Iğdır Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 13(3), 2105-2119, 2023
Journal of the Institute of Science and Technology, 13(3), 2105-2119, 2023

ISSN: 2146-0574, eISSN: 2536-4618

Matematik / *Mathematics* **DOI: 10.21597/jist.1278267**

Araştırma Makalesi / *Research Article*

Geliş tarihi / *Received*: 06.04.2023 **Kabul tarihi / *Accepted*: 04.05.2023**

Atıf İçin: Yalaz, F., Uğur, T. ve Elmalı, C. S. (2023). Düğümlerle Eşlenen Bitopolojiler Üzerine Bir Çalışma. *Iğdır Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, 13(3), 2105-2119.

To Cite: Yalaz, F., Uğur, T. & Elmalı, C. S. (2023). A Work On Bitopologies Associated With Knots. *Journal of the Institute of Science and Technology*, 13(3), 2105-2119.

Düğümlerle Eşlenen Bitopolojiler Üzerine Bir Çalışma

Ferit YALAZ[1*], Tamer UĞUR[2], Ceren Sultan ELMALI[3]

Öne Çıkanlar:

- Bitopolojiler ile eşlenen düğümler
- Düğüm digraf notasyonunun tersi
- Bir düğümün regüler diyagramının elde edilmesi

Anahtar Kelimeler:

- Düğüm
- Digraf
- Bitopoloji
- Zayıf Küme
- Güçlü Küme

ÖZET:

Literatürde, düğüm digraf notasyonu olarak isimlendirilen bir yöntem yardımıyla bazı düğümlerle bitopolojiler eşlendi. Bu bitopolojileri elde etmek için düğüm grafları ve quasi pseudo metrik uzaylar kullanıldı. Quasi pseudo metrikler yardımıyla bir küme üzerinde iki yeni topoloji elde edildi. Bu sayede bazı düğümler ile bitopolojiler arasında bir eşleme kurulmuş oldu.Yazarlar "Düğümlerle eşlenen bitopolojiler verildiğinde, düğümün kendisi elde edilebilir mi?" sorusuna cevap aradılar ve bir yöntem verdiler. Bu bahsedilen yöntem 6 adımdan oluşmaktadır. Bu çalışmada ise düğüm digraf notasyonun tersinin, Alexander-Briggs notasyonuna göre, 3_1, 5_1, 5_2, 6_1, 6_2, 7_1, 7_2, 7_3,8_1 , 8_2, 8_3, 9_1, 9_2, 9_3, 10_1, 10_2, 10_3 düğümleri için sağlandığı detaylı bir şekilde gösterilmektedir.

A Work On Bitopologies Associated With Knots

Highlights:

- Bitopologies associated with knots
- Reverse of knot digraph notation
- Obtaining the regular diagram of a knot

Keywords:

- Knot
- Digraph
- Bitopology

ABSTRACT:

The bitopologies have been associated with some knots in the literature with the help of a method called the knot digraph notation. The knot graphs and quasi pseudo metric spaces were used to obtain these bitopologies. With the help of quasi pseudo metrics, two topologies were obtained on a set. In this way, an association between some knots and bitopologies was established. The authors sought an answer to the question "Given the bitopologies associated with knots, can the knot itself be obtained ?" and they gave a method. This mentioned method consists of 6 steps.. In this work, it is shown in detail that according to the Alexander-Briggs notation, the reverse of the knot digraph notation is provided for the knots 3_1, 5_1, 5_2, 6_1, 6_2, 7_1, 7_2, 7_3,8_1 , 8_2, 8_3, 9_1, 9_2, 9_3, 10_1, 10_2, 10_3..

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This study was produced from Ferit Yalaz's Master's thesis..

INTRODUCTION

Using quasi pseudo-metric and its conjugate, the concept of bitopology was first introduced by Kelly (Kelly, 1963). Quasi-pseudo metric spaces are a generalization of well-known metric spaces. Kelly corresponded a quasi-pseudo metric space to a new pseudo-metric space. These quasi-metric spaces are called conjugates of each other. Two different topologies are obtained from these two conjugate quasi-pseudo metrics, similar to well-known metric space. Together with the set on which these quasi-pseudo metrics are defined, these two topologies are called bitopological spaces. Moreover, Kelly gives new separation axioms and generalizations of many theorems on this structure.

Baby Girija and Pilakkat (Girija & Pilakkat, 2013) define two quasi-pseudo metrics on the vertices set of a given digraph, which are conjugates. So they have defined a bitopology with Kelly's method on the vertices of the digraph.

A knot with a crossing divides the plane into regions. These regions are open discs that are homomorphic to each other. With the help of these regions, the knot graphs and the knot dual graphs are obtained. Moreover, the knot itself is obtained with the help of the knot's signed graph. This method is called the "Tait method" (For detail (Yajima & Kinoshita,1957; Murasugi, 1993)).

In (Kunduracı, 2017; Elmalı et al., 2018), some knots correspond to a bitopological space, and this method is called knot digraph notation.

We consider bitopologies associated with knots which are in (Kunduracı, 2017; Elmalı el al., 2018). In (Yalaz, 2017), given the bitopologies associated with knots, a method is given to obtain some knots themselves, and it is shown that this method is provided for the knots (m=3,..,10, n=1,2,3) (except 6_3). Nevertheless, it is only presented for 4_1 (Uğur et al., 2018). Therefore, in this work, we show one by one that this method is provided for these mentioned knots.

MATERIALS AND METHODS

The following table is obtained from the results (Kunduracı, 2017; Elmalı et al., 2018). This table shows that bitopologies are associated with some knots in Alexander-Briggs notation.

3_1	$X=\{1,2,\varepsilon_1^{12}\},\ \tau_1=\{X,\emptyset,\{1\},\{1,2\}\},\tau_2=\{X,\emptyset,\{2,\varepsilon_1^{12}\},\{\varepsilon_1^{12}\}\}$
4_1	$X=\{1,2,\varepsilon_1^{12}\},\tau_1=\{X,\emptyset,\{1\},\{1,2\},\{1,2,3\}\},\tau_2=\{X,\emptyset,\{2,3,\varepsilon_1^{12}\},\{3,\varepsilon_1^{12}\},\{\varepsilon_1^{12}\}\}$
5_1	$X=\{1,2,\varepsilon_1^{12},\varepsilon_2^{12}\},\ \tau_1=\{X,\emptyset,\{1\},\{1,2\}\},\tau_2=\{X,\emptyset,\{2,\varepsilon_1^{12},\varepsilon_2^{12}\},\{\varepsilon_1^{12},\varepsilon_2^{12}\}\}$
5_2	$X=\{1,2,3,4,\varepsilon_1^{14}\},\quad \tau_1=\{X,\emptyset,\{1\},\{1,2\},\{1,2,3\},\{1,2,3,4\}\},$ $\tau_2=\{X,\emptyset,\{2,3,4,\varepsilon_1^{14}\},\{3,4,\varepsilon_1^{14}\}\{4,\varepsilon_1^{14}\},\{\varepsilon_1^{14}\}\}$
6_1	$X=\{1,2,3,4,5,\varepsilon_1^{15}\},\quad \tau_1=\{X,\emptyset,\{1\},\{1,2\},\{1,2,3\},\{1,2,3,4\},\{1,2,3,4,5\}\}$ $\tau_2=\{X,\emptyset,\{2,3,4,5,\varepsilon_1^{15}\},\{3,4,5,\varepsilon_1^{15}\},\{4,5,\varepsilon_1^{15}\},\{5,\varepsilon_1^{15}\},\{\varepsilon_1^{15}\}\}$
6_2	$X=\{1,2,3,\varepsilon_1^{23},\varepsilon_1^{13}\},\tau_1=\{X,\emptyset,\{1\},\{1,2\},\{1,2,3\}\},$ $\tau_2=\{X,\emptyset,\{2,3,\varepsilon_1^{23},\varepsilon_1^{13}\},\{3,\varepsilon_1^{23},\varepsilon_1^{13}\},\{\varepsilon_1^{23},\varepsilon_1^{13}\}\}$
7_1	$X=\{1,2,\varepsilon_1^{12},\varepsilon_2^{12},\varepsilon_3^{12}\},\ \tau_1=\{X,\emptyset,\{1\},\{1,2\}\},$ $\tau_2=\{X,\emptyset,\{2,\varepsilon_1^{12},\varepsilon_2^{12},\varepsilon_3^{12}\},\{\varepsilon_1^{12},\varepsilon_2^{12},\varepsilon_3^{12}\}\}$
7_2	$X=\{1,2,3,4,5,6,\varepsilon_1^{16}\},\tau_1=\{X,\emptyset,\{1\},\{1,2\},\{1,2,3\},\{1,2,3,4\},\{1,2,3,4,5\},\{1,2,3,4,5,6\}\},$ $\tau_2=\{X,\emptyset,\{2,3,4,5,6,\varepsilon_1^{16}\},\{3,4,5,6,\varepsilon_1^{16}\},\{4,5,6,\varepsilon_1^{16}\},\{5,6,\varepsilon_1^{16}\},\{6,\varepsilon_1^{16}\},\{\varepsilon_1^{16}\}\}$
7_3	$X=\{1,2,3,4,\varepsilon_1^{14},\varepsilon_2^{14}\},\tau_1=\{X,\emptyset,\{1\},\{1,2\},\{1,2,3\},\{1,2,3,4\}\},$

	$\tau_2 = \{X, \emptyset, \{2,3,4,\varepsilon_1^{14},\varepsilon_2^{14}\}, \{3,4,\varepsilon_1^{14},\varepsilon_2^{14}\}, \{4,\varepsilon_1^{14},\varepsilon_2^{14}\}, \{\varepsilon_1^{14},\varepsilon_2^{14}\}\}$
$\mathbf{8_1}$	$X = \{1,2,3,4,5,6,7,\varepsilon_1^{17}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,4,5\}, \{1,2,3,4,5,6\}, \{1,2,3,4,5,6,7\}\}, \tau_2 = \{X, \emptyset, \{2,3,4,5,6,7,\varepsilon_1^{17}\}, \{3,4,5,6,7,\varepsilon_1^{17}\}, \{4,5,6,7,\varepsilon_1^{17}\}, \{5,6,7,\varepsilon_1^{17}\}, \{6,7,\varepsilon_1^{17}\}, \{7,\varepsilon_1^{17}\}, \{\varepsilon_1^{17}\}\}$
$\mathbf{8_2}$	$X = \{1,2,3,\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_1^{13}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}\}$ $\tau_2 = \{X, \emptyset, \{2,3,\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_1^{13}\}, \{3,\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_1^{13}\}, \{\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_1^{13}\}\}$
$\mathbf{8_3}$	$X = \{1,2,3,4,5,\varepsilon_1^{15},\varepsilon_2^{15}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,4,5\}\},$ $\tau_2 = \{X, \emptyset, \{2,3,4,5,\varepsilon_1^{15},\varepsilon_2^{15}\}, \{3,4,5,\varepsilon_1^{15},\varepsilon_2^{15}\}, \{4,5,\varepsilon_1^{15},\varepsilon_2^{15}\}, \{5,\varepsilon_1^{15},\varepsilon_2^{15}\}, \{\varepsilon_1^{15},\varepsilon_2^{15}\}\}$
$\mathbf{9_1}$	$X = \{1,2,\varepsilon_1^{12},\varepsilon_2^{12},\varepsilon_3^{12},\varepsilon_4^{12}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}\},$ $\tau_2 = \{X, \emptyset, \{2,\varepsilon_1^{12},\varepsilon_2^{12},\varepsilon_3^{12},\varepsilon_4^{12}\}, \{\varepsilon_1^{12},\varepsilon_2^{12},\varepsilon_3^{12},\varepsilon_4^{12}\}\}$
$\mathbf{9_2}$	$X = \{1,2,3,4,5,6,7,8,\varepsilon_1^{18}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,4,5\}, \{1,2,3,4,5,6\}, \{1,2,3,4,5,6,7\}, \{1,2,3,4,5,6,7,8\}\}, \tau_2 = \{X, \emptyset, \{2,3,4,5,6,7,8,\varepsilon_1^{18}\}, \{3,4,5,6,7,8,\varepsilon_1^{18}\}, \{4,5,6,7,8,\varepsilon_1^{18}\}, \{5,6,7,8,\varepsilon_1^{18}\}, \{6,7,8,\varepsilon_1^{18}\}, \{7,8,\varepsilon_1^{18}\}, \{8,\varepsilon_1^{18}\}, \{\varepsilon_1^{18}\}\}$
$\mathbf{9_3}$	$X = \{1,2,3,4,\varepsilon_1^{14},\varepsilon_2^{14},\varepsilon_3^{14}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}\},$ $\tau_2 = \{X, \emptyset, \{2,3,4,\varepsilon_1^{14},\varepsilon_2^{14},\varepsilon_3^{14}\}, \{3,4,\varepsilon_1^{14},\varepsilon_2^{14},\varepsilon_3^{14}\}, \{4,\varepsilon_1^{14},\varepsilon_2^{14},\varepsilon_3^{14}\}, \{\varepsilon_1^{14},\varepsilon_2^{14},\varepsilon_3^{14}\}\}$
$\mathbf{10_1}$	$X = \{1,2,3,4,5,6,7,8,9,\varepsilon_1^{19}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,4,5\}, \{1,2,3,4,5,6\}, \{1,2,3,4,5,6,7\}, \{1,2,3,4,5,6,7,8\}, \{1,2,3,4,5,6,7,8,9\}\},$ $\tau_2 = \{X, \emptyset, \{2,3,4,5,6,7,8,9,\varepsilon_1^{19}\}, \{3,4,5,6,7,8,9,\varepsilon_1^{19}\}, \{4,5,6,7,8,9,\varepsilon_1^{19}\}, \{5,6,7,8,9,\varepsilon_1^{19}\}, \{6,7,8,9,\varepsilon_1^{19}\}, \{7,8,9,\varepsilon_1^{19}\}, \{8,9,\varepsilon_1^{19}\}, \{9,\varepsilon_1^{19}\}, \{\varepsilon_1^{19}\}\}$
$\mathbf{10_2}$	$X = \{1,2,3,\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_3^{23},\varepsilon_1^{13}\}, \tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}\}$ $\tau_2 = \{X, \emptyset, \{2,3,\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_3^{23},\varepsilon_1^{13}\}, \{3,\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_3^{23},\varepsilon_1^{13}\}, \{\varepsilon_1^{23},\varepsilon_2^{23},\varepsilon_3^{23},\varepsilon_1^{13}\}\}$
$\mathbf{10_3}$	$X = \{1,2,3,4,5,6,7,\varepsilon_1^{17},\varepsilon_2^{17}\}$ $\tau_1 = \{X, \emptyset, \{1\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,4,5\}, \{1,2,3,4,5,6,7\}\},$ $\tau_2 = \{X, \emptyset, \{2,3,4,5,6,7,\varepsilon_1^{17},\varepsilon_2^{17}\}, \{3,4,5,6,7,\varepsilon_1^{17},\varepsilon_2^{17}\}, \{4,5,6,7,\varepsilon_1^{17},\varepsilon_2^{17}\}, \{5,6,7,\varepsilon_1^{17},\varepsilon_2^{17}\}, \{6,7,\varepsilon_1^{17},\varepsilon_2^{17}\}, \{7,\varepsilon_1^{17},\varepsilon_2^{17}\}, \{\varepsilon_1^{17},\varepsilon_2^{17}\}\}$

Let X be a finite set and (X, τ_1, τ_2) be any bitopological space. The members with the most number of the elements in the family $\tau_1 \backslash \{X\}$ are called a strong set of τ_1. A similar definition may be given for τ_2. The members with the least number of elements in the family $\tau_2 \setminus \{\emptyset\}$ are called a weak set of τ_2. A similar definition may be given for τ_2. Since in the above table bitopologies have only one strong set and one weak set, we denote the strong set of τ_1 and the weak set of τ_2 as $\overline{\tau_1}$ and $\underline{\tau_2}$, respectively. Moreover, we show the number of elements of these sets as s$(\overline{\tau_1}) = V$ and s$(\underline{\tau_2}) = B$, respectively.

Now, let us restate the method given in (Yalaz, 2017; Uğur et al., 2018).

Let any bitopology be given from the table:

1) As many points as the number of elements of the strong set of the bitopology τ_1 are marked on the plane, that is, s$(\overline{\tau_1}) = V$ points. All placed points are numbered clockwise:

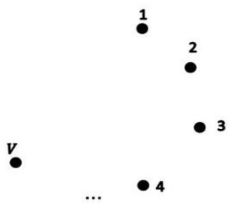


Figure 1

2) The elements ε_k^{ij} of the weak set $\underline{\tau_2}$ of the topology τ_2 indicate that there are k deformations between the vertices i and j. These deformations are placed between the points in the 1st item, first straight and then cycle as follows:

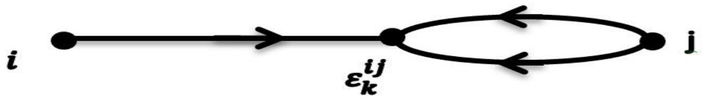


Figure 2

3) Let the number of elements of the weak set $\underline{\tau_2}$ of the topology τ_2 be B. The formula $V + 2B - 1$ gives the number of the crossing of the knot whose bitopology is given. This shows how many edges the digraph to be obtained has.

4) After the deformations are placed, $V - 1$ edges are added clockwise from point numbered 1. The added edges are given a clockwise orientation.

5) Deformations are converted to cycles as follows:

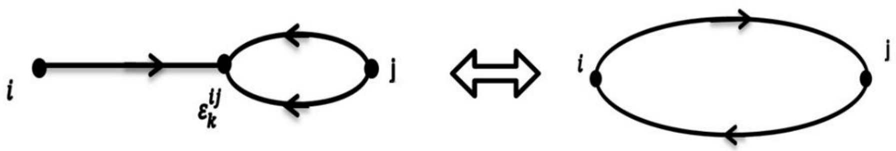


Figure 3

6) By accepting each edge of the digraph obtained in the 5th step as a "+" sign, the regular diagram of the knot whose bitopology is given is obtained with the help of the Tait method (see (Murasugi, 1993) for more details on the Tait's method).

RESULTS AND DISCUSSION

In this section, we show that the inverse operation of knot digraph notation is provided for knots $3_1, 5_1, 5_2, 6_1, 6_2, 7_1, 7_2, 7_3, 8_1, 8_2, 8_3, 9_1, 9_2, 9_3, 10_1, 10_2, 10_3$. The topologies obtained for these knots are given in Table 1.

According to the bitopology associated with knot $\mathbf{3_1}$ in Table 1:

1) The strong set of the topology τ_1 is $\overline{\tau_1} = \{1,2\}$, so the number of elements of this set is $V = 2$. This $V = 2$ number shows us that the knot whose given bitopology are two vertices. So let's place two points on the plane and number as 1,2.

Figure 4

2) The weak set of the topology τ_2 is $\underline{\tau_2} = \{\varepsilon_1^{12}\}$, so the number of elements of this set is $B = 1$. This $B = 1$ number shows us that the knot whose given bitopology one deformation applied. Also, the element $\varepsilon_1^{12} \in \underline{\tau_2}$ tells us that there is one deformation between vertices 1 and 2. If this information is applied to the points where we are placed on the plane in the first item, we obtain:

Figure 5

3) Since the number of elements of the strong set of the topology τ_1 is $V = 2$, and the number of elements of the weak set of τ_2 is $B = 1$,

$V + 2B - 1 = 2 + 2.1 - 1 = 3.$

The knot whose bitopology is given has 3 crossings. that is, it says that it is obtained from a digraph with 3 sides.

4) With $V = 2,\ V - 1 = 2 - 1 = 1$. It says that from the corner numbered 1, we need to add one edge clockwise. Also, the added edge is oriented clockwise.

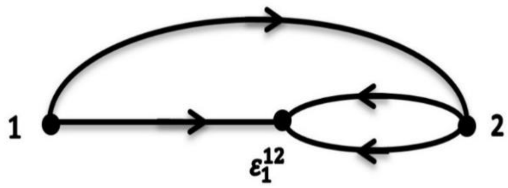


Figure 6

5) The digraph is converted to its equivalent digraph as follows:

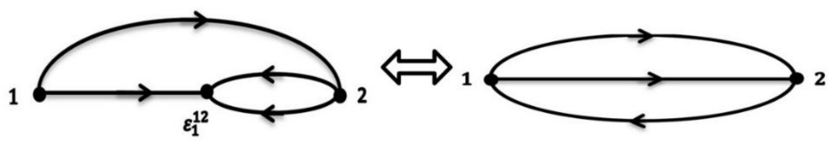


Figure 7

6) The digraph which is found by applying the first five steps, gives us when the Tait method is applied, as seen in the following, to find the regular diagram of the knot whose bitopology is given.

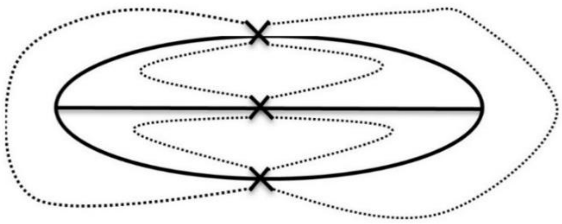


Figure 8

Here we consider each edge as a sign "+". The knot we obtained is the one in (a). We know that the knots obtained from its graph and the dual graph are equivalent. The knot we obtained is equivalent to the trefoil knot as seen in (b).

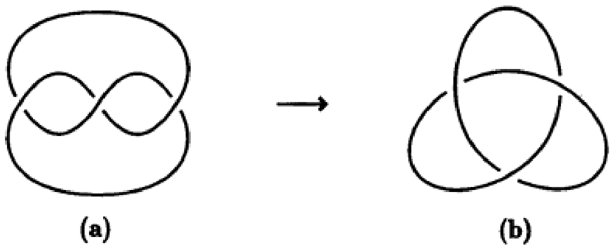


Figure 9

According to the bitopology associated with knot $\mathbf{5_1}$ in Table 1:

1) $\overline{\tau_1} = \{1,2\}$, $s(\overline{\tau_1}) = V = 2$. So, let us place 2 points on the plane and enumerate these points in a clockwise direction (Figure 10-(a))

2) $\underline{\tau_2} = \{\varepsilon_1^{12}, \varepsilon_2^{12}\}$, $s\left(\underline{\tau_2}\right) = B = 2$. Then two deformations are placed between points 1 and 2 (Figure 10-(b)).

3) Since $V + 2B - 1 = 2 + 4 - 1 = 5$, the digraph drawn has 5 sides.

4) Since $V - 1 = 2 - 1 = 1$, one edge is added clockwise from the numbered 1 vertex (Figure 10-(c)).

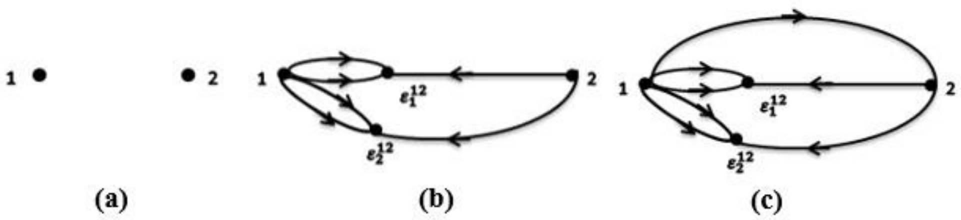
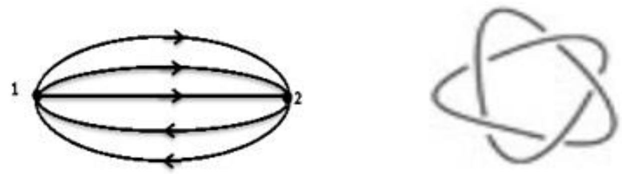


Figure 10

5) Deformations between points 1 and 2 are converted to cycles (Figure 11-(a)).

6) By accepting the edges of the digraph in Figure 11-(a) as sign "+", the regular diagram of the following knot 5_1 is obtained with the help of the Tait method.



(a) (b)

Figure 11

According to the bitopology associated with knot $\mathbf{5_2}$ in Table 1:

1) $\overline{\tau_1} = \{1,2,3,4\}, s(\overline{\tau_1}) = V = 4$. So, let us place 4 points on the plane and enumerate these points in a clockwise direction.

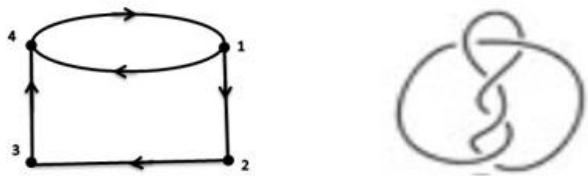
2) $\underline{\tau_2} = \{\varepsilon_1^{14}\}, s\left(\underline{\tau_2}\right) = B = 1$. Then a deformation is placed between points 1 and 4.

3) Since $V + 2B - 1 = 4 + 2 - 1 = 5$, the digraph be drawn has 5 sides.

4) Since $V - 1 = 4 - 1 = 3$, three edges are added clockwise from the numbered 1 vertex.

5) Deformations between points 1 and 4 are converted to cycles(Figure 13-(a)).

6) By accepting the edges of the digraph in Figure 13-(a) as "+", the regular diagram of knot 5_2 is obtained with the help of the Tait method.



(a) (b)

Figure 12

According to the bitopology associated with knot $\mathbf{6_1}$ in Table 1:

1) $\overline{\tau_1} = \{1,2,3,4,5\}, s(\overline{\tau_1}) = V = 5$. So, let us place 5 points on the plane and enumerate these points in a clockwise direction (Figure 14-(a)).

2) $\underline{\tau_2} = \{\varepsilon_1^{15}\}, s\left(\underline{\tau_2}\right) = B = 1$. Then one deformation are placed between points 1 and 5 (Figure 14-(b)).

3) Since $V + 2B - 1 = 5 + 2 - 1 = 6$, the digraph drawn has 6 sides.

4) Since $V - 1 = 5 - 1 = 4$, one edge is added clockwise from the numbered 1 vertex (Figure 14-(c)).

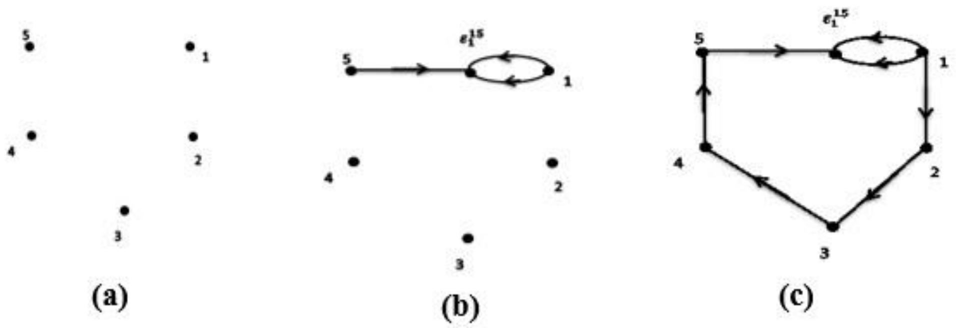


Figure 13

5) Deformations between points 1 and 5 are converted to cycles (Figure 15-(a)).

6) By accepting the edges of the digraph in Figure 15-(a) as the sign "+", the regular diagram of the following knot 6_1 is obtained with the help of the Tait method (Figure 15-(b)).

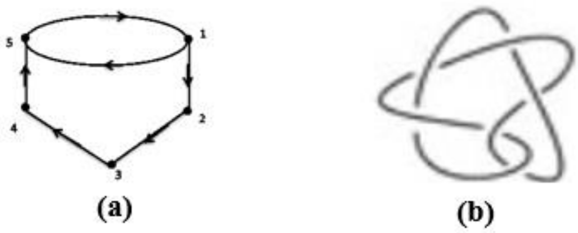


Figure 14

According to the bitopology associated with knot $\mathbf{6_2}$ in Table 1:.

1) $\overline{\tau_1} = \{1,2,3\}, s(\overline{\tau_1}) = V = 3$. So, let us place 3 points on the plane and enumerate these points in a clockwise direction.

2) $\underline{\tau_2} = \{\varepsilon_1^{23}, \varepsilon_1^{13}\}$, $s\left(\underline{\tau_2}\right) = B = 2$. One deformation is placed between points 1-3 and one deformation is placed between points 2-3.

3) Since $V + 2B - 1 = 3 + 4 - 1 = 6$, the digraph is drawn has 6 sides.

4) Since $V - 1 = 3 - 1 = 2$, two edges are added clockwise from the numbered 1 vertex.

5) Deformations between points 1-3 and 2-3 are converted to cycles (Figure 17-(a)).

6) By accepting the edges of the digraph in Figure 17-(a) as the sign "+", the regular diagram of the following knot 6_2 is obtained with the help of the Tait method (Figure 17-(b)).

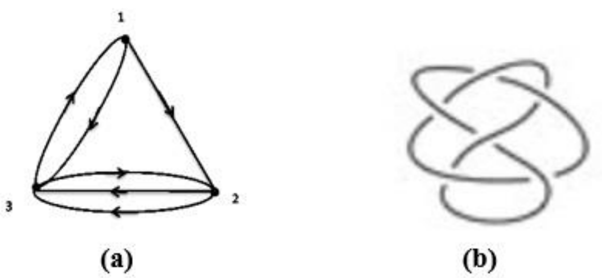


Figure 15

According to the bitopology associated with knot $\mathbf{7_1}$ in Table 1:

1) $\overline{\tau_1} = \{1,2\}, s(\overline{\tau_1}) = V = 2$. So, let us place 2 points on the plane and enumerate these points in a clockwise direction (Figure 18-(a)).

2) $\underline{\tau_2} = \{\varepsilon_1^{12}, \varepsilon_2^{12}, \varepsilon_3^{12}\}$, $s\left(\underline{\tau_2}\right) = B = 3$. Three deformations are placed between points 1 and 2 (Figure 18-(b)).

3) Since $V + 2B - 1 = 2 + 3 - 1 = 4$, the digraph drawn has 4 edges.

4) Since $V - 1 = 2 - 1 = 1$, three edges are added clockwise from the numbered 1 vertex (Figure 18-(c)).

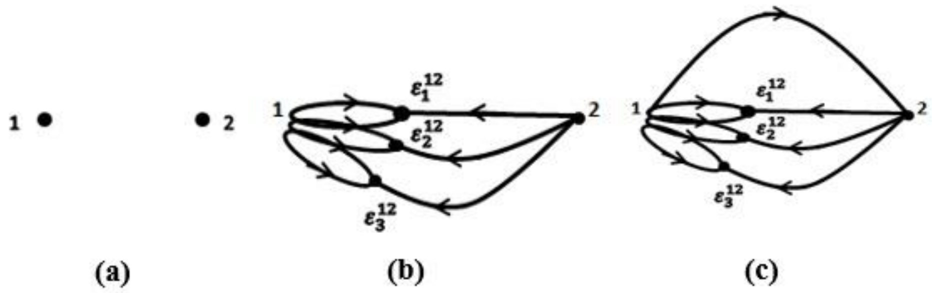


Figure 16

5) Deformations between points 1-2 are converted to cycles (Figure 19-(a)).

6) By accepting the edges of the digraph in Figure 19-(a) as the sign "+", the regular diagram of the following knot 7_1 is obtained with the help of the Tait method (Figure 19-(b)).

Figure 17

According to the bitopology associated with knot $\mathbf{7_2}$ in Table 1:

1) $\overline{\tau_1} = \{1,2,3,4,5,6\}, s(\overline{\tau_1}) = V = 6$. So, let us place 6 points on the plane and enumerate these points in a clockwise direction.

2) $\{\varepsilon_1^{16}\}$, $s\left(\underline{\tau_2}\right) = B = 1$. One deformation are placed between points 1 and 6 .

3) Since $V + 2B - 1 = 6 + 2 - 1 = 7$, the digraph drawn has 7 edges.

4) Since $V - 1 = 6 - 1 = 5$, five edges are added clockwise from the numbered 1 vertex .

5) Deformations between points 1 and 6 are converted to cycles (Figure 21-(a)).

6) By accepting the edges of the digraph of Figure 21-(a) as sign "+", the regular diagram of the following knot 7_2 is obtained with the help of the Tait method (Figure 21-(b)).

(a) (b)

Figure 18

According to the bitopology associated with knot $\mathbf{7_3}$ in Table 1:

1) $\overline{\tau_1} = \{1,2,3,4\}, s(\overline{\tau_1}) = V = 4$. So, let us place 4 points on the plane and enumerate these points in a clockwise direction.
2) $\underline{\tau_2} = \{\varepsilon_1^{14}, \varepsilon_2^{14}\}$, $s\left(\underline{\tau_2}\right) = B = 2$. Two deformations are placed between points 1 and 4.
3) Since $V + 2B - 1 = 4 + 4 - 1 = 7$, the digraph drawn has 7 edges.
4) Since $V - 1 = 4 - 1 = 3$, three edges are added clockwise from the numbered 1 vertex.
5) Deformations between the points 1 and 4 are converted to cycles (Figure 23-(a)).
6) By accepting the edges of the digraph in Figure 23-(a) as the sign "+", the regular diagram of the following knot 7_3 is obtained with the help of the Tait method (Figure 23-(b)).

(a) (b)

Figure 19

According to the bitopology associated with knot $\mathbf{8_1}$ in Table 1:

1) $\overline{\tau_1} = \{1,2,3,4,5,6,7\}, s(\overline{\tau_1}) = V = 7$. So, let us place 7 points on the plane and enumerate these points in a clockwise direction (Figure 24-(a)).
2) $\{\varepsilon_1^{17}\}$, $s\left(\underline{\tau_2}\right) = B = 1$. One deformation are placed between points 1 and 7 (Figure 24-(b)).
3) Since $V + 2B - 1 = 7 + 2 - 1 = 8$, the digraph drawn has 8 edges.
4) Since $V - 1 = 7 - 1 = 6$, six edges are added clockwise from the numbered 1 vertex (Figure 24-(c)).

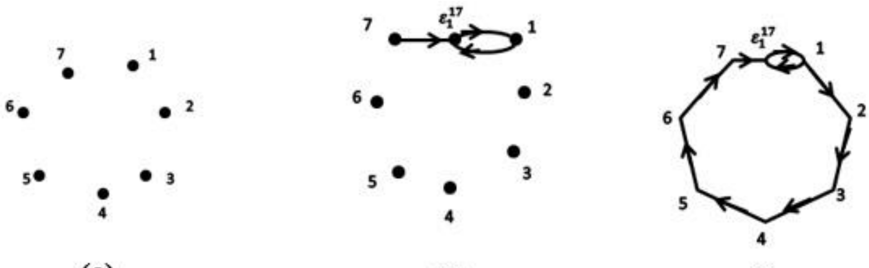


Figure 20

5) Deformations between points 1 and 7 are converted to cycles (Figure 25-(a)).
6) By accepting the edges of the above digraph as the sign "+", the regular diagram of knot 8_1 in Figure 25-(b) is obtained with the help of the Tait's method.

Figure 21

According to the bitopology associated with knot 8_2 in Table 1:.

1) $\overline{\tau_1} = \{1,2,3\}, s(\overline{\tau_1}) = V = 3$. So, let us place 3 points on the plane and enumerate these points in a clockwise direction.
2) $\underline{\tau_2} = \{\varepsilon_1^{23}, \varepsilon_2^{23}, \varepsilon_1^{13}\}$, $s\left(\underline{\tau_2}\right) = B = 3$.. Three deformations are placed between points 2-3 and 1-3.
3) Since $V + 2B - 1 = 3 + 6 - 1 = 8$, the digraph drawn has 8 edges.
4) Since $V - 1 = 3 - 1 = 2$, two edges are added clockwise from the numbered 1 vertex.
5) Deformations are converted to cycles.
6) By accepting the edges of the digraph as the sign "+", the regular diagram of knot 8_2 is obtained with the help of the Tait's method.

According to the bitopology associated with knot $\mathbf{8_3}$ in Table 1:.

1) $\overline{\tau_1} = \{1,2,3,4,5\}, s(\overline{\tau_1}) = V = 5$. So, let us place 5 points on the plane and enumerate these points in a clockwise direction.
2) $\underline{\tau_2} = \{\varepsilon_1^{15}, \varepsilon_2^{15}\}$, $s\left(\underline{\tau_2}\right) = B = 2$. Two deformations are placed between points 1 and 5.
3) Since $V + 2B - 1 = 5 + 4 - 1 = 8$, the digraph drawn has 8 edges.
4) Since $V - 1 = 5 - 1 = 4$, four edges are added clockwise from the numbered 1 vertex.
5) Deformations between points 1 and 5 are converted to cycles.

6) By accepting the edges of the digraph as the sign "+", the regular diagram of knot 8_3 is obtained with the help of the Tait's method.

According to the bitopology associated with knot $\mathbf{9_1}$ in Table 1:

1) $\overline{\tau_1} = \{1,2\}, s(\overline{\tau_1}) = V = 2$. So, let us place 2 points on the plane and enumerate these points in a clockwise direction (Figure 30-(a)).

2) $\underline{\tau_2} = \{\varepsilon_1^{12}, \varepsilon_2^{12}, \varepsilon_3^{12}, \varepsilon_4^{12}\}$, $s\left(\underline{\tau_2}\right) = B = 4$. Four deformations are placed between points 1 and 2 (Figure 30-(b)).

3) Since $V + 2B - 1 = 2 + 8 - 1 = 9$, the digraph drawn has 9 edges.

4) Since $V - 1 = 2 - 1 = 1$, one edge is added clockwise from the numbered 1 vertex (Figure 30-(c)).

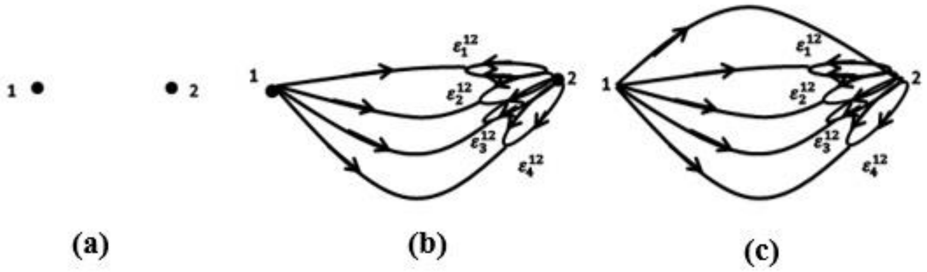


Figure 22

5) Deformations between the points 1 and 2 are converted to cycles (Figure 31-(a)).

6) By accepting the edges of the digraph in Figure 31-(a) as the sign "+", the regular diagram of knot 9_1 in Figure 31-(b) is obtained with the help of the Tait method.

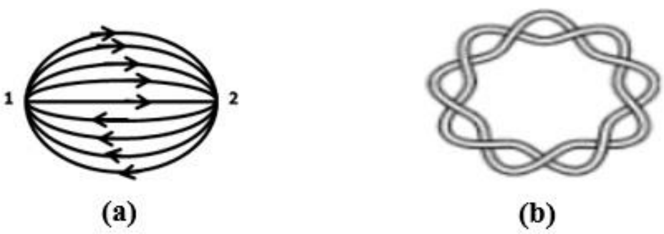


Figure 23

According to the bitopology associated with knot $\mathbf{9_2}$ in Table 1:.

1) $\overline{\tau_1} = \{1,2,3,4,5,6,7,8\}, s(\overline{\tau_1}) = V = 8$. So, let us place 8 points on the plane and enumerate these points in a clockwise direction.

2) $\underline{\tau_2} = \{\varepsilon_1^{18}\}$, $s\left(\underline{\tau_2}\right) = B = 1$. One deformation are placed between points 1 and 8.

3) Since $V + 2B - 1 = 8 + 2 - 1 = 9$, the digraph drawn has 9 edges.

4) Since $V - 1 = 8 - 1 = 7$, seven edges are added clockwise from the numbered 1 vertex.

5) Deformations between points 1 and 8 are converted to cycles.

6) By accepting the edges of the digraph as the sign "+", the regular diagram of knot 9_2 is obtained with the help of the Tait's method.

According to the bitopology associated with knot $\mathbf{9_3}$ in Table 1:.

1) $\overline{\tau_1} = \{1,2,3,4\}, s(\overline{\tau_1}) = V = 4$. So, let us place 4 points on the plane and enumerate these points in a clockwise direction.

2) $\underline{\tau_2} = \{\varepsilon_1^{14}, \varepsilon_2^{14}, \varepsilon_3^{14}\}$, $s\left(\underline{\tau_2}\right) = B = 3$. Three deformations are placed between points 1 and 4.

3) Since $V + 2B - 1 = 4 + 6 - 1 = 9$, the digraph drawn has 9 edges.

4) Since $V - 1 = 4 - 1 = 3$, three edges are added clockwise from the numbered 1 vertex.

5) Deformations between points 1 and 4 are converted to cycles.

6) By accepting the edges of the digraph as the sign "+", the regular diagram of knot 9_3 is obtained with the help of the Tait's method.

According to the bitopology associated with knot $\mathbf{10_1}$ in Table 1:.

1) $\overline{\tau_1} = \{1,2,3,4,5,6,7,8,9\}, s(\overline{\tau_1}) = V = 9$. So, let us place 9 points on the plane and enumerate these points in a clockwise direction (Figure 36-(a)).

2) $\underline{\tau_2} = \{\varepsilon_1^{19}\}$, $s\left(\underline{\tau_2}\right) = B = 1$. Three deformations are placed between points 1 and 9 (Figure 36-(b)).

3) Since $V + 2B - 1 = 9 + 2 - 1 = 10$, the digraph drawn has 10 edges.

4) Since $V - 1 = 9 - 1 = 8$, eight edges are added clockwise from the numbered 1 vertex (Figure 36-(c)).

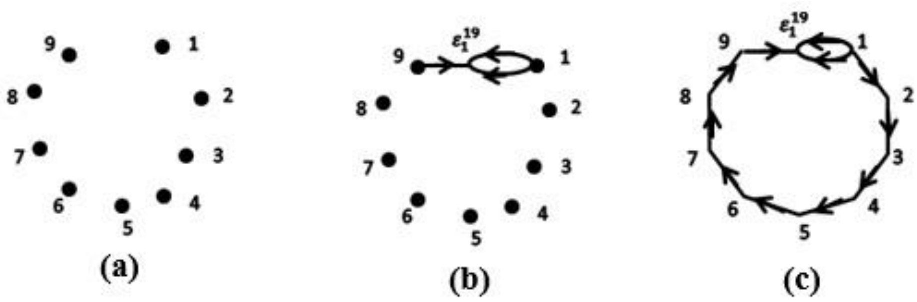


Figure 24

5) Deformations between the points 1 and 9 are converted to cycles (Figure 37-(a)).

6) By accepting the edges of the digraph in Figure 37-(b) as the sign "+", the regular diagram of knot 10_1 in Figure 37-(b) is obtained with the help of the Tait method.

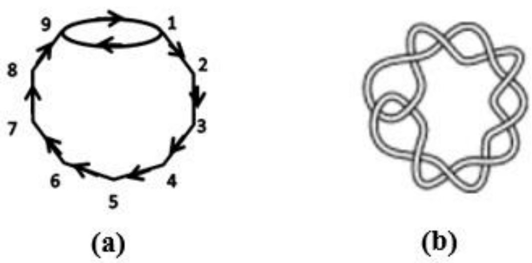


Figure 25

According to the bitopology associated with knot $\mathbf{10_2}$ in Table 1:

1) $\overline{\tau_1} = \{1,2,3\}, s(\overline{\tau_1}) = V = 3$. So, let us place 3 points on the plane and enumerate these points in a clockwise direction.

2) $\underline{\tau_2} = \{\varepsilon_1^{23}, \varepsilon_2^{23}, \varepsilon_3^{23}, \varepsilon_1^{13}\}$, $s\left(\underline{\tau_2}\right) = B = 4$. Four deformations are placed between points 1-3 and 2-3 .

3) Since $V + 2B - 1 = 3 + 8 - 1 = 10$, the digraph to be drawn have 10 edges.

4) Since $V - 1 = 3 - 1 = 2$, two edges are added clockwise from the numbered 1 vertex .

5) Deformations are converted to cycles.

6) By accepting the edges of the digraph as the sign "+", the regular diagram of knot 10_2 is obtained with the help of the Tait's method.

According to the bitopology associated with knot $\mathbf{10_3}$ in Table 1:.

1) $\overline{\tau_1} = \{1,2,3,4,5,6,7\}, s(\overline{\tau_1}) = V = 7$. So, let us place 7 points on the plane and enumerate these points in a clockwise direction.

2) $\underline{\tau_2} = \{\varepsilon_1^{17}, \varepsilon_2^{17}\}$, $s\left(\underline{\tau_2}\right) = B = 2$. Two deformations are placed between points 1 and 7.

3) Since $V + 2B - 1 = 7 + 4 - 1 = 10$, the digraph to be drawn have 10 edges..

4) Since $V - 1 = 7 - 1 = 6$, six edges are added clockwise from the numbered 1 vertex.

5) Deformations between the points 1 and 7 are converted to cycles.

6) By accepting the edges of the digraph in as the sign "+", the regular diagram of knot 10_3 is obtained with the help of the Tait's method.

CONCLUSION

With the help of the reverse of knot digraph notation, the regular diagrams of the knots $3_1, 5_1, 5_2, 6_1, 6_2, 7_1, 7_2, 7_3,\ 8_1, 8_2, 8_3, 9_1, 9_2, 9_3, 10_1, 10_2, 10_3$ are obtained using bitopologies associated with these knots .

Conflict of Interest

The article authors declare that there is no conflict of interest between them.

Author's Contributions

The authors declare that they have contributed equally to the article.

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