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Low-computational adaptive MPC algorithmization strategy for overshoots and undershoots in instantaneous water heater stability

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ABSTRACT

Tankless gas hot water users' perceived comfort is severely affected by sudden changes deviating from the desired temperature. Water temperature instability due to overshoots and undershoots is the most common issue that appears mainly due to sudden changes in users' water flow demands and response delays inherent to the heating system. Classical controllers for heat cells have difficulties responding to temperature instability in a timely manner because they lack the capacity to anticipate the effects of sudden variations in water flow rate. Previous studies have reported the model predictive controller (MPC) with adaptive function strategy to provide the best response for stabilizing temperature, and its performance is a result of the predictive nature that allows for anticipating and correcting the negative effects on temperature from sudden flow rate variations. The present study aims to employ this strategy to a low-computational algorithm that can be embedded in low-cost hardware with limited computational and memory resources. The study's motivation is to fill the space manufacturers have left in this regard by implementing low-cost optimal-performance microcontrollers for water heaters. The algorithm results show good agreement for the responses in temperature stabilization with experimental data.

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1. INTRODUCTION

The use of hot water contributes to about 40% of the energy consumed in residential dwellings (Bourke et al., 2014) and is responsible for an important percentage of domestic energy consumption. Instantaneous gas heating systems are widely used for their advantages of not needing a reservoir and competitive use/consumption ratio. Tankless gas water heaters (TGWHs) have the highest sales and have become an efficient means of heating water with low carbon emissions (Bourke et al., 2014). Their advantages compared to storage heaters are their smaller size, continuous hot water flow, and longer estimated useful life (Yuill et al., 2010). However, they require more power to provide the proper flow capacity, which makes their control quite complex. Furthermore, users' perceived comfort is severely affected by sudden changes in water temperature deviating from the desired temperature (Costa et al., 2016). Hot water temperature instability due to overshoots and undershoots is generally the most common drawback and mainly results from sudden changes in water flow and response delays inherent to the hot water device's inability to make predictions using classical controllers (Costa et al., 2016). Figure 1 demonstrates the temperature overshoots and undershoots of a 58kW nominal power TGWH with respect to sudden changes in the water flowrate. In particular, this figure uses real data from an experimental laboratory test performed by the manufacturer with the use of a feed-forward proportional–integral–derivative (FFPID) controller for stabilizing the outlet hot water temperature at 60°C. Classical controllers rely on current and previous

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measurements to regulate the system (Ehtiwesh & Durović, 2009). However, temperature overshoots and undershoots are neither acceptable nor comfortable for users regarding unpredictable changes in the hot water flow rate and have become a safety issue in extreme scenarios.

Henze et al. (2009) addressed the development of a strategy aimed at water temperature control in tankless hot water devices. Their strategy uses a model-based predictive controller to reduce outlet temperature errors. A dynamic heat transfer model for an electric tank water heater was implemented within the predictive controller of the model. The controller was connected to a physical tank water heater prototype and showed effective control of the output temperature. An artificial neural network (ANN) controller was embedded into a low profile microcontroller for a commercial electric instantaneous water heater and resulted in a lower temperature peaks and recovery times compared to the classic PID controller (Laurencio-Molina and Salazar-Garcia 2018). Takács et al.'s (2016) study showed the embedment of a model with predictive controller feedback laws into Python applications and developed a code-generation module for MATLAB's Multi-Parametric Toolbox. The study reported that the Python algorithm can be encoded within just a couple of lines in the MATLAB environment. Wang et al. (2011) proposed a few controller schemes for improving the outlet temperature stabilization of TGWH systems with a fuzzy control system intended as a black box gain scheduler regarding the parameters of a PID controller. Haissig and Woessner (2000) studied an adaptive fuzzy control code that would adapt to changing conditions such as water flow rate and inlet water temperature and automatically adjust the feed-forward curves of the gas valve control. Xu et al. (2008) studied a dynamic neuro-fuzzy control system as a controller in gas water heaters. The controller comprises a fuzzy logic controller in the feedback configuration and two dynamic neural networks in the forward path. Ehtiwesh et al.'s (2021) previous study developed a classical controller (i.e., FFPID) and model predictive controller (MPC) for controlling TGWH systems. The classic design of MPC controllers is more complicated due to the behavior of the dominant nonlinear dynamics, which can lead to performance drops (Aliskan 2018). An adaptive predictive control strategy that provides a new linear model for each time interval under dynamic operating conditions has also been implemented. Adaptive MPCs make more accurate predictions for the next time interval in contrast to classic MPCs that employ a fixed internal model. However, TGWH manufacturers are open to implementing new controllers within low-cost microcontrollers possessing limited computational abilities and memory resources. Therefore, the present study aims to address the strategy of employing a low computational code that can be embedded in low-cost hardware following the developed adaptive MPC strategy.

2. MODELING

The developed model (Ehtiwesh et al., 2021) is based on a real residential and commercial tankless water heater: the Hydro

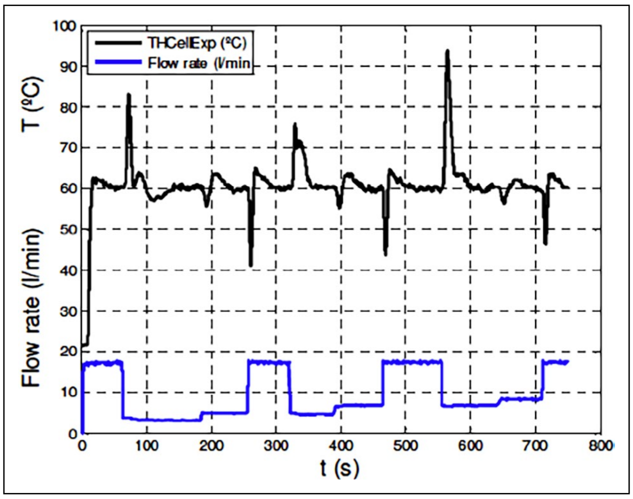

Figure 1. Experimental data of a 58kW TGWHs heat cell (Costa et al., 2016).

TGWH: Tankless gas water heater.

4600 F WTD10-4KME 23 JU, a non-condensing model rated at 22kW of thermal power and a thermal efficiency of 0.86. The system incorporates several sensors such as a carbon monoxide detector, type K thermocouples and RTD Pt100 temperature probes, a pressure sensor, and a water flow meter that enables measuring of the variables utilized in the feedback loop of the control systems. The heat cell contains a gas combustion burner and heat exchanger to heat up the water using water condensation in the flue gases. The system is a semi-empirical nonlinear model with the following energy balance equation regarding the distributed parameter model:

$$C\, dT/dt = \dot{Q} + \dot{m}\, c_{p,w}\, (T_{in} - T) \tag{1}$$

where C is the thermal capacitance defined as a coupling constant based on the energy conservation law:

$$C = (m_w\, c_{p,w} + m_m\, c_{p,m}) \tag{2}$$

$\dot{Q}$ is the thermal power utilized in the heat cell, T is the heat cell temperature, $\dot{m}$ is the mass flow rate ($\rho\dot{V}$), $\dot{V}$ is the volumetric flowrate, m_w is mass, and c_p is the heat capacity ρ is water density. With this simplification, Eq. 1 can be rewritten as follows:

$$\mathrm{dT/dt} = (\dot{Q} + \rho\dot{V}\, c_{p,w}\, (T_{in} - T))/(m_w\, c_{p,w} + m_m\, c_{p,m}) \tag{3}$$

The model inputs are the thermal power (controller output) and the disturbance in the water flowrate, with the produced water temperature being the model output. The delay associated with thermal power delivery is considered to be a constant input time delay. The outlet delay varies with time based on the velocity of the water and the pipe section as presented in Eq. 4, where r_i is the orifice radius and L_i is circuit length inside the heat exchanger. Implicit dead-time compensation is utilized to employ the time delay, which varies with the flowrate as:

$$t_{delay} = \pi\, r_i^2\, L_i / \dot{V} \tag{4}$$

2.1. Case studies

Water temperature instability from overshoots and undershoots is the most common disadvantage of TGWHs and mainly occur due to sudden changes in users' water flow demand and the response delays inherent to the heating system. Classical controllers for heat cells have difficulties responding to temperature instability in a timely manner because they lack the ability to anticipate the effects of sudden variations in the water flow rate. Ehtiwesh et al.'s (2021) previous study carried out a comparative analysis of model-based predictive controls with and without adaptive function and classical controllers (i.e., FFPIDs) with regard to TGWH controllers. Model predictive control (MPC) is a feedback control scheme that relies on a model, an optimization solver, a receding horizon control, and optimization of a quadratic programming (QP) problem (Li et al., 2015). The future outputs of a decided horizon (prediction horizon) are predicted at each instant. These predicted outputs depend on the preceding output and input and on future control signals. The linear quadratic function is employed as a controller performance criterion in order to obtain a smooth and rapid response with minimal error and limited strain on operation. The quadratic programming problem optimizes the objective (i.e., cost function) as a nonnegative measure. The weights are adjusted to tune the controller, and the constraints are the physical bounds on the manipulated variables (MVs) and the model output parameters, and a discrete linear time-invariant (LTI) state space model is used to predict the response within the prediction horizon. The parameter estimation and subsequent validation are performed with an associated optimization platform using a discrete state space model. The undisclosed parameters have been identified using the experimental virtual test (Quintã et al., 2022). In addition, by reason of the dominant nonlinear dynamics in TGWH behavior, an adaptive predictive control strategy has also been studied (Ehtiwesh et al., 2021). The adaptive strategy provides a novel linear model at each time step regarding changes in dynamic operating conditions. Therefore, adaptive MPC offers more accurate predictions for the new time step. Altogether, the adaptive function resizes and updates the state space system of the model's parts according to flow rate changes and integrates time delays, which are absorbed as discrete states. The adaptive MPC presents superior performance regarding temperature stability in the event of sudden water flow variations.

2.2. Algorithm approach

Undoubtedly, the adaptive MPC strategy outputs the best behavior compared to the other aforementioned strategies that have been studied. Notwithstanding, TGWH manufacturers demand these controller strategies be implementable in low-cost microcontrollers with limited computational and memory resources. Therefore, the present study aims to address the potential for developing a low computational code that can be embedded into low-cost hardware based on the developed state-of-the-art adaptive MPC. The algorithm has been encoded without using pre-made functions and toolboxes in order to be easily encoded within any compiler, with Python possessing the embedded computational capability in minimal hardware devices based on device limitations. Embedding provides applications with the capability to perform some of the functionality, Python more so than Matlab, C, or C++. This can be utilized for many goals, such as allowing users to tailor the application to their requirements by creating certain scripts within Python. The algorithm encompasses two main parts: the first aims to create the plant model in which the mathematical model is defined, linearized, and discretized and to define the parameters, with uncertain parameters being defined explicitly. The second part involves configuring the MPC controller, which encompasses the following three steps including the adaptive function:

1. Measure values,
2. Solve the constrained optimization problem, and
3. Update the states and the controller output.

The first step (i.e., measure values) calls for the ordinary subroutine to use the information obtained by the plant model to define the values being measured. Three values are required: 1) current time (t_0), which is calculated as a function of the sample time (T_s) and the current iteration; 2) the manipulated variable (u_0), which is the vector containing the predicted thermal power in the previous iteration (i.e., system input) where the new vector will be an initial state in the optimization problem; and 3) the system response (x_0), which is the current state of the system (i.e., outlet temperature). In addition, the TGWH plant needs further information to simulate real dynamics, such as:

i. *Flow rate*. The differential equation used in the MPC model for calculating the future thermal power is based on the flow rate. The current flow rate value is read by the differential equation and adapts its dynamics like the real system.

ii. *Delays*. Input and output delays are simulated using the variables' input delay and predicted values.

Due to this study's aim of embedding an MPC controller into low-computational microcontrollers, ensuring a linear model is a preferred strategy. Therefore, the study considers approximating the nonlinear functions in the proximity of the steady state operating point using a first-order Taylor series expansion that neglects the terms after the first partial derivatives. Eq. 3 is linearized around the operation point, namely, (Q_0, $\dot{V}_0$, dT_0) and converted to a state-space representation (Eq. 5). The inputs are the power (manipulated variable) and the flow rate (measured disturbance), with the temperature deviation ($dT=T-T_{in}$) being the output.

$$\dot{x}=\left[-\frac{\rho c_{p,w}\dot{V}_0}{c}\right]x+\left[\frac{1}{c}\ -\frac{\rho c_{p,w}dT_0}{c}\right]\begin{bmatrix}u_1\\u_2\end{bmatrix},\ y=x \tag{5}$$

The constrained optimization problem (Step 2) is solved using the MPC model, where the cost function is the quadratic error between the reference signal (50°C) and the MPC response using the constrained power inputs. The constraints over the action control are that the input power is limited from 0 to 1 (1 implying 100%). The solver calculates the coming sequence of manipulated variables via the control horizon, and the first value of the sequence is directed to the actuator for the following step. During the following time step, the state values are recalculated and advanced based on sensory information and applied manipulated variables. Optimization is repeated to estimate the optimal future sequence of the manipulated variables using the prediction horizon. The final step aims to prove the predicted input and uses the optimization function “fmincon”, which is an inbuilt function for solving optimization problems by defining the minimum of a constrained nonlinear multivariable function. The function toolbox defined as a nonlinear programming solver (Eq. 6), realizes the minimum of a specified problem (MathWorks, 2021) as:

$$\min_{x} \; f(x) \quad \text{subject to}$$

$$C(x) \le 0,\ ceq(x)=0,\ A\,x \le b,\ Aeq\,x = beq,\ lb \le x \le ub \qquad (6)$$

where b and beq are null initializing vectors, A and Aeq are null initializing matrices, $C(x)$ and $ceq(x)$ are functions that return vectors, $f(x)$ is a cost function that returns a scalar, lb and ub are the lower and upper limit constraints vectors whose values are narrowed by the function, and $f(x)$, $C(x)$, and $ceq(x)$ are nonlinear functions. The final step also aims to update the states and the controller's output alongside the incoming iterations in order to prove the predicted inputs by including the system delays. The adaptive function resizes and updates the state space system of the model elements according to the flow rate changes and integrates the time delays that have been absorbed as discrete states. The time delay is calculated using Eq. 4 and absorbed in the discretization LTI state-space model by replacing time delays with poles at the phase shift (z=0) and a delay of the sampling period using the same sampling poles at z=0. The capability to keep track of delays makes the state space the best suited one for the model and for analyzing the delay effects in control systems. Assuming that the model is described by the subsequent uncertain discrete time-linear system with dead-time:

$$x(k+1) = A\,x(k) + B\,u(k\text{-}d) + w(k),\ y(k) = C\,x(k) \qquad (7)$$

where $x(k) \in Rn$ is the current state, $u(k) \in Rm$ is the current control input, $w(k) \in Rn$ is a bounded vector of disturbance, $y(k) \in Rp$ is a linear combination of the states that identifies the desired output, k denotes the current sampling instant and d represents the nominal dead-time. Mostly, for simple cases, the time invariant linear systems without disturbances ($w(k)$=0) and dead-time (d=0) (Santos et al. 2012), an augmented used model is presented in (Astrom and Wittenmark 1997), incorporating the dead-time effect such as a dead-beat dynamics to achieve a “dead-time free” using implicit dead-time compensation as:

$$\Gamma(k+1)=A_\Gamma\,\Gamma(k)+B_\Gamma\,u(k),\ y(k)=C_\Gamma\,\Gamma(k) \qquad (8)$$

With

$$\Gamma(k) = (x(k)'\ u(k\text{-}d)'\ u(k\text{-}d+1)'\ \ldots\ u(k\text{-}2)\ u(k\text{-}1)')';$$

$$A_\Gamma = \begin{bmatrix} A & B & 0 & 0 & \ldots & 0 \\ 0 & 0 & I & 0 & \ldots & 0 \\ 0 & 0 & 0 & I & \ldots & 0 \\ . & . & . & & & . \\ . & . & . & & & . \\ . & . & . & & & . \\ 0 & 0 & 0 & 0 & \ldots & I \\ 0 & 0 & 0 & 0 & \ldots & 0 \end{bmatrix}, \qquad B_\Gamma = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ I \end{bmatrix}, \qquad C'_\Gamma = \begin{bmatrix} C' \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

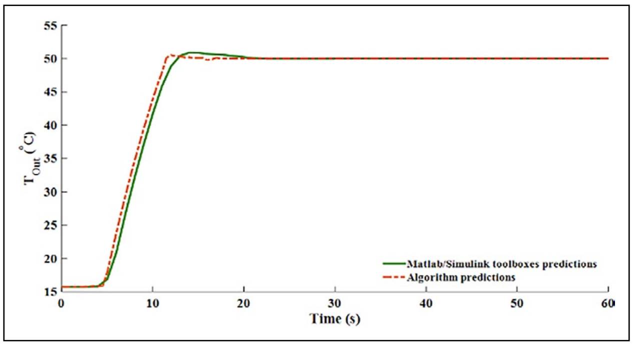

Figure 2. Comparison of the algorithm and MATLAB model predictions at a constant flow rate (10L/m).

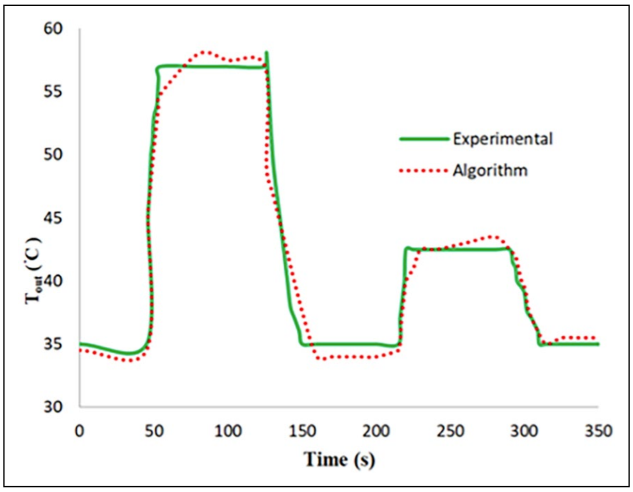

Figure 3. Experimental and algorithm predictions (water temperature) for a sequence of thermal.

The strategy is to store the past control actions in Γ(k) Rn+d.m until the time they can actually be considered. Therefore, Γ(k+1) depends only on Γ(k) and u(k) being able to directly describe the stabilizing elements.

3. RESULTS

The algorithm predictions have been validated using the results from the model developed in Ehtiwesh et al.'s (2021) previous study. The comparison has been implemented at the linearized flow rate of 10 L/m and sample time of 1 sec. The predictions are in extremely close agreement (Fig. 2).

Figure 3 presents a comparison between the experimental results Quinta et al. (2022) presented and the algorithm predictions of the experiment carried out for open-loop tests at constant flowrate with a sequence of fast changes in the applied thermal power (32%, 50%, and 100%). The predictions are in a good agreement, with the steady-state values being essentially coincident, despite small differences being observed during the transient segments. Figure 4 shows the overshoots and undershoots in temperature stability that appear between the 40s-60s mark in the event of sudden variations in water flow around the 34s mark, with a total simulation time of 100s. The flow rate varies sharply from 10 to 3 L/min, and the water temperature causes negligible overshoots and undershoots. Figure 5

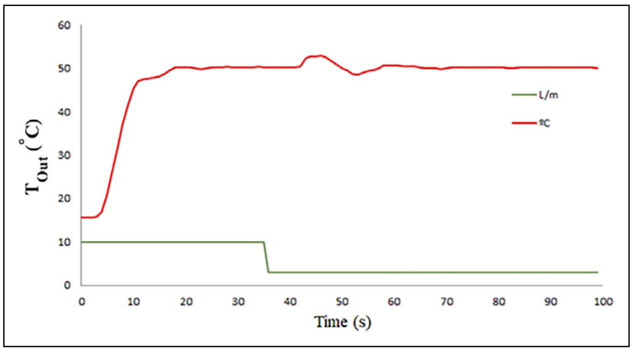

Figure 4. The algorithm predictions for a cold start at T_s=250ms.

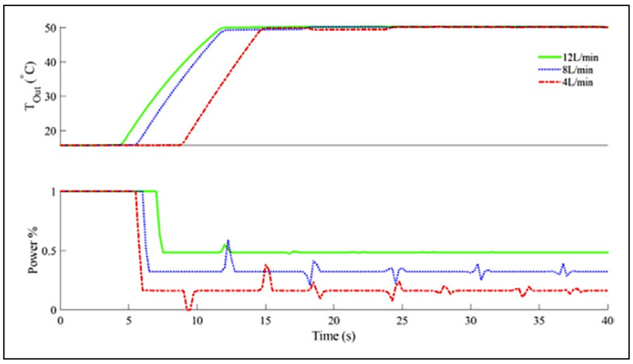

Figure 5. The algorithm predictions for a cold start at different flow rates and T_s=250ms.

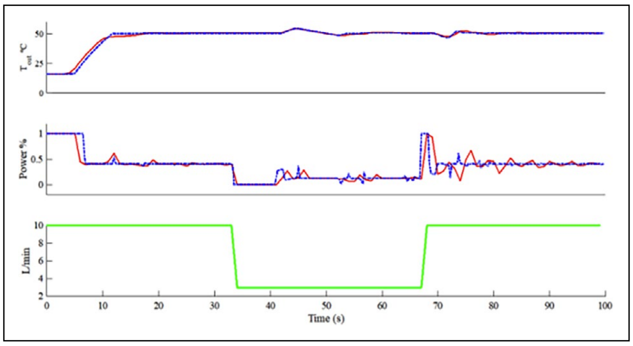

Figure 6. The algorithm predictions for a cold start at variable flow rates and different sampling intervals.

presents the simulation at various flow rates, with the manipulated variable of power being maximized in the initial stage before being switched to a steady-state value.

The power behavior shows an almost step-like transition from the upper value of the power to the steady-state value in all cases, resulting in faster temperature stabilization without overshoots or oscillations. Figure 6 presents the simulation at variable flow rates for sampling times of 250ms and 1000ms. The water flow rate changes sharply from 10L/min to 3L/min and back to 10 L/min, and the water temperature again shows negligible overshoots and undershoots.

The differences in the quality is presented in the controller signals (i.e., power). A tradeoff occurs between that frequency and T_s, with the frequency of the control signal being greater at T_s=250ms compared to T_s=1000ms. The reason for the zero signals between 33s-41s is due to the model having input and output delays where the internal delay is neglected. In fact, this behavior is the most difficult to overcome. The delay is approximately 10 seconds. This behavior matches the incline in the temperature output at the 42s mark.

6. CONCLUSION

Water temperature instability takes place mainly because of the nonlinearities and time-varying delays associated with TGWH systems. The MPC controller with adaptive function displayed excellent performance regarding temperature stabilization for sudden changes in flow rate. However, its employment required high computational resources and memory space and therefore needed expensive hardware. Because of this, the current study has developed a low-computational algorithm with the ability to be embedded in low-cost microcontrollers. The results of the findings demonstrate good agreement with the previously developed MATLAB/Simulink model and other experimental data. In conclusion, an adaptive model predictive control strategy can be a good solution for fulfilling the objective of reducing the time sampling to the 250ms rate that generally used by manufacturers with regard to improving water temperature instability due to overshoots and undershoots in TGWH devices.

DATA AVAILABILITY STATEMENT

The published publication includes all graphics and data collected or developed during the study.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

FINANCIAL DISCLOSURE

The authors declared that this study has received no financial support.

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