1 to 7. Level 1 returns an error message. Figure 9 illustrates the RMSE computed over the last 23 in-sample years of the database (forecasts versus observed data) of Saudi Real GDP growth rates.

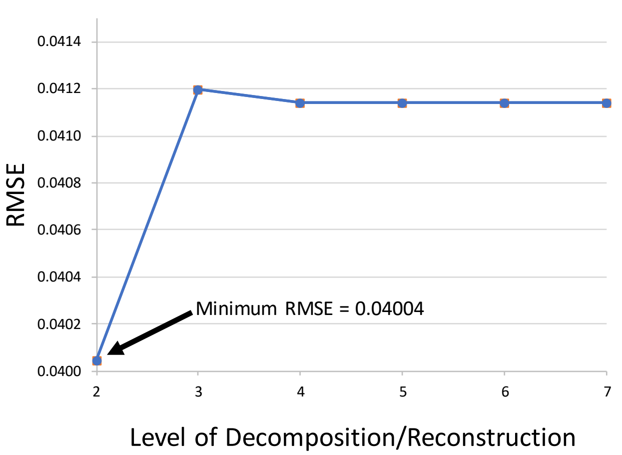


Figure 9: RMSE computed over the last 23 in-sample years of the database (forecasts versus observed data) of Saudi Real GDP Growth Rates.

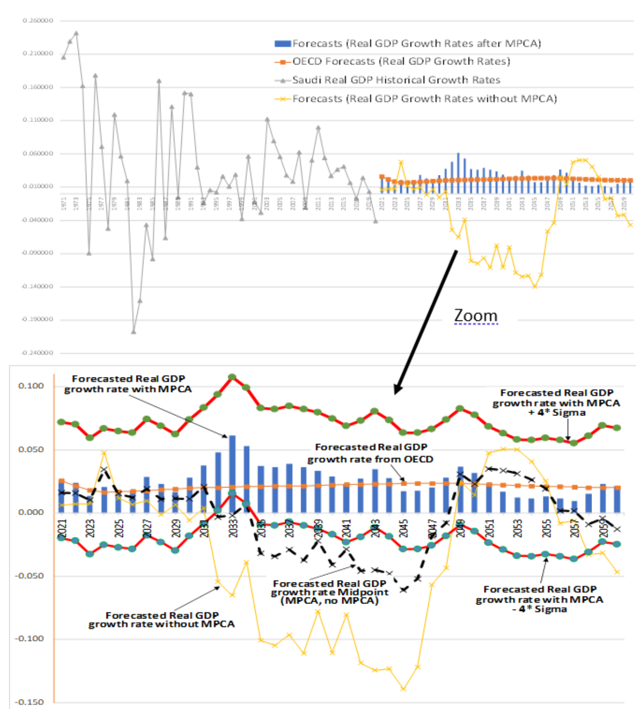
At level 2-decomposition/reconstruction, the RMSE is at its minimum (i.e. 0.04004). Level 2 is therefore the optimal level of decomposition/reconstruction of Saudi annual real GDP growth rate forecasts and is used to generate forecasts in the Results section.

4. Results

The objective of the paper is to forecast expansions and contractions of the Saudi Economy by forecasting real GDP annual growth rate at constant national prices, not seasonally adjusted until 2060. The Saudi economy being oil-driven, the special relationship between oil price and Saudi GDP is unfolded with Multiscale Principal Component Analysis (MPCA). The transformed Real GDP after MPCA is decomposed in simpler signals called approximations and details in the framework of the one-dimensional discrete wavelet analysis. The simplified signals are recomposed after Burg extension.

Figure 10 illustrates 40 forecasts of the Saudi Real GDP annual growth rate from 2021 to 2060 with spectral analysis combined with Multiscale Principal Component Analysis (MPCA). The spectral growth rate forecasts are benchmarked to OECD forecasted growth rates obtained from Real GDP long-term forecasts expressed in Million US dollars (Source: https://data.oecd.org/gdp/gdp-long-term-forecast.htm). OECD forecasts are based on an assessment of the economic climate in individual countries and the world economy, using a combination of model-based analyses and expert judgement. Real GDP is measured in USD at 2010 Purchasing Power Parities.

2021-2060 Saudi Real GDP annual growth rate forecasts



Figures 10: Observed (1971 to 2020) and Forecasted (2021 to 2060) Saudi Real GDP annual growth rates. Source of historical data: World Bank, https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=SA and OECD https://data.oecd.org/gdp/gdp-long-term0-forecast.htm

From Figures 10, 2021-2060 spectral analysis forecasts of the Real GDP annual growth rate are on average more optimistic than OECD forecasts, both models forecasting expansions for the 2021-2060 period: 2021-2060 annual average forecast with spectral analysis is equal to 2.62% versus 2.11% with OECD. In the case the MPCA (Multiscale Principal Component Analysis) is not applied to the time series of Saudi Real GDP annual growth rate, 2021-2060 projections are more pessimistic with an annual average forecast of -3.35%. Assuming that the model assumption without MPCA represents the worst-case scenario and the model assumption with MPCA the best case scenario, the more likely scenario stands between the two values with an annual average forecast of -0.37%, which is illustrated in Figures 10 (bottom figure) with the Forecasted Real GDP Midpoint time series.

A plus or minus 4 times the standard deviation interval is built within which 95% of the forecasted data of the Saudi Real GDP annual growth rate should be included based on the findings of the methodology section "Assessing the Forecasting Ability of Wavelet Analysis" applied to in-sample data. The population standard deviation is computed from the forty 2021-2060 forecasted data with MPCA and spectral analysis and is equal to 1.64%. This interval is centered on the estimates of the Saudi Real GDP annual growth rate forecasted with the MPCA method. The interval is drawn to help improving the interpretation of the forecasts since 95% of the forecasted data should be within the interval. Table 1 gathers the values of the three scenarios. Based on Figures 10 (bottom figure) and the Midpoint scenario of the Real GDP growth rate in Table 1, two periods of contraction are forecasted between 2032 and 2048 and 2058 and 2060. Periods of expansions will occur the remaining years.

Table 1: 2021-2060 Saudi Real GDP annual growth rate forecasted values based on three scenarios: forecasts with MPCA, without MPCA and Midpoint.

Year	With MPCA	Midpoint	Without MPCA	Year	With MPCA	Midpoint	Without MPCA
2021	2.60%	1.61%	0.61%	2041	2.28%	-2.88%	-8.04%
2022	2.39%	1.56%	0.73%	2042	2.70%	-4.59%	-11.87%
2023	1.33%	1.03%	0.74%	2043	3.45%	-4.50%	-12.45%
2024	2.07%	3.43%	4.78%	2044	2.75%	-4.80%	-12.34%
2025	1.87%	1.52%	1.17%	2045	1.74%	-6.11%	-13.95%
2026	1.75%	1.22%	0.69%	2046	1.76%	-5.22%	-12.19%
2027	2.83%	1.90%	0.97%	2047	2.02%	-1.82%	-5.66%
2028	2.30%	1.09%	-0.13%	2048	2.80%	-0.79%	-4.37%
2029	1.62%	1.14%	0.67%	2049	3.66%	3.08%	2.49%
2030	2.80%	1.12%	-0.56%	2050	3.16%	2.30%	1.43%
2031	3.75%	2.08%	0.41%	2051	2.26%	3.51%	4.75%
2032	4.80%	-0.32%	-5.44%	2052	1.69%	3.38%	5.06%
2033	6.14%	-0.19%	-6.52%	2053	1.21%	3.13%	5.04%
2034	5.31%	0.72%	-3.88%	2054	1.16%	2.62%	4.07%
2035	3.70%	-3.20%	-10.10%	2055	1.33%	1.93%	2.53%
2036	3.62%	-3.43%	-10.48%	2056	1.16%	0.17%	-0.81%
2037	3.88%	-2.89%	-9.66%	2057	0.93%	0.16%	-0.62%
2038	3.61%	-3.75%	-11.11%	2058	1.50%	-0.91%	-3.32%
2039	3.35%	-2.22%	-7.78%	2059	2.32%	-0.42%	-3.15%
2040	2.89%	-4.09%	-11.06%	2060	2.12%	-1.29%	-4.69%

2021-2060 Saudi Real GDP annual forecasts

Figure 11 illustrates the projections of the Saudi Real GDP with and without MPCA and the midpoint scenario of the Saudi Real GDP forecasts. It also represents the 2021-2060 projections of Annual Average Nominal oil prices of Illinois

Crude.

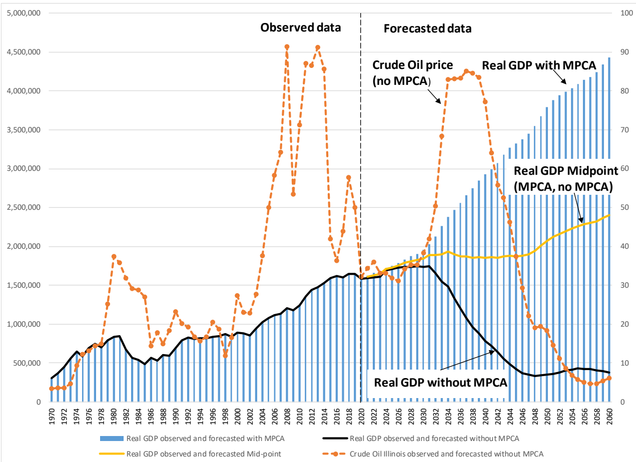


Figure 11: Observed (1971 to 2020) and Forecasted (2021 to 2060) Saudi Real GDP, annual frequency, using 3 scenarios: MPCA, no MPCA, Mid-point. Source of historical data: World Bank, https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=SA and OECD https://data.oecd.org/gdp/gdp-long-term0-forecast.htm, Observed (1971 to 2020) and Forecasted (2021 to 2060) Annual Average Nominal oil prices of Illinois Crude, not using MPCA Source of historical data: Inflationdata.com, https://inflationdata.com/articles/inflation-adjusted-prices/historical-crude-oil-prices-table/

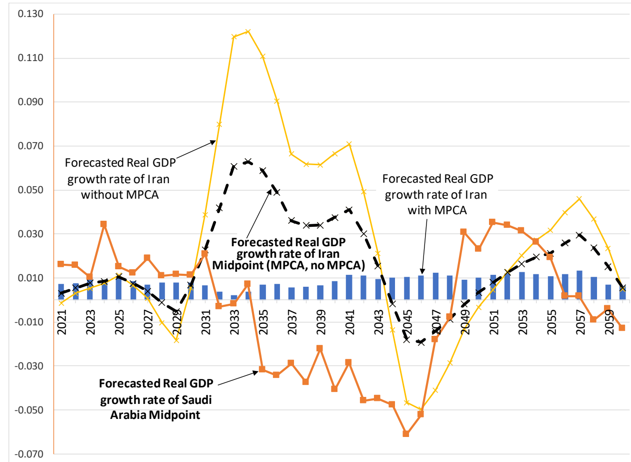
Left axis: Real GDP in Millions of 2017 U.S. Dollars

Right axis: Crude Oil Price in USD

Based on the 2021-2060 projections of Figure 11, the 2021-2060 annual growth rate of the midpoint scenario of the Saudi Real GDP forecasts is +2.61% and the 2021-2060 annual growth rate of the Illinois Crude oil price is -4.08%. The oil projections are pessimistic, which is a strong signal for the Saudi government to diversify its economy away from petroleum products, which is the main objective of the Vision 2030 plan implemented in 2016.

2021-2060 Saudi Real GDP annual forecast Midpoints benchmarked to Iran Real GDP annual forecasts

In a final exercise, 2021-2060 Saudi Real GDP annual forecast Midpoints are benchmarked to Iran Real GDP annual forecasts, which are illustrated in Figure 12. Why Iran? First, Iran is an oil-producing country like Saudi Arabia, Iran being ranked 9th with a crude oil production of 2,546,336 barrels per day in 2021 versus 9,313,145 barrels per day in Saudi Arabia, ranked 3rd (OPEC, 2022). Second, Iran's economy was 3rd in terms of current US$ GDP (231.55 Billion USD estimate) among the Persian Gulf economies positioning after Saudi Arabia (733.37 Billion USD GDP estimate, World Bank, 2023) and UAE. Third, Iran is a slightly more diversified economy than Saudi Arabia, the 2020 percentage estimate of GDP of Revenue minus production cost of oil was equal to 17.15% for Iran when it was 17.7% for Saudi Arabia (The Global Economy, 2022a and 2022b).



Figures 12: Forecasted (2021 to 2060) Real GDP annual growth rates of Saudi Arabia and Iran. Source of historical data: World Bank, https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=SA and https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=IR

Commenting on the impact of the COVID-19 pandemic on Saudi Arabia and Iran, the 2020 Real GDP annual growth rate were respectively -4.11% for Saudi Arabia and +1.66% for Iran (World Bank, 2023). Iran has better coped with the pandemic due to its more diversified economy when oil price fell in 2020, the annual average Crude Oil Illinois price per barrel decreasing from $50.01 to $32.25 (-35.51%), following a collapse in the global demand for oil and an oversupplied industry due to the pandemic. From Figure 12, 2021 midpoint forecasts of Real GDP growth rate look better for Saudi Arabia (+1.6%) than Iran (+0.3%). This positive trend in 2021 for Saudi Arabia was confirmed by Saudi Aramco, the Saudi world's biggest energy company, which announced a surge in profit in 2021, following positive results of big oil rivals (Wallace and Martin, 2021). 2021 second quarter net profit rose to $25.5b, highest since end of 2018. Figure 11 illustrates a rebound of oil price after 2020 that will pull up Saudi Arabia's GDP. However, in 2031, Saudi Arabia's and Iran's GDP growth rates will diverge, Iran's growth rate remaining in positive territory until 2044, whereas Saudi economy will endure a lengthy recession until 2048. After 2048, the two economies will reemerge from recession but will eventually return into it before 2060.

5. Conclusion

The objective of the paper is to identify future contractions and expansions of the Saudi economy between 2021 and 2060 with spectral analysis combined with Multiscale Principal Component Analysis (MPCA). Spectral analysis has stirred interest for its ability to analyze changing transient physical signals. Extending the analysis to complex-behavior economic time series, the originality of this paper is to apply MPCA and spectral analysis to economic variables subject to common dynamics such as real GDP and oil price time series. The Saudi economy being oil-driven, the special relationship between oil price and Saudi Real GDP is unfolded with Multiscale Principal Component Analysis (MPCA). The transformed Real GDP annual growth rate time series after MPCA is decomposed in simpler signal called approximations and details in the framework of the one-dimensional discrete wavelet analysis. The simplified signals are recomposed after Burg extension. 2021-2060 Real GDP annual growth rate forecasts with spectral analysis and MPCA are on average more optimistic than OECD forecasts, both models forecasting expansions for the 2021-2060 period. 2021-2060 annual average forecast with spectral analysis is equal to 2.62% versus 2.11% with OECD. In the case the MPCA is not applied to the time series of Saudi Real GDP annual growth rate, 2021-2060 projections are more pessimistic with an annual average forecast of -3.35%. Assuming that the model assumption without MPCA represents the worst case scenario and the model assumption with MPCA the best case scenario, the more likely scenario stands between the two values with an annual average forecast of -0.37%. A plus or minus 4 times the standard deviation interval is built where 95% of the forecasted data of the Saudi Real GDP annual growth rate should be included based on the finding of in-sample data. The standard deviation population is computed from the forty 2021-2060 forecasted data. The interval is centered on the estimates of the Saudi Real GDP annual growth rate forecasted with the MPCA method.

In addition, Saudi Arabia's economy is benchmarked to the one of Iran. Based on the 2020 Real GDP annual growth rate respectively equal to negative 4.11% for Saudi Arabia and plus 1.66% for Iran (World Bank, 2023), Iran has better coped with the pandemic due to its more diversified economy when oil price fell in 2020, following a collapse in the global demand for oil and an oversupplied industry. 2021-2060 estimates of oil price forecast a rebound of oil price after 2020 that will pull up Saudi Arabia's GDP. However, in 2031, Saudi Arabia's and Iran's GDP growth rates will diverge, Iran's growth rate remaining in positive territory until 2044, whereas Saudi economy will endure a lengthy recession until 2048. After 2048, the two economies will reemerge from recession but will eventually return into it before 2060.

With this research, Saudi government policymakers and planners, economists and investors should have with Real GDP annual growth rate forecasts a better insight, understanding and outlook of future periods of contractions and expansions of the Saudi economy.

Finally, the benefit of adding Multiscale Principal Component Analysis to spectrum analysis for the forecast of GDP is illustrated with two examples that show the relevance of adding spectral analysis combined with Multiscale Principal Component Analysis to the battery of tools used by econometricians and quantitative analysts for the forecast of economic time series.

References

Abdel-Latif, H., R.H. Osman, & Ahmed, H. (2018). Asymmetric impacts of oil price shocks on government expenditures: Evidence from Saudi Arabia. Cogent Economics and Finance, 6(1), Retrieved from: https://doi.org/10.1080/23322039.2018.1512835 .

Al-Nakib, O. (2015). Saudi Arabia: Economy resilient but growth slowing amid the oil price slump. Macroeconomic Outlook, Economic Update series, NKB Saudi Arabia.

Al Rasasi, M., Qualls, J.H., & Almutairi, S. (2019). Testing for Causality between Oil Prices and Money Supply in Saudi Arabia. Saudi Arabian Monetary Authority working paper,

Retrieved from: http://www.sama.gov.sa/en-US/EcomicResearch/WorkingPapers/Testing%20for%20Causality%20Between%20Oil%20Prices%20and%20Money%20Supply%20in%20Saudi%20Arabia.pdf .

Arouri, M.E.H., & Nguyen. D.K. (2010). Oil prices, stock markets and portfolio investment: Evidence from sector analysis in Europe over the last decade. Energy Policy, 38 (8), 4528-4539.

Baillie, R., & Bollerslev, T. (1992). Prediction in Dynamic Models with Time-Dependent Conditional Variances. Journal of Econometrics, 52(9), pp. 1-113.

Berger, T. (2016). A wavelet analysis: Forecasting based on decomposed financial return series. Journal of Forecasting, 35(5), 419-433, doi:10.1002/for.2384

Boyer, M.M., & Filion, D. (2009). Common and fundamental factors in stock returns of Canadian oil and gas companies. Energy Economics, 29, 428-453.

Box, G.E.P., & Jenkins, G.M. (1976). Time Series Analysis: Forecasting and Control, Revised Edition. Holden Day, San Francisco, CA.

Box, G.E.P., Jenkins, G.M., & Reinsel, G.C. (1994). Time Series Analysis: Forecasting and Control, 3rd ed. Prentice Hall, Englewood Cliffs.

Broadstock, D.C., Cao, H., & Zhang, D. (2012). Oil shocks and their impact on energy related stocks in China. Energy Economics, 34(6), 1888-1895.

Burg, J.P. (1975). Maximum Entropy Spectral Analysis, Retrieved from: http://sepwww.stanford.edu/theses/sep06/.

Castells, F., Laguna, P., Sörnmo, L., Bollmann, A. & Roig, J.M. (2007). Principal Component Analysis in ECG Signal Processing. EURASIP Journal on Advances in Signal Processing, doi: 10.1155/2007/74580.

Ciner, C. (2001). On the long run relationship between gold and silver prices: A note. Global Finance Journal, 12(2), 299-303.

Conejo, A.J., Plazas, M.A., Espila, R., & Molina, A.B. (2005). Day-ahead electricity price forecasting using the wavelet transform and ARIMA models. IEEE Transactions on Power Systems, 20(2), 1035-1042, doi:10.1109/TPWRS.2005.846054

Corinthios, M. (2009). Signals, Systems, Transforms, and Digital Signal Processing with MATLAB, Taylor and Francis Group, LLC CRC Press, Boca Raton, FL.

Daubechies, I. (1994). Ten lectures on wavelets, CBMS, SIAM, 61, 198-202 and 254-256.

Diebold, F., & Li, C. (2006). Forecasting the term structure of government bond yields, Journal of Econometrics, 130, 337–364.

Durbin, J. (1960). The fitting of time series models. Revue de l'Institut International de Statistique, 28, 233-44.

ElKholy, L. (2017). What we kw about the place in Egypt where Prophet Yusuf lived, Alarabiya News. Retrieved from: https://english.alarabiya.net/features/2017/07/14/Kw-the-place-where-Prophet-Joseph-lived-in-Egypt .

El-Sharif, I., Brown, D., Burton, B., Nixon, B., & Russell, A. (2005). Evidence on the nature and extent of the relationship between oil prices and equity values in the UK. Energy Economics, 27(6), 819-830.

Elyasiani, E., Mansur, I., & Odusami, B. (2011). Oil price shocks and industry stock returns. Energy Economics, 33(5), 966-974.

Faff, R.W., & Brailsford, T.J. (1999). Oil price risk and the Australian stock market. Journal of Energy Finance and Development, 4(1), 69-78.

Fang, C-R, & You, S-Y (2014). The impact of oil price shocks on the large emerging countries' stock prices: Evidence from China, India and Russia. International Review of Economics and Finance, 29, 330-338.

FHI. (2021). Overview of Economic Forecasting Methods, Retrieved from: http://www.fhi.sk/files/katedry/kove/predmety/Progstické_modely/Methods_basics.pdf .

Filis, G. (2010). Macro economy, stock market, and oil prices: Do meaningful relationships exist among their cyclical fluctuations? Energy Economics, 32(4), 877-886.

Forbes. (2018). Best Countries for Business: Saudi Arabia, Retrieved from: https://www.forbes.com/places/saudi-arabia/

Fred (Federal Reserve Bank of St. Louis). (2023). Federal Reserve Economic Data, Retrieved from: https://fred.stlouisfed.org .

Gupta, K. (2016). Oil price shocks, competition, and oil and gas stock returns — Global evidence. Energy Economics, 57, 140-153. ISSN 0140-9883, Retrieved from: https://doi.org/10.1016/j.eneco.2016.04.019 .

He, K., Wang, L., Zou, Y., & Lai, K. (2014). Exchange rate forecasting using entropy optimized multivariate wavelet denoising model. Mathematical Problems in Engineering, 2014, 1-9, doi:10.1155/2014/389598

Hemrit, W., & Benlagha, N. (2018). The impact of government spending on ilgdp in Saudi Arabia (multiplier analysis). International Journal of Economics and Business Research, 15(3), 350–372, doi:10.1504/IJEBR.2018.091050

Huang, R.D., Masulis, R.W., & Stoll, H.R. (1996). Energy shocks and financial markets. Journal of Futures Markets, 16(1), 1-38.

Holy Quran. (2023). Surah Yusuf. Retrieved from: https://www.clearquran.com/012.html

IEEE. (2019), IEEE Transactions on Signal Processing, Retrieved from: https://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=78

Kang, W., Ratti, R.A. & Yoon, K.H. (2015). Time-varying effect of oil market shocks on the stock market. Journal of Banking and Finance, 61(2), 150-S163, ISSN 0378-4266, Retrieved from: https://doi.org/10.1016/j.jbankfin.2015.08.027. .

Kao, L., Chiu, C., Lu, C., & Chang, C. (2013). A hybrid approach by integrating wavelet-based feature extraction with MARS and SVR for stock index forecasting. Decision Support Systems, 54(3), 1228-1244, doi:10.1016/j.dss.2012.11.012

Kriechbaumer, T., Angus, A., Parsons, D., & Casado, M. (2014). An improved wavelet-ARIMA approach for forecasting metal prices. Resources Policy, 39, 32-41, doi:10.1016/j.resourpol.2013.10.005

Lee, D.T.L., & Yamamoto, A. (1994). Wavelet Analysis, theory and applications. Hewlett-Packard Journal, 44-52.

Levinson, N. (1946). The Wiener RMS (root mean square) error criterion in filter design and prediction. Journal of Mathematical Physics, 25, 261-78.

Misiti, M., Misiti, Y., Oppenheim, G. & Poggi, J.M. (2015). Wavelet Toolbox For Use with MATLAB, User's guide. The MathWorks, Natick, MA.

Moshashai, D., Leber, A.M., & Savage, J.D. (2018). Saudi Arabia plans for its economic future: Vision 2030, the National Transformation Plan and Saudi fiscal reform.

British Journal of Middle Eastern Studies, doi: 10.1080/13530194.2018.1500269

Mseddi, S., & Benlagha, N. (2017). Linkage between energy consumption and economic growth: Evidence from Saudi Arabia. The Empirical Economics Letters, 16(10).

OECD. (2021). Real GDP long-term forecast, Million US dollars, 2020 – 2060. OECD Economic Outlook: Statistics and Projections: Long-term baseline projections, 103. Retrieved from: https://data.oecd.org/gdp/real-gdp-long-term-forecast.htm#indicator-chart .

OPEC. (2022). Members countries facts and figures. Retrieved from: https://www.opec.org/opec_web/en/about_us/25.htm .

Ortega, L., & Khashanah, K. (2014). A Neuro wavelet model for the Short-Term forecasting of High Frequency time series of stock returns. Journal of Forecasting, 33(2), pp. 134-146, doi:10.1002/for.2270

Prochazka, A., Mudrova, M., Vysata, O., & Araujo, C.P.S. (2010). Multi-Channel EEG Signal Segmentation and Feature Extraction. Proceedings of Intelligent Engineering Systems (INES), doi: 10.1109/INES.2010.5483824

Renaud, O., Starck, J.L., & Murtagh, F. (2002). Wavelet-based Forecasting Short and Long Memory Time Series. Cahiers du departement d'econometrie, Universite de Geneve, 4.

Riedel, B. (2020). As a global economic crisis wreaks havoc on Saudi Arabia, the kingdom should reduce military spending", Brookings, Retrieved from: https://www.brookings.edu/blog/order-from-chaos/2020/05/27/as-a-global-ecomic-crisis-wreaks-havoc-on-saudi-arabia-the-kingdom-should-reduce-military-spending/ .

Rostan, P., Belhachemi, R., & Rostan, A. (2015). Appraising the financial sustainability of a pension system with signal processing", Studies of Applied Economics, 33(3), 801-816, doi: https://doi.org/10.25115/eea.v33i3.3134, Retrieved from: https://ojs.ual.es/ojs/index.php/eea/article/view/3134 .

Rostan, P., Belhachemi, R., & Racicot, F.E. (2017). Forecasting the yield curve with the Burg model. Journal of Forecasting, 36(1), 91-99, doi: https://doi.org/10.1002/for.2416, Retrieved from: https://onlinelibrary.wiley.com/doi/abs/10.1002/for.2416 .

Rostan, P., & Rostan, A. (2017). Population Projections and Pension System Sustainability. Saarbrücken, Germany: Lambert Academic Publishing. ISBN978-620-2-06479-8

Rostan, P., & Rostan, A. (2018a). The versatility of spectrum analysis for forecasting financial time series. Journal of Forecasting, 37(3), 327-339, doi: https://doi.org/10.1002/for.2504, Retrieved from: http://onlinelibrary.wiley.com/doi/10.1002/for.2504/abstract .

Rostan, P., & Rostan, A. (2018b). Will Saudi Arabia Get Older? Will its pension system be sustainable? Spectral Answers. PSU Research Review, 2(3), doi: https://doi.org/10.1108/PRR-12-2017-0045, Retrieved from: https://www.emeraldinsight.com/doi/full/10.1108/PRR-12-2017-0045 .

Rostan, P., & Rostan, A. (2018c). Forecasting Spanish nominal and real GDPs with Spectral Analysis", Studies of Applied Economics, 36(1), 217-234, Retrieved from: https://dialnet.unirioja.es/servlet/articulo?codigo=6283924 .

Rostan, P., & Rostan, A. (2018d). Where is Greek's Economy Heading? International Journal of Management and Applied Science (IJMAS), 4(3), 28-31, Retrieved from: http://ijmas.iraj.in/paper_detail.php?paper_id=11490andname=Where_is_Greece%E2%80%99s_Ecomy_Heading?_A_Spectral_Perspective .

Rostan, P., & Rostan, A. (2019). When will European Muslim Population be majority and in which country. PSU Research Review, 3(2), doi: https://doi.org/10.1108/PRR-12-2018-0034,

Retrieved from: https://www.emerald.com/insight/content/doi/10.1108/PRR-12-2018-0034/full/html .

Rostan, P., & Rostan, A. (2020). Where is Austria's Economy Heading? Economic and Business Review, 22(1), 105-130, doi: https://doi.org/10.15458/ebr97, Retrieved from: https://www.ebrjournal.net/uploads/ebr/public/document/13-ebr_221_d_rostan_barvni_en.pdf .

Rostan, P., & Rostan, A. (2021a). Where is Saudi Arabia's Economy Heading?, International Journal of Emerging Markets, 16(8), 2009-2033. doi: https://doi.org/10.1108/IJOEM-08-2018-0447).

Rostan, P. & Rostan A. (2021b). Where are fossil fuels prices heading?, International Journal of Energy Sector Management, 15(2), 309-327. doi: https://doi.org/10.1108/IJESM- 07-2019-0009.

Rostan, P., & Rostan, A. (2022a). 2050 Projections of the Persian Gulf Economies, Iranian Economic Review, 26(2), 269-288. doi: 10.22059/ier.2022.88164. https://ier.ut.ac.ir/article_88164.html

Rostan, P. & Rostan A. (2022b). Assessing the Resilience of Turkey's Economy during the Covid-19 Pandemic with its 2050 Projections, Journal of Emerging Economies &

Policy, 7(2), 38-49. https://dergipark.org.tr/en/download/article-file/2595010

Rostan, P. & Rostan A. (2022c). Assessing the Resilience of UK's Economy After The Covid-19 Pandemic And Brexit, Online Journal Modelling The New Europe, 40, 47-77. (doi: 10.24193/OJMNE.2022.40.03) http://neweurope.centre.ubbcluj.ro/wp-content/uploads/2022/12/3.pdf

Rostan, P. & Rostan A. (2023a). How South Korea's economy gained momentum because of Covid-19, Studies of Applied Economics, article in press.

Schlüter, S., & Deuschle, C. (2010). Using wavelets for time series forecasting: Does it pay off? Working Paper, IWQW discussion paper series, in Cooperation with: Friedrich-Alexander University Erlangen-Nuremberg, Institute for Economics, 4.

Soyemi, A., Ore-Oluwa, R., & Akingula, J.O. (2017). Effects of oil price shock on stock returns of energy firms in Nigeria. Kasetsart Journal of Social Sciences, ISSN 2452-3151, Retrieved from: https://doi.org/10.1016/j.kjss.2017.09.004. .

Statista, (2022). Share of the gross domestic product generated from oil rents in the Gulf Cooperation Council in 2019, by country. Retrieved from: https://www.statista.com/statistics/1303697/gcc-oil-rent-share-of-gdp-by-country/ .

Stoica, P., & Moses, R. (2005). Spectral Analysis of Signals, Prentice Hall, Upper Saddle River.

Tan, Z., Zhang, J., Wang, J., & Xu, J. (2010). Day-ahead electricity price forecasting using wavelet transform combined with ARIMA and GARCH models. Applied Energy, 87(11), 3606-3610, doi:10.1016/j.apenergy.2010.05.012

The Global Economy. (2022a). Revenue minus production cost of oil, percent of GDP in Iran. Retrieved from: https://www.theglobalecomy.com/Iran/oil_revenue/ .

The Global Economy. (2022b) Revenue minus production cost of oil, percent of GDP in Saudi Arabia. Retrieved from: https://www.theglobalecomy.com/Saudi-Arabia/ .

Valens, C. (1999). A Really Friendly Guide to Wavelets, Retrieved from: http://agl.cs.unm.edu/~williams/cs530/arfgtw.pdf .

Wallace, P., & Martin, M. (2021). Saudi Aramco Follows Big Oil Competitors as Profit Surges. Bloomberg. Retrieved from: https://www.bloombergquint.com/business/saudi-aramco-follows-big-oil-rivals-with-a-profit-surge .

World Bank. (2023). Real GDP at Constant National Prices for Saudi Arabia and Iran, Millions of 2017 U.S. Dollars, Annual, not Seasonally Adjusted. Retrieved from: https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=SA

https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=IR

World Bank. (2023). GDP (current US$). Retrieved from: https://data.worldbank.org/indicator/Ny.Gdp.Mktp.Cd?most_recent_value_desc=true .