Mathematical Modelling and Numerical Simulation with Applications, 2022, 2(4), 211–220

https://www.mmnsa.org
ISSN Online: 2791-8564 / Open Access
https://doi.org/10.53391/mmnsa.2022.017

RESEARCH PAPER

A new analytical approach to the (1+1)-dimensional conformable Fisher equation

Gülnur Yel [1,*,‡], Miraç Kayhan [2,‡] and Armando Ciancio [3,‡]

[1]Faculty of Education, Final International University, Kyrenia, 10 Mersin, Türkiye, [2]Department of Mathematics, University of Inonu, 44280 Malatya, Türkiye, [3] Department of Biomedical and Dental Sciences and Morphofunctional Imaging, University of Messina, 98125 Messina, Italy

[*]Corresponding Author
[‡]gulnur.yel@final.edu.tr (Gülnur Yel); mirackayhan@yandex.com (Miraç Kayhan); aciancio@unime.it (Armando Ciancio)

Abstract

In this paper, we use an effective method which is the rational sine-Gordon expansion method to present new wave simulations of a governing model. We consider the (1+1)-dimensional conformable Fisher equation which is used to describe the interactive relation between diffusion and reaction. Various types of solutions such as multi-soliton, kink, and anti-kink wave soliton solutions are obtained. Finally, the physical behaviours of the obtained solutions are shown by 3D, 2D, and contour surfaces.

Key words: Rational sine-Gordon expansion method; (1+1)-dimensional Fisher equation; conformable derivative; travelling wave solutions
AMS 2020 Classification: 35A25; 35C07; 35R11

1 Introduction

Mathematical modelling of physical systems leads to linear and nonlinear differential equations in physics, engineering, and other fields. Understanding physical processes described by nonlinear equations necessitates finding exact solutions. Aside from that, exact solutions can be used to calculate specific critical physical quantities analytically and for simulation [1]. There are some methods to calculate analytically including the Hirotas' direct method [2], the tanh method [3], the extended tanh-function method [4], the multiple exp-function method [5], the transformed rational function method [6], the first integral method [7], the modified simplest equation method [8], the improved Bernoulli sub-ODE method [9], the Sine-Gordon expansion method [10, 11, 12], the modified exponential method [13]. Besides these analytical methods, there are many efficient numerical techniques have been submitted to the literature by mathematicians. For example, the q-homotopy analysis transform technique [14], the trapezoidal base homotopy perturbation method [15], and others [16, 17, 18]. The paper aims to find exact soliton solutions of the conformable (1+1)-dimensional Fisher equation [19] by using the rational sine-Gordon expansion method. The (1+1)-dimensional Fisher equation

$$u_t = \alpha^2 u_{xx} + pu - \beta u^3, \tag{1}$$

describes the process of diffusion and reaction interacting. Fisher presented this equation as a model for mutant gene propagation, with $u(x,t)$ denoting the density of favourable mutations, α^2 as diffusion factor [20]. This equation is used in chemical kinetics and population dynamics and is also used to solve problems like the nonlinear evolution of a population in one-dimensional habitual space or the neutron population in a nuclear reaction [21]. Zhou et al. have applied the improved $tan(\varphi(\xi)/2)$-expansion

➤ **Received:** 18.08.2022 ➤ **Revised:** 15.12.2022 ➤ **Accepted:** 24.12.2022 ➤ **Published:** 27.12.2022

method, the generalized Kudryashov method and the extended (G'/G)-expansion method, and gained the bright-like, dark-like and singular-like solitary wave solutions [19]. Triki and Wazwaz have used the trial equation method to a generalized Fisher equation, and gained some new wave solutions [22]. As a different approach, Matinfar et al. [23] focused on the numerical solution. In this context, they obtained solutions compatible with the exact solution by applying the generalized two-dimensional differential transform method. In this article, we will apply the rational sine-Gordon expansion method to the (1+1)-dimensional Fisher equation with conformable derivative to construct wave solutions. The conformable derivative operator overcomes some limitations of other fractional operators (Caputo, Riemann-Liouville, Caputo-Fabrizio and etc.) and provides basic properties of classical calculus such as the quotient rule, the chain rule, the product of two functions, Rolle's and mean value theorems. The application of the conformable derivative is simpler and very effective.

The rest of the paper is organized as follows: In Section 2, we describe the conformable derivative and its fundamental properties. In Section 3, the basic steps of the rational sine-Gordon expansion method, which is the novelty of the paper are presented. In Section 4, we apply the proposed method to the (1+1)-dimensional Fisher equation. Several conclusions are given in the last section.

2 Preliminary remarks on the conformable derivative

Definition 1 *Given a function $h : [0, \infty) \rightarrow R$. Then the conformable derivative of h order γ is defined by*

$$L_{\gamma}(h)(t) = \lim_{\varepsilon \rightarrow 0} \frac{h\left(t + \varepsilon t^{1-\gamma}\right) - h(t)}{\varepsilon},$$

for all $t > 0, \gamma \in (0, 1]$ [24].

Theorem 1 *Let L_{γ} be the derivative operator with order $\gamma \in (0, 1]$ and h, k be γ- differentiable at a point $t > 0$. Then we have the following properties [24, 25]:*

i. $L_{\gamma}(ah + bk) = aL_{\gamma}(h) + bL_{\gamma}(k), \forall a,\ b \in R.$

ii. $L_{\gamma}(t^{p}) = pt^{p-\gamma}, \forall p \in R.$

iii. $L_{\gamma}(hk) = hL_{\gamma}(k) + kL_{\gamma}(h).$

iv. $L_{\gamma}(\frac{h}{k}) = \frac{kL_{\gamma}(h) - hL_{\gamma}(k)}{k^2}.$

v. $L_{\gamma}(\lambda) = 0$, *for all constant functions $h(t) = \lambda$.*

vi. *If h is differentiable, then $L_{\gamma}(h)(t) = t^{1-\gamma} \frac{dh}{dt}(t)$.*

■

3 Fundamental structure of the RSGEM

Before giving the rational sine-Gordon expansion method (RSGEM) [26, 27, 28], we will explain the sine-Gordon expansion method (SGEM). Let us suppose the sine-Gordon equation

$$\varphi_{xx} - \varphi_{tt} = m^2 \sin(\varphi), \tag{2}$$

where $\varphi = \varphi(x, t)$, m is a real constant. Considering the wave transform $\varphi = \varphi(x, t) = \Phi(\xi),\ \xi = \mu(x - ct)$ into Eq. (2), gives the nonlinear ordinary differential equation (NODE) as,

$$\Phi'' = \frac{m^2}{\mu^2(1 - c)} \sin(\Phi), \tag{3}$$

where $\Phi = \Phi(\xi)$, μ is the amplitude and c is the velocity of the travelling wave, respectively.
We find as follows after full simplification of Eq. (3);

$$\left[\left(\frac{\Phi}{2}\right)'\right]^2 = \frac{m^2}{\mu^2(1 - c^2)} \sin^2\left(\frac{\Phi}{2}\right) + C, \tag{4}$$

where C is the constant of integration. Replacing $C = 0, \omega(\xi) = \frac{\Phi}{2}$ and $A^2 = \frac{m^2}{\mu^2(1-c^2)}$ in Eq. (4), gives;

$$\omega' = A \sin(\omega). \tag{5}$$

Setting $A = 1$ in Eq. (5), gives

$$\omega' = \sin(\omega). \tag{6}$$

Solving Eq. (6) by variables separable, we get the two significant properties of trigonometric functions as follows;

$$sin(w) = sin[w(\xi)] = \frac{2pe^{\xi}}{p^2e^{2\xi}+1}\Big|_{p=1} = sech(\xi), \tag{7}$$

$$cos(w) = cos[w(\xi)] = \frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\Big|_{p=1} = tanh(\xi), \tag{8}$$

where $p \neq 0$ is the integration constant. Let's consider the nonlinear partial differential equation (NPDE) of the form below, for which the solution is searched;

$$P\left(\varphi, \varphi_x, \varphi_t, \varphi_{xx}, \varphi_{tt}, \varphi_{xt}, \varphi_{xxx}, \varphi_{xxt}, \varphi^2, \ldots\right) = 0. \tag{9}$$

We apply the wave transformation, $\varphi = \varphi(x,t) = \Phi(\xi)$, $\xi = kx + w\frac{t^\gamma}{\gamma}$ into Eq. (9), it gives the following equation,

$$N\left(\Phi, \frac{d\Phi}{d\xi}, \frac{d^2\Phi}{d\xi^2}, \ldots\right) = 0, \tag{10}$$

where N is a nonlinear ordinary equation (NODE) that has partial derivatives of Φ depending on ξ. The SGEM, the solution of Eq. (9) is considered in the following form

$$\Phi(\xi) = \sum_{i=1}^{n} \tanh^{i-1}(\xi)\left[b_i \operatorname{sech}(\xi) + a_i \tanh(\xi)\right] + a_0. \tag{11}$$

Eq. (11) can be rearranged considering Eqs. (7) and (8) as follows;

$$\Phi(\omega) = \sum_{i=1}^{n} \cos^{i-1}(\omega)\left[b_i \sin(\omega) + a_i \cos(\omega)\right] + a_0. \tag{12}$$

As we know, rational functions are more general functions than polynomial functions. We can obtain significantly general forms of wave solutions which are including polynomial function solutions by this way. The different point of the method is the solution functions have two auxiliary functions, viz. $\operatorname{sech}(\xi), \tanh(\xi)$ We consider the following solution form

$$\Phi(\xi) = \frac{\sum_{i=1}^{M} \tanh^{i-1}(\xi)\left[a_i \operatorname{sech}(\xi) + c_i \tanh(\xi)\right] + a_0}{\sum_{i=1}^{M} \tanh^{i-1}(\xi)\left[b_i \operatorname{sech}(\xi) + d_i \tanh(\xi)\right] + b_0}, \tag{13}$$

which is also written as

$$\Phi(\omega) = \frac{\sum_{i=1}^{M} \cos^{i-1}(\omega)\left[a_i \sin(\omega) + c_i \cos(\omega)\right] + a_0}{\sum_{i=1}^{M} \cos^{i-1}(\omega)\left[b_i \sin(\omega) + d_i \cos(\omega)\right] + b_0}, \tag{14}$$

where $a_i, b_i, c_i, d_i, a_0, b_0$ are constants that will be determined later. a_i, b_i, c_i, d_i values are not all zero at the same time. The value of M is determined using the balance principle between the highest power nonlinear term and the highest derivative in NODE. After equating the coefficients of $\sin^i(\omega)\cos^j(\omega)$ to zero, we find a set of algebraic equations. $a_i, b_i, c_i, d_i, a_0, b_0$ values are found in solving the set of algebraic equations by Wolfram Mathematica 12. At the end, we substitute these values into Eq. (13) and get the new travelling wave solutions of Eq. (9).

4 Application of RSGEM

The (1+1)-dimensional conformable Fisher equation is given as

$$u_t^\gamma = \alpha^2 u_{xx} + pu - \beta u^3, \tag{15}$$

where γ is the order of the conformable derivative between $0 < \gamma \leq 1$.
We use the wave transformation as given below,

$$u(x,t) = U(\xi), \xi = kx + w\frac{t^\gamma}{\gamma}, \tag{16}$$

where k, w are constants that will be determined. Getting partial derivatives of $U(\xi)$ the function with respect to x, t, we find a non-linear ordinary differential equation as

$$wU' - \alpha^2k^2U'' - pU + \beta U^3 = 0. \tag{17}$$

According to the homogeneous balance principle, we obtain a relationship between U'' and U^3 in Eq. (17), $M = 1$. For $M = 1$, Eq. (13) turns to the below form.

$$U(\xi) = \frac{a_0 + a_1 \operatorname{sech}(\xi) + c_1 \tanh(\xi)}{b_0 + b_1 \operatorname{sech}(\xi) + d_1 \tanh(\xi)}. \tag{18}$$

We put Eq. (18) and its first and second-order derivatives into Eq. (17) and can obtain a system of algebraic equations. By using Wolfram Mathematica 12, $a_0, a_1, b_0, b_1, c_1, d_1$ and the other parameters can be found. Finally, we put these coefficients into Eq. (13) and obtain new travelling wave solutions of Eq. (1).

Case-1

$$a_1 = \frac{ib_1\sqrt{w\beta} - \sqrt{\beta\left(6\beta a_0^2 - wb_1^2\right)}}{\beta\sqrt{6}}, d_1 = \frac{ia_0\sqrt{6\beta}}{\sqrt{w}}, c_1 = -a_0, p = -\frac{2w}{3}, k = \frac{i\sqrt{w}}{\alpha\sqrt{3}}, b_0 = 0.$$

Putting the above coefficients into Eq. (13), yields

$$u_1(x,t) = -\left\{\frac{\operatorname{Sec}\left[\frac{\sqrt{w}x}{\sqrt{3}\alpha} - \frac{it^{\gamma}w}{\gamma}\right]\left(-i\sqrt{w\beta}b_1 + \sqrt{\beta\left(6\beta a_0^2 - wb_1^2\right)}\right)}{\sqrt{6}\beta} - a_0\left(-1 + i\operatorname{Tan}\left[\frac{\sqrt{w}x}{\sqrt{3}\alpha} - \frac{it^{\gamma}w}{\gamma}\right]\right)\right\}$$
$$/\operatorname{Sec}\left[\frac{\sqrt{w}x}{\sqrt{3}\alpha} - \frac{it^{\gamma}w}{\gamma}\right]b_1 - \sqrt{\frac{6\beta}{w}}a_0\operatorname{Tan}\left[\frac{\sqrt{w}x}{\sqrt{3}\alpha} - \frac{it^{\gamma}w}{\gamma}\right]. \tag{19}$$

where $i^2 = \sqrt{-1}$. Considering the suitable values of parameters, we can find wave simulations for Eq. (19) as following Figures 1 and 2:

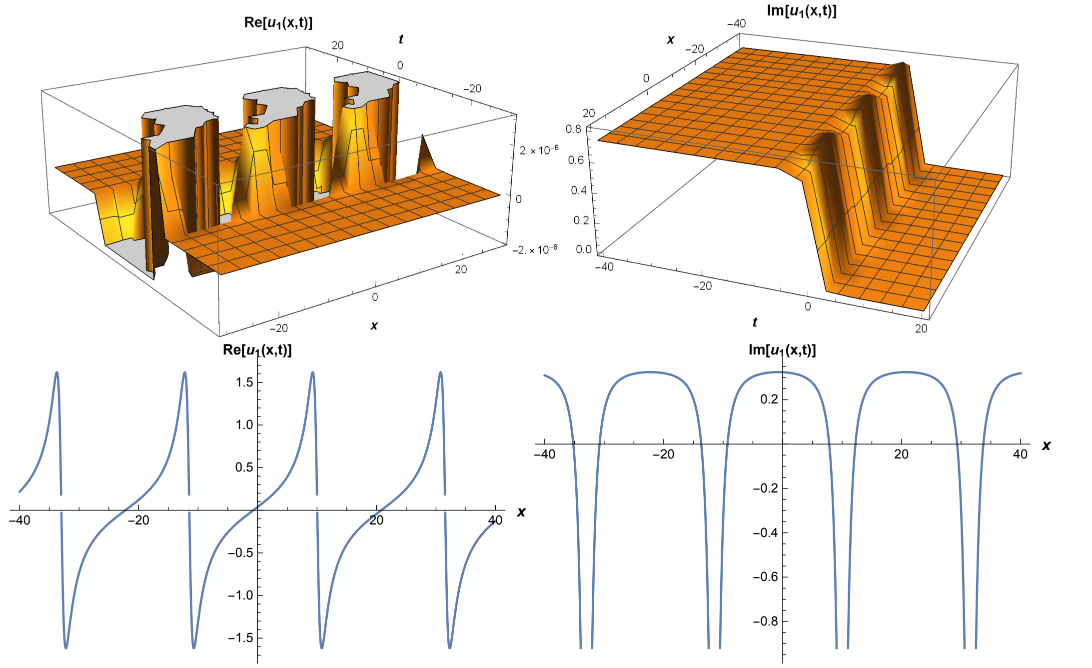

Figure 1. The 3D and 2D surfaces of the wave solution (19) by considering the values $\gamma = 0.9, a_0 = 2.1, b_1 = 1.2, \alpha = 2.5, w = 1.6, \beta = 2, t = 0.1$.

Case-2

$$a_1 = \frac{a_0 b_1}{b_0} - \sqrt{a_0^2\left(-1 + \frac{b_1^2}{b_0^2}\right)}, c_1 = -a_0, w = -\frac{6\beta a_0^2}{b_0^2}, \alpha = \frac{\sqrt{2\beta}a_0}{kb_0}, p = \frac{4\beta a_0^2}{b_0^2}, d_1 = 0.$$

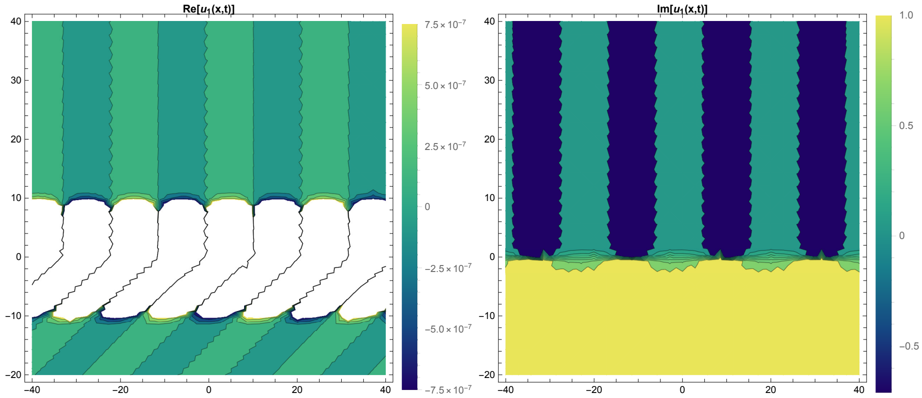

Figure 2. The contour plot surfaces of the wave solution Eq. (19) by considering the values $\gamma = 0.9, a_0 = 2.1, b_1 = 1.2, \alpha = 2.5, w = 1.6, \beta = 2, t = 0.1$.

Putting the above coefficients into Eq. (13), yields

$$u_2(x,t) = \frac{-\operatorname{Sech}\left[kx - \frac{6t^{\gamma}\beta a_0^2}{\gamma b_0^2}\right]\sqrt{a_0^2\left(-1+\frac{b_1^2}{b_0^2}\right)} + a_0\left(1 + \frac{\operatorname{Sech}\left[kx - \frac{6t^{\gamma}\beta a_0^2}{\gamma b_0^2}\right]b_1}{b_0} - \operatorname{Tanh}\left[kx - \frac{6t^{\gamma}\beta a_0^2}{\gamma b_0^2}\right]\right)}{b_0 + \operatorname{Sech}\left[kx - \frac{6t^{\gamma}\beta a_0^2}{\gamma b_0^2}\right]b_1}. \tag{20}$$

When we consider the suitable values of parameters, we can find wave simulations for Eq. (20) as following figures:

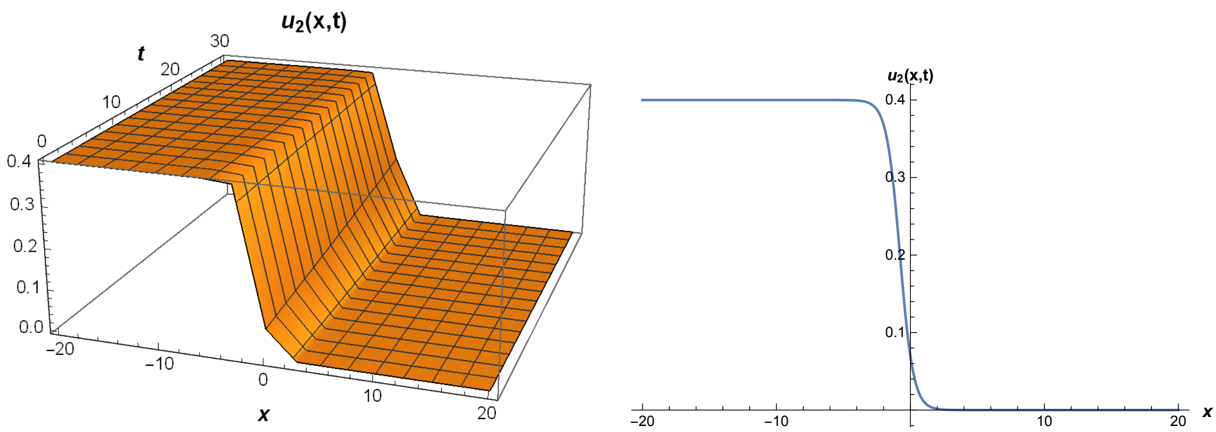

Figure 3. The 3D and 2D surfaces of the wave solution Eq. (20) by considering the values $\gamma = 0.9, a_0 = 0.2, b_1 = 2.5, b_0 = 1, k = 2, \beta = 0.2, t = 0.1$.

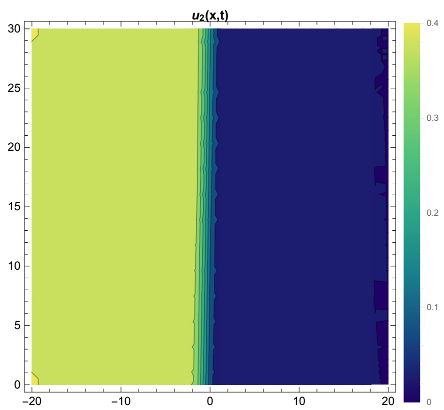

Figure 4. The contour plot surfaces of the wave solution (20) by considering the values $\gamma = 0.9, a_0 = 0.2, b_1 = 2.5, b_0 = 1, k = 2, \beta = 0.2, t = 0.1$.

Case-3

$$a_1 = -\sqrt{\frac{\beta a_0^2 + k^2\alpha^2 b_1^2 - \sqrt{2k^2\alpha^2\beta a_0^2 b_1^2 + k^4\alpha^4 b_1^4}}{\beta}}, d_1 = -\frac{\sqrt{2\beta}a_0}{k\alpha}, b_0 = 0, c_1 = -\frac{a_1\left(k^2\alpha^2 b_1^2 + \sqrt{2k^2\alpha^2\beta a_0^2 b_1^2 + k^4\alpha^4 b_1^4}\right)}{\sqrt{2\beta}k\alpha a_0 b_1},$$

$$p = 2k^2\alpha^2, \quad w = \frac{3c_1}{a_0}.$$

Putting these coefficients into Eq. (13), yields

$$u_3(x,t) = \frac{k\alpha\left(a_0 + a_1 \operatorname{Sech}\left[kx + \frac{3c_1 t^{\gamma}}{a_0\gamma}\right] + c_1 \operatorname{Tanh}\left[kx + \frac{3c_1 t^{\gamma}}{a_0\gamma}\right]\right)}{k\alpha b_1 \operatorname{Sech}\left[kx + \frac{3c_1 t^{\gamma}}{a_0\gamma}\right] - \sqrt{2\beta}a_0 \operatorname{Tanh}\left[kx + \frac{3c_1 t^{\gamma}}{a_0\gamma}\right]}. \tag{21}$$

When we consider the suitable values of parameters, we can find wave simulations for Eq. (21) as following figures:

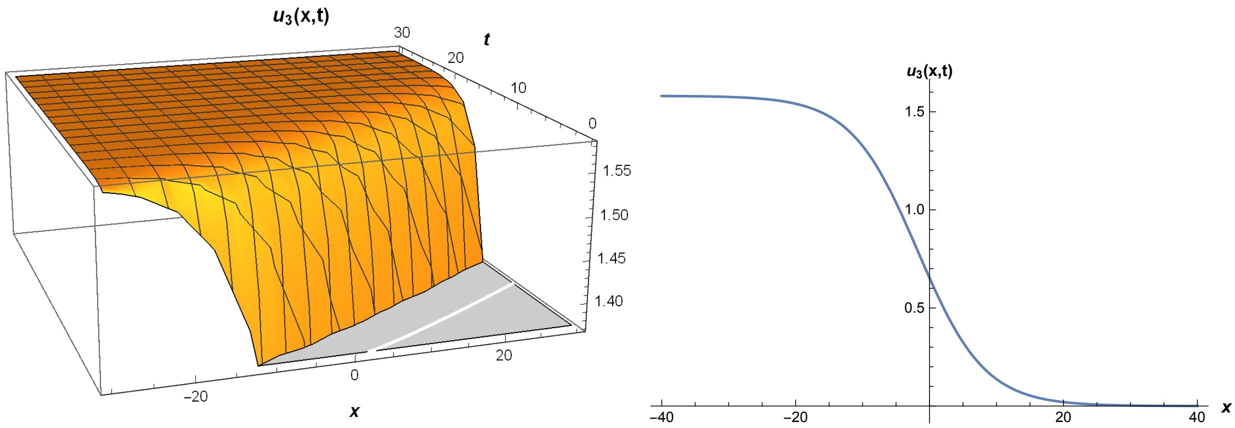

Figure 5. The 3D and 2D surfaces of the wave solution (21) by considering the values $\gamma = 0.9, a_0 = 2.5, b_1 = 1.5, \alpha = 2.5, k = 0.2, \beta = 0.2, t = 0.1$

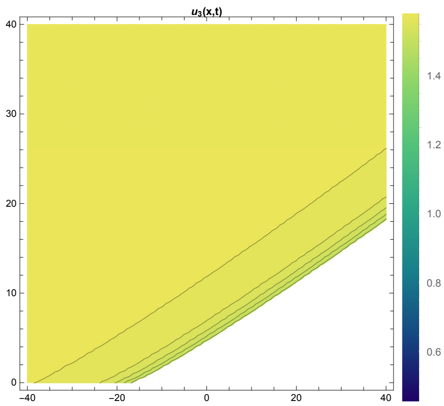

Figure 6. The contour plot surfaces of the wave solution (21) by considering the values $\gamma = 0.9, a_0 = 2.5, b_1 = 1.5, \alpha = 2.5, k = 0.2, \beta = 0.2, t = 0.1$.

Case-4

$$a_1 = -\frac{a_0 b_1 + \sqrt{a_0^2\left(b_1^2 + d_1^2\right)}}{d_1}, c_1 = -a_0, \alpha = -\frac{\sqrt{p}}{\sqrt{2k}}, \beta = \frac{p d_1^2}{4a_0^2}, w = -\frac{3p}{2}, b_0 = 0,$$

Putting these values into Eq. (13), yields

$$u_4(x,t) = -\frac{a_0\left(b_1 - \mathrm{e}^{-kx+\frac{3pt^{\gamma}}{2\gamma}}d_1\right) + \sqrt{a_0^2\left(b_1^2+d_1^2\right)}}{d_1\left(b_1 + \mathrm{Sinh}\left[kx - \frac{3pt^{\gamma}}{2\gamma}\right]d_1\right)}. \tag{22}$$

When we consider the suitable values of parameters, we can find wave simulations for Eq. (22) as following figures:

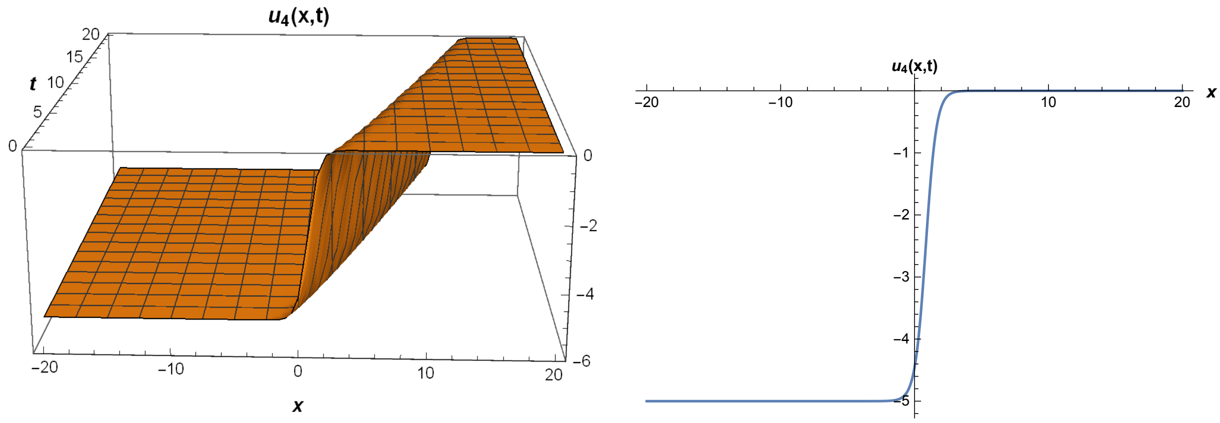

Figure 7. The 3D and 2D surfaces of the wave solution (22) by considering the values $\gamma = 0.9, a_0 = 2.5, b_1 = 3.1, d_1 = 1, p = 1.2, k = 2.5, t = 0.1$.

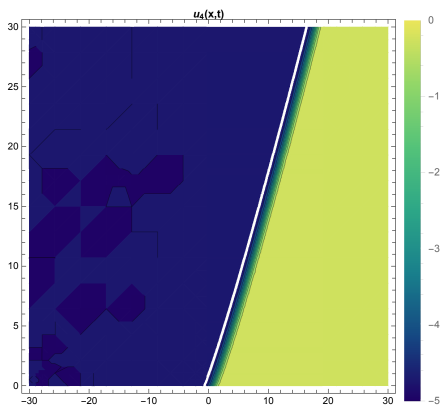

Figure 8. The contour plot surfaces of the wave solution (22) by considering the values $\gamma = 0.9, a_0 = 2.5, b_1 = 3.1, d_1 = 1, p = 1.2, k = 2.5, t = 0.1$.

Case-5

$$c_1 = \frac{-a_1b_1 + \sqrt{a_1^2\left(-b_0^2+b_1^2\right)}}{b_0}, a_0 = \frac{a_1b_1 - \sqrt{a_1^2\left(-b_0^2+b_1^2\right)}}{b_0}, w = -3k^2\alpha^2, \beta = \frac{k^2\alpha^2\left(-a_1\left(b_0^2-2b_1^2\right) + 2b_1\sqrt{a_1^2\left(-b_0^2+b_1^2\right)}\right)}{2a_1^3},$$

$$p = 2k^2\alpha^2, d_1 = 0.$$

Putting these values into Eq. (13), yields

$$u_5(x,t) = \frac{2\left(a_1\left(e^{kx-\frac{3k^2\alpha^2 t^\gamma}{\gamma}} b_0 + b_1\right) - \sqrt{a_1^2\left(-b_0^2+b_1^2\right)}\right)}{\left(1+e^{2\left(kx-\frac{3k^2\alpha^2 t^\gamma}{\gamma}\right)}\right) b_0\left(b_0 + \mathrm{Sech}\left[kx-\frac{3k^2\alpha^2 t^\gamma}{\gamma}\right] b_1\right)}. \tag{23}$$

When we consider the suitable values of parameters, we can find wave simulations for Eq. (23) as following figures:

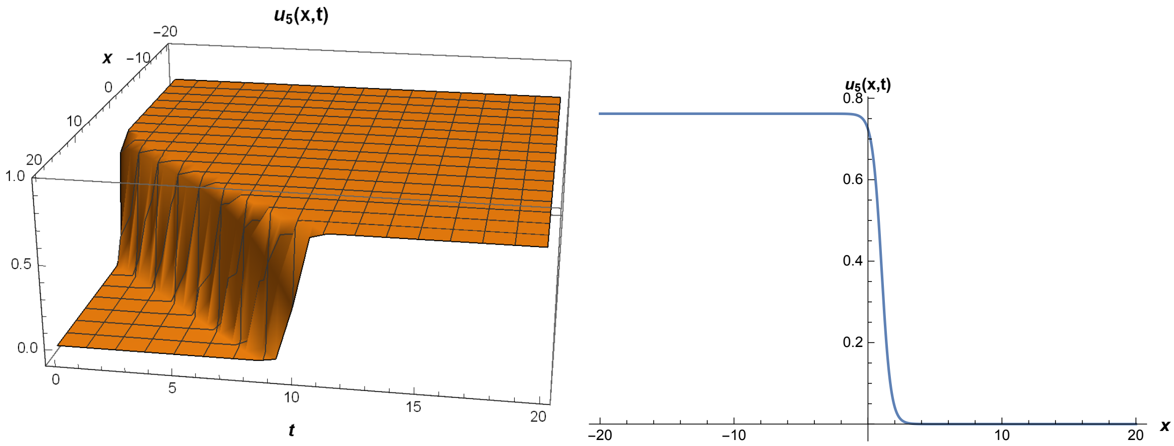

Figure 9. The 3D and 2D surfaces of the wave solution (23) by considering the values $\theta = 0.9, a_1 = 1.5, b_1 = 2, b_0 = 0.5, \alpha = 0.5, k = 2.9, t = 0.1.$

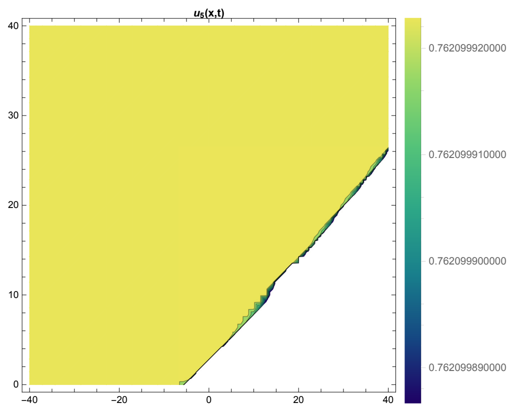

Figure 10. The contour plot surfaces of the wave solution (23) by considering the values $\theta = 0.9, a_1 = 1.5, b_1 = 2, b_0 = 0.5, \alpha = 0.5, k = 2.9, t = 0.1.$

5 Conclusion

In this paper, the rational sine-Gordon expansion method has been applied to the (1+1)-dimensional conformable Fisher equation. We have obtained some new wave solutions including hyperbolic and trigonometric functions. Figure 1 shows multi-soliton solution surfaces both imaginary and real parts of Eq. (19). Figure 3, Figure 5, and Figure 9 show the anti-kink soliton surfaces for Eq. (20), Eq. (21) and Eq. (23), respectively. Figure 7 shows the kink soliton surface for Eq. (22). Kink-type solitons are travelling wave solutions that climb up or climb down from one phase to another, and kink soliton reaches a constant at infinity. The mentioned model is used for modelling the relationship between the rate of inflation and both real and nominal interest rates, population dynamics in nonlinear media, and logistic population growth models, as well [20, 29, 30]. Fisher's model has been investigated by a numerical technique which is the q-homotopy analysis transform method (q-HATM) in [31]. They considered the time-fractional Fisher's model in Caputo's sense. Besides, they assumed special values of the coefficients in the model. The main advantage of the proposed method is the derived solutions include many other analytical techniques. According to new results and all figures, it has been observed that this method is a powerful tool for obtaining analytical solutions of nonlinear partial differential equations such as governing models. We hope that the provided solutions may be useful for scientists in mathematical biology, neurophysiology, chemical reactions, and economy.

Declarations

Consent for publication

Not applicable

Conflicts of interest

The authors declare that they have no conflict of interest.

Funding

Not applicable.

Author's contributions

G.Y.: Conceptualization, Methodology, Software, Writing - Original Draft, M.K.: Conceptualization, Methodology, Writing - Original Draft A.C.: Methodology, Writing - Review and Editing. All authors discussed the results and contributed to the final manuscript.

Acknowledgements

Not applicable.

References

[1] Cooper, F., Khare, A., Mihaila, B., & Saxena, A. Exact solitary wave solutions for a discrete $\lambda\phi^4$ field theory in (1+1)-dimensions. *Physical Review E*, 72(3), 036605, (2005). [CrossRef]
[2] Hirota, R. Exact solution of the Korteweg–de Vries equation for multiple collision of solitons. *Physical Review Letters*, 27(18), 1192–1194, (1971). [CrossRef]
[3] Malfliet, W. Solitary wave solutions of nonlinear wave equations. *American Journal of Physics*, 60(7), 650–654, (1992). [CrossRef]
[4] Fan, E. Extended tanh-function method and its applications to nonlinear equations. *Physics Letters A*, 277(4-5), 212–218, (2000). [CrossRef]
[5] Ma, W.X., Huang T., & Zhang, Y. A multiple exp-function method for nonlinear differential equations and its application. *Physica Scripta*, 82(6), 065003, (2010). [CrossRef]
[6] Ma, W.X., & Lee, J.H. A transformed rational function method and exact solutions to the (3+1)-dimensional Jimbo–Miwa equation. *Chaos, Solitons & Fractals*, 42(3), 1356–1363, (2009). [CrossRef]
[7] Feng, Z.S., & Wang, X.H. The first integral method to the two dimensional Burgers–Korteweg–de Vries equation. *Physics Letters A*, 308(2-3), 173–178, (2003). [CrossRef]
[8] Jawad, A.J.A.M., Petkovic, M.D., & Biswas, A. Modified simple equation method for nonlinear evolution equations. *Applied Mathematics and Computation*, 217(2), 869–877, (2010). [CrossRef]
[9] Aksan, E.N., Bulut, H., & Kayhan, M. Some wave simulation properties of the (2+1) dimensional breaking soliton equation. *ITM Web Of Conferences*, 13, 01014, (2017). [CrossRef]
[10] Bulut, H., Aksan, E.N., Kayhan, M., & Sulaiman, T.A. New solitary wave structures to the (3+1) dimensional Kadomtsev–Petviashvili and Schrödinger equation. *Journal of Ocean Engineering and Science*, 4(4), 373-378, (2019). [CrossRef]
[11] Yel, G., Baskonus, H.M., & Bulut, H. Novel archetypes of new coupled Konno-Oono equation by using sine-Gordon expansion method. *Optical and Quantum Electronics*, 49(9), 285, (2017). [CrossRef]
[12] Yan, L., Baskonus, H.M., Cattani, C., & Gao, W. Extractions of the gravitational potential and high-frequency wave Perturbation properties of nonlinear (3+1)- dimensional Vakhnenko-Parkes equation via novel approach. *Mathematical Methods in the Applied Sciences*, 1-10, (2022). [CrossRef]
[13] Chen, Q., Baskonus, H.M., Gao, W., & Ilhan, E. Soliton theory and modulation instability analysis: The Ivancevic option pricing model in economy. *Alexandria Engineering Journal*, 61(10), 7843-7851, (2022). [CrossRef]
[14] Veeresha, P., Yavuz, M., & Bhaishya, C. A computational approach for shallow water forced Korteweg-De Vries equation on critical flow over a hole with three fractional operators. *An International Journal of Optimization and Control: Theories & Applications*, 11(3), 52-67, (2021). [CrossRef]
[15] Jayaprakasha, P.C., & Bhaishya, C. Numerical analysis of predator-prey model in presence of toxicant by a novel approach. *Mathematics in Computer Science*, 11(4), 3963-3983, (2021). [CrossRef]
[16] Bhaishya, C. A new application of Hermite collocation method. *International Journal of Mathematical, Engineering and Management Sciences*, 14(1), 182-190, (2019). [CrossRef]
[17] Bhaishya, C., & Jaipala. Comparative study of homotopy perturbation method and Genocchi polynomial method for first order fractional differential equation. *Journal of Computer and Mathematical Sciences*, 10(1), 197-206, (2019). [CrossRef]
[18] Bhaishya, C., & Veeresha, P. Laguerre polynomial-based operational matrix of integration for solving fractional differential equations with non-singular kernel. *Proceedings of the Royal Society A*, 477(2253), 20210438, (2021). [CrossRef]
[19] Zhou, Q., Ekici M., Sonmezoglu, A., Manafian, J., Khaleghizadeh, S., & Mirzazadeh, M. Exact solitary wave solutions to the generalized Fisher equation. *Optik*, 127(24), 12085-12092, (2016). [CrossRef]

[20] Fisher, R.A. The advance of advantageous genes. *Annals of Eugenics*, 7(4), 355-369, (1937). [CrossRef]
[21] Wazwaz, A.M. The extended tanh method for abundant solitary wave solutions of nonlinear wave equations. *Applied Mathematics and Computation*, 187(2), 1131-1142, (2007). [CrossRef]
[22] Triki, H., & Wazwaz, A.M. Trial equation method for solving the generalized Fisher equation with variable coefficients. *Physics Letters A*, 380(13), 1260-1262, (2016). [CrossRef]
[23] Matinfar, M., Bahar, S.R., & Ghasemi, M. Solving the Generalized Fisher's equation by differential transform method. *Journal of Applied Mathematics and Informatics*, 30(3-4), 555–560, (2012). [CrossRef]
[24] Khalil, R., Al Horani, M., Yousef, A., & Sababheh, M. A new definition of fractional derivative. *Journal of Computational and Applied Mathematics*, 264, 65–70, (2014). [CrossRef]
[25] Atangana, A., Baleanu, D., & Alsaedi, A. New properties of conformable derivative. *Open Mathematics*, 13(1), 889–898, (2015). [CrossRef]
[26] Yan, L., Yel, G., Baskonus, H.M., Bulut, H., & Gao, W. Newly developed analytical method and its applications of some mathematical models. *International Journal of Modern Physics B*, 36(04), 2250040, (2022). [CrossRef]
[27] Yan, L., Yel, G., Kumar, A., Baskonus, H.M., & Gao, W. Newly developed analytical scheme and its applications to the some nonlinear partial differential equations with the conformable derivative. *Fractal and Fractional*, 5(4), 238, 1-15, (2021). [CrossRef]
[28] Yamgoué, S.B., Deffo, G.R., & Pelap, F.B. A new rational sine-Gordon expansion method and application to nonlinear wave equations arising in mathematical physics. *The European Physical Journal Plus*, 134(8), 380, (2019). [CrossRef]
[29] Tyson, J.J., & Brazhnik, P.K. On travelling wave solutions of Fisher's equation in two spatial dimensions. *SIAM Journal on Applied Mathematics*, 60(2), 371-391, (2000). [CrossRef]
[30] Murray, J.D. *Mathematical Biology: I. An Introduction* (3rd Edition). Springer (2002).
[31] Veeresha, P., Prakasha, D.G., & Baskonus, H.M. Novel simulations to the time fractional Fisher's equation. *Mathematical Sciences*, 13(1), 33-42, (2019). [CrossRef]