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Convergence Properties of a Kantorovich Type of Szász Operators Involving Negative Order Genocchi Polynomials

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Abstract

The goal of this research is to construct a generalization of a Kantorovich type of Szász operators involving negative-order Genocchi polynomials. With the aid of Korovkin's theorem, modulus of continuity, Lipschitz class, and Peetre's K-functional the approximation properties and convergence rate of these operators are established. To illustrate how operators converge to a certain function, we present some examples.

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1. INTRODUCTION

In analytic number theory, the generating functions method has an important place because this method provides to construct many useful and significant results, identities, and theorems for special polynomials and numbers (Simsek, 2008; 2012; 2013; 2017; 2018; Kucukoglu et al., 2019; Kucukoglu, 2022; Kilar & Simsek, 2020). The following is a definition of the Genocchi polynomials' generating function:

$$\left(\frac{2t}{e^t+1}\right)e^{xt}=\sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!}, |t|<\pi, \tag{1}$$

Horadam (1992) defined negative-order Genocchi polynomials and studied on their properties such as summation formula and complementary arguments. The generating functions of negative-order Genocchi polynomials are defined to be

$$\sum_{n=0}^{\infty}G_n^{-k}(x)\frac{t^n}{n!}=\left(\frac{1+e^t}{2t}\right)^k e^{tx}, \tag{2}$$

where $k\in\mathbb{N}=\{1,2,3,\dots\}$ (Horadam, 1992).

Some Genocchi polynomials ,$G_n^{-1}(x)$, were given by A. F. Horadam as follows:

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$$G_0^{-1}(x) = x + \frac{1}{2},$$
$$G_1^{-1}(x) = \frac{1}{2}\left(x^2 + x + \frac{1}{2}\right),$$
$$G_2^{-1}(x) = \frac{1}{3}\left(x + \frac{1}{2}\right)(x^2 + x + 1),$$
$$G_3^{-1}(x) = \frac{1}{4}\left(x^4 + 2x^3 + 3x^2 + 2x + \frac{1}{2}\right),$$

where $x \in [0, \infty)$. For more information on Genocchi polynomials and their applications, follow these references (Cangul et al., 2009; Kilar & Simsek, 2021; Srivastava & Choi, 2001; Srivastava et al., 2012)

An example of the applied disciplines of generating functions of unique polynomials is approximation theory (Jakimovski & Leviatan, 1969; Davis, 1975; Lupas, 1995 Gupta & Rassias, 2019). Varma et al. (2012) provided a new generalization of the Szász type operators that are described using Brenke-type polynomials. Through the use of Korovkin's theorem, continuity's second modulus, and Peetre's K-functional, they could able to determine the approximation properties of their operators as well as the order of convergence (Varma et al., 2012). İçöz et al. (2016) presented the definition and proof of a new sort of approximation theorem for a series of type operators that includes generalized Appell polynomials. Menekşe Yilmaz (2022) provided an operator form that makes use of the generating function of order α Apostol-Genocchi type polynomials. and reached the approximation of the operator by applying the Korovkin's theorem and using moments and central moments. Many techniques, including the K-functional, continuity's modulus, and continuity's second modulus, were used to calculate the operator's rate of convergence (Menekşe Yılmaz, 2022). Mursaleen et al. (2018) constructed a generalize Chlodowsky type Szász type operators involving Boas-Buck type polynomials and studied their some approximation properties such as Korovkin type theorem. Atakut and Büyükyazıcı (2016) presented some approximation properties of a generalization Kantorovich- Szász type operator including Brenke-type polynomials. Agyuz (2021a; 2021b; 2022; 2023) defined positive linear operators of Szász type and Kantorovich-Szász type by using generator functions of various family of special polynomials and examined the approximation these operators' characteristics.

By the inspired above studies, we offer a generalization Kantorovich type of Szász operators involving negative-order Genocchi type polynomials by way of their generating functions of when $k = 1$ because negative-order Genocchi polynomials are positive for $k = 1$. The operator is defined in the following definition:

Definition 1.1. For all $x \in [0, \infty)$, we have

$$\mathcal{H}_n^*(f, x) = \frac{2}{e+1} n e^{-nx} \sum_{k=0}^{\infty} \frac{G_k^{-1}(nx)}{k!} \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt. \tag{3}$$

In this study, we explore the convergence properties of $\mathcal{H}_n^*(f, x)$. First, we construct the Korovkin's theorem by using moment functions for $\mathcal{H}_n^*(f, x)$. Second, we calculate the rate of convergence applying Peetre's K-functional, the local Lipschitz class's constituents, and the continuity's modulus. Finally, we use the Maple to provide numerical examples to prove error estimate of our operator.

2. MAIN RESULTS

In this part, we examine the convergence properties of $\mathcal{H}_n^*(f, x)$ using approximation methods such as modulus of continuity, Korovkin's theorem, Peetre's K-functional and local Lipschitz class. To demonstrate these properties, first the moment and central moment functions are given for the operator $\mathcal{H}_n^*(f, x)$.

Consider the following definition of the class E:

$$\mathrm{E}:=\left\{f: x \in [0,\infty), \frac{f(x)}{1+x^2}\ is\ convergent\ as\ x \to \infty\right\}.$$

The moment functions of $\mathcal{H}_n^*(f,x)$ are given at the subsequent lemma:

Lemma 2.1. Let $\forall x \in [0,\infty)$, the $\mathcal{H}_n^*(f,x)$ yields at the following equations:

$$\mathcal{H}_n^*(1,x) = 1, \tag{4}$$

$$\mathcal{H}_n^*(s,x) = x + \frac{5e+3}{2n(e+1)}, \tag{5}$$

$$\mathcal{H}_n^*(s^2,x) = x^2 + \frac{6e+4}{n(e+1)}x + \frac{6e+2}{n^2(e+1)}. \tag{6}$$

Proof: Let $t=1$ and $x \to nx$. If we take $f=1$ at Eq. (3), we give

$$\mathcal{H}_n^*(1,x) = \frac{2}{e+1} ne^{-nx} \sum_{k=0}^{\infty} \frac{G_k^{-1}(nx)}{k!} \int_{\frac{k}{n}}^{\frac{k+1}{n}} dt. \tag{7}$$

By taking an integral and applying the Eq. (2) at Eq. (7), we obtain

$$\mathcal{H}_n^*(1,x) = \frac{2}{e+1} ne^{-nx} \left(\left(\frac{1+e}{2}\right) e^{nx}\right)\left(\frac{k+1}{n} - \frac{k}{n}\right) = 1. \tag{8}$$

Let $f=s$. The $\mathcal{H}_n^*(s,x)$ is described to be

$$\mathcal{H}_n^*(s,x) = \frac{2}{e+1} ne^{-nx} \sum_{k=0}^{\infty} \frac{G_k^{-1}(nx)}{k!} \int_{\frac{k}{n}}^{\frac{k+1}{n}} tdt. \tag{9}$$

By taking an integral and applying the Eq. (2) at Eq. (9), we obtain

$$\begin{aligned}
\mathcal{H}_n^*(s,x) &= \frac{2}{e+1} ne^{-nx} \sum_{k=0}^{\infty} \frac{G_k^{-1}(nx)}{k!} \left(\frac{\left(\frac{k+1}{n}\right)^2}{2} - \frac{\left(\frac{k}{n}\right)^2}{2}\right) \\
&= \frac{2}{e+1} ne^{-nx} \sum_{k=0}^{\infty} \frac{G_k^{-1}(nx)}{k!} \left(\frac{2k+1}{2n^2}\right), \\
&= \frac{2}{e+1} ne^{-nx} \frac{1}{2n^2} \left(\sum_{k=0}^{\infty} \frac{2kG_k^{-1}(nx)}{k!} + \sum_{k=0}^{\infty} \frac{G_k^{-1}(nx)}{k!}\right) \\
&= x + \frac{5e+3}{2n(e+1)},
\end{aligned} \tag{10}$$

where $\sum_{k=0}^{\infty} \frac{kG_k^{-1}(nx)}{k!}$ is the first derivative of -1 order Genocchi polynomials in terms of their generating function for $t=1$ and $x \to nx$ and is defined to be

$$\sum_{k=0}^{\infty}\frac{kG_k^{-1}(nx)}{k!}=\frac{1}{2}e^{nx}(nx+e(nx+2)+1).$$

Let $f=s^2$. The $\mathcal{H}_n^*(s^2,x)$ is defined to be

$$\mathcal{H}_n^*(s^2,x)=\frac{2}{e+1}ne^{-nx}\sum_{k=0}^{\infty}\frac{G_k^{-1}(nx)}{k!}\int_{\frac{k}{n}}^{\frac{k+1}{n}}t^2dt. \tag{11}$$

By taking an integral and applying the Eq. (2) at Eq. (11), we obtain

$$\begin{aligned}\mathcal{H}_n^*(s^2,x)&=\frac{2}{e+1}ne^{-nx}\sum_{k=0}^{\infty}\frac{G_k^{-1}(nx)}{k!}\left(\frac{\left(\frac{k+1}{n}\right)^3}{3}-\frac{\left(\frac{k}{n}\right)^3}{3}\right)\\&=\frac{2}{e+1}ne^{-nx}\sum_{k=0}^{\infty}\frac{G_k^{-1}(nx)}{k!}\left(\frac{3k^2+3k+1}{3n^3}\right),\\&=\frac{2}{e+1}ne^{-nx}\frac{1}{3n^3}\left(\sum_{k=0}^{\infty}\frac{3k^2G_k^{-1}(nx)}{k!}+\sum_{k=0}^{\infty}\frac{3kG_k^{-1}(nx)}{k!}\right.\\&\quad\left.+\sum_{k=0}^{\infty}\frac{G_k^{-1}(nx)}{k!}\right)\\&=x^2+\frac{6e+4}{n(e+1)}x+\frac{6e+2}{n^2(e+1)},\end{aligned} \tag{12}$$

where $\sum_{k=0}^{\infty}\frac{k^2G_k^{-1}(nx)}{k!}$ is part of the second derivative of -1 order Genocchi polynomials in terms of their generating function for $t=1$ and $x\to nx$ is defined to be

$$\sum_{k=0}^{\infty}\frac{k^2G_k^{-1}(nx)}{k!}=\frac{1}{2}e^{nx}(nx(nx+2)+e^{nx}(nx+1)(nx+3))+\frac{1}{2}e^{nx}(nx+e(nx+2)+1).$$

Therefore, the desired results are obtained.

We need central moments to estimate for our operator's rate of convergence. The central moments of $\mathcal{H}_n^*(f,x)$ are given at the subsequent lemma:

Lemma 2.2. For all $x\in[0,\infty)$, the $\mathcal{H}_n^*(f,x)$ provides at the following equations:

$$\mathcal{H}_n^*((s-x),x)=\frac{5e+3}{2n(e+1)}, \tag{13}$$

$$\mathcal{H}_n^*((s-x)^2,x)=\frac{x}{n}+\frac{6e+2}{n^2(e+1)}. \tag{14}$$

Proof: Via the use of linearity property of $\mathcal{H}_n^*(f,x)$, we discover

$$\mathcal{H}_n^*((s-x),x)=\mathcal{H}_n^*(s,x)-x\mathcal{H}_n^*(1,x)=\frac{5e+3}{2n(e+1)}, \tag{15}$$

and

$$\begin{aligned}\mathcal{H}_n^*((s-x)^2,x) &= \mathcal{H}_n^*(s^2,x)-2x\mathcal{H}_n^*(s,x)+x^2\mathcal{H}_n^*(1,x)\\ &=\frac{x}{n}+\frac{6e+2}{n^2(e+1)}.\end{aligned} \tag{16}$$

In view of the above equations, we obtain the desired results.

Korovkin-type theorems offer elementary and effective methods for assessing whether an accepted series of positive linear operators acting on a function space is functioning approximatively or, alternatively, if it greatly converges to the identity operator. In general, these theorems offer a variety of test subsets of functions that, if they are true, warranty that the approximation (or convergence) feature is true across the board.

The phrase "Theorems of the Korovkin kind" alludes to P. P. Korovkin, who showed in 1953 that the functions $1, x$, and x^2 have such a property in the collection of all continuous functions on the real interval $[0,1]$ known as $C([0,1])$. (Korovkin, 1953; 1960; Altomare, 2010).

Now, we give a theorem to show uniformly convergence of $\mathcal{H}_n^*(f,x)$ as below:

Theorem 2.3. Let $f \in C[0,\infty) \cap E$. We give,

$$\lim_{n\to\infty} \mathcal{H}_n^*(f,x) = f(x), \tag{17}$$

uniformly on all of the compact subsets of $[0,\infty)$.

Proof: We know for a fact that

$$\begin{aligned}&\lim_{n\to\infty} \mathcal{H}_n^*(1,x) = \lim_{n\to\infty} 1 = 1\\ &\lim_{n\to\infty} \mathcal{H}_n^*(s,x) = \lim_{n\to\infty}\left(x+\frac{5e+3}{2n(e+1)}\right) = x\\ &\lim_{n\to\infty} \mathcal{H}_n^*(s^2,x) = \lim_{n\to\infty}\left(x^2+\frac{6e+4}{n(e+1)}x+\frac{6e+2}{n^2(e+1)}\right) = x^2\end{aligned}$$

The Korovkin's theorem can be used to obtain the desired result.

As stated in its definition, continuity's modulus is shown by

$$\omega(\mathrm{f},\delta) := \underbrace{sup}_{\substack{x,y\in[0,\infty)\\ |x-y|\le\delta}} |f(x)-f(y)|, \tag{18}$$

where f is a function that is continuous throughout $[0,\infty)$, and that $\delta > 0$. A property of the modulus of continuity is given at the subsequent inequality:

$$|f(x)-f(y)| \le \omega(\mathrm{f},\delta)\left(\frac{|x-y|}{\delta}+1\right). \tag{19}$$

By using the definition and property of the continuity's modulus, we have a theorem for $\mathcal{H}_n^*(f,x)$ as follows:

Theorem 2.4. Let $f \in C_B[0,\infty) \cap E$, then

$$|\mathcal{H}_n^*(f,x)-f(x)| \le 2\omega(\mathrm{f},\delta_n), \tag{20}$$

where $\delta_n(x) = \sqrt{\mathcal{H}_n^*((s-x)^2,x)}$.

Proof Lemma 2.1 and the monotonicity property of $\mathcal{H}_n^*(f,x)$ lead to this conclusion

$$|\mathcal{H}_n^*(f,x)-f(x)| \leq \mathcal{H}_n^*(|f(x)-f(y)|;x). \tag{21}$$

By using Eq. (19), what Eq. (21) reveals to us is as follows:

$$|\mathcal{H}_n^*(f,x)-f(x)| \leq \omega(\mathrm{f},\delta_n)\left(1+\frac{1}{\delta}\mathcal{H}_n^*(|x-y|,x)\right). \tag{22}$$

When we consider Eq. (22)'s right side and apply the Cauchy-Schwarz inequality, we obtain

$$|\mathcal{H}_n^*(f,x)-f(x)| \leq \omega(\mathrm{f},\delta_n)\left(1+\frac{1}{\delta}\sqrt{\mathcal{H}_n^*((x-y)^2,x)}\right). \tag{23}$$

The evidence is concluded if the answer is $\delta \coloneqq \delta_n(x)=\sqrt{\mathcal{H}_n^*((s-x)^2,x)}$ in Eq. (23).

An estimate of the approximation error of $\mathcal{H}_n^*$ operators to f, similarly the modulus of continuity, is given by the Lipschitz class, which is defined below:

$$Lip_M(\alpha) := \{f \in C_B[0,\infty) : |f(t)-f(x)| \leq M|t-x|^\alpha; t,x\epsilon[0,\infty)\},$$

where $C_B[0,\infty)$ is the set of spaces of continuous and bound functions, $M>0$, and $\alpha\in(0,1]$.

The subsequent theorem satisfies a prediction for the error of the operator $\mathcal{H}_n^*$ to a function f belonging to the Lipschitz class of order α by above equation.

Theorem 2.5. We suppose that $f \in C_B[0,\infty)$. For $x\geq 0$, we give

$$|\mathcal{H}_n^*(f,x)-f(x)| \leq \mathrm{M}\delta_n^\alpha(x), \tag{24}$$

where $\delta_n(x)=\sqrt{\mathcal{H}_n^*((s-x)^2,x)}$.

Proof: According to the monotonicity characteristics of the operators $\mathcal{H}_n^*$, we obtain

$$|\mathcal{H}_n^*(f,x)-f(x)| \leq \mathrm{M}\mathcal{H}_n^*(|s-x|^\alpha;x). \tag{25}$$

The following can be written using the Hölder inequality and from (25),

$$|\mathcal{H}_n^*(f,x)-f(x)| \leq \mathrm{M}\left(\mathcal{H}_n^*((s-x)^2,x)\right)^{\frac{\alpha}{2}}.$$

Consequently, the theorem's proof is complete.

In approximation theory, the Peetre's *K*-functional proved to be a highly useful tool for calculating the error. The Peetre's *K* functional is given to be as follows:

$$\mathcal{K}(f,\delta)=\inf\left\{\|f-h\|_{C_B[0,\infty)}+\delta\|h\|_{C_B^2[0,\infty)}\right\},$$

where $\delta>0$, $f\in C_B[0,\infty)$ and $C_B^2[0,\infty) \coloneqq \{h \in C_B[0,\infty): h', h'' \in C_B[0,\infty)\}$, here the norm is defined to be as

$\|h\|_{C_B^2[0,\infty)} \coloneqq \|h\|_{C_B[0,\infty)}+\|h'\|_{C_B[0,\infty)}+\|h''\|_{C_B[0,\infty)}$ (DeVore & Lorentz, 1993).

We will use the definition of Peetre's K-functional at the subsequent theorem to assess the degree of approximation for this purpose.

Theorem 2.6. Let $f \in C_B[0,\infty)$ and $x \in [0,\infty)$. The inequality that follows is true

$$|\mathcal{H}_n^*(f,x) - f(x)| \leq 2\mathcal{K}\big(f, \varphi_u(x)\big), \tag{26}$$

where $\varphi_u(x) = \frac{1}{4n}x + \frac{2e[(2n+1)(e+1)+(e-1)]}{8n^2(e+1)^2}$.

Proof: Suppose that $h \in C_B^2[0,\infty)$. Using the linearity property of $\mathcal{H}_n^*$ operators and Taylor's expansion, we bring

$$\mathcal{H}_n^*(h,x) - h(x) = h'(x)\mathcal{H}_n^*(s-x,x) + \frac{h''(\tau)}{2}\mathcal{H}_n^*\big((s-x)^2,x\big), \tau \in (x,s).$$

The aforementioned equality allows for the writing

$$|\mathcal{H}_n^*(f,x) - f(x)| \leq \left(\frac{1}{4n}x + \frac{2e[(2n+1)(e+1)+(e-1)]}{8n^2(e+1)^2}\right)\|h\|_{C_B^2[0,\infty)} \tag{27}$$

As opposed to that, applying Lemma (2.1) and expression (27), we obtain

$$\begin{aligned} |\mathcal{H}_n^*(f,x) - f(x)| &\leq |\mathcal{H}_n^*(f-h,x)| + |\mathcal{H}_n^*(h,x) - h(x)| + |f(x) - h(x)| \\ &\leq 2\|f-h\|_{C_B[0,\infty)} + |\mathcal{H}_n^*(h,x) - h(x)| \\ &\leq 2\left(\|f-h\|_{C_B[0,\infty)} + \varphi_u(x)\|h\|_{C_B^2[0,\infty)}\right) \end{aligned} \tag{28}$$

Catching the upper limit to the right of Eq. (28) over all $h \in C_B^2[0,\infty)$, we obtain the subsequent inequality.

$$|\mathcal{H}_n^*(f,x) - f(x)| \leq 2\mathcal{K}\big(f, \varphi_u(x)\big).$$

Therefore, the proof is completed.

Now, we provide a few examples to help we obtain a higher limit for the error $f(x) - \mathcal{H}_n^*(f,x)$ by means of the continuity modulus. Maple2023TM was used to complete the computations for this paper.

Example 2.7. The approximation of $\mathcal{H}_n^*(f,x)$ to $f(x) = sin(\pi x)$ depends on $[0,\infty)$ is illustrated in the Table 1.

Table 1. *Modulus of continuity-based error estimate for function $f(x) = sin(\pi x)$*

n	$sin(\pi x) - \mathcal{H}_n^*(f,x)$
10	0.9037367966
10^2	0.06591390272
10^3	0.006314114694
10^4	0.0006286279188
10^5	0.00006283494702
10^6	0.000006283216246
10^7	0.0000006283188400

Example 2.8. The approximation of $\mathcal{H}_n^*(f,x)$ to $f(x)=\frac{x}{\sqrt{x^2+1}}$ depends on $[0,\infty)$ is illustrated in the Table 2.

Table 2. *Modulus of continuity-based error estimate for function* $f(x)=\frac{x}{\sqrt{x^2+1}}$

n	$\boldsymbol{\frac{x}{\sqrt{x^2+1}} - \mathcal{H}_n^*(f,x)}$
10	0.2952150886
10^2	0.02098369184
10^3	0.002009847454
10^4	0.0002000984836
10^5	0.00002000098484
10^6	0.000002000009848
10^7	0.0000002000000984

In these two examples, we use continuity's modulu to numerically determine the approximations of $\mathcal{H}_n^*(f,x)$ to the functions, respectively, $f(x)=sin(\pi x)$ and $f(x)=\frac{x}{\sqrt{x^2+1}}$. We found that a tiny quantity of inaccuracy was produced when using ω. According to Table 1 and Table 2, we observe that the amount of error when using ω gets smaller as n increases.

3. CONCLUSION

Many mathematicians, physicists, engineers, and other experts have extensively studied the generating functions method. Particularly, the generating functions of Genocchi type polynomials have found widespread application in a wide range of fields. Due to this, we have constructed a generalization of Kantorovich type of Szász linear positive operator, $\mathcal{H}_n^*(f,x)$, using generating functions of -1 order Genocchi polynomials.

We investigated convergence properties of $\mathcal{H}_n^*(f,x)$. Firstly, we obtained moment and central moment functions of our operator. Secondly, we gave Korovkin's theorem for $\mathcal{H}_n^*(f,x)$ by using moment functions. By the help of this theorem, we satisfied uniformly convergence property of our operator. And then, we investigated to estimate rate of convergence of $\mathcal{H}_n^*(f,x)$ by using some well-known approximation devices such as modulus of continuity, Lipshitz class, and Peetre's K-functional. Finally, by means of the modulus of continuity, we have discovered a higher limit for the error $f(x)-\mathcal{H}_n^*(f,x)$ for particular functions.

The study's methods were all employed to look at the created operator's characteristics, including convergence rate and uniform convergence. These methods have demonstrated that our operator smoothly converges to all functions under favorable conditions, and the approximation speed is adequate.

In this work, we describe a generalization of positive linear operators involving -1-order Genocchi polynomials that have important applications, particularly in analytical number theory. This study can be shown as an important example of defining the special polynomial families with the help of generator functions and forming linear positive operators. As a result, numerous fields, including operator theory, mathematics, and engineering, may benefit from this study's findings.

The original results obtained in this study may inspire the use of special polynomial families defined in q- and (p,q)-analysis to construct positive linear operators in approximation theory.

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CONFLICT OF INTEREST

The author declares no conflict of interest.

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