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RESEARCH PAPER

Ion temperature gradient modes driven soliton and shock by reduction perturbation method for electron-ion magneto-plasma

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Abstract

In our observation, we have used an easy and reliable approach of the reduction perturbation method to obtain the solution of the ion temperature gradient mode driven linear and nonlinear structures of relatively small amplitude. One can use that methodology in the more complex environment of the plasma and can obtain a straightforward approach toward his studies. We have studied different parameter impacts on the linear and nonlinear modes of the ITG by using data from tokamak plasma. Hence, our study is related to the tokamak plasma and one that can apply to the nonlinear electrostatic study of stiller and interstellar regimes where such types of plasma environment occur.

Key words: Ion temperature gradient; soliton; shock; electron-ion plasma; reduction perturbation method; linear and nonlinear structures
AMS 2020 Classification: 70K60; 35A09; 35G20

1 Introduction

Most of the research work has been done on the linear and nonlinear structures over the last few decades and its applications are compared with the stiller and interstellar spaces where the medium is plasma [1, 2, 3]. For that purpose, many researchers investigated the electron temperature gradient (ETG) and ion temperature gradient (ITG) drift mode in which some of them used the simplest slab geometry [4, 5] and the other used some complex geometry like toroidal geometry [6]. Mathematically ITG coefficient is defined as $\eta_i = L_n/L_T$ [10] while $L_T = 1/\partial x \ln T_{io}(x)$ and $L_n = 1/\partial x \ln n_{io}(x)$ are the ion temperature and ion density scale lengths. For the first time, ITG driven mode was studied by Sagdeev and Rudakov [4], then the work extended to the nonuniform number density of plasma species with a shear magnetic field where ion kinetic effect was also introduced in their calculation. The same research was extended further with inhomogeneous plasma configuration for the instability limits in the toroidal geometry. Further, the pressure effect in the same geometry was also observed [5]. Under the external magnetic field applied to the plasma, some of the new properties of the ITG mode were introduced by Hahm and Tang [8]. Jerman et al. [10] using heat flux effect in the energy balancing equation and Braginskii's equation to derive ITG mode equation for the simple Maxwellian electron-ion plasma. The ITG and toroidal ITG modes were studied and coupled by Shukla [11, 12] the same scientists also obtained theoretical calculations for the dipolar vortices. Zakir et al. [13] calculated the nonlinear structure of dipolar vortices in the

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plasma where electrons species were considered to be super-thermal. Adnan et al. [14] observed low-frequency electrostatic waves in an inhomogeneous plasma. Whether the instability is of η_e type or η_i type these both are very strong as compared to the gyro-radius (ρ_e/ρ_i) and driven fluxes effects [18]. In the ITG mode-driven instability both temperature gradient and number density fluctuations are out of phase, and those types of modes are robust in the non-thermal regime. η_i mode instabilities are produced due to the free energy that is stored in the form of ITG mode [16, 15].

Fluid-like plasma is complex and nonlinear where the nonlinear structures like solitary shock waves can transport heat energy, mass, and momentum inside the fluid from one to another, bringing instability in the [19, 20, 21, 22]. The nonlinear collision-less structures were studied by Sabry et al. [23] in a plasma whose constituents are electron-positron and ions. Nonlinear solitary waves were studied by many authors considering various models of plasma [24, 25], shocks [26] and vortices of the two dimensional by the authors [27, 28, 29]. For the first time, Zakir et al. [17] studied the linear and nonlinear solitary and shock waves structure in the ITG driven mode instability by considering electron to be Maxwellian and ion dynamic. Khan et al. [30] extended the work by incorporating the entropy drift in the momentum equation of the fluid and the effect of entropy in the ITG mode, his study revealed that entropy is an essential factor in the transportation of instability in the plasma. Javed et al. [32] theoretically obtained the solitary wave potential solution from the kdv equation in the ITG mode by homotopy perturbation method (HPM) and compare the solution of the analytical and HPM method and gives that both types of solutions agree with each other if the time interval is taken very small. Aziz et al. [33] observed ITG mode soliton and shock in electron-positron-ion magneto-plasma by taking electron and positron species as Maxwellian; the same work is carried out by Rehan et al. [34] and investigated the linear and nonlinear mode in (e-p-i) plasma taking electron to be super-thermal. Zakir et al. [17] studied the effects on the shock and solitary structure by taking the heat flux effect in the energy balancing equation of the ITG mode. Aziz et al. in [35] studied electron-positron-ion magneto-plasma by considering the entropy effect has study shows that it is one of the dominant factors in plasma parameters that can change the various linear and nonlinear structures magnificently in the fluid.

To observe different nonlinear structures like a soliton, shock, etc., in various compositions and models of a plasma, we can use the reduction perturbation technique (RPT). The reduction perturbation technique was first introduced theoretically to the problem's solution by [36, 37]. RPT has advantages like flexibility and algorithmic methodology to solve different problems in various fields of physics. Taniuti and Wei [38, 39] suggested RPT to be a generalized technique for obtaining the nonlinear partial differential equation of the corresponding waves in a model plasma [40]. Different types of waves to which that technique has been successfully applied are ion-acoustic in a hot and cold plasma, magnetosonic waves in both hot and cold plasma, etc. [41, 42]. As the literature shows us that no one has yet solved the shock and solitary waves solution in ITG mode by reduction perturbation technique so we for the first time investigating the problem of ITG mode driven soliton and shock formation in the electron-ion plasma by reduction perturbation method (RPT). This article is divided into the following sections: Section 2 gives MHD equations and the linear root calculation by the RPM method. In sections 3 and 4, we study the solitary and shock waves profiles; Section 5 concludes the related article.

2 Theory related to the model

We consider a nonuniform plasma consisting of two species as electron and ion, with a background magnetic field along the z-axis i.e., $B_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ represents the unit vector along the z-axis and $\mathbf{B}_0$ is the magnitude of the magnetic field. We also considered the temperature and number density gradients in the $x-$direction to simplify the calculation of the ion temperature gradient modes driven linear and nonlinear study i.e., $d_x n_{i0} \neq 0$ and $d_x T_{i0} \neq 0$ for ions, here n_{i0}, T_{i0} are the equilibrium number density and ions temperature. The inertial mass for an electron in comparison to the ion is so small, therefor ions are taken dynamic while electrons are subjected to have Maxwellian distribution. We assume here low-frequency ITG mode i.e., $\partial_t \ll \omega_{ci} = (eB/m_i c)$, (here e stands for the ion charge, m_i is for the ion mass, B is taken for the magnitude of the magnetic field and c denote the speed of light). The fluctuations are considered to be electrostatic in nature, so we have taken $\nabla \times \mathbf{E} = 0$ in our calculation. The first equation of our model plasma for the ion temperature gradient mode is the ion momentum equation that is [13, 17],

$$(\partial_t + v_i.\nabla)\mathbf{v}_i = -\frac{e}{m_i}\mathbf{E} - \frac{1}{m_i n_i}(\nabla p_i), \tag{1}$$

where $\mathbf{E} = -\nabla\phi$. Under the action of some external forces plasma species are driven so the inhomogeneity occurs in different parameters of the plasma i.e., $n_i = n_{i0} + n_{i1}$, $T_i = T_{i0} + T_{i1}$ with $n_{i1} \ll n_{i0}$ and $T_{i0} \ll T_{i1}$ (here the quantities with subscript 0 denote the unperturbed parameters while those with subscript 1 denotes the perturbed plasma parameters). Ion velocity in the limit $\partial_t \ll \omega_{ci}$, superposed by different drifts that is given as [13, 17]

$$\mathbf{v}_i = \mathbf{v}_E + \mathbf{v}_{Di} + \mathbf{v}_{pi} + v_{ix}\widehat{x}, \tag{2}$$

where $\mathbf{v}_E = \frac{c}{B_0}(\hat{\mathbf{z}} \times \nabla\Phi)$, $\mathbf{v}_{Di} = \frac{c}{eB_0 n_i}(\hat{\mathbf{z}} \times \nabla P_i)$ and $\mathbf{v}_{pi} = -\frac{c}{B_0\omega_{ci}}(\partial_t + \mathbf{v}_i.\nabla)\hat{\mathbf{z}} \times \mathbf{v}_i$ are the $\mathbf{E} \times \mathbf{B}$ drift, ion diamagnetic drift and ion polarization drift. Here Φ, P_i represents the normalized electrostatic potential, ions pressure, and v_{ix}, the drift velocity's x—component. Here $P_i = n_i T_i$ is the ion pressure. The ion continuity equation is given by [13, 17]

$$\partial_t n_i + \nabla.(n_i \mathbf{v}_i) = 0. \tag{3}$$

The energy balance equation is given as [13, 17]

$$\frac{3}{2}(\partial_t + \mathbf{v}_i \cdot \nabla)T_i + T_i(\nabla \cdot \mathbf{v}_i)n_i = -\frac{1}{n_i}\nabla.\left[(5T_i/2eB_0)\hat{z} \times \nabla T_i\right], \tag{4}$$

$(5T_i/2eB_0)\hat{z} \times \nabla T_i$ is known as Righi-Leduce heat flux term for ions due to the ion temperature gradient. Now at the last Poission equation (that is based on the Gauss's law for electric flux) is as [13, 17]

$$\nabla^2\phi = 4\pi e(n_i - n_e). \tag{5}$$

Now to incorporate drift speed of ion v_i in the equations of (1-5) and after a little manipulation we can get the continuity equation as [13, 17]

$$D_t^i N + \tau \mathbf{v}_{ni}\nabla\Phi - \frac{1}{2}\rho_i^2\tau^{-1}\partial_t\nabla^2(T+N+\Phi) + \partial_z v_{iz} = 0. \tag{6}$$

In above expression, the new terms introduced are defined as $D_t^i = (\partial_t + \mathbf{v}_E.\nabla)$ [13, 17], $\mathbf{v}_{ni} = \left(\frac{cT_{io}}{eB_o}\right)\nabla \ln n_{io} \times \hat{\mathbf{z}}$, $\tau = \frac{T_{eo}}{T_{io}}$, $T = \frac{T_{i1}}{T_{io}}$, $N = \frac{n_{io1}}{n_{io}}$ and $\Phi = \frac{e\phi}{T_{eo}}$. The momentum equation obtained is as

$$(\partial_t + v_{iz}\partial_z)v_{iz} + c_s^2\nabla\left[\Phi + \tau^{-1}(T+N)\right] = 0. \tag{7}$$

here $c_s = \rho_s\omega_{ci}$. Using the drift approximation in Eq. (4) and neglected the Righi-Leduce heat flux term in the same equation we can get the energy balance equation as

$$\partial_t T - \frac{2}{3}\partial_t N = \tau\left(\eta_i - \frac{2}{3}\right)\mathbf{v}_{ni}.\nabla\Phi. \tag{8}$$

While the Poisson equation in the form of

$$\nabla^2\Phi = \alpha_1\left(\frac{n_{eo}}{n_{io}}\Phi - N\right), \tag{9}$$

where $\alpha_1 = \frac{4\pi e^2 n_{io}}{T_e}$ in the Poisson equation which is based on the electric flux according to Gauss's law.

Phase velocity

To get dispersion relation for the ITG mode, we use a compelling reduction perturbation method (RPM). From the dispersion relation, we then can extract phase velocity for the same mode that will reveal the linear behavior of the ITG mode. To proceed further, we first introduce the stretching coordinate to express all the differential equations of Eqs. (1)-(5) in terms of ξ– the coordinate system as did by [44]. The stretching coordinates are given as $\xi = \epsilon^{\frac{1}{2}}(\frac{x}{u} - t)$ and $\ell = \epsilon^{\frac{3}{2}}t$ where the parameter ϵ has a very small value that represents, the weakness of the mode amplitude and u is the phase velocity of the mode. We write equations (6)-(9) in the stretching coordinate and then use the following power series for the different normalized quantities i.e.,

$$\begin{pmatrix} N \\ v_{ix} \\ T \\ \Phi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \sum_{n=1}^{m}\epsilon^n\begin{pmatrix} N^{(n)} \\ v_{ix}^{(n)} \\ T^{(n)} \\ \Phi^{(n)} \end{pmatrix}, \quad m \text{ is a higher order of the perturbation,} \tag{10}$$

which gives a number of equations. To express the lowest order of different normalized quantities in terms of each other we compare ϵ power one to both sides of each equation in the form of

$$N^{(1)} = \frac{\tau\mathbf{v}_{ni}}{u}\Phi^{(1)} + \frac{v_{ix}^1}{u}, \tag{11}$$

$$v_{ix}^{(1)} = \frac{c_s^2}{u}\left[\Phi^{(1)} + \tau^{-1}\left(T^{(1)} + N^{(1)}\right)\right], \tag{12}$$

$$T^{(1)} = \frac{2}{3}N^{(1)} - \tau\left(\eta_i - \frac{2}{3}\right)\frac{\mathbf{v}_{ni}}{u}\Phi^{(1)}, \tag{13}$$

$$N^1 = \Phi^1. \tag{14}$$

When coupled equations of $(11-14)$, we can get

$$1 - \frac{\tau\mathbf{v}_{ni}}{u} - \frac{c_s^2}{u^2}\left\{1 + \frac{5\tau^{-1}}{3} - \left(\eta_i - \frac{2}{3}\right)\frac{\mathbf{v}_{ni}}{u}\right\} = 0. \tag{15}$$

Eq. (15) is a cubic root equation w.r.t u, where u is the phase velocity for the ITG mode. By a little algebraic calculation, we can find easily the roots of Eq. (15). As the phase velocity is obtained from the linear algebraic equations, we can describe the linear properties of the mode from the roots. One root of Eq. (15) is as

$$u = \frac{1}{6\tau}\left[\frac{(2v_{ni}\tau^2 + (2\times\sqrt[3]{2}\tau(v_{ni}^2\tau^3 + c_s^2(5+3\tau))))}{\Gamma} + 2^{\frac{2}{3}}\times\Gamma\right], \tag{16}$$

where $\Gamma = (s_1+s_2)^{\frac{1}{3}}$, $s_1 = 33c_s^2 v_{ni}\tau^3 - 27c_s^2 v_{ni}\eta_i\tau^3 + 9c_s^2 v_{ni}\tau^4 + 2v_{ni}^3\tau^6$ and $s_2 = \sqrt{\tau^3(-4(v_{ni}^2\tau^3 + c_s^2(5+3\tau))^3 + v_{ni}^2\tau^3(2v_{ni}^2\tau^3 + c_s^2(33 - 27\eta_i + 9\tau))^2)}$. In Fig. (1), the graph shows the phase velocity against the electron to ion temperature ratio τ and the ion temperature gradient coefficient η_i. Observation of the graph shows that by enhancing the electron to ion temperature ratio, the phase velocity of the ITG mode decreases to the negative values, but the effect reverses as η_i value of the plasma enlarge in value. On the other side, the phase velocity increases with the ion temperature coefficient η_i. These observations

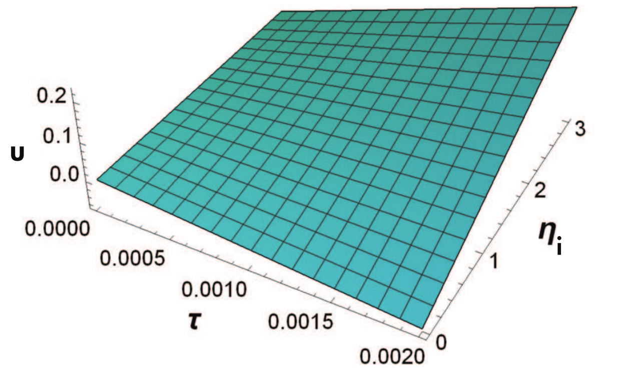

Figure 1. ITG driven mode phase velocity against τ and η_i.

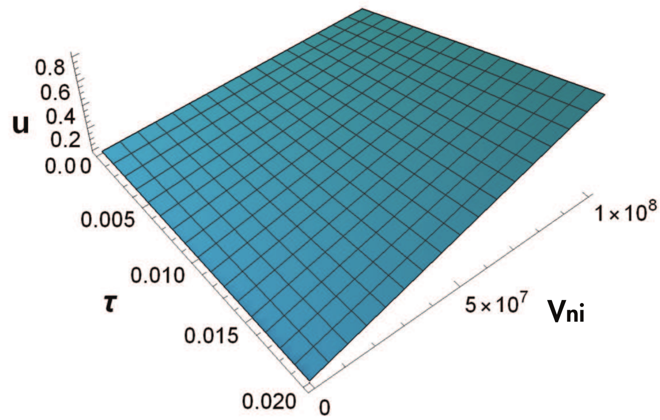

Figure 2. ITG driven mode phase velocity against τ and v_{ni}.

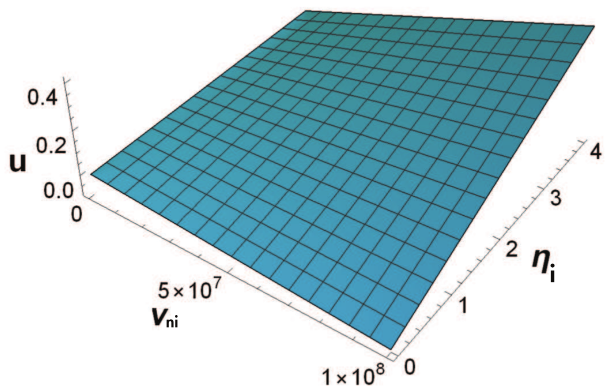

Figure 3. ITG driven mode phase velocity against τ and η_i.

remain valid with the mathematical reasoning because τ is related inversely with the ion temperature of the plasma so τ value increase means that the ion temperature decreases in the plasma, therefore, the mobility of the ion species decreases and also the phase velocity. On the other hand, the ion temperature coefficient is related directly to the change of ion temperature of the plasma, so an increase in the η_i value means increasing the value of the ion temperature in the plasma and the mobility of the ions phase velocity. Fig. (2) shows us that with v_{ni} and τ value the phase velocity of the linear mode can be enhanced here again. The ion drift speed v_{ni} related directly with the ion temperature of the plasma by increasing the v_{ni} value means to increase the value of the ion temperature and we can see from the Fig. (2) that τ value changes by minimal factor but with a significant chance of the v_{ni} the phase velocity of the mode change abruptly. Now Fig. (3) reveals the same variation for the phase velocity against v_{ni} and η_i as in the graph first of the article for τ and η_i.

3 Solitary waves

Now to obtain a nonlinear structure in the ITG driven mode (i.e., solitary and shock waves). We compare the next higher power of ϵ in the magnetohydrodynamics equations (such as in the continuity equation, momentum equation, energy balancing equation, and Poisson equation). We obtained linear and nonlinear differential equations in the form

$$\partial_\xi N^{(2)} - \frac{\tau \mathbf{v}_{ni}}{u}\partial_\xi \Phi^{(2)} - \tau \mathbf{v}_{ni}\partial_\ell \Phi^{(1)} - \frac{1}{2}\frac{\rho_i^2\tau^{-1}}{u^2}\partial_\xi^3\left(T^{(1)} + N^{(1)} + \Phi^{(1)}\right) + \frac{1}{u}\partial_\xi v_{ix}^{(2)} = 0, \tag{17}$$

$$-\partial_\xi v_{ix}^{(2)} + \frac{v_{ix}^{(1)}}{u}\partial_\xi v_{ix}^{(1)} + \frac{c_s^2}{u}\partial_\xi\left[\Phi^{(2)} + \tau^{-1}\left(T^{(2)} + N^{(2)}\right)\right] + c_s^2\partial_\ell\left[\Phi^{(1)} + \tau^{-1}\left(T^{(1)} + N^{(1)}\right)\right] = 0, \tag{18}$$

$$-\partial_\xi T^{(2)} + \frac{2}{3}\partial_\xi N^{(2)} = \tau\left(\eta_i - \frac{2}{3}\right)\mathbf{v}_{ni}\left(\frac{1}{u}\partial_\xi \Phi^2 + \partial_\ell \Phi^1\right), \tag{19}$$

$$\frac{1}{u^2\alpha_1}\partial_\xi^2\Phi^1 = (N^2 - \Phi^2), \tag{20}$$

where, $N^{(1)}$, $N^{(2)}$ are the normalized ion-number density of order first and second, $\Phi^{(1)}$, $\Phi^{(2)}$ are the normalized perturbed potential of order first and second, $T^{(1)}$, $T^{(2)}$ are the normalized ion-temperature of order first and second, v_{ix}, $v_{ix}^{(1)}$ are the ion-drift x-component of order first and second, v_{ni} ion-number density drift, u phase velocity of the mode, ρ_i ion gyro-radius, c_s acoustic speed, η_i ion-temperature gradient coefficient, $\alpha_1 = (4\pi e^2 n_{io})/T_e$, and $\tau = T_{eo}/T_{io}$. Now, combining Eqs. (17)-(20) we get the following nonlinear Korteweg-de-Vries (KdV) type of equation as

$$A_1\partial_\ell\Phi^1 + A_2\Phi^1\partial_\xi\Phi^1 + A_3\partial_\xi^3\Phi^1 = 0, \tag{21}$$

where $A_1 = \left\{\tau\mathbf{v}_{ni} + u\,(1 + \tau\mathbf{v}_{ni}) - \frac{2c_s^2}{u^2}\left(\eta_i - \frac{2}{3}\right)\mathbf{v}_{ni} + \frac{c_s^2}{u}\left(1 + \frac{5\tau^{-1}}{3}\right)\right\}$, $A_2 = -u\,(u - \tau\mathbf{v}_{ni})^2$ and $A_3 = \left\{\frac{u}{\alpha_1} + \frac{1}{2}\rho_i^2\tau^{-1}u\left(\frac{8}{3} - \tau\left(\eta_i - \frac{2}{3}\right)\frac{\mathbf{v}_{ni}}{u}\right) + \frac{5}{3}\frac{c_s^2\tau^{-1}}{u^3\alpha_1}\right\}$. Dividing both sides of Eq. (21) by A_1 coefficient we get

$$\partial_\ell\Phi^1 + A\Phi^1\partial_\xi\Phi^1 + B\partial_\xi^3\Phi^1 = 0, \tag{22}$$

where $A = \frac{A_2}{A1}$ and $B = \frac{A_3}{A1}$. The solution of Eq. (22) can be written (using a new variable as $\Omega = \xi - u\ell$ where u is the speed of the solitory waves in the ITG mode) as

$$\Phi = \Phi_0 \sec h^2\left[\frac{\Omega}{W}\right], \tag{23}$$

where $\frac{3u}{A} = \Phi_0$ and $\sqrt{\frac{4B}{u}} = W$.

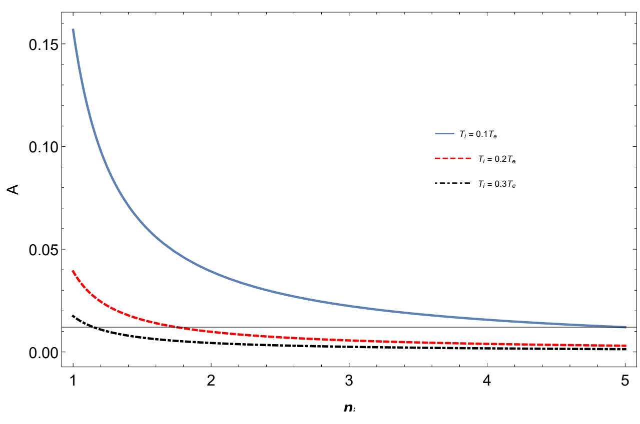

Figure 4. ITG driven mode KDV equation nonlinear coefficient versus η_i. under the effect of ion to electron temperature ratio.

Figs. (4) and (5) show that the nonlinear A and dispersion B coefficients of the kdv equation become smaller in value with the ion to electron temperature ratio T_i/T_e of the electron-ion plasma. So, we can observe the effects of different plasma parameters on the nonlinear and dispersion coefficients that will affect the magnitude as well as the sign of the coefficients hence the solitary and shock wave structure in the plasma can be changed from the compressional to the refractional type of soliton/shock.

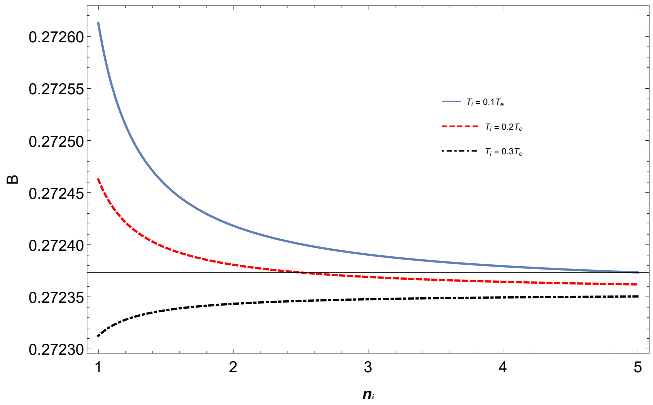

Figure 5. ITG driven mode KDV equation dispersion coefficient versus η_i under the effect of ion to electron temperature ratio.

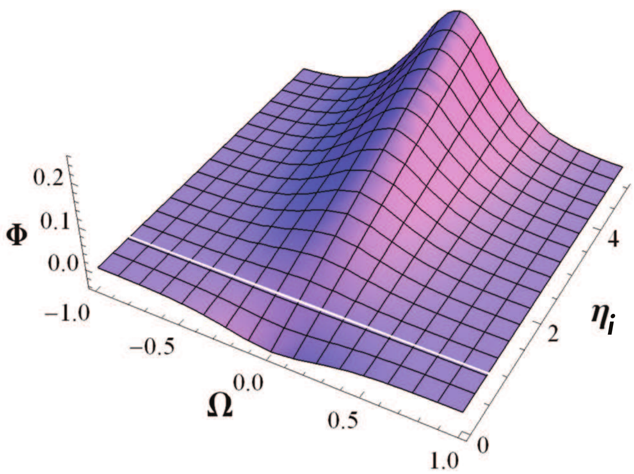

Figure 6. ITG driven mode solitary wave potential against the phase of the soliton η and η_i.

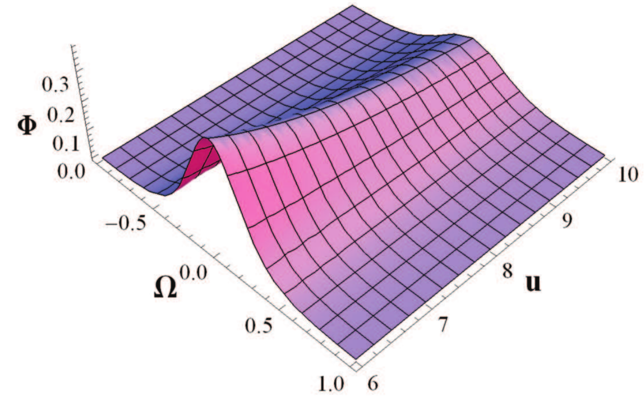

Figure 7. ITG driven mode solitan potential against phase of the soliton Ω and τ.

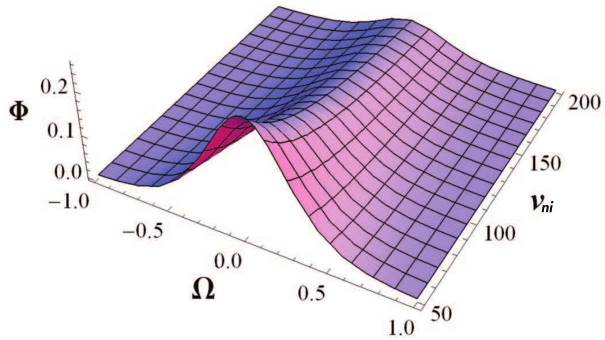

Figure 8. ITG driven mode solitary wave potential against phase of the soliton Ω and v_{ni}.

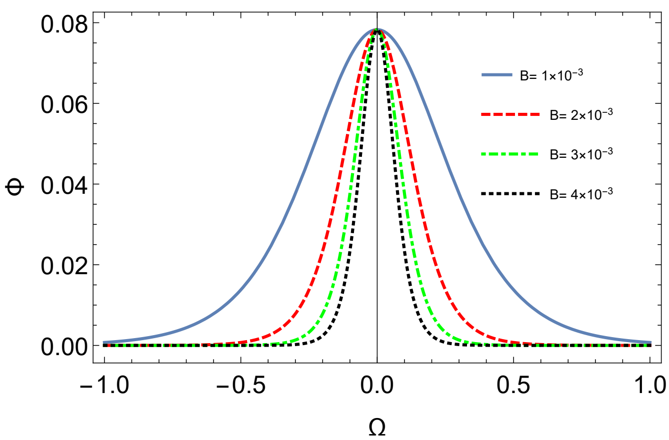

Figure 9. ITG driven mode solitary wave potential against phase of the soliton Ω.

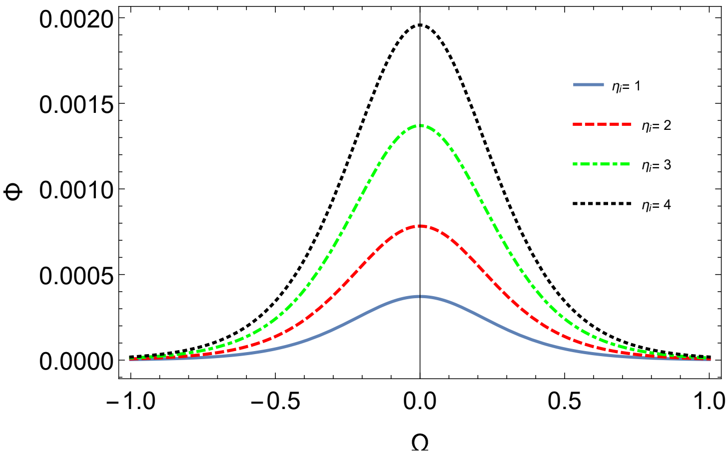

Figure 10. ITG driven mode solitary wave potential against phase of the soliton Ω.

That observation shows that both types of solitary waves can exist in the ITG mode at the low value of η_i−mode plasma a depth type of solitary waves are generated. In contrast, for $\eta_i \geq 1$ hump type of solitary waves are generated in the plasma it depends on the sign of the nonlinear coefficient of the KdV type of equation for a low value of ion temperature gradient coefficient its value is negative so refractive solitary waves are produced. Still, when the ion temperature gradient coefficient value is more significant than one, the compressive type of solitary waves is generated in the plasma. Also, dispersive properties of the solitary waves increase with the lowering of the η_i value while its amplitude is decreased by decreasing η_i value. Fig.(7) has been sketched among the soliton potential Φ against the soliton phase Ω, and its phase velocity u, which show that the solitary wave potential enhances in amplitude for low phase velocity while diminishing for the high phase velocity and also the dispersion properties of the soliton increases with the high phase velocity of the solitary waves in the electron-ion plasma. Fig. (8) is the graph of solitary wave potential against its phase and ion number density drift vni that shows the same situation as the previous graph i.e., with the drift velocity of the ion number density soliton potential decreasing in amplitude but its dispersive properties increases. Maybe the decrease in the amplitude of the solitary wave is due to the ion temperature and greater ion number density of the plasma because these plasma parameters can change its viscosity and bring more dissipation in the plasma. In Fig. (9) we have investigated the solitary wave potential against its phase Ω , which shows that the amplitude of the solitary wave is independent of the external magnetic field applied to the ITG mode driven electron-ion magnetoplasma. Still, the dispersion properties of the small amplitude solitary waves decrease with the background magnetic field's strength. While Fig. (9) shows the relation of the solitary waves against the soliton phase, with the ion temperature coefficient η_i the amplitude of the solitary wave becomes enhanced, and the dispersion properties of the waves is also increased with η_i. We can obtain the electric field from the solitary wave potential using a basic definition, i.e., $E = -\nabla\Phi$.

4 Shock wave

The shock wave can be generated in a fluid only when the dissipation effect is larger in a medium as compared to the dispersion. So, including the dissipative terms (i.e., $\eta_1 \frac{\partial^2 v_i}{\partial_x^2}$) in the ion momentum equation, we will get a nonlinear Burger-like differential equation whose solution gives us the shock structure in the medium

$$A_1\partial_\ell\Phi^1 + A_2\Phi^1\partial_\xi\Phi^1 - A_4\partial_\xi^2\Phi^1 = 0. \tag{24}$$

$A_1 = \left\{\tau\mathbf{v}_{ni} + u\left(1 + \tau\mathbf{v}_{ni}\right) - \frac{2c_s^2}{u^2}\left(\eta_i - \frac{2}{3}\right)\mathbf{v}_{ni} + \frac{c_s^2}{u}\left(1 + \frac{5\tau^{-1}}{3}\right)\right\}$,
$A_2 = -u(u - \tau\mathbf{v}_{ni})^2$, and
$A_4 = \eta_1(u - \tau\mathbf{v}_{ni})$ dividing both sides of Eq. (24) we get the nonlinear partial differential equation in the form as

$$\partial_\ell\Phi^1 + A\Phi^1\partial_\xi\Phi^1 - C\partial_\xi^2\Phi^1 = 0, \tag{25}$$

where $A = \frac{A_2}{A_1}$ and $C = \frac{A_4}{A_1}$ the solution of Eq (25). By using a new variable as $\Omega = \xi - u\ell$ is given as

$$\Phi = \Phi_0\left[1 - \tan h\left(\frac{\Omega}{\frac{4C}{u}}\right)\right], \tag{26}$$

here $\frac{u}{A} = \Phi_0$ and $\frac{4C}{u} = Z$.

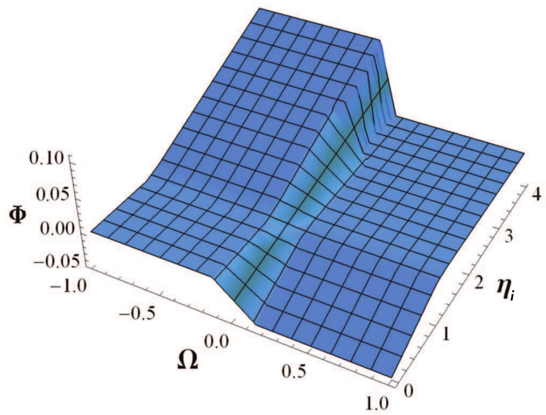

Figure 11. ITG driven mode Shock wave potential against phase of the shock Ω and η_i.

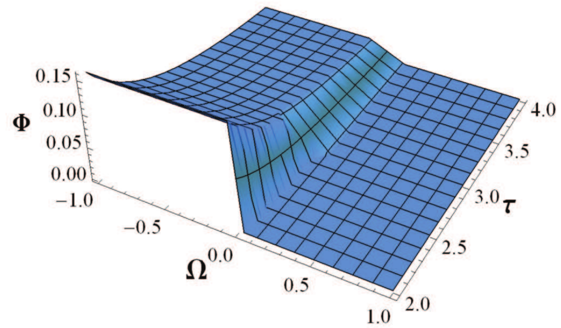

Figure 12. ITG driven mode Shock wave potential against phase of the shock Ω and τ.

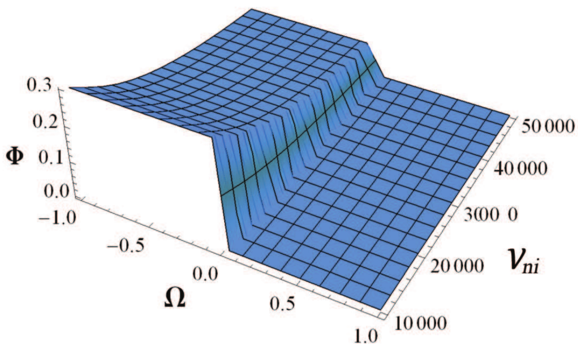

Figure 13. ITG driven mode Shock wave potential against phase of the shock Ω and ν_{ni}.

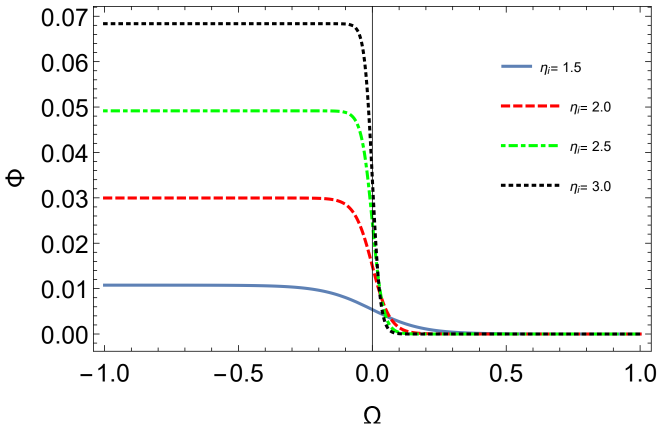

Figure 14. ITG driven mode Shock wave potential against phase of the shock Ω.

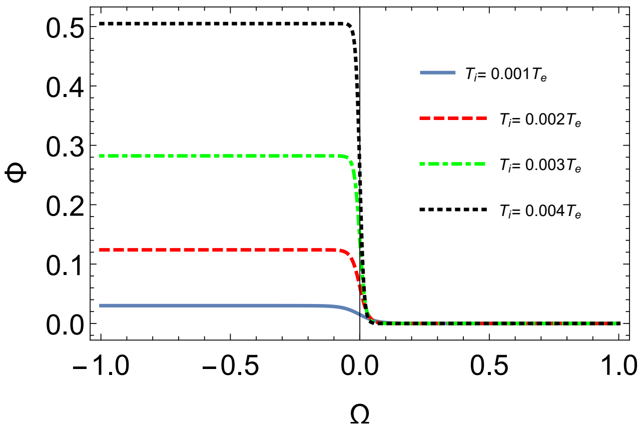

Figure 15. ITG driven mode Shock wave potential against phase of the shock Ω.

Fig. (11) is a graph of the shock wave potential against the phase of the shock and ion temperature gradient coefficient η_i. That figure gives a very interesting observation about the shock wave profile that for $\eta_i < 1$ then a rarefaction type of shock waves is produced in the plasma while compression type of the shock waves is produced when $\eta_i > 1$. Here the reason is the same as for the solitary waves because the nonlinear and dissipation coefficients of the kdv-Burger equation Eq. (25) can change its sing by changing the values of the plasma parameters. Fig. (12) reveals that the shock wave amplitude becomes smaller with the electron to ion temperature ratio, possibly, the high temperature species electron of the plasma presents opposition to the shock wave in the plasma due to the ion species. Similarly, the effect observed in Fig. (13) where the rise in the drift velocity of the ion can enlarge the shock wave amplitude and vice versa, may be the high ion number density in the fluid offer resistance to the production of the shock wave. In Fig. (14) we have compared the shock wave against its phase the 2-dimensional plot, here we can see that the amplitude of the shock with the ion temperature coefficient η_i increases and the same variation observed in Fig. (15) for the shock wave potential against its phase for the different values of the ion to electron temperature ratios, here we see the vibration of the shock wave amplitude is larger as compare to the previous Fig. (14). We have used the following parameters in analyzing the linear dispersion relation, nonlinear shock and solitory wave $n = 10^{14} cm^{-3}$, $B_0 = 1 \times 10^{-4}$, $T_i = 0.1 T_e$, $n_p = 0.001 n_e$, $\eta_i = 2$, $c_s = 10^6 \frac{cm}{s}$, ion gyro-frequency $\omega_{ci} = 10^4 \frac{rad}{s}$, in ξ-coordinates $u = 10^6 \frac{cm}{s}$, $\alpha = 0.1 rad$. These values are in agreement with the previous literature [13, 15, 17, 33, 35].

5 Conclusion

We have studied here the linear and nonlinear properties related to the ion temperature gradient (ITG) driven mode in the electron-ion plasma. Ions are observed to have dynamics while electrons follow the Maxwellian distribution in our consideration. We have derived the linear and nonlinear ITG modes by using a set of MHD equations for electron-ion plasma and then using the reduction perturbation method to derive the phase velocity for the mode that was independent of the wavenumber k of the wave, as has been shown in the calculation. Then we obtained a nonlinear structure in the form of solitary and shock waves in the same electron-ion magneto-plasma. We have shown in our calculation that different parameters like ion temperature, ion number density, magnetic field, etc., can affect the phase velocity as well as the shock and solitary waves in the electron-ion plasma.

Abbreviations

Electron temperature gradient (ETG), ion temperature gradient (ITG), Reduction Perturbation Method (RPM), electron-positron-ion (e-p-i), homotopy perturbation method (HPM), reduction perturbation method (RPT), Korteweg–De Vries (kdv), Magneto–hydrodynamics (MHD)

Declarations

Consent for publication

Not applicable.

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Author's contributions

Aziz Khan: Supervision, Validation, Conceptualization, Methodology, Software. Abbas Khan: Writing-Original draft preparation, methodology. Muhammad Sinan: Visualization, Methodology, Investigation, Software, Writing-Reviewing and Editing. All authors discussed the results and contributed to the final manuscript.

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