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The Fractional Integral Inequalities Involving Kober and Saigo–Maeda Operators

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Abstract
This work uses the Marichev-Saigo-Maeda (MSM) fractional integral operator to achieve certain special fractional integral inequalities for synchronous functions. Compared to the previously mentioned classical inequalities, the inequalities reported in this study are more widespread. We also looked at several unique instances of these inequalities involving the fractional operators of the Saigo, Erdelyi, and Kober, and Riemann-Liouville types.

Keywords: Fractional Integral Inequalities, Saigo–Maeda Operators, Synchronous functions
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1. Introduction

Over the past forty years, the idea of fractional calculus has grown significantly in popularity and significance. This is because it has distinguished uses in so many different branches of engineering and research. Mathematicians also use the operators of the fractional calculus to break down the classical special functions into some more fundamental, well-known special or elementary functions. This approach was followed by Samko et al. [1], and Kiryakova [2]. Following the ideas of Lavoie, Osler and Tremblay [3], Kiryakova [4, 5] further demonstrated that almost all the special functions of mathematical physics, can be represented as (generalized) fractional integrals or derivatives of the three elementary functions. Relations of this kind also provide some alternative definitions for the special functions by means of Poisson type and Euler type integral representations and Rodrigues type differential formulas. Mariusz Ciesielski [6] studied the fractional eigenvalue problem by a numerical method when the fractional Sturm-Liouville equation is subjected to the mixed boundary conditions. The non-integer order differential equation is discretized to the scheme with the symmetric matrix representing the action of the numerically expressed composition of the left and the right Caputo derivatives. Kiryakova [7] pointed out few basic classical results, combined with author's ideas and developments, that show how one can do the task at once, in the rather general case: for both operators of generalized fractional calculus and generalized hypergeometric functions. Thus, great part of the results are well predicted and fall just as special cases of the discussed general scheme. Saigo et al. [8] demonstrated fractional calculus operator associated with the H-function. Jahanshahi S. [9] introduced an algorithm for computing fractional integrals and derivatives and applied it for solving problems of the calculus of variations of fractional order.

Fractional integral inequalities (FIIs) and its applications have received considerable attention from researchers and mathematicians during the past few decades. Recent research uses a variety of fractional integral operators and focuses on different forms of Fractional integral inequalities (FIIs). (see, e.g., [10]-[16], [21]). Here the authors have established various types of inequalities and some other results by utilizing the Saigo–Maedafractional integral operator.
Recently, Purohit and Raina [16] used Saigo fractional integral operators to investigate several integral inequalities of the Chebyshev type and established the q-extensions of the major discoveries. In this paper, a few generalised integral inequalities for synchronous functions connected to the Chebyshev functional are shown using the fractional hypergeometric operator developed by Curiel and Galue [17]. The results attributed to Purohit and Raina [16] and Belarbi and Dahmani [18] are shown below as particular cases of our findings.

2. Preliminaries

Definition: On [a,b], the two functions f and g are synchronous. if

$$\Big(f(x)-f(y)\Big)\Big(g(x)-g(y)\Big) \geq 0, \quad for \;\; any \;\; x,y \in [a,b] \tag{2.1}$$

Riemann-Liouville fractional integral operator:
Joseph Liouville (1832) introduced the Riemann Liouville integral operator which included the definition given by Bernhard Riemann [15]. It is first significant definition which fulfilled almost all the requirements of a fractional calculus operator. Named in honour of Riemann and Liouville, this operator is defined as

$$_aI_x^{\alpha} f(x) = {}_a D_x^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)dt \tag{2.2}$$

where a is arbitrary but fixed point.
The Riemann-Liouville operator [15] has its importance in physical science where it exists in the theory of linear ordinary differential equations.

Weyl Fractional Integral Operator:
The Weyl fractional integral operator [1] is defined as:

$$_xW_{\infty}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^{\infty} (t-x)^{\alpha-1} f(t)dt; \;\; -\infty < x < \infty. \tag{2.3}$$

and

$$_{-\infty}W_x^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{x} (x-t)^{\alpha-1} f(t)dt; \;\; -\infty < x < \infty. \tag{2.4}$$

Generalizing both the Riemann-Liouville operator [15] and the Weyl-operator [1], Oldham and Spanier [19] defined the familiar differ-integral operator $_aD_x^{\alpha}$ as follows:

$$_aD_x^{\alpha} = \frac{1}{\Gamma(-\alpha)} \int_a^x (x-t)^{-\alpha-1} f(t)dt; \;\; Re(\alpha) < 0$$

$$= \frac{d^m}{dx^m} \, {}_aD_x^{\alpha-m} f(x); \qquad 0 < Re(\alpha) < a \tag{2.5}$$

here ‘m’ is positive integer, α is complex and $Re(\alpha) > 0$.
Erdelyi-Kober Operators:
Erdelyi and Hermann (1940) [20] introduced the fractional integral operators namely Erdelyi-Kober operators. These operators are defined as follows:

$$E_{0,x}^{\alpha,\eta} f(x) = I_{0,x}^{\alpha,0,\eta} f(x) = \frac{x^{-\alpha-\eta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} t^{\eta} f(t)dt, \;\; Re(\alpha) > 0. \tag{2.6}$$

and

$$K_{x,\infty}^{\alpha,\eta}f(x)=J_{x,\infty}^{\alpha,0,\eta}f(x)=\frac{x^{\eta}}{\Gamma(\alpha)}\int_{x}^{\infty}(t-x)^{\alpha-1}t^{-\alpha-\eta}f(t)dt,\ \ Re(\alpha)>0. \tag{2.7}$$

when $\eta=0$, [21] reduces to Riemann-Liouville operator, that is

$$I_{x}^{0,\alpha}f(x)=x_{0}^{-\alpha}I_{x}^{\alpha}f(x). \tag{2.8}$$

and for $\eta=0$, [16] reduces to Weyl operator, that is

$$K_{x}^{0,\alpha}f(x)=x_{0}^{-\alpha}W_{x}^{\alpha}f(x). \tag{2.9}$$

The Saigo's Operator:
Saigo [22] introduced this operator after studying the Euler-Darboux equation [23], which is a partial differential equation with boundary conditions.
For real numbers $\alpha>0,\beta$ and η the Saigo operator, involving hypergeometric function is defined as [22]:

$$I_{0,x}^{\alpha,\beta,\eta}f(x)=\frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_{0}^{x}(x-t)_{2}^{\alpha-1}F_{1}(\alpha+\beta,\eta;\alpha;1-\frac{t}{x})f(t)dt$$

$$\left(I_{0+}^{\alpha,\beta,\eta}f\right)(x)=\left(\frac{d}{dx}\right)^{k}\left(I_{0+}^{\alpha+k,\beta-k,\eta-k}f\right)(x)$$

For $Re(\alpha)\leq 0,k=[-Re(\alpha)+1]$, it takes the form

$$J_{x,\infty}^{\alpha,\beta,\eta}f(x)=\frac{1}{\Gamma(\alpha)}\int_{x}^{\infty}(t-x)^{\alpha-1}t_{2}^{-\alpha-\beta}F_{1}(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t})f(t)dt,\ \ Re(\alpha)>0. \tag{2.10}$$

$$\left(I_{0-}^{\alpha,\beta,\eta}f\right)(x)=\left(\frac{-d}{dx}\right)^{k}\left(I_{0-}^{\alpha+k,\beta-k,\eta}f\right)(x),\quad Re(\alpha)\leq 0,k=[-Re(\alpha)+1] \tag{2.11}$$

$$\left(D_{0+}^{\alpha,\beta,\eta}f\right)(x)=\left(I_{0+}^{-\alpha,-\beta,\alpha+\eta}f\right)(x)=\left(\frac{d}{dx}\right)^{k}\left(I_{0+}^{-\alpha+k,-\beta-k,\alpha+\eta-k}f\right)(x) \tag{2.12}$$

$Re(\alpha)>0,k=[Re(\alpha)+1]$

$$\left(D_{0-}^{\alpha,\beta,\eta}f\right)(x)=\left(I_{0-}^{-\alpha,-\beta,\alpha+\eta}f\right)(x)=\left(\frac{-d}{dx}\right)^{k}\left(I_{0-}^{-\alpha+k,-\beta-k,\alpha+\eta}f\right)(x) \tag{2.13}$$

$Re(\alpha)>0,k=[Re(\alpha)+1]$

The Saigo-Maeda Operator:
In 1996 Saigo-Maeda [23] extended the fractional integral operators defined by Saigo [22]. The generalized fractional integral operators are defined as:

$$I_{0,x}^{\mu,\mu',\nu,\nu',\eta}f(x)=\frac{x^{-\mu}}{\Gamma(\eta)}\int_{0}^{x}(x-t)^{\eta-1}t^{-\mu'}F_{3}(\mu,\mu',\nu,\nu';\eta;1-\frac{t}{x};1-\frac{x}{t})f(t)dt \tag{2.14}$$

where $\mu,\mu'>0,Re(\eta)>0,\nu,\nu'$ are real number.

$$\left(I_{0+}^{\mu,\mu',\nu,\nu',\eta}f\right)(x)=\left(\frac{d}{dx}\right)^{k}\left(I_{0+}^{\mu,\mu',\nu+k,\nu',\eta+k}f\right)(x) \tag{2.15}$$

$Re(\eta)>0,k=[-(Re(\eta)+1]$

$$I_{x,\infty}^{\mu,\mu',\nu,\nu',\eta}f(x)=\frac{x^{-\mu}}{\Gamma(\eta)}\int_{x}^{\infty}(t-x)^{\eta-1}t^{-\mu}F_{3}(\mu,\mu',\nu,\nu';\eta;1-\frac{x}{t};1-\frac{t}{x})f(t)dt, \tag{2.16}$$

where $\mu, \mu' > 0, Re(\eta) > 0, \nu, \nu'$ are real number.

$$\left(I_{0-}^{\mu,\mu',\nu,\nu',\eta} f\right)(x) = \left(\frac{-d}{dx}\right)^k \left(I_{0-}^{\mu,\mu',\nu,\nu'+k,\eta+k} f\right)(x) \tag{2.17}$$

where $Re(\eta) \leq 0, k = [-(Re(\eta)+1]$

$$\left(D_{0+}^{\mu,\mu',\nu,\nu',\eta} f\right)(x) = \left(I_{0+}^{-\mu',-\mu,-\nu',-\nu,\eta} f\right)(x) = \left(I_{0+}^{-\mu',-\mu,-\nu'+k,-\nu,\eta+k} f\right)(x) \tag{2.18}$$

where $R(\eta) > 0, k = [Re(\eta)+1]$

$$\left(D_{0-}^{\mu,\mu',\nu,\nu',\eta} f\right)(x) = \left(I_{0-}^{-\mu',-\mu,-\nu',-\nu,\eta} f\right)(x) = \left(\frac{-d}{dx}\right)^k \left(I_{0+}^{-\mu',-\mu,-\nu',-\nu+k,-\eta+k} f\right)(x) \tag{2.19}$$

where $\mu, \mu', \nu, \nu', \eta \in C, (R(\eta) > 0)$ and $x > 0$, $Re(\eta) > 0, k = [Re(\eta)+1]$ and $F_3(.)$ *is Appell's function.*

3. Main Results

Theorem 3.1. *Assume u and v are two positive integrable and synchronous mappings on* $[0,\infty]$. *Suppose there exists four positive integrable mappings* m_1, m_2, n_1 *and* n_2 *such that:*

$$0 < m_1(t) \leq u(t) \leq m_2(t), \quad and \quad 0 < n_1(t) \leq v(t) \leq n_2(t) \qquad (t \in [0,x], x > 0) \tag{3.1}$$

then the following inequality holds true:

$$K_{0,y}^{a,b}\{n_1 n_2 u^2\}x \times K_{0,y}^{a,b}\{m_1 m_2 v^2\}x \leq \frac{1}{4}\left(K_{0,y}^{a,b}\{(m_1 n_1 + m_2 n_2)uv\}\right)^2. \tag{3.2}$$

Proof: By using the relations that are given in (3.1), for $t \in [0,x]$, for all $x > 0$, we can easily have:

$$\left(\frac{m_2(t)}{n_1(t)} - \frac{u(t)}{v(t)}\right) \geq 0. \tag{3.3}$$

$$\left(\frac{u(t)}{v(t)} - \frac{m_1(t)}{n_2(t)}\right) \geq 0. \tag{3.4}$$

On multiplying equations (3.3) and (3.4) we get,

$$\left(m_1(t)n_1(t) + m_2(t)n_2(t)\right)u(t)v(t) \geq n_1(t)n_2(t)u^2(t) + m_1(t)m_2(t)v^2(t). \tag{3.5}$$

Consider the following function $F(x,t)$ defined by:

$$F(x,t) = \frac{y^{-a-b}}{\Gamma(a)} t^b (y-t)^{a-1} \tag{3.6}$$

Then multiplying both sides of (3.5) by $F(y,t)$ and integrate w.r.t to t from 0 to x and using definition (2.6)

$$E_{0,x}^{a,b}\{(m_1 n_1 + m_2 n_2)uv\}x \geq E_{0,x}^{a,b}\{n_1 n_2 u^2\}x + E_{0,x}^{a,b}\{m_1 m_2 v^2\}x.$$

Using A.M-G.M inequality , we get

$$E_{0,x}^{a,b}\{(m_1 n_1 + m_2 n_2)uv\}x \geq 2\sqrt{E_{0,x}^{a,b}\{n_1 n_2 u^2\}x \times E_{0,x}^{a,b}\{m_1 m_2 v^2\}x}.$$

Further on simplifying above equation, we get

$$E_{0,y}^{a,b}\{n_1n_2u^2\}x \times E_{0,y}^{a,b}\{m_1m_2v^2\}x \leq \frac{1}{4}\left(E_{0,y}^{a,b}\{(m_1n_1+m_2n_2)uv\}\right)^2.$$

This complete proof of the theorem.

Theorem 3.2. *Assume u and v are two positive integrable and synchronous mappings on* $[0,\infty]$. *Suppose there exists four positive integrable mappings* m_1, m_2, n_1 *and* n_2 *such that:*

$$0 < m_1(t) \leq u(t) \leq m_2(t), \quad and \quad 0 < n_1(t) \leq v(t) \leq n_2(t) \qquad (t \in [0,x], x > 0) \tag{3.7}$$

then the following inequality holds true:

$$I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{n_1n_2u^2\}x \times I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{m_1m_2v^2\}x \leq \frac{1}{4}\left(I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{(m_1n_1+m_2n_2)uv\}\right)^2. \tag{3.8}$$

Proof: By using the relations that are given in (3.7), for $t \in [0,x], \forall x > 0$,we can easily have:

$$\left(\frac{m_2(t)}{n_1(t)} - \frac{u(t)}{v(t)}\right) \geq 0. \tag{3.9}$$

$$\left(\frac{u(t)}{v(t)} - \frac{m_1(t)}{n_2(t)}\right) \geq 0. \tag{3.10}$$

On multiplying equations (3.9) and (3.10) we get,

$$\left(m_1(t)n_1(t) + m_2(t)n_2(t)\right)u(t)v(t) \geq n_1(t)n_2(t)u^2(t) + m_1(t)m_2(t)v^2(t). \tag{3.11}$$

Consider the following function $F(x,t)$ defined by:

$$F(x,t) = \frac{x^{-\alpha}}{\Gamma(\gamma)}(x-t)^{\gamma-1}F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t},1-\frac{t}{x}\right). \tag{3.12}$$

Then multiplying both sides of (3.11) by $F(x,t)$ and integrate w.r.t to t from 0 to x and using definition (2.14)

$$I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{(m_1n_1+m_2n_2)uv\}x \geq I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{n_1n_2u^2\}x + I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{m_1m_2v^2\}x. \tag{3.13}$$

Using A.M-G.M inequality, that is

$$\frac{(a+b)}{2} \geq \sqrt{ab}, \qquad a,b \in R$$

we get

$$I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{(m_1n_1+m_2n_2)uv\}x \geq 2\sqrt{I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{n_1n_2u^2\}x \times I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{m_1m_2v^2\}x}.$$

Further on simplifying above equation, we get

$$I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{n_1n_2u^2\}x \times I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{m_1m_2v^2\}x \leq \frac{1}{4}\left(I_{0^+}^{\alpha,\alpha',\beta,\beta',\gamma}\{(m_1n_1+m_2n_2)uv\}\right)^2.$$

This completes the proof.

4. Special Cases

In this section, we discuss some of the important special cases of the main results established above.
Corollary (1): If we take $\beta' = \gamma = 0$ in the theorems (3.2) we get well known results based on Saigo type fractional operator reported in [24], which is as follows:

$$I_{0^+}^{\alpha,\alpha',\beta}\{n_1n_2u^2\}x \times I_{0^+}^{\alpha,\alpha',\beta}\{m_1m_2v^2\}x \leq \frac{1}{4}\left(I_{0^+}^{\alpha,\alpha',\beta}\{(m_1n_1+m_2n_2)uv\}\right)^2 \tag{4.1}$$

Corollary (2): If we take, $\alpha' = 0$ in the Corollary (1), we get well known results based on Erdelyi-Kober type fractional operator reported in [11].

$$K_{0^+}^{\alpha,\beta}\{n_1n_2u^2\}x \times K_{0^+}^{\alpha,\beta}\{m_1m_2v^2\}x \leq \frac{1}{4}\left(K_{0^+}^{\alpha,\beta}\{(m_1n_1+m_2n_2)uv\}\right)^2 \tag{4.2}$$

Corollary (3): If we take, $\beta = 0$ in the Corollary (2), we get well known results based on Riemann-Liouville type fractional operator reported in [24].

$$R_{0+}^{\alpha}\{n_1n_2u^2\}x \times R_{0+}^{\alpha}\{m_1m_2v^2\}x \leq \frac{1}{4}\left(R_{0+}^{\alpha}\{(m_1n_1+m_2n_2)uv\}\right)^2 \tag{4.3}$$

5. Results and Discussions

We conclude our investigation by stating that the findings presented in this paper are all original and significant. First, using a Saigo-Maeda type fractional integral operator, we have created a number of inequalities and generated a number of special cases for the operators namely Saigo type fractional operator, Erdelyi - Kober type fractional operator and Riemann-Liouville type fractional operator.

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