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Abstract

The present study investigated the effect of the flipped classroom model, which was practiced in an undergraduate mathematics class, on pre-service primary school teachers' ability to solve problems related to real life and modelling. This study was designed as an action research study exploring the implementation of a new teaching technique. The participants were 16 pre-service teachers who voluntarily took part in the study. The implementation period, which lasted for five weeks, included the following topics: ordered pairs, Cartesian product, relation and its characteristics, the concept of function, types of functions, and linear relation. The data in this study consisted of worksheets that had problem-solving processes, video records of classroom practices, participant journals, and field notes taken by the researcher. The data were analyzed through content analysis. It was observed that the flipped classroom model in the study enabled students to develop their mathematics language, conceptual knowledge, and the use of multiple representations by providing them with an opportunity to work on modelling and real-life problems. This model allowed students to take responsibility for their learning and also provided the teacher with an opportunity to prevent mistakes and misconceptions of students. In addition, it was observed in the data collected from classroom practices and student journals that student–student and student–teacher interactions developed as a result of the classes taught in the flipped classroom model.

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Research Article

Implications from a Modelling based Flipped Mathematics Course *

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Abstract

The present study investigated the effect of the flipped classroom model, which was practiced in an undergraduate mathematics class, on pre-service primary school teachers' ability to solve problems related to real life and modelling. This study was designed as an action research study exploring the implementation of a new teaching technique. The participants were 16 pre-service teachers who voluntarily took part in the study. The implementation period, which lasted for five weeks, included the following topics: ordered pairs, Cartesian product, relation and its characteristics, the concept of function, types of functions, and linear relation. The data in this study consisted of worksheets that had problem-solving processes, video records of classroom practices, participant journals, and field notes taken by the researcher. The data were analyzed through content analysis. It was observed that the flipped classroom model in the study enabled students to develop their mathematics language, conceptual knowledge, and the use of multiple representations by providing them with an opportunity to work on modelling and real-life problems. This model allowed students to take responsibility for their learning and also provided the teacher with an opportunity to prevent mistakes and misconceptions of students. In addition, it was observed in the data collected from classroom practices and student journals that student–student and student–teacher interactions developed as a result of the classes taught in the flipped classroom model.

Keywords: Flipped classroom, real-life problems, modelling problems, relation, function

1. INTRODUCTION

As a result of technological development over the years, mobile devices have become a huge part of our lives, and learning that is independent of time and place has become widespread around the world. The flipped classroom concept is a teaching model that allows this opportunity, and it is considered a technique that enables students to receive course content through electronic devices while they are at home, thus, allowing the classroom time to be used for applied activities (Bergmann & Sams, 2012). The flipped classroom approach does not restrict learning to the classroom environment and enables a range of activities to be conducted in the classroom through tasks and responsibilities undertaken and fulfilled by students.

In flipped classrooms, the time spent out of the classroom should be structured to be able to prepare students for classroom activities in the best way possible, and students should spend their time in the classroom participating in cooperative problem-solving and discussion activities rather than on listening or note-taking (McGivney-Burelle & Xue, 2013). In addition to increasing student–student and student–teacher interaction, flipped classrooms transform traditional teaching environments into student-centered and inquiry-based settings (Bergmann & Sams, 2012). In this model, the teacher

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walks around the classroom and interacts with students who work individually or in small groups and asks questions, answers them, and provides explanations to issues that s/he thinks the class is in need of (McGivney-Burelle & Xue, 2013). Flipped classrooms are considered to be suitable for transforming students' focus on taking notes and encouraging them to engage in sense-making and exploratory activities (Bergmann & Sams, 2012).

Jungić, Kaur, Mulholland, and Xin (2015) consider flipped classrooms as a valuable model because they balance technology and human interaction in the teaching process. This model aids in increasing students' motivation to learn the course content before attending class, providing opportunities for interaction among peers, along with the benefit of receiving immediate feedback from teachers (Zainuddin, 2017). Bhagat, Chang, and Chang (2016), highlighted that low-achieving students receive more attention from teachers in flipped classrooms and have opportunities to discuss their problems in understanding mathematical concepts.

In mathematics education, it has been highlighted that the flipped classroom method eases the transition between conceptual and relational understanding (Bergmann & Sams, 2012). In addition, it was stated that the teachers in flipped classrooms can better identify students who are eager to tackle the problems discussed in the class (Kirvan, Rakes & Zamora, 2015), and this was considered to be critical for the mathematical learning process (Hiebert & Grouws, 2007). In traditional classrooms, while the teachers' attention is on students with high academic performance, the remaining students sit passively. In contrast, the teachers in flipped classrooms are more helpful toward students who struggle with the materials presented to them (Bergmann & Sams, 2012).

Quite a few number of studies were conducted on the use of flipped classrooms for various courses at the undergraduate level in universities (Cronhjort, Filipsson & Weurlander, 2017; McGivney-Burelle & Xue, 2013; Sherrow, Lang & Corbett, 2016; Strayer, 2007). It is especially noteworthy that courses such as Introduction to Statistics (Heuett, 2017; Strayer, 2007); Calculus (Anderson & Brennan, 2015; Cronhjort, et al., 2017; Jungić, et al., 2015; McGivney-Burelle & Xue, 2013; Schroeder, McGivney-Burelle & Xue, 2015); Linear Algebra (Murphy, Chang & Suaray, 2016; Wright, 2015); Algebra (Kirvan, et al., 2015; Lesseig & Krouss, 2017; Van Sickle, 2015); and Finite Mathematics (Guerrero, Beal, Lamb, Sonderegger & Baumgartel, 2011), which are related to mathematics and other disciplines with a link to mathematics, were taught in flipped classrooms. The analysis of relevant studies revealed that McGivney-Burelle and Xue's (2013) study on flipped classrooms showed that students particularly emphasized the roles of videos in achieving better results in homework and tests and classroom time in solving problems. In his study where he compared the use of traditional and flipped classrooms in Introduction to Statistics classes offered at the undergraduate level, Strayer (2007) noted that when students in flipped classrooms were less content with the guidance offered in the classroom, they became more open to cooperative learning and innovative teaching approaches. Similarly, in her research, Van Sickle (2015) emphasized that flipped classrooms was a good method to develop skills in algebra classes. Moreover, she added that it is a convenient model for teaching algebra classes that students were already acquainted with.

Although pre-service classroom teachers are considered to be qualified in mathematics, in terms of using information that is based on operations and rules, especially in arithmetic, it is also known that they experience problems when they are asked to explain why algorithms and/or operations work (Ball, 1990). The starting point of the present study is the fact that when the undergraduate mathematics course was taught to develop pre-service classroom teachers' problem-solving skills and performances, the time given was not sufficient. These circumstances created the need to try out a practice-based teaching model such that theoretical parts are supported with pre-class preparations. Most studies in the literature, while showing that flipped classrooms contribute to students' mathematical performances in quantitative terms (Murphy, et al., 2016; Petrillo, 2016), highlighted the need for in-depth studies to find out whether flipped classrooms are effective in developing certain mathematical skills (Cronhjort, et al., 2017). It is important to conduct an in-depth investigation of both the application of this model and how the model facilitates student learning (Naccarato & Karakok, 2015), just like it is done for any new teaching model.

Prior to conducting this study, it was observed that the classroom time was not enough to teach the content required to solve real-life-related problems in the classroom. This was because of the intensity of the contents of the mathematics course offered at undergraduate level pre-service classroom teacher education programs planned for two hours per week. Therefore, the researchers have designed an action research study based on the flipped classroom model, which they found to be practical in terms of giving more room to practice real-life and modelling problems in the classroom. In this context, the present study aimed to investigate students' performances in solving real-life and modelling problems in the classroom and the learning outcomes of the process. For this reason, the following question makes up the research problem statement in this study: "What are the benefits of flipped classroom model for students, and how did the students' real-life and modelling problem-solving processes develop?"

1.1. Real-Life Problems

It was seen that individuals' needs changed in line with science and advancing technology in the 21st century and that individuals are in need of skills such as decision making, reasoning, and creativity to solve problems (OECD, 2010). However, it is also known that traditional teaching/learning approaches are not sufficient to equip individuals with such and similar skills (English & Watters, 2004; Ministry of National Education Turkey [MoNE], 2016; Zawojewski, Lesh & English, 2003). Although the history of mathematics carries many traces of mathematical subjects being born out of real-life problems, a large number of students' experiences in mathematics courses cause them to consider mathematical subjects to be abstract and irrelevant to real life (Yoon, Dreyfus & Thomas, 2010). At this point, the responsibility of teachers of mathematics classes is to introduce teaching and learning approaches that help set up the link between the mathematics world and the real world in their classes. Model-eliciting activities and application problems are considered to be the two most common ways of setting up the link between the mathematics and the real world (Yoon, Dreyfus & Thomas, 2010). Thanks to such problems, students interpret a case in their real life and mathematize it in a way they are able to understand (Lesh & Doerr, 2003), and thus, mathematical models are structured to seek real-life solutions (Geiger & Kaiser, 2014). These problems also include developing helpful strategies to interpret the nature of the order and patterns, the data that is not clearly seen, aims, and potential solution strategies (Gravemeijer & Doorman, 1999).

The models, which are based on hypotheses developed through mathematical modeling processes, can take the form of mathematical representations in which the relationship between two or more variables is explained (e.g., functions, graphics, tables, equations, inequalities, a system of equations, and geometrical shapes; Bukova-Güzel, 2016). Additionally, a modeling activity generally includes mathematization through various means such as "quantifying, dimensionalizing, coordinatizing, categorizing, algebraizing, and systematizing relevant objects, relationships, actions, and patterns and regularities" (Lesh & Doerr, 2003, p. 5). Model developing activities, which include all these processes, pave the way for students to learn mathematical concepts by relating them to real life (Siller, Çevikbaş, Geiger & Greefrath, 2022). Even though model-eliciting activities are mathematizing the world in a more productive way when compared to application problems, researchers have claimed that model-eliciting activities take more time than other types of problems that are used (Lesh & Doerr, 2003; Lesh, Yoon & Zawojewski; 2007). The need for giving time to real-life problems and mathematical applications in a mathematics course offered at the undergraduate level motivated the researchers to integrate the flipped classroom model into the course and leave the responsibility of learning theoretical mathematics to students.

2. METHOD

The present study adopted an action research design, which is a data-driven research design that teacher-researchers use to make sense of the practices they conduct and to analyse and develop those practices (Cohen, Manion & Morrison, 2007). Action research is defined as a systematic data collection and analysis approach that is directly conducted by the researcher undertaking the practice to overcome process-related problems (Yıldırım & Şimşek, 2016). The present study adopted the scientific action research model, which is a type of action research that is specifically focused on testing and evaluating a certain practice. The study adopted the four-stage procedure described by Lewin (1948), which included planning, acting, observing, and reflecting.

2.1. Participants

The present study was conducted with 16 pre-service classroom teachers registered in an undergraduate mathematics course over five weeks. Around 110 students who were enrolled in an undergraduate elementary teacher education program (Grade 1 to 4) were contacted, and the research process was explained to them. Those who wanted to take the mathematics course among the contacted students were identified, and detailed explanations about what the students were expected to work on within and outside the classroom were provided. Lastly, other responsibilities that they were expected to take on was explained to those students who volunteered to take part in the study.

The participants in the study consisted of individuals who completed their high-school education in various cities, had different socioeconomic backgrounds, were between the ages of 18 and 22, and studied in the same department. The whole group consisted of 16 preservice teachers. There were 11 women and 5 men from a elementary teacher education program (Grade 1 to 4) who agreed to participate voluntarily. They were asked to work in groups of their preference with three to four other students. Further, the students, from the beginning of the first week onward, were asked to work with the same team members in the problem-solving environment during the teaching process to enhance group dynamics by creating a peer learning environment and to enable them to gain the maximum benefit. Participants' real names were not used in the study, and the groups were coded as follows based on the group names the participants chose: Yeni Nesil (YN), Maviş (MVS), Teşkilat-ı Cebir (TC), Son (SON), and Sıralı Dörtlüler (SD).

2.2. Procedures

During the research process, students' learning activities inside and outside of class were designed by the researchers. First, the course materials to be used in out-of-class activities as part of the flipped classroom model were designed. In line with this, the massive open online courses available in the literature were examined by the researchers. Each of the sources accessed was examined by a mathematics expert, two mathematics teaching experts, and an information and education technologies expert in terms of criteria such as their fitness with respect to the learning outcomes of the course, the suitability and structure of the mathematical language used in the videos, the technological characteristics of the videos, and the adequacy of the mathematical content. Following the analysis of the online content, video(s) were edited to align with the learning outcomes of the course, and necessary additions were made. Within the 5-week implementation period, we shared 35 videos (total 158 minutes) with the preservice teachers. The course materials were delivered to the preservice teachers through WhatsApp. During the research period, students were asked to come to the class after watching the videos prepared by the researchers. Thus, students were able to spend more time on real-life and modeling problems during classroom activities. Throughout the study we conducted classroom activities with volunteer students. During the study, there was only one participant could not watch the videos before arriving the in class activities.

The classroom activities that were part of the research study included a number of problem-solving activities that consisted of real-life problems and modeling activities. Each of the problems used in classroom activities was developed by the researchers on the basis of real life. The nature of the problems were directed at developing students' mathematical competence and their use of mathematical language and, at the same time, to help them set up links between real life and mathematics. Topics such as ordered pair, Cartesian product, relation and its characteristics, the concept of function, types of function, and linear relation models, which were part of the undergraduate mathematics course that was the research setting, were included in the study. The topics included in the research study (e.g., relation, functions, and graphics) and their sub-concepts are presented in Table 1. The detailed information regarding the activities designed were given in a drive folder (https://drive.google.com/drive/folders/1IhYE5P-gNugW-KRKQNs6KwcGq8GdtfLa?usp=sharing).

Table 1. The activities in the research study and their content

Week	Activity	Content
Week 1	Cabin Crew	Ordered pair, Cartesian Product & Relation
Week 2	Cabin Control List	Cartesian Product & Relation
	Valentine's Day	Relation, and the characteristics of relation (reflection)
Week 3	Blood Transfusion	The characteristics of relation and the concept of function
Week 4	Yoga Course	Functions & linear relation
Week 5	Call Center	Linear relation, inverse function, graphs of functions and the linear functions

As a part of an action research, after every sessions of in-class activities, researchers conduct a negotiation meeting to evaluate the effectiveness of the activities and to rearrange the forthcoming week's plan. Thus, even if the researchers design all the process before the process, they also rearrange and redesign activities and videos in accordance with group progression and group dynamics.

2.3. Data Collecting Process

The data sources in this study consisted of worksheets that contained problem-solving processes, video records, participant journals, and field notes taken by the researcher. At the beginning of each class, students were given worksheets that explained problems from real life. Those worksheets were the main source to assess groups' performances in a given week. Audio and video recordings relating to groups' problem-solving processes were made in order to understand the strategies and ways of thinking that each group utilized when finding solutions. In addition, field notes were created by recording the feedback given to students and the common mistakes made by them. Triangulation was achieved in the study by using field notes and audio-video records to support the data collected from the worksheets with problems.

2.4. Data Analysis

The analysis of the data took place in two stages: 1) through micro analyses conducted at the end of the class taught each week to analyze the weekly effectiveness of the plans and 2) through macro analyses conducted at the end of the study. Action plans, which were implemented each week, were evaluated by the researchers after implementation. The microanalyses at the end of each week were conducted by two mathematics education experts and a mathematics expert and then, due to the nature of the action research design, new decisions were made to develop action plans to improve the practice.

As for the macro analyses conducted at the end of the study, experts, after coding the data through open coding, met to decide on their agreement with the codes, and they developed themes based on the codes by consensus. Based on the themes that were developed, weekly development in each group was observed procedurally, and similar and different processes between groups were presented through diagrams. After analyzing the whole process, the inter-coder reliability (Miles &

Huberman, 2014) between the coders are found .86. After the codes were negotiated, researchers were constructed the themes and visualize the data as findings.

3. FINDINGS

The results of the present study, which was designed as an action research study, were analyzed in terms of the topics covered and in the context of real-life and modelling problems prepared in relation to those topics.

3.1. First Week: Ordered Pair, Cartesian Product, and Relation

In the cabin crew problem that was presented in the first week of the study, students were asked to use their knowledge on ordered pair, Cartesian product, and relation topics, which they had been learning since high school, in solving real-life problems. In the context presented in the problem, students were, first, asked to identify the convenient seats, then, to match passengers with the dishes they could eat based on their special diet categories, and, finally, identify all the conditions relating to these matchings. The groups showed the solution using various representation forms such as lists and Venn diagrams and by placing the plane in Cartesian coordinates. Additionally, YN, MVS, and TC preferred graphic representation whereas SD, YN, and TC preferred representation through common characteristics in addition to other forms of representation (Figure 1).

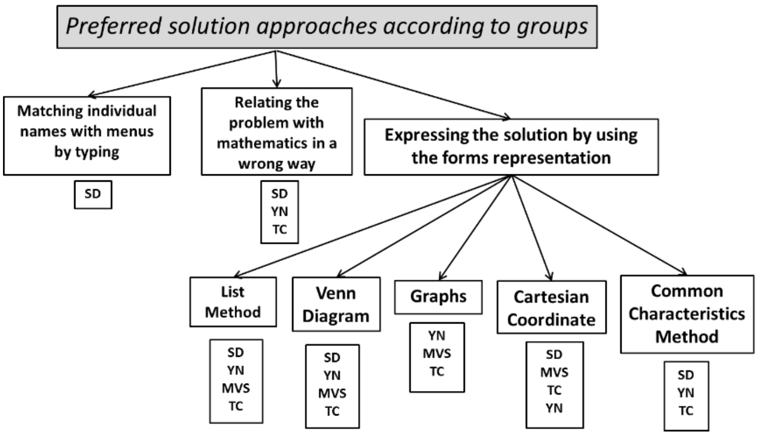
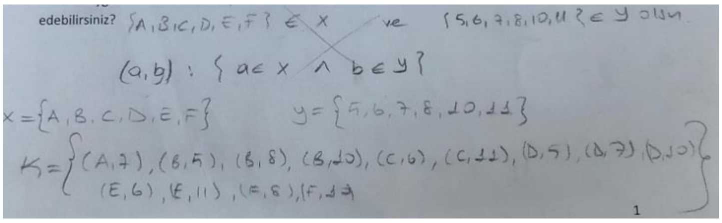


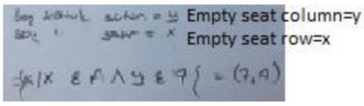
Figure 1. General performances in Week 1

Most of the participants expressed that they encountered such real-life problems for the first time and experienced difficulties in applying their knowledge of ordered pair and Cartesian product, which constitute the basis for the concept of relation, to real-life problems, as shown in researchers' field notes.

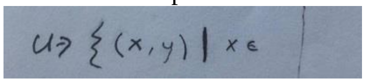
When the preferred solution approaches among the groups were analyzed, it was observed that students tried to express the solution without using the mathematical language appropriately (representations, expressions, notation e.g.). Samples of common characteristics and list methods, which were among the representation forms used by SD, can be found in Figure 2. It has been observed that students were not able to associate the topic with the real-life problems using either one of the representation methods, and they were not able to adequately understand representation using the common characteristics method. It was seen that MVS, like SD, repeated the same mistake in using mathematical language to represent relation with the common characteristics method.



Group SD



Group MVS



Group YN

Figure 2. Samples of mistakes in using mathematical language inadequately in Week 1

When the groups' journals were analyzed, it was found that all the groups liked the videos they watched and perceived teamwork as joyful and educational.

> *"Real-life problems were problems that helped us comprehend the topic. They were questions that were open for reasoning. We found the solutions through discussion and cooperation with our teacher and group members. Classroom environment was quite open for discussion, and it was joyful." (SD)*
>
> *"…Today, watching videos beforehand was extremely helpful in solving the problems our teacher gave us. Our teamwork was joyful and educational. It was an environment suitable for discussion, and we found common ground by listening to each other and transferred it on to paper…" (TC)*

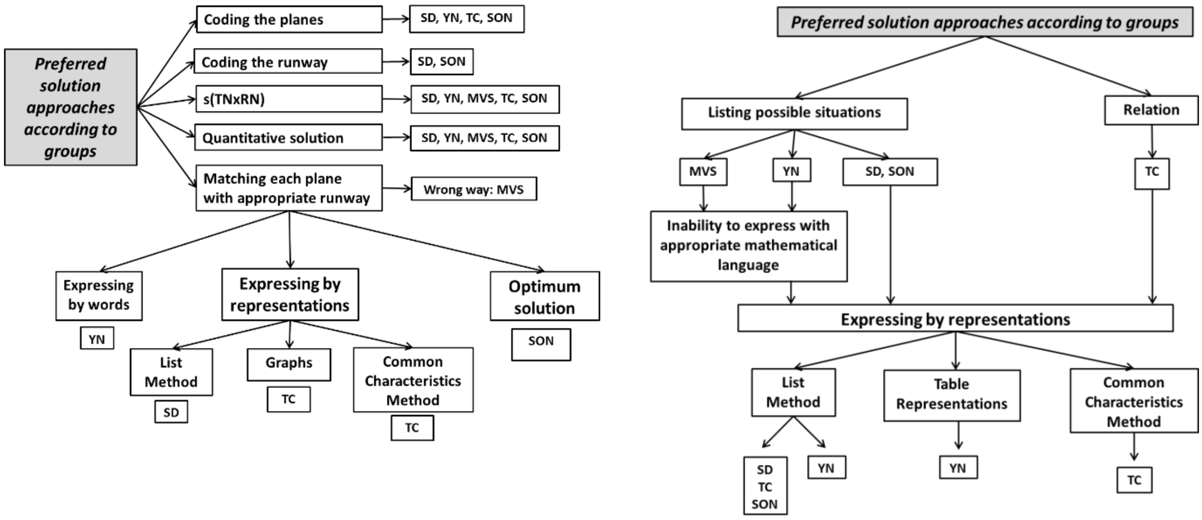
At the end of the first week, although all researchers agreed that relevant concepts were learned based on the worksheets and participant journals, they decided on an action plan for the next classes to provide feedback to students for developing their use of mathematical language.

3.2. Second Week: Cartesian product, Relation, and Characteristics of Relation

In the second week of the study, students' learning processes regarding the Cartesian product, relation, and the characteristics of relation (reflection) were analyzed through the Cabin Control List and Valentine's Day problems.

The process of solving the first problem touched upon the Cartesian product, which students were considered to have difficulty in explaining through mathematical language. The problem included details of planes' tail numbers (TN) and the smallest runway distance (RD) necessary for takeoff. Students were asked to calculate how many matches there could be without matching TN and runway numbers (RN). This task was assigned to students to assess their knowledge of the Cartesian product. Later, groups were able to match TN and runways on the basis of takeoff-landing RDs for all flights appropriately. This matching task was assigned to students to make them use the concept of relation in real-life problems.

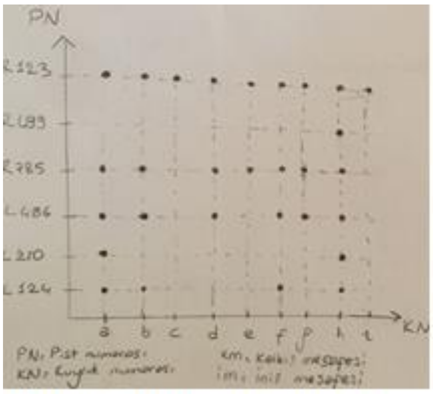
In the Valentine's Day problem, which was the second problem of the week, groups were expected to use relation and its characteristics (reflection) in a real-life problem; the case is that Valentine's Day is being organized in a shopping mall, and the context is a raffle for couples. When the groups' approaches to the solution of the second problem were analyzed, few groups (MVS & YN) were found to have listed all possible conditions without using formal mathematical language related to the characteristics of relation. Figure 3 presents an overview of the groups' performances in the second week.



a. Problem 1 **b.** Problem 2

Figure 3. General performances in Week 2

When the groups' approaches to solving the problems were analyzed (Figure 3a), it was observed that all groups preferred to code the planes; additionally, it was seen that SD and SON groups coded RN as well. All groups were able to find the numerical solution, write the Cartesian product correctly with the help of the feedback they received, and calculate the number of figures in a set without listing all the figures. It was observed that MVS was not able to solve the problem of landing the planes; YN was only able to explain the solution in words and could not explain it mathematically; and SD presented the solution using the list method. However, TC, approached the problem differently from other groups by using graphics (Figure 4) and modeling the solution through the common characteristics method.

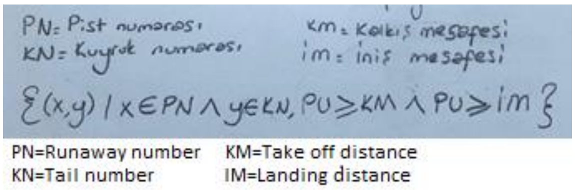


PN=Runway number KM=take-off distance
KN=Tail number IM=Landing distance

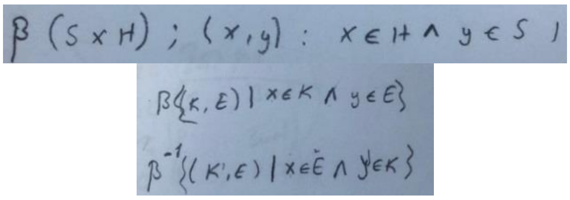
Figure 4. Performance of the TC Group

When the groups' problem-solving approaches for the second problem (Figure 3b) were analyzed, it was found that all groups except TC started the solution by listing all the possible conditions. In this process, TC noted a direct relation; MVS was found to not have used proper mathematical language in this process and did not complete the solution; and YN was found to experience problems in the mathematical representation of domains and codomains. While YN presented the solution using lists and tables, SD and SON used the list method to reach the solution of the problem. Additionally, TC was found to use the list and common characteristics method.

Although students experienced difficulties in using formal mathematical language in expressing the characteristics of relation and, more specifically, domain and codomains in Week 2, it can be understood from the solutions presented in Figure 5 that they were able to develop their mathematical language when compared to the previous week because of the feedback they received regarding mathematical language.



Group TC



Group YN

Figure 5. Samples of the groups' use of mathematical language about the concept of relation

As shown in the participant journals, the researchers' approach in the classroom with regard to the use of mathematical language had an impact on the development of the mathematical language that the students used:

> *"..The questions required more knowledge and thinking skills in terms of content…We have realized that our mathematical language improved due to the nature of the questions' content." (SD)*
>
> *"We have many problems in expressing with mathematical language. We have seen that the research has already been useful. This is because it is now easier to identify our weaknesses." (SON)*
>
> *.".Today… we have developed our thinking skills, we are doing a better job each week…The questions are similar to those we may encounter in real life, and this allows us to solve the problems with pleasure." (TC)*
>
> *These questions allowed us to better express our analytical thinking structure compared to last week…" (MVS)*

Although the researchers had an agreement that the concept of relation was learned at the end of the activities of Week 2, an action plan was made for the next week, which included showing new videos about the characteristics of relation and other videos on the concept of functions for revision.

3.3. Third Week: Characteristics of Relations and the Concept of Functions

The process of learning the characteristics of relation and the concept of function was analyzed in the third week of the research through the Blood Transfusion problem. Details about the blood types of individuals and Rh antigens in their blood were provided, and the students were asked to create a model for blood transfusion. The problem, within a scenario in the context of blood transfusion, aimed to identify the characteristics of relation in the first two questions and the conditions that change a relation into a function as well as the domains and codomains in the following six questions. Figure 6 presents an overview of the group performances in the third week.

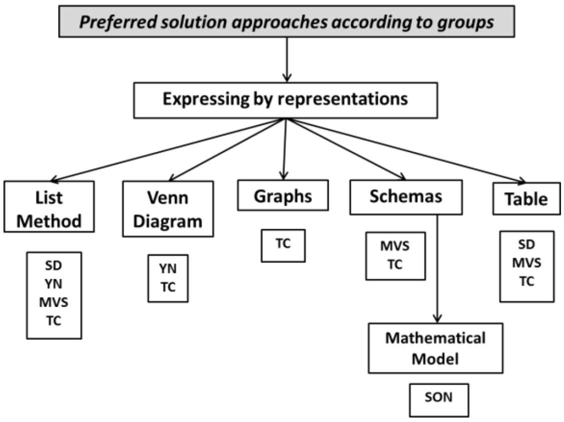


Figure 6. General performances in Week 3

When the solution approaches were analyzed, it was found that almost all groups preferred the list method. It can be seen in Figure 6 that groups presented the solution through multiple forms of representation. All groups except for SON analyzed the characteristics of relation and used proper mathematical language relatively accurately. However, SON focused on identifying the type of relation without referring to the characteristics of relation and wrote all characteristics using proper mathematical language with the guidance of the researchers. All groups except for YN were found to have identified the type of relation (ordered/equivalence) even though it was not requested for in the scenario.

In the next part of the scenario, SD and TC were found to have expressed the characteristics of a relation that meet and that do not meet the criteria of being a function using proper mathematical language. In this part, SD used mathematical representations present in the videos, but TC preferred the use of Venn diagrams. However, YN, MVS, and SON were found not to have linked the solution with the concept of function and to have only identified the condition that was asked for in the scenario. In identifying domains and codomains, SON and TC groups were found to have used proper mathematical language, but the other groups were not able to identify those domains.

In general, tables or diagrams were preferred for presenting solutions. Specifically, SD and YN, as presented in Figure 7, found the solution using tables, and SON put forward two different model proposals as presented in Figure 8. The fact that students tried to develop different models in the subject of blood transfusion, which was a topic they had learned in the past, and that some of them even tried to use models that were non-existent in the literature were noted as striking findings.

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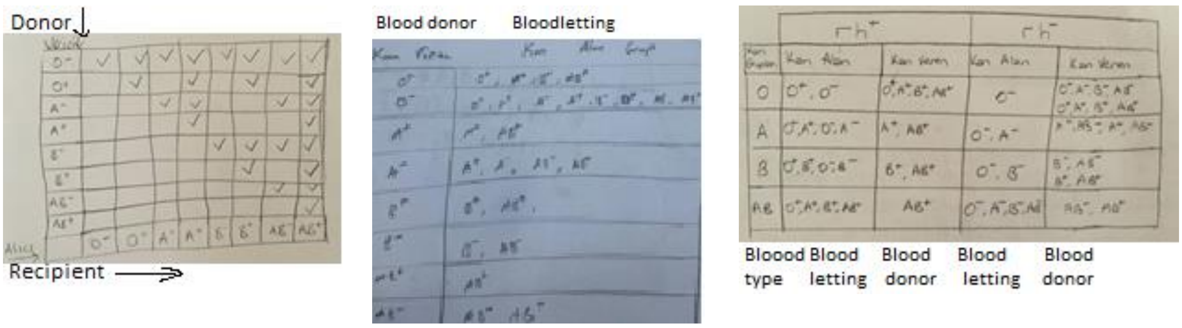
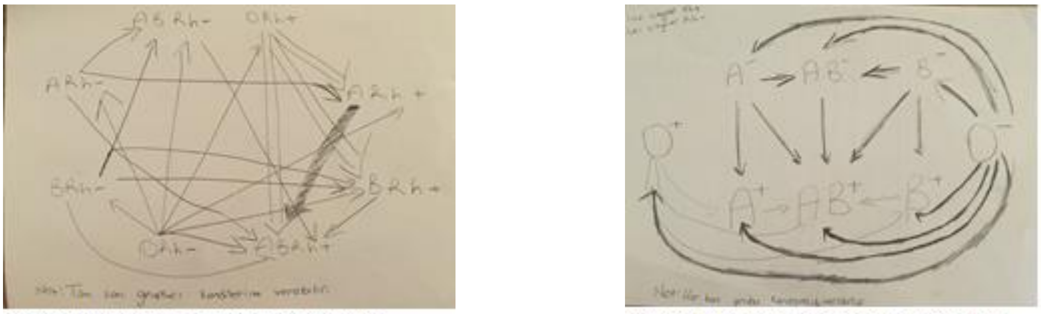


a. Group SD **b.** Group YN **c.** Group TC

Figure 7. Tables used by SD, YN, and TC Groups

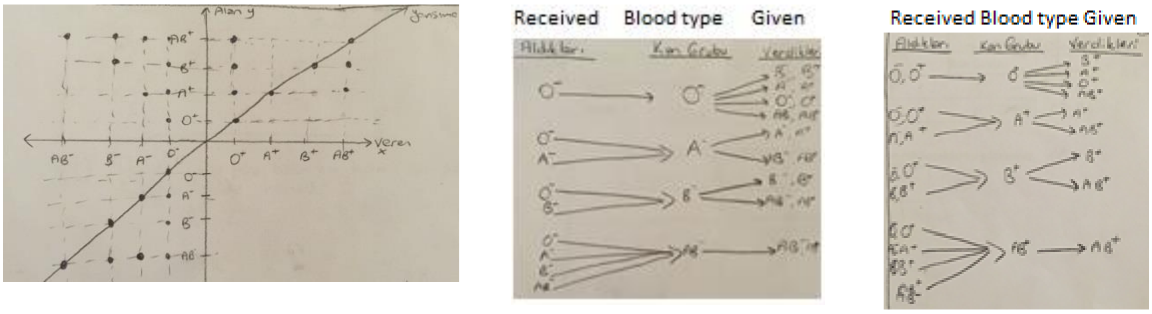


Note: All blood groups can give blood to themselves.

Note: All blood groups can give blood to themselves.

Figure 8. Models developed by the SON Group

While a number of groups preferred the use of more than one form of representation, TC was found to have used all forms of representation, which were used by other groups (Figure 9). Apart from preparing tables and diagrams/schemas, they also presented a solution that was placed on an analytic platform as can be seen in Figure 9.

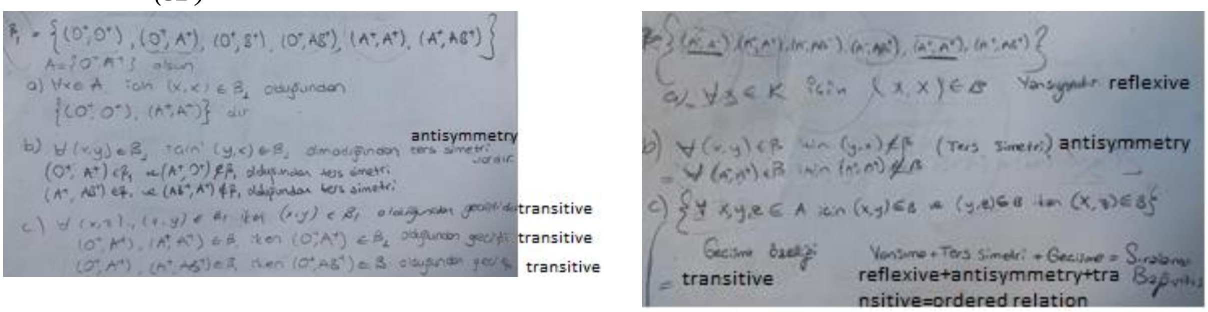


a **b**

Figure 9. Schemas used by the TC Group

It was observed that students showed progress in the third week in relation to presenting the characteristics of relation using proper mathematical language, which was a topic that students experienced problems with in previous weeks (Figure 10). Moreover, it can be understood from the journals of the groups that real-life problems had positive effects on students' understanding.

> *"… the fact that the question was related to real life helped us better understand the characteristics of relation. The week was tiring, but we learned while having fun again…" (SD)*



a. Group SD **b.** Group TC

Figure 10. Samples of mathematical language use for the characteristics of relation

At the end of the third week, it was decided that the characteristics of relation had been practiced enough, and new decisions were made for the next action plan, such as continuing practice work on the concept of functions in the videos that were to be prepared, decreasing the length of activities considering the volume of the content, and adding credits to the videos.

3.4. Fourth Week: Concepts of Linear Relation and Function

In the fourth week of the study, the process of learning the concept of function and linear relation was analyzed through the Yoga Course problem. The problem included the details of costs for members and non-members in a yoga course, and students were asked to find the most attractive alternative. The first two questions in the scenario focused on linear relation and the last two on whether the relation among variables formed a function. An overview of the problem-solving process is presented in Figure 11.

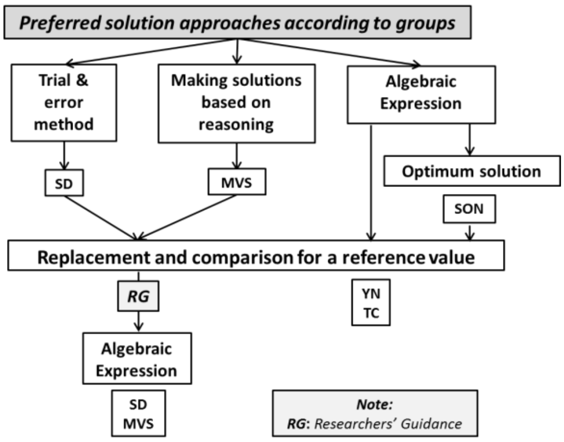


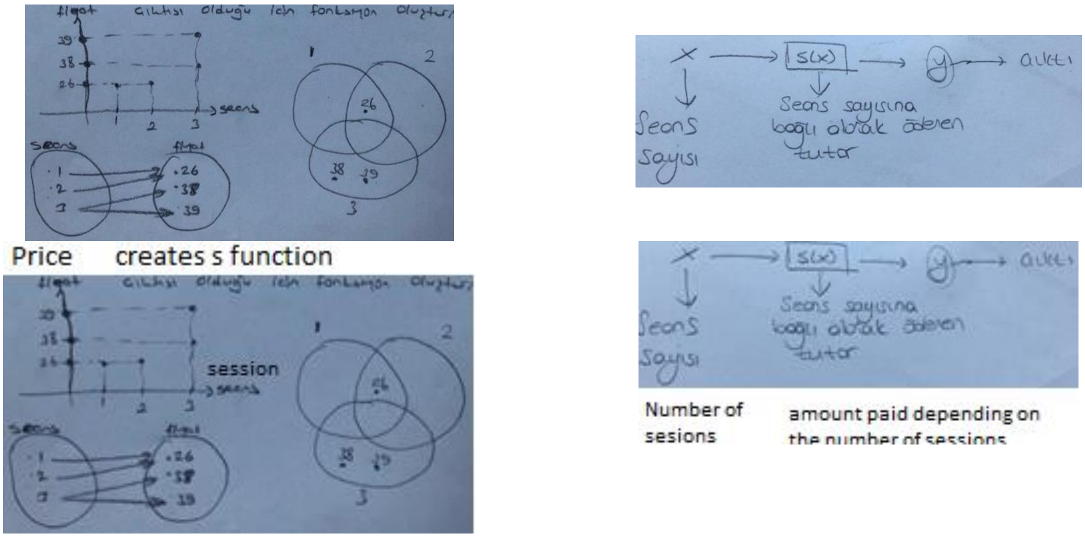
Figure 11. General performances in Week 4

In solving the problem, it was observed that the SD discovered the system that was more attractive by providing numerical examples, and the MVS did so through reasoning. YN, TC, and SON, were observed to, first, prefer writing down the algebraic expression. In the next stage, YN and TC were found to investigate the algebraic expression they wrote for its reference value; they compared it with the results. At this stage, the SON was seen to have reached the best solution by writing the algebraic expression. An important finding was that SD and MVS were able to express the linear relation algebraically only after receiving guidance from the researchers. In the next stage, in comparing membership and non-membership conditions, all groups were found to have correctly interpreted the conditions of whether the relationship among variables indicated a function or not.

The fact that YN, TC, and SON came up with the algebraic expression that they were expected to find in the last stage of solving the problem and that they were able to express it using formal language was considered a finding that indicated that students' mathematical language use had developed. Moreover, the fact that the SD started to diversify their answers by using representations from the videos and that the MVS, which showed low performance in previous weeks when compared to other groups, used multiple representation forms was also considered as a proof of students' development in terms of mathematical language (Figure 12).

When student statements in the journals of the groups during Week 4 were analyzed, the materials prepared in advance for flipped classrooms were found to be effective: *"The videos helped me 100 % in reaching the solution."* (YN)

Since the researchers were in agreement that the identified concepts were taught and students had enough knowledge of linear relation at the end of the fourth week, it was decided that the content of videos would be on the concepts of function and functional relation.



a. Group MVS b. Group SD

Figure 12. Samples of groups' mathematical language use in Week 4

3.5. Fifth Week: Linear relation, Function, and Inverse Function

In the fifth week of the study, the process of discovering linear relation in a real-life problem and expressing this relation in formal mathematical language was analysed through the Call Center problem. The problem included details of customers waiting to be served in a call center and their estimated waiting time, and students were asked to identify the relation between these variables and to calculate the waiting time for customers who were given a sequence number, and similarly, calculate the sequence numbers of customers who were informed of length of the duration they had to stay on the line. In the last part of the problem, students were expected to question whether there was an inverse function for the working principles of the call center. An overview of the problem-solving process is presented in Figure 13.

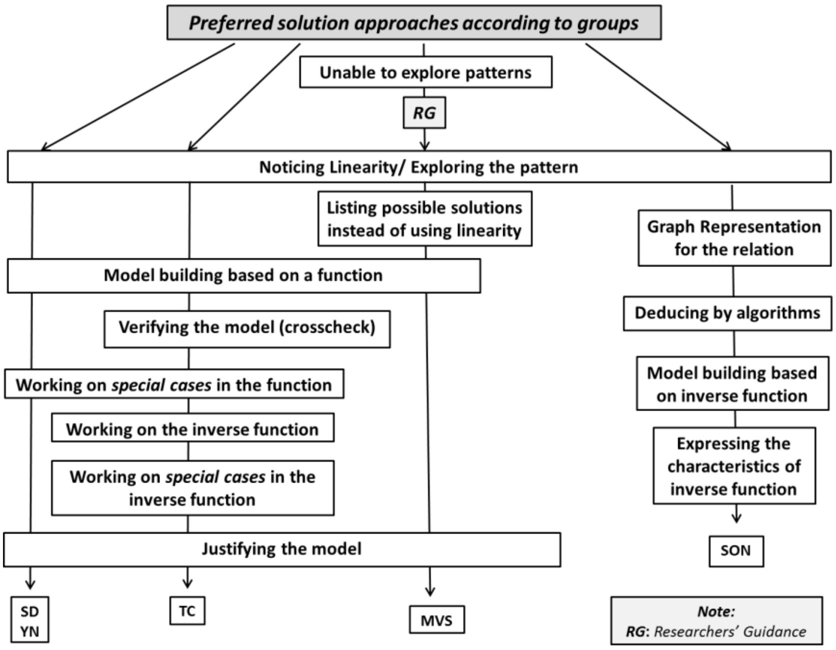
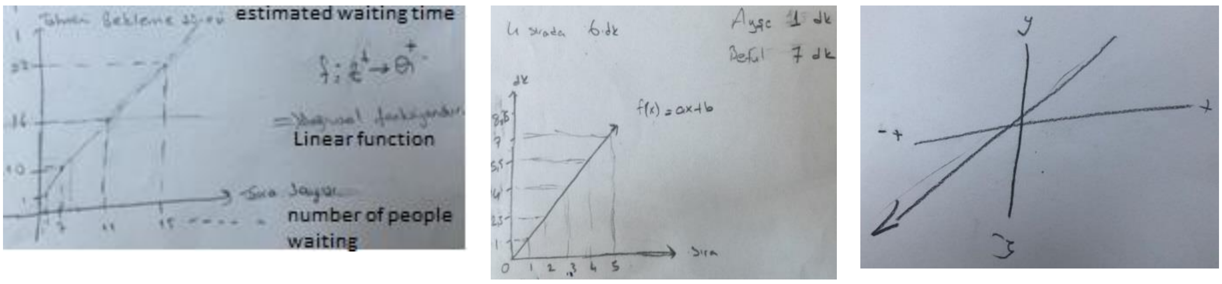


Figure 13. Group performances in Week 5

During the fifth week, it was observed that all groups except for one were able to easily identify the linear relation and had started solving the problem. MVS experienced difficulties at the beginning and was then able to realize the linear relation with guidance from the researchers.

MVS, SD, YN, and TC, after realizing the functional relation, were found to continue to the stage of expressing this relation with a model. At this stage, MVS worked persistently on listing potential conditions that exemplify the linear relation, experienced difficulties in developing the model later, and continued with the validation process. TC was found to have worked on special conditions after confirming the model. SD and YN focused on special conditions right after developing the model. In the next stage, although MVS, SD, and YN had defended their models, TC defended their model by working on the inverse function and the special conditions existing in its inverse form. As for SON, after realizing the linear relation, they tried to represent it using graphics, made inferences through algorithmic approaches and, unlike other groups, developed the inverse version of the function and explained its characteristics.

Except for TC, none of the groups mentioned domains or codomains during the process of defining function and inverse function; they paid attention to sets intuitively and did not feel the need to express them through mathematical language. In this process, only TC managed to express the domain correctly, but they made a mistake in writing the codomain. YN, TC, and SON tried to represent the function about the working principle of the call center through a graphic during the solution process. Figure 14 presents the mistakes that the groups made in representing the solution through graphs. Taking domain and range into consideration, it was found that the group TC's graph includes discrete points starting from index number 1; the Group SON's graphed a line starting from 0; and the Group YN graphed a line too, but it includes negative values as well, which was out of the context.



Group TC Group SON Group YN

Figure 14. The graphs drawn by the groups for the problems of Week 5

Considering the groups' performances and because it was the last week in which the concept of function was covered, immediate feedback was provided to the groups to resolve mistakes (e.g., showing domains and codomains in functions, representation of linear functions via graphics, etc.). At the end of the fifth week's practice session, from the groups' worksheets and their statements in the journals, it was clear that concepts related to linear functions were learned.

> *"We have completely understood linear function" (YN)*
> *"…the activity in the classroom allowed the use of the information in the videos…" (SD)*
> *"…it was enjoyable to find the questions' relation to the topic (e.g., developing a formula); the questions are like reinforcements in terms of mastering the topic…" (MVS)*
> *"Today, we had difficulty in graphics; our mathematical language is not as bad as it was when we first started. We have developed ourselves quite a bit. I find this study helpful." (SON)*

4. DISCUSSION and CONCLUSION

The present study evaluated students' problem-solving processes together with their mathematical language, concept development, and the difficulties they experienced in flipped undergraduate mathematics classes. Research indicated that the nature of flipped classroom provide active learning (Novak, Kensington-Miller & Evans, 2017), peer instruction (Novak, Kensington-Miller & Evans, 2017), cooperative learning (Foldnes, 2016), scaffolding (Witt, Trivedi & Aminalroayae, 2021) opportunities for both the learners and the instructors. In this study, by creating opportunities for students to work on real-life problems and model development activities, the learning process was differentiated and enriched with the help of flipped classroom model.

The flipped classroom model used in the study provided the researchers with many opportunities. First, classroom time has become more convenient for cooperative and practice-based activities. As a result, the real-life and modeling problems that were used allowed students to develop various perspectives. Similarly, most of the research (Bukova-Güzel; 2016; Geiger & Kaiser, 2014; Siller, et al., 2022) noted that students who worked on modeling activities learned important mathematical concepts and structures present in complex real-life situations. In our study, the participants stated that they were in such a learning environment for the first time and experienced difficulties in solving the problems since it was the first time that they had encountered real-life problems and model development activities.

According to the conventional perspective regarding the learning and problem-solving process, solving real-life problems, unlike model development and modeling perspectives, are accepted to be more difficult than problems encountered in exams and books (Lesh & Doerr, 2003). In the present study, although the participants stated they had difficulties at the beginning, they were found to consider this method effective later. Therefore, if flipped classrooms are to be applied in a class for the first time, considering the difficulties that both the teacher and the learners may experience, to start with, a subject in which students are less likely to experience problems may be selected. The analysis of the difficulties experienced by students in solving problems indicated that they had the most difficulty in linking the characteristics of function with real life. The students, until they took part in the study, stated that in traditional learning environments throughout their academic life; they were assessed through standardized tests and focused on passing those tests. Likewise, because standardized tests do not allow for satisfactory feedback on students' mathematical skills (Niss, 1999), the use of such activities in assessing students during the learning process is necessitated. Research (Bredow, Roehling, Knorp, & Sweet, 2021) also support that the flipped learning model is more effective than traditional learning setting. Especially, in the field of mathematics education Heuett (2017) and Sen (2021) argued that students' performances in the flipped classroom were better especially in terms of solving mathematical problems when compared to others in traditional learning environments. Strelan, Osborn, and Palmer (2020) state that the most important factor affecting this result is that the flipped classroom model provides opportunities for structured, active learning and problem solving.

An important advantage of practicing the flipped classroom model was the fact that it enhanced the student-teacher and student-student interaction in the classroom. Similar studies in the literature highlighted that flipped classrooms improved student-teacher and student-student interaction (Guerrero, et al., 2011). As Wright (2015) mentioned in her study, one of the reasons why this interaction increased was the fact that students cooperated and helped one another during group work and while learning concepts. Earlier research showed that flipped classrooms equipped students with skills such as cooperative problem-solving, out-of-class study habits, and learning from their peers in both in-class and out-of-class processes (Schroeder, McGivney-Burelle & Xue, 2015). Additionally, Kirvan, et al., (2015) highlighted that while this method saves time for diversifying teaching, it also provides an opportunity to help students who have difficulties and enable them to work both individually and cooperatively.

In this study, another reason why student-student interactions increased was the fact that the teachers were able to pay more attention to students. This situation is underlined in a study conducted by Anderson and Brennan (2015), which showed that the students found the teachers' approach positive and explained that they received more attention. Moreover, a similar situation was evaluated through the perspectives of peer-learning, participation in content, and participation in the learning process in Novak, Kensington-Miller, and Evans' (2016) study. In this context, looking through the perspective of students, it was emphasized that the flipped classrooms model can develop mathematical interaction.

In the present study, the flipped classroom model allowed researchers to focus on preventing student mistakes and misconceptions by decreasing the number of theoretical mathematics topics, with the help of real-life and modeling problems used in classroom activities. Similarly, Heuett (2017) mentioned the advantage of flipped classrooms in responding to students' questions on time and eliminating misconceptions. In our study, the flipped classroom practice provided the opportunity to support students' use of mathematical language as a result of the time it saved. In the present study, it was found that there was generally a tendency among the participant groups to present the solution in words or with any form of representation, and they experienced problems in using mathematical symbols. However, during the weeks of the practice, they were found to have made developments in using mathematical language more effectively. In this process, it was observed that the use of representation forms increased, and students often used multiple forms of representation even if they were not required to do so. Researchers highlighted that flipped classrooms enabled students to develop their mathematical communication skills (Schroeder, McGivney-Burelle & Xue, 2015) and generate more comprehensive and detailed answers to questions requiring the use of complicated mathematical terms, reasoning, and exemplifying (Murphy, et al., 2016).

The structure of flipped classrooms has a number of advantages such as providing students with an opportunity to watch the same video over and over or watching videos at home (Cronhjort, et al., 2017). The videos in the present study guided the students in solving the problems given in the classroom, and they made use of the videos during class without being guided to. Further, they tried to make use of videos that were not presented to them for course preparation. Similarly, in Lesseig and Krouss' (2017) study, the students highlighted the usefulness of watching videos to the learning process. Some students prepared notes while watching the videos to prepare for the classroom and made frequent use of those notes while solving problems as a group in the class. In spite of this, from time to time, students were found to ignore the characteristics of number systems, which provide the input for concepts such as domain, codomain, Cartesian product, relation, and function, which were present in the videos that they watched. Some students ignored this information because they considered it a detail, similar to the attitude in a traditional classroom environment. Therefore, special precautions should be taken to develop students' sensitivity for such concepts. In this sense, Kirvan, et al., (2015) underline the importance of planning classroom activities and videos to be used in flipped classroom practices in a way that would increase students' concentration on conceptual understanding. Although the participants in the study considered the videos to be sufficient, researchers may consider presenting (teaching) concepts in future studies because students can perceive as being more familiar and effective.

Although the study was conducted with volunteer students, the fact that occasionally there were students who could not participate in activities (those who could not join classes on the day of activities) decreased others' motivation. Similarly, that a number of individuals in the group who arrived without watching the videos were included in activities only after they watched the videos negatively affected students' motivation. In Heuett's (2017) study, it was explained that students who arrived in the class without watching the videos had difficulties in adapting to activities because they were not prepared, and this situation prevented them from understanding the concepts well. In the present study, there was only one student who was in a similar situation. Kensington-Miller, Novak, and Evans (2016) stated that the flipped classroom model enabled students to take responsibility for their own learning processes. It is important to develop strategies to get students to undertake this responsibility by planning studies in which the whole classroom takes part for a longer period of time.

In the present study, the students were asked to work in the same group for five weeks to support group dynamics in solving problems. According to Novak, Kensington-Miller, and Evans (2017), active learning and group work are the fundamentals of the flipped learning model. In future studies, groups can be recreated in each class based on the students who are present in the class on a given day.

The analysis of national and international research on flipped classroom are quite relevant to our research. Especially, the research on mathematics education is considered, it is stated in the systematic literature review (Şen, 2022) and meta-analysis study (Sopamena et al., 2023) that mathematics teaching with flipped classroom is more effective than the traditional method. In addition, review studies (meta-analysis, systematic review and thematic analysis) report that this model is also effective on learning outcomes when it is conducted in different disciplines regardless of the course type, both in the Turkish perspective and internationally (Aydın, Ökmen, Şahin & Kılıç, 2021;; Karagöl & Esen, 2019; Strelan, Osborn & Palmer, 2020; Şen, 2022).

Ethics Committee Decision

Ethical approval and written permission for this study were obtained from the Adnan Menderes University with the decision dated 10/03/2017 and numbered 15428.

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