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Development of mathematical models to predict the load-carrying capacity of tractor tires on rigid surface

Traktör lastiklerinin rijit yüzeydeki taşıma kapasitesinin tahminlenmesi için matematiksel modellerin geliştirilmesi

Fırat Kömekçi [1*]

Adnan Değirmencioğlu [1]

[1] Ege University Faculty of Agriculture, Department of Agricultural Machinery and Technologies Engineering, 35100, Bornova, İzmir, Türkiye

* Corresponding author (Sorumlu yazar)

firat.komekci@ege.edu.tr

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ABSTRACT

Objective: The objective of this study was to develop the load-carrying capacity models of tractor tires on the rigid surface for both, radial and bias tires.

Material and Methods: In order to develop functions, two tire constructional variables (section width and overall diameter), inflation pressure, and forward speed for the load-carrying capacity model were considered. The models developed in the literature for the load-carrying capacity of tires on rigid and soil surfaces were built on some limited data and as a result of this, the predominant variable and contribution of other variables still remain unknown. This study has the widest range of tractor tire sizes (width and diameter). The load-carrying capacity models developed in this study were verified with the theoretical models and experimental measurement values in the literature.

Results: As a result of the study, it was found that the appropriate mathematical models were in non-linear (power) form and the coefficient of determination of the models was greater than 0.95.

Conclusion: The models were verified against published data in the literature and found that the predictions from the models are in good agreement with the measured values.

Keywords: Bias tires, inflation pressure, modelling, model selection, radial tires, tire width

ÖZ

Amaç: Bu çalışmanın amacı hem radyal hem de diyagonal traktör lastiklerinin rijit yüzey üzerindeki yük taşıma kapasitesi modellerini geliştirmektir.

Materyal ve Yöntem: Radyal ve diyagonal lastikler için yük taşıma kapasitesi modellerini geliştirmek amacıyla iki lastik yapısal değişkeni (kesit genişliği ve toplam çap), şişirme basıncı ve ilerleme hızı dikkate alınmıştır. Lastiklerin rijit ve toprak yüzeylerdeki yük taşıma kapasiteleri için literatürde geliştirilen modeller sınırlı veriler üzerine inşa edilmiştir ve bunun sonucunda baskın değişken ve diğer değişkenlerin katkısı hala bilinmemektedir. Bu çalışma oldukça geniş aralıktaki traktör lastiği boyutlarıyla (genişlik ve çap) gerçekleştirilmiştir. Çalışmada geliştirilen yük taşıma kapasitesi modelleri, literatürdeki teorik modeller ve deneysel ölçüm değerleriyle doğrulanmıştır.

Araştırma Bulguları: Çalışma sonucunda uygun matematiksel modellerin doğrusal olmayan (üstel) formda ve belirleme katsayısının 0.95 'den büyük olduğu görülmüştür.

Sonuç: Geliştirilmiş modeller, literatürde yayınlanmış verilerle doğrulanmıştır ve modellerden elde edilen tahminlerin ölçülen değerlerle iyi bir uyum içinde olduğu belirlenmiştir.

Anahtar sözcükler: Diyagonal lastikler, şişirme basıncı, modelleme, model seçimi, radyal lastikler, lastik genişliği

INTRODUCTION

Tires are one of the most important components of tractors since they determine the level of soil compaction and traction. The load-carrying capacity of tires is important in order to determine the amount of load to be carried safely. It is a well-known fact that the behavior of radial and bias tires in terms of load-carrying capacity differs due to their constructional differences. Radial tires have plies that run at right angles to the tread and may have one or more layers or plies. The stability and strength are provided by a belt around the radial-ply tire. This design, with plies running 90 degrees to the tread, allows radial sidewalls more flexible than bias tires and produces a larger and more stable ground contact area (Grubaugh et al., 1982).

As the basis of agricultural machinery and farm tractor management, it is a suitable solution to use ballast on the axles so that rubber-wheeled tractors can develop higher drawbar power in field conditions. In the case of ballast adding to the axles and working with the machines connected by towing or three-point hitch, load transfer takes place during the operation. Furthermore, extra ballast must be placed on the front of the tractor in order to prevent the tractor from rearing. Prediction of the tire load carrying capacity by the tractor user means determining the ballast usage limit.

The studies concerning load-carrying capacity and its modeling in the literature are very limited. Perdok & Arts (1987) developed some mathematical functions for the load-carrying capacity. They used conventional tire data (bias tires) to develop the models. Their first empirical model included the tire loading capacity at the (low) level of 100 kPa inflation pressure and tire width. This model was valid at a constant inflation pressure of 100 kPa. Their further analysis using a company data set developed an equation that is valid between 75 and 250 kPa inflation pressure. Perdok & Arts (1987) as a result of their study concluded that the tire load-carrying capacity is proportional to the square of the tire width and reducing the tire inflation pressure leads to an exponential decrease in tire loading capacity.

As a conclusion of the literature search, it can be stated that the models were built on some limited data with a narrow range of variables. Hence, a study was conducted and the objective of this study was to develop mathematical functions to predict load-carrying capacity on a rigid surface with a wide range of variables considered. Only two tire constructional variables (section width and overall diameter), inflation pressure, and forward speed were used in this study. Although some tire manufacturers provide load-carrying capacity information under certain conditions in their catalogs, not all do so. Load-carrying capacity models to be developed can be used in two ways. The first is the prediction of load under the use of certain tires and forward speed while the other one could be the prediction of either inflation pressure or forward speed with the same tire that carries a certain load.

MATERIALS AND METHOD

The data to develop mathematical models to predict load-carrying capacity for both, radial and bias tires were obtained from the published catalogs of some national and international tire manufacturing companies. Even though an extensive search was made on the internet but it was found that only three different companies revealed the necessary data for load-carrying capacity for radial tires and two companies for bias tires. The number of data and the manufacturers to develop mathematical functions for contact area and load-carrying capacity are tabulated in Table 1.

Table 1. Data used to develop prediction models for load-carrying capacity for radial and bias tires from the different tire manufacturers

Çizelge 1. *Radyal ve diyagonal lastiklerde yük taşıma kapasitesi için tahmin modelleri geliştirmek amacıyla kullanılan farklı lastik üreticilerinin veri seti*

Model	Company code	Number of data points	Total
Load-carrying capacity for radial tires	A	2878	25133
	B	20829	
	C	1426	
Load-carrying capacity for bias tires	B	1792	3342
	C	1550	

The basic two dimensions of the tire (section width and overall diameter) and inflation pressure were used along with load to develop prediction functions for load-carrying capacity. The load-carrying capacity (F) models are theoretically considered in the following:

$$F = f\,(I_p, w, D, V) \quad (1)$$

where F is load-carrying capacity (kN), I_p is inflation pressure (kPa), w is section width (m), D is overall diameter (m) and V is the forward speed (km h^{-1}). The data were first organized in Excel and then transferred to Minitab® V19 for stepwise analysis at a probability level of α=0.05.

Even though many different model types with different with or without transformations to the variables were developed but the predictions of such models were not acceptable level in terms of model selection criteria (Equ. 2,3 and 4) that described below. Hence, only three different competitive models for load-carrying capacity (Table 2) were considered in this paper.

Table 2. Theoretical model forms considered for the prediction of load-carrying capacity for radial and bias tires

Çizelge 2. *Radyal ve diyagonal lastiklerde yük taşıma kapasitesinin tahminlenmesi amacıyla dikkate alınan teorik model formları*

Model no	Model description	Theoretical form of the model
I	Square root transformed linear model	$\sqrt{F}= b_1+b_2I_p+b_3w+b_4D+b_5V$
II	Cubic root transformed linear model	$\sqrt[3]{F} = b_1+b_2I_p+b_3w+b_4D+b_5V$
III	Power model	$F= b_1I_p^{b2}\, w^{b3}\, D^{b4}\, V^{b5}$
b_1,b_2 ……b_5 are model constants.		

The model selection criteria are the coefficient of correlation (r) and the lower E_{RMS} and χ^2 as used in many mathematical modeling studies (Ertekin & Yaldız. 2000; Karayel D et al. 2004; Taskin et al. 2021; Thota et al. 2021; Demir et al. 2022; Shing-Hong Liu et al. 2022). Additionally, percentage difference values between the predicted and measured values were calculated and used for comparison purposes. The model selection criteria are as follows (Equations 2, 3 and 4):

$$E_{RMS} = \sqrt{\left[\frac{1}{N}\sum_{i=1}^{N}\left(Y_{pred.,i} - Y_{mea.,i}\right)^2\right]} \quad (2)$$

$$\chi 2 = \frac{\sum_{i=1}^{N}\left(Y_{pred.,i} - Y_{mea.,i}\right)^2}{N - n_1} \quad (3)$$

$$Percentage\ difference\ = \frac{Y_{pred}-Y_{mea}}{Y_{mea}}100 \quad (4)$$

Where E_{RMS} is root mean square error, χ^2 is khi square, $Y_{pred.}$ is predicted contact area or load-carrying capacity, $Y_{mea.}$ is measured contact area or load-carrying capacity, N is the number of measurements, n is the number of model constants. The higher the coefficient of correlation (r) and the lower the E_{RMS} and χ^2 are, the better the models predict.

RESULTS AND DISCUSSION

The models considered for load-carrying capacity for radial tires are given below.

$$\sqrt{F_{cr}} = -0.894 + 0.0084\, I_p + 4.947\,\mathrm{w} + 1.992\,\mathrm{D} - 0.0168\,\mathrm{V} \qquad r^2=0.95 \quad (5)$$

$$\sqrt[3]{F_{cr}} = 0.67 + 0.003019\, I_p\ +\ 1.724\, w\ +\ 0.805\, D\ -\ 0.0608\, V \qquad r^2=0.96 \quad (6)$$

$$F_{cr} = 5.216\, I_p^{0.51125}\, w^{0.92679}\, D^{1.142}\, V^{-0.198} \qquad r^2=0.984 \quad (7)$$

As understood from the above-given models, an increase in inflation pressure, width, and overall diameter increases the load-carrying capacity while it goes down if the speed increases.

The models are valid under the following conditions for radial tires:

$$40 \leq I_p \leq 480 \text{ kPa}$$
$$0.197 \leq w \leq 1.22 \text{ m}$$
$$0.69 \leq D \leq 2.322 \text{ m}$$
$$9.654 \leq V \leq 90 \text{ km/h}$$

The mathematical models for predicting the load-carrying capacity for radial and bias tires have built on the widest range of variables given above as compared to other models that are available in the literature.

The comparison results obtained from three models in terms of model selection criteria and percentage differences are tabulated in Table 3.

Table 3. Results from the model comparisons to predict the load-carrying capacity (F_{cr}) for radial tires (Equations 5 thru 7) based on r, E_{RMS}, χ^2, and percentage difference

Çizelge 3. *Radyal lastiklerin yük taşıma kapasitesini (F_{cr}) tahmin etmek için model karşılaştırmalarından elde edilen r, E_{RMS}, χ^2 ve % farklılık temelli sonuçlar (Eşitlik 5,6 ve 7)*

Model no	Model description	r	E_{RMS}	χ^2	Percentage difference
I	Square root transformed linear model	0.967	5.679	32.259	-67.44 and +74.24
II	Cubic root transformed linear model	0.968	5.608	31.457	-32.26 and +75.92
III	Power model	0.988	3.448	11.890	-18.79 and +21.82

As seen from Table 3, the power model is most appropriate model when the three evaluation criteria along with the percentage differences between the measured and predicted model. The power range model makes better predictions than the other two models. The percentage differences range of the power model indicate a good agreement between the measured and predicted models. The results from the stepwise analysis for the power model (Equation 7) are tabulated in Table 4.

Table 4. The results from the stepwise regression analysis (Equation 7) for the load-carrying capacity (F_{cr}) for radial tires

Çizelge 4. *Radyal lastiklerin yük taşıma kapasitesi (F_{cr}) için adımsal regresyon analizinin (Eşitlik 7) sonuçları*

Variable	Contribution (%)	P value
Inflation pressure (I_p; kPa)	12.05	0.002
Tire width (w; m)	65.93	<0.001
Overall diameter (D; m)	16.83	<0.001
Forward speed (V; km/h)	3.67	<0.001
Model r^2	98.4	

As seen from the Table 4, tire width is the most important variable for the load carrying capacity. Overall diameter also is the second important variable that contributes to load carrying capacity model.

The comparison of measured and predicted load-carrying capacity for radial tires (correlation coefficient, r=0.992) is depicted in Figure 1.

The mathematical functions developed to predict the load-carrying capacity of bias tires as a function of four variables are as follows:

$$\sqrt{F_{cb}} = -0.839 + 0.0058\, I_p + 10.176\, \mathrm{w} + 0.859\, \mathrm{D} - 0.0266\, \mathrm{V} \qquad r^2=0.962 \qquad (8)$$

$$\sqrt[3]{F_{cb}} = 0.34 + 0.002495\, I_p + 4.169\, w + 0.457\, D - 0.01075\, V \qquad r^2=0.97 \qquad (9)$$

$$F_{cb} = 8\, I_p^{0.625}\, w^{1.6137}\, D^{0.5513}\, V^{-0.2833} \qquad r^2=0.989 \qquad (10)$$

The results from the calculations made to select the best model among the three are tabulated in Table 5.

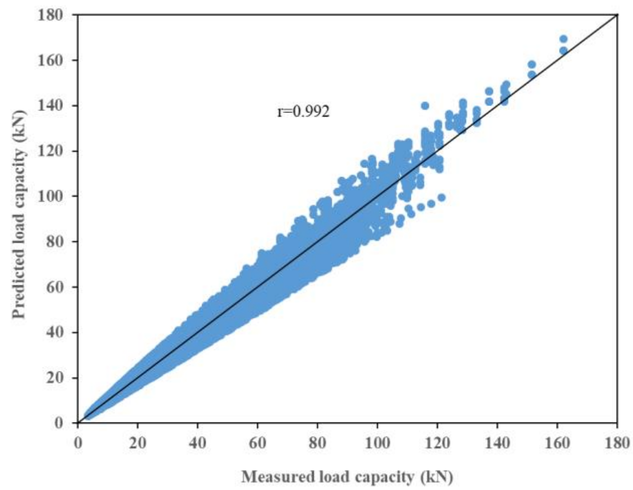

Figure 1. Comparison of measured (all data used) and predicted (Equation 7) load-carrying capacity for radial tires.

Şekil 1. *Radyal lastikler için lastik üreticileri tarafından ölçülen ve eşitlik 7'ye göre tahminlenen yük taşıma kapasitesinin karşılaştırılması.*

Table 5. Results from the model comparisons to predict the load-carrying capacity (F_{cb}) for bias tires (Equations 9 thru 11) based on r, E_{RMS}, χ^2, and percentage difference (Equations 8 thru 10)

Çizelge 5. *Diyagonal lastiklerin yük taşıma kapasitesini (F_{cb}) tahmin etmek için model karşılaştırmalarından elde edilen r, E_{RMS},* χ^2 ve *% farklılık temelli sonuçlar (Eşitlik 8, 9, ve 10)*

Model no	Model description	r	E_{RMS}	χ^2	Percentage difference
I	Square root transformed linear model	0.976	2.416	5.847	-95.56 and +108.65
II	Cubic root transformed linear model	0.977	2.313	5.360	-43.74 and +128.14
III	Power model	0.987	1.752	3.077	-20.82 and +22.90

As seen from Table 5, the model in power form is better than the other two models is the results are examined in terms of correlation coefficient (*r*), E_{RMS} and χ^2. It is worth stating here that percentage differences range is especially narrow and better as compared to other two models. The range for square root and cubic root transformed models made in a wider percentage difference range and the predictions are not as good as the ones made by power model. Hence, the power model is the appropriate model for predicting the load carrying capacity of bias tires. The detailed results from stepwise analysis are given in Table 6.

Table 6. The results from the stepwise regression analysis (Equation 10) for the load-carrying capacity (F_{cb}) for bias tires

Çizelge 6. *Diyagonal lastiklerin yük taşıma kapasitesi (F_{cb}) için adımsal regresyon analizinin (Eşitlik 10) sonuçları*

Variable	Contribution (%)	P value
Inflation pressure (Ip; kPa)	7.92	0.002
Tire width (w; m)	87.62	<0.001
Overall diameter (D; m)	0.17	<0.001
Forward speed (V; km/h)	3.19	<0.001
Model r^2	98.9	

As seen from the Table 6, the contribution of the tire width is much higher than the other three variables while the contribution of overall diameter is less than inflation pressure.

The boundary conditions for load-carrying capacity for bias tires are:

$$60.0 \le I_p \le 500 \text{ kPa}$$

$$0.196 \le w \le 0.587 \text{ m}$$

$$0.328 \le D \le 1.75 \text{ m}$$

$$10 \le V \le 50 \text{ km/h}$$

The comparison of the measured and predicted load-carrying capacity results are shown in Figure 2. As seen from the figure the measured values are in good agreement with the predicted ones with a correlation coefficient of 0.994.

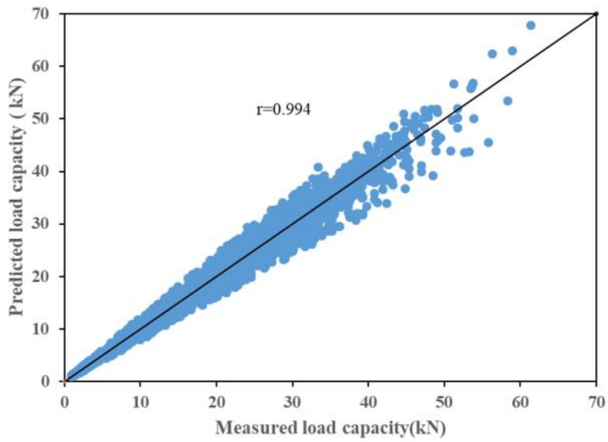

Figure 2. Comparison of measured (all data used and predicted load carrying capacity (calculated using Equ. 10) for bias tires.

Şekil 2. *Diyagonal lastikler için lastik üreticileri tarafından ölçülen ve eşitlik 10'a göre tahminlenen yük taşıma kapasitesinin karşılaştırılması.*

The analysis using the model predictions ranges of percentage difference for load carrying capacity was carried out and the results are tabulated in Table 7. As seen from the Table 7, the differences mostly accumulated between -10 and +10 % for both models.

Table 7. Distribution of percentage differences for the load-carrying capacity comparisons between the measured and predicted (Equations 7 and 10 for radial and bias tires, respectively) values

Çizelge 7. *Yük taşıma kapasitesinin ölçülen ve tahmin edilen değerler arasındaki karşılaştırmaları için % farklarının dağılımı (sırasıyla radyal ve diyagonal lastikler için Eşitlik 7 ve 10)*

Model	Ranges for percentage difference*				
	-18.79 and -10	-10 and 0	0 and +10	+10 and +21.82	Total
F_{cr}	10% (2523)	37.7% (9464)	41.63% (10463)	10.7% (2683)	100% (25133)
		79.33% (19927)			
	-20.82 and -10	-10 and 0	0 and +10	+10 and +22.90	Total
F_{cb}	11.7% (391)	35.8% (1195)	41.4% (1384)	11.1% (372)	100% (3342)
		77.2% (2579)			

*The numbers in parenthesis are the number of data points.

The comparison of the developed models for radial and bias tires in terms of load-carrying capacity was achieved. But the analysis using the data to create mathematical models did not show an advantage neither radial nor bias tires. The information in the literature such that the advantage of bias tires (McGee, 2021) was not validated with this analysis. In some combinations of variables, radial tires have higher load-carrying capacity than bias ones while in some cases bias tires have the advantage under the same conditions. This could be attributed to the fact that the data used for building mathematical functions in this study belong to new-generation tires, especially for radial tires in terms of the number of plies that make the tire stiffer and carry more load. This hypothesis is supported by Diserens et al. (2011) since they stated that the rapid development of new tires is continually in progress, particularly with respect to their load-carrying capacity.

Verification of load-carrying capacity models against other theoretical models and experimentally measured values

The load-carrying capacity models developed for radial and bias tires were verified with a small group of data published by two companies and with some data obtained experimentally. It could be stated that the data from these two companies were not included in the data pool of radial and bias tires.

The load-carrying capacity model developed for radial tires as compared to data published by two companies is depicted in Figure 3. The percent differences for these data sets ranged between -15.11 and +13.9 while the correlation coefficient was calculated to be 0.991.

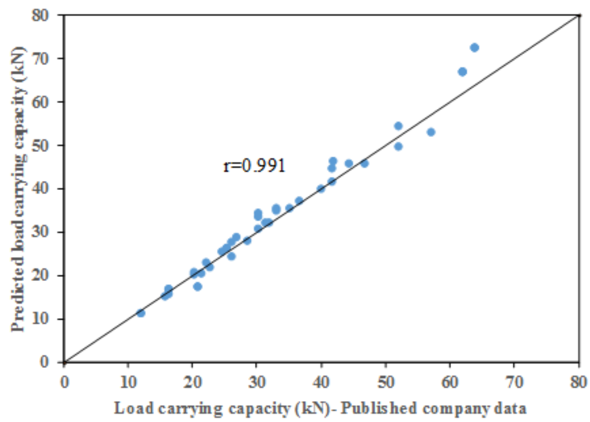

Figure 3. Comparison of measured (data not included to develop model) and predicted load carrying capacity (calculated using Equation 7) for radial tires.

Şekil 3. *Radyal lastikler için lastik üreticileri tarafından ölçülen ve eşitlik 7'ye göre tahminlenen yük taşıma kapasitesinin karşılaştırılması.*

The results from the load-carrying capacity model developed for bias tires as compared to data published by two companies are shown in Figure 4. The percent differences for these data sets ranged between -9.88 and +15.53% while the correlation coefficient was calculated to be 0.992.

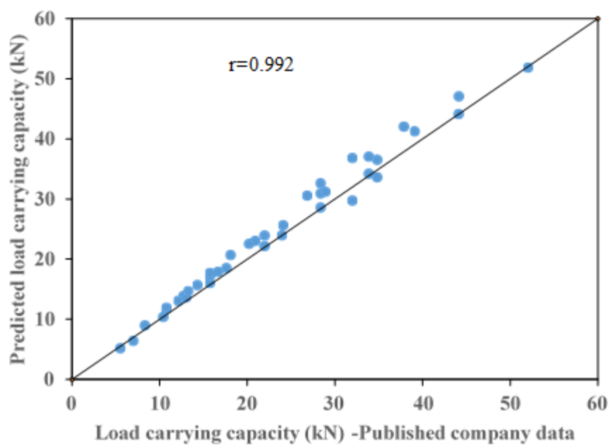

Figure 4. Comparison of measured (data not included to develop the model) and predicted load-carrying capacity (calculated using Equation 10) for bias tires.

Şekil 4. *Diyagonal lastikler için lastik üreticileri tarafından ölçülen ve eşitlik 10'a göre tahminlenen yük taşıma kapasitesinin karşılaştırılması.*

Additionally, a comparison between Perdok & Arts model (1987) and load-carrying capacity for bias tires was carried out. The comparison results are depicted in Figure 5. For the comparisons, only front and rear-driven tractor tire data along with data for tires used in row crop tractors were used. As seen from the figure, there is a good agreement between the two models. The developed model even makes better predictions for the data used by Perdok & Arts (1987). Another point here is that Perdok & Arts model (1987) is only valid between 75 and 250 kPa inflation pressure while the developed model makes predictions between 60 and 500 kPa range.

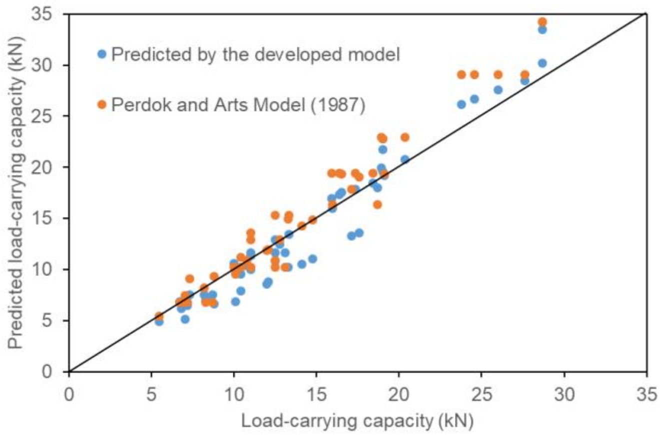

Figure 5. Measured and predicted load-carrying capacity by the developed model (Equation 10) for bias tires in this study and Perdok & Arts (1987) model.

Şekil 5. *Çalışma kapsamında diyagonal lastikler için geliştirilen model (Eşitlik 10) ve Perdok & Arts (1987) modeli ile ölçülen ve tahmin edilen yük taşıma kapasitesi.*

CONCLUSIONS

- The mathematical models for predicting the load-carrying capacity for radial and bias tires have built on the widest range of variables as compared to other models that are available in the literature.
- The load-carrying capacity models developed for radial and bias tires use only four variables that farmers can easily obtain from catalogs, such as tire width, overall diameter, inflation pressure, and speed.
- Many different model approaches were made and their coefficient of determination (r^2) values were obtained then three promising models based on their (r^2) were selected to be the candidate model for the load-carrying capacity predictions.
- The width of the tire that makes the highest contribution to the models was found to be the predominant variable in predicting the load-carrying capacity.
- Increasing the tire width, overall diameter, and inflation pressure increased the load-carrying capacity of radial and bias tires while hauling heavy loads. But the speed effect was inversely correlated with load-carrying capacity.

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