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Research Article

MHD Nanofluidic Flow Past a Nonlinear Exponentially Stretched Plate with Enhanced Thermal Source/Sink and Thermo-migration

Uchenna A. UKA[1], Digbo I. IKU[2], Great E. ESEKHAIGBE[3]

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Nanofluidics
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Series Approximation Scheme
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Abstract: The study investigates the heat transfer characteristics of a nanofluidic flow past a non-linear exponentially stretched plate in the presence of enriched heat generation/absorption through the application of the Standard series approximation technique. The significance of the study includes but is not limited to drug targeting, food processing industries, manufacturing firms, solar power technology and nuclear mechanizations etc. The mathematical models governing the fluid flow was modelled through the Navier-Stokes equations. Thus, such partial differential expressions (PDE) were transformed into coupled ordinary differential models (CODM) through the application of adequate similarity transformation variables. Thereafter, the resulting equations were solved by the use of the aforementioned technique with appropriate boundary conditions. However, the Wolfram Mathematica package has been applied for the numerical solutions. Thus, the results showed that the presence of nanoparticles and thermal source/sink significantly affects the velocity, temperature and mass concentration. It was found that an increase in the Hartman number leads to a decline in the velocity of the flow whereas the velocity distribution surges as radiation and Grashof parameters appreciate in values. Similarly, a rise in the thermo-migration factor breeds an upsurge in temperature and nanoparticle concentration respectively. The results also showed that an improvement in the values of Prandtl and Schmidt numbers led to a reduction in the thermal and mass boundary layer thicknesses. Therefore, this study provides an insight into the heat transfer characteristics of nanofluidic flow and can be used in various engineering applications such as cooling of electronic devices and nuclear reactors.

1. Introduction

Magnetohydrodynamics is the study of the motion of electrically conducting fluids in the presence of magnetic fields. Recently, the studies of heat transfer in nanofluidics have attracted the interest of many scholars due to its significance in the area of technological advancement. However, this type of fluids consist of colloidal immersions of finely divided nanoparticles of metals and their oxides, and etc. in a base fluid such as water,

[1] Uchenna A. Uka 1 Babcock University, School of Science and Engineering, Basic Science Department, 121003, Ilishan-Remo, Ikenne, Nigeria.
[2] Digbo I. Iku 2 Babcock University, School of Science and Engineering, Basic Science Department, 121003, Ilishan-Remo, Ikenne, Nigeria.
[3] Great E. Esekhaigbe 3 University of Africa, Department of Mathematics and Computer Science, 561101, Bayelsa State, Nigeria.

Corresponding Author and e–mail address: Uchenna A. Uka ukau@babcock.edu.ng

engine oil, etc. and have appeared as a capable alternative to conventional coolants due to their unique properties that make them suitable for many industrial applications and in the radiator of an automobile system. Often, a surface-active agent like a 'surfactant' is being added in order to maintain the composition's stability and reduces surface tension, thereby improving its thermal conductivity. Meanwhile, their application in such system has the prospective of reducing the negative environmental impacts. This is because the traditional coolants such as ethylene glycol (EG) and propylene glycol usually comprise harmful chemicals, which can pose dangers to human health and the environment. Conversely, nanofluids tend to be environmentally friendly by decreasing the negative effects linked with coolant discarding and leakage. Thus, scientifically nanofluids remain the best for refining the performance and sustainability of automobile cooling systems. Hence, the thermal conductivity of the nanofluid is enhanced

Sharma et al., [1] investigated MHD slip flow and temperature transference along an exponentially extending leaky piece fixed in a spongy material. Mahdi et al. [2] reviewed nanofluid flow and heat transfer passing through porous media. They laid emphasis on thermo-physical properties of the nanofluid and the form of the convective heat transfer. Rashidi et al. [3] analyzed the buoyancy effect on MHD flow of Nanofluid over a stretching sheet in the presence of thermal radiation. Mustafa et al. [4] investigated stagnation spot flow of nanofluids along an extending sheet while an analysis involving exploration of thermal transmission developments with viscous fluid flow over enlarging sheet in the presence of power law velocity distribution and nonlinear expanding proportion is conducted [5]. Miklavcic and Wang [6] reported the viscous flow over a shrinking sheet with suction effect at the boundary. They opined that the vorticity of the lessening sheet is unrestricted within a boundary layer. However, Uka et al. [7] explored chemical reaction and radiation impact on MHD Nanofluid flow over an exponentially widening sheet. From the results of their work inferred that a rise in the stretching and radiation parameters lead to an enhancement of the velocity. The analyses of heat and mass transfer with applications have been investigated by many researchers [8, 9]. Furthermore, Mansour et al. [10] affirmed the applications of heat and mass transport on magneto-hydrodynamic stream. They found out that the micro polar fluids exhibited a reduction in strain with heat transfer rate in comparison with Newtonian fluids. The first investigation of improving thermal conductivity of fluids containing nanoadditives was done by Choi and Eastman [11]. Hence, due to the overall scientific significance of nanofluid in terms of heat transmission on MHD flow, various theories have been propounded. Thus, Boungiorno [12] analyzed different theories surrounding the transferal of heat and mass in nanofluids. He considered some mechanisms that can produce a relative velocity between the nanoparticles and base fluid by testing the validity of the assumptions in his study. Similarly, the analysis of a convective nanofluid over a vertical sheet was conducted by Kuznetsov and Nield [13]. From their result, it was noted that Schmidt number wielded an obvious effect on the heat and fluid flow machinery around the exterior sphere involving velocity, temperature, and concentration distributions. The study of boundary layer flow past a continuous rigid surface with stable speed due to the fluid's movement in the surrounding area was facilitated by Sakiadios [14]. The effect of free and forced convection with Jeffery fluid in the presence of a non-isothermal segment was studied by Gaffar et al. [15]. The investigation of heat transmission in a fluid flow surrounded by a penetrable medium was carried out by Tamayol et al. [16]. According to Khani et al. [17] the examination of fluid flow in a soaking non-Darcy porous media under thermal transportation is determined. Consequently, Hayat et al. [18] presented unsteady 3-D movement of combined pressure fluid over a stretching surface with chemical reaction impact. They noted that the concentration field is a decreasing function of Schmidt number and shows opposite results for destructive $(\gamma > 0)$ and generative $(\gamma < 0)$ chemical reactions. Similarly, Awucha and Okechukwu [19] maintained that the rate at which energy flows is of paramount concern and importance due to its numerous industrial applications in the areas of cooling of nuclear reactors, power generating gadgets as well as its mechanisms. The examination of nonlinear unsteady MHD viscous, incompressible fluid distribution over an upright permeable medium due to thermal radiation and chemical reaction [20] was undertaken. The dual term perturbation approach was deployed in the solution of the problem. It was found that velocity shrinks as chemical reaction number rises. The hall current impact on the MHD stream of Newtonian fluid over a spongy channel was considered [21]. It was observed that the resultant velocity enhances as the Reynolds number improves. The influence of thermal source on mixed convection stream in nanofluids past a horizontal spherical cylinder [22] was discussed. The solutions were achieved by applying the Keller-box technique. The result showed that velocity distribution enhances and temperature distribution declines with the enhancement of the mixed convection factor.

It is important to mention that the importance of the current study applies to nuclear power machines, earth (soil) sciences, cooling of electronics, industrial metal processing, coating of cables and fibers, aerodynamic extrusion of plastic sheets, repeated casting, rolling, annealing as well as the tinning of copper cables. This study gives a direction into the behavior of MHD nanofluids flow and thermal transmission with improved thermal source/sink and thermo-migration effects. Similarly, its findings have prospective applications in designing of microfluidic devices and in the optimization of industrial processes which involve the stream of electrically conducting fluids. It is also viable in contributing to the growing body of knowledge in the field of fluid mechanics and MHD nanofluidic drifts and provides a better understanding of the complex physics involved in these systems which can serve as a basis for further research in this area.

2. Formulation of the Problem, Materials and Method of Solution

A magneto-hydrodynamic nanofluid flow past an expanding exponentially plate in two dimensions $x \wedge y$ is considered. While $y - axis$, is normal to the plate $x - axis$ is horizontal to it as depicted in Figure 1.

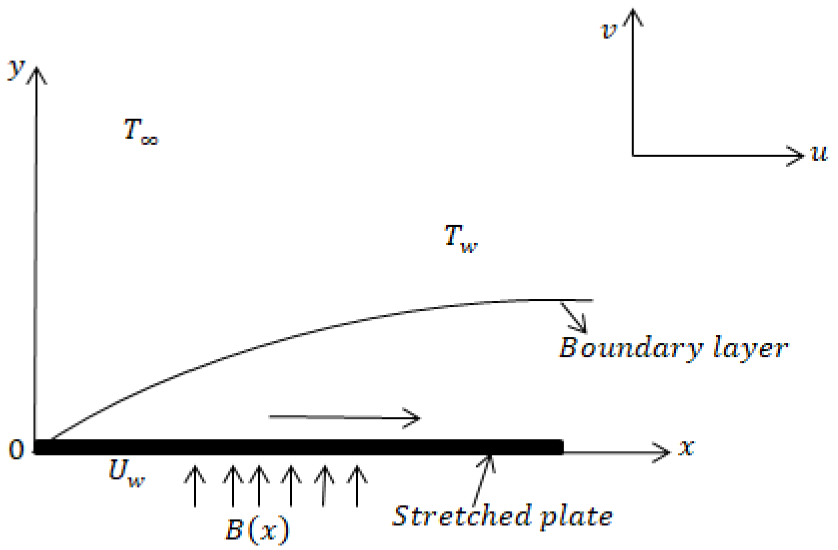

Fig. 1 Physical flow representation

The fluid flow is along the horizontal direction in such a way that the wall is expanded as the origin remains unchanged. We assume that the surface of the plate is stretched towards $x - axis$ with velocity, $U = U_w(x) = ce^{\frac{x}{L}}$. The application of a magnetic field of strength B_0^2 is also in path of $x - axis$ in such a way that the induced magnetic Reynolds factor $\Re \ll 1$, is of negligible value. Hence, the applied magnetic field and non-uniform heat source are considered. Therefore, the boundary layer equations are:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_\infty \frac{du_\infty}{dx} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\frac{\mu \partial u}{\partial y}\right) - \frac{\sigma B^2}{\rho} u + \frac{g\beta_T(T - T_\infty)}{v} \tag{2}$$

Energy conservation equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{(\rho c)_f}\frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_p}{(\rho c)_f}\left[D_B(C - C_\infty)(T - T_\infty) + \frac{D_T}{T_\infty}(T - T_\infty)\right] - \frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y} - \frac{Q}{\rho C_p}(T - T_\infty) \tag{3}$$

Mass conservation equation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\left(\frac{T - T_\infty}{T_w - T_\infty}\right) - R(C - C_\infty) \tag{4}$$

With the boundary conditions as

$$u = U_w(x), \quad v = -k_0, \quad T = T_w, \quad C = C_\infty \qquad \text{at } y = 0$$
$$u = v \to 0, \quad T \to T_\infty, \quad C \to C_\infty \qquad \text{as } y \to \infty \tag{5}$$

with,
ρ_f =base fluid density, $\sigma =$ electrical conductivity parameter, $\beta =$ volumetric heat enlargement coefficient of the base fluid, $g =$ acceleration under gravity, $k =$ heat conduction, $\mu =$ fluid viscosity, $D_T =$ thermophoresis diffusion coefficient, $q_r =$ radiative heat flux, $D_m =$ Brownian motion coefficient, Temperature, T at the boundary (y = 0) is T_w, and far from the plate is T_∞; local nanoparticle Concentration is C, $\frac{\rho c_f}{\rho c_p} = \tau$ is ratio of heat capacity of the nanofluid to nanoparticle; Q_s is heat number, and the additive inverse sign in the boundary condition above indicates that the occurrence of suction takes the same path as the plate.

We shall introduce the following variables in order to convert equations (2) – (4) into ODEs.

$$\text{Let } \eta = y\sqrt{\frac{a}{v}}e^{\frac{x}{2L}}, \ \varphi = g(\eta)\sqrt{vae^{\frac{x}{2L}}}, \ \theta = \frac{T-T_\infty}{T_w-T_\infty}, \quad \emptyset = \frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

$$v = \frac{-\partial\varphi}{\partial x}, u = \frac{\partial\varphi}{\partial y} \tag{7}$$

Substituting equation (7) into equation (1), we obtain

$$\frac{\partial^2\varphi}{\partial x\partial y} - \frac{\partial^2\varphi}{\partial x\partial y} = 0$$

This shows that equation (1) is satisfied.

In using equations (6) and (7) to transform equations (2) – (4), we realize the following equations:

$$g'''(\eta) + g(\eta)g''(\eta) - Hg'(\eta) + J_t\theta(\eta) = 0$$
$$(1 + \tfrac{4S}{3})\theta''(\eta) + Prg(\eta)\theta'(\eta) + Nb\theta(\eta)\emptyset(\eta) + Nt\theta(\eta) + Q_s\theta(\eta) = 0 \tag{8}$$

$$\emptyset''(\eta) + \frac{N_t}{N_b}\theta(\eta) + g(\eta)\emptyset'(\eta)Sc - R_cSc\emptyset(\eta) = 0 \tag{9}$$

The converted form of equation (5) follows as:

$$g(0) = g_0, g'(0) = 1, g'(\infty) = 0, \theta(0) = 1, \theta(\infty) = 0, \emptyset(0) = 1, \emptyset(\infty) = 0 \tag{10}$$

With, Hartman number $H = \frac{2\sigma B^2 L}{\rho U_w}$, local Grashof number $J_t = \frac{g\beta_T(T_w - T_\infty)l^3}{v^2}$,
Prandtl number $Pr = \frac{\mu c_p}{k}$, thermo-migration parameter $N_t = \frac{\tau D_T(T_w - T_\infty)}{T_\infty v}$, radiation term $S = \frac{4T_\infty^3\sigma}{3k \quad k}$, Brownian diffusion parameter $N_b = \frac{\tau D_B(C_w - C_\infty)}{v}$, Schmidt number $Sc = \frac{v}{D_m}$, modified rate of chemical reaction $R_c = \frac{2Lkr}{U_w}$, heat number $Q_s = \frac{2vl\rho C_f Q_s}{kU_w}$, parameter for suction $g_0 = \frac{z_w}{\sqrt{\square}}$ and the primes relates to differentiation, $fo > 0$ implies suction at the surface of the plate while $fo < 0$ is indicative of injection or blowing. The series approximation method [23] has been proved to be reliable and efficient. The method is simple in terms of its application just as its convergent rate is faster. Meanwhile, this method is a mathematical technique used to approximate the solution to a problem that involves a small parameter. It is mainly beneficial when faced with coupled and complex equations that cannot be solved directly. The methodology involves expanding the solution as a series in powers of the small parameter and iteratively solving the resulting equations to obtain successive

approximations. The presence of a small parameter given by ϖ, quantifies the magnitude of the method. Then, the solution to the problem is assumed and can be expressed as a power series in the small parameter ϖ. i.e., Solution = $\text{Solution}_0 + \varpi\,\text{Solution}_1 + \varpi^2\,\text{Solution}_2 + \ldots$

Where Solution_0 is the zeroth-order approximation, Solution_1 entails the first-order approximation, etc. Thereafter, substitutions of the expansions into the original equation(s) are carried out. Thus, expansion of the equations and grouping of terms with similar powers of the term ϖ follow. Then, the coefficients of the same power of ϖ are equated on both sides of the expanded equations and each equation is solved separately with its transformed physical boundary constraints to obtain the solution at each order. Finally, the solutions obtained are simulated with the Wolfram Mathematica package for the realization of the numerical solutions.

In order to apply the aforementioned procedure of solution, we need to make the following definitions:

Let $\eta = \vartheta g_0, g(\eta) = g_0 G(\eta), \theta(\eta) = \Theta(\eta), \emptyset(\eta) = \emptyset(\eta), \varpi = \frac{1}{(g_0)^2}$ (11)

Such that differentiating equation (11) accordingly yields

$$g' = (g_0)^2 G',\ g'' = (g_0)^3 G'',\ g''' = (g_0)^4 G''',\ g''' = (g_0)^4 G''',\ \theta' = g_0 \Theta',$$
$$\theta'' = (g_0)^2 \Theta'', \emptyset'(\eta) = g_0 \emptyset'(\eta), , \emptyset''(\eta) = (g_0)^2 \emptyset''(\eta) \tag{12}$$

By substituting equations (11) and (12) into equations (7) – (9), we have

$$G'''(\eta)(g_0)^4 + G(\eta)G''(\eta)(g_0)^4 - HG'(\eta)(g_0)^2 + J_t\Theta(\eta) = 0 \tag{13}$$

$$(1+\tfrac{4S}{3})\Theta''(\eta)(g_0)^2 + PrG(\eta)\Theta'(\eta)(g_0)^2 + Nb\theta(\eta)\emptyset(\eta) + Nt\Theta(\eta) + Q_s\Theta(\eta) = 0 \tag{14}$$

$$\emptyset''(\eta)(g_0)^2 + ScG\emptyset'(\eta)(g_0)^2 - ScG'\emptyset(\eta)(g_0)^2 + \frac{Nt}{Nb}\Theta(\eta) - R_c Sc\emptyset(\eta) = 0 \tag{15}$$

Then, multiplying equations (13) by $\frac{1}{(g_0)^4}$, (14) and (15) by $\frac{1}{(g_0)^2}$, we obtain equations (16) – (18).

$$G'''(\eta) + G(\eta)G''(\eta) - \varpi HG'(\eta) + J_t\emptyset(\eta)\varpi^2 = 0 \tag{16}$$

$$(1+\tfrac{4S}{3})\Theta''(\eta) + PrG(\eta)\Theta'(\eta) + Nb\Theta(\eta)\emptyset(\eta)\varpi + Nt\Theta(\eta)\varpi + Q_s\Theta(\eta)\varpi = 0 \tag{17}$$

$$\emptyset''(\eta) + ScG\emptyset'(\eta) - ScG'(\eta)\emptyset(\eta) + \frac{Nt}{Nb}\Theta(\eta)\varpi - R_c Sc\emptyset(\eta)\varpi = 0 \tag{18}$$

However, due to large suction i.e. $\varpi \ll 1$, the series approximation solution is defined as:

$$\begin{aligned} G &= 1 + \varpi G_1 + \varpi^2 G_2 + \cdots \\ \Theta &= \Theta_0 + \varpi\Theta_1 + \cdots \\ \emptyset &= \emptyset_0 + \varpi\emptyset_1 + \cdots \end{aligned} \tag{19}$$

Taking the first, second and third derivatives of equation (19) with respect to η produces

$$\begin{aligned} G'(\eta) &= \varpi G_1' + \varpi^2 G_2' + \cdots \\ G''(\eta) &= \varpi G_1'' + \varpi^2 G_2'' + \cdots \\ G'''(\eta) &= \varpi G_1''' + \varpi^2 G_2''' + \cdots \\ \Theta' &= \Theta_0' + \varpi\Theta_1' + \cdots \\ \Theta'' &= \Theta_0'' + \varpi\Theta_1'' + \cdots \\ \emptyset' &= \emptyset_0' + \varpi\emptyset_1' + \cdots \\ \emptyset'' &= \emptyset_0'' + \varpi\emptyset_1'' + \cdots \end{aligned} \tag{20}$$

By putting equations (19) and (20) into equations (16) – (18), we simplify and equate corresponding powers of ϖ to have the following:

$$(1+\tfrac{4}{3}S)\Theta_0''(\eta)+Pr\Theta_0'(\eta)=0; \quad \Theta_0(0)=1,\ \Theta_0(\infty)=0 \tag{21}$$

$$\emptyset_0''(\eta)+Sc\emptyset_0'(\eta)=0; \quad \emptyset_0(0)=1, \quad \emptyset_0(\infty)=0 \tag{22}$$

$$G_1'''(\eta)+G_1''(\eta)=0; \quad G_1(0)=0,\ G_1'(0)=1, \quad G_1'(\infty)=0 \tag{23}$$

$$(1+\tfrac{4}{3}S)\Theta_1''(\eta)+Pr\Theta_1'(\eta)+PrG_1(\eta)\Theta_0'(\eta)+Nb\Theta_0(\eta)\emptyset_0(\eta)+Nt\Theta_0(\eta)+Q_s\Theta_0(\eta)=0$$
$$\Theta_1(0)=0, \Theta_1(\infty)=0 \tag{24}$$

$$\emptyset_1''(\eta)+Sc\emptyset_1'(\eta)+ScG_1(\eta)\emptyset_0'(\eta)-ScG_1'(\eta)\emptyset_0(\eta)+\frac{Nt}{Nb}\Theta_0(\eta)-R_cSc\emptyset_0(\eta)=0$$
$$\emptyset_1(0)=0, \quad \emptyset_1(\infty)=0 \tag{25}$$

$$G_2'''(\eta)+G_2''(\eta)+G_1(\eta)G_1''(\eta)-HG_1'(\eta)+J_t\Theta_0(\eta)=0 \ G_2(0)=0, \quad G_2'(0)=0, \quad G_2'(\infty)=0 \tag{26}$$

However, equations (21) – (26) have been solved numerically and the results have been obtained.

After solving equations (21) – (26) analytically, the following solutions for velocity $f'(\eta)$, heat $\theta(\eta)$ and nanoparticle concentration $\emptyset(\eta)$ are obtained:

$$f'(\eta)=e^{-\eta}+\varpi\left(\frac{-1}{2}e^{-2\eta}-H\eta e^{-\eta}-\frac{Jt}{M-1}e^{-M\eta}+\frac{1}{2}e^{-\eta}+\frac{Jt}{M-1}e^{-\eta}\right) \tag{27}$$

$$\theta(\eta)=e^{-M\eta}+\varpi\Big(-M\eta e^{-M\eta}-\frac{M^2}{M+1}e^{-(1+M)\eta}-\frac{3Nb}{Sc(+4S)(M+Sc)}e^{-(Sc+M)\eta}+\frac{3Nt}{M(3+4S)}\eta e^{-M\eta}+$$
$$\frac{3Q_s}{M(3+4S)}\eta e^{-M\eta}+\frac{M^2}{M+1}e^{-M\eta}+\frac{3Nb}{Sc(+4S)(M+Sc)}e^{-M\eta}\Big) \tag{28}$$

$$\emptyset(\eta)=e^{-Sc\eta}+\varpi\Big(-Sc\eta e^{-Sc\eta}-\frac{(Sc)^2}{1+Sc}e^{-(1+Sc)\eta}+\frac{Sc}{1+Sc}e^{-(1+Sc)\eta}-\frac{Nt}{MNb(M-Sc)}e^{-M\eta}-R_c\eta e^{-Sc\eta}+$$
$$\frac{(Sc)^2}{1+Sc}e^{-Sc\eta}-\frac{Sc}{1+Sc}e^{-Sc\eta}+\frac{Nt}{MNb(M-Sc)}e^{-Sc\eta}\Big) \tag{29}$$

where

$$\varpi=0.1, M=\frac{3Pr}{3+4S}. \tag{30}$$

3. Results and Discussion

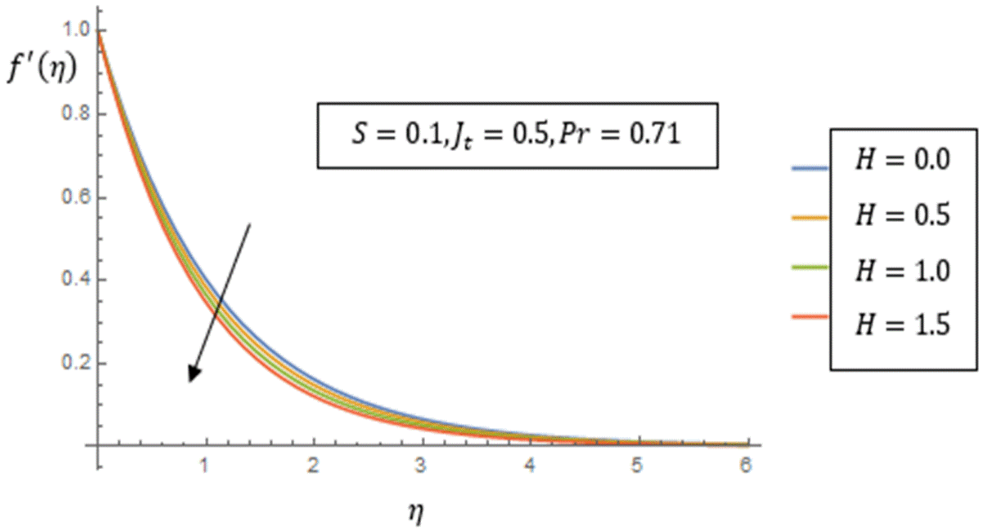

Fig 2. Influence of Hartmann factor on velocity

The effect of the Hartmann parameter on velocity is shown in Figure 2. Meanwhile, increase in its values leads to a decrease in the velocity. This is as a result of the fact that an opposing force drags the rate of the fluid flow backward thereby retarding the velocity. Thus, the Lorentz force is responsible for this phenomenon. Figure 3 illustrates the impact of the modified thermal Grashof number J_t on the fluid velocity near the stretching plate. This number J_t involves the influence of buoyancy force to viscous force and its enhancement entails an increasing trend in velocity distribution which breeds cooling at the surface of the plate.

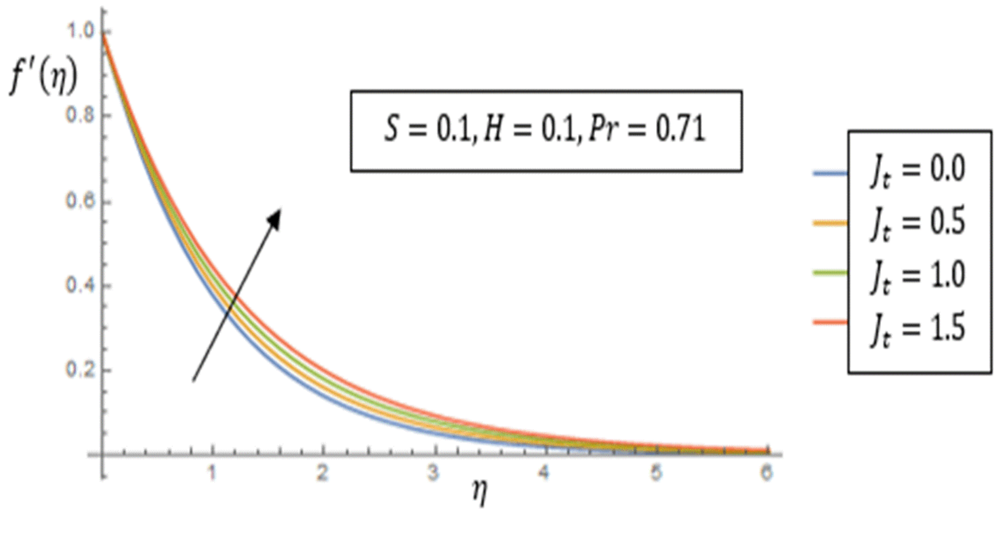

Fig. 3. Influence of Grashof number on velocity

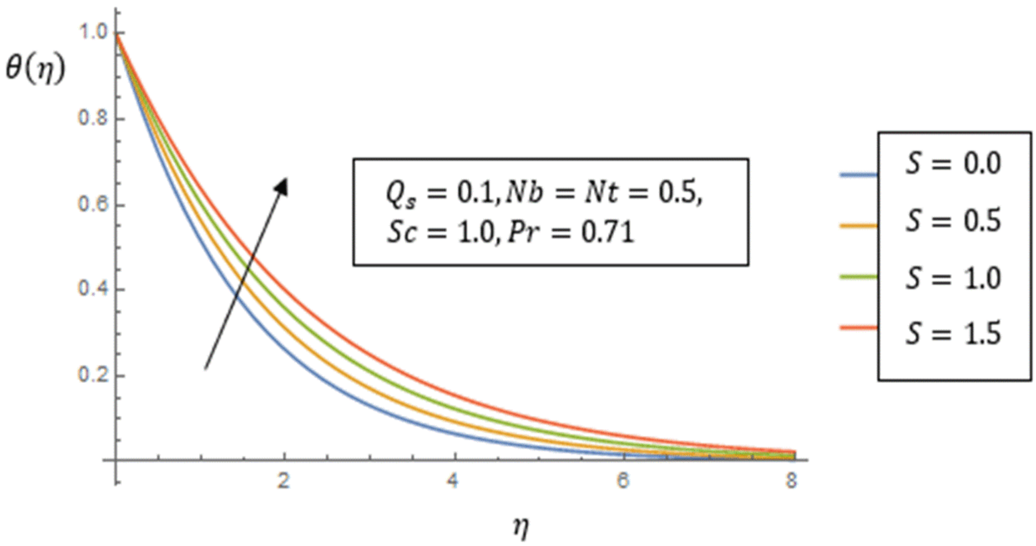

Fig. 4. Influence of radiation parameter on temperature

The evolution of radiation parameter S on temperature and concentration is displayed in Figures 4 and 5. Appreciating values of S causes an improvement of thermal boundary layers. This is due to the fact that the surface of the plate which is already heated releases more energy in form of heat. Thus, this leads to the augmentation of the temperature. Similarly, radiation transfers energy to the molecules immersed in a solution, leading to the generation of free radicals. These free radicals react with the dissolved metal ions to lessen them to their metallic status, resulting in the formation of nanoparticles and leading to an increase in their concentration.

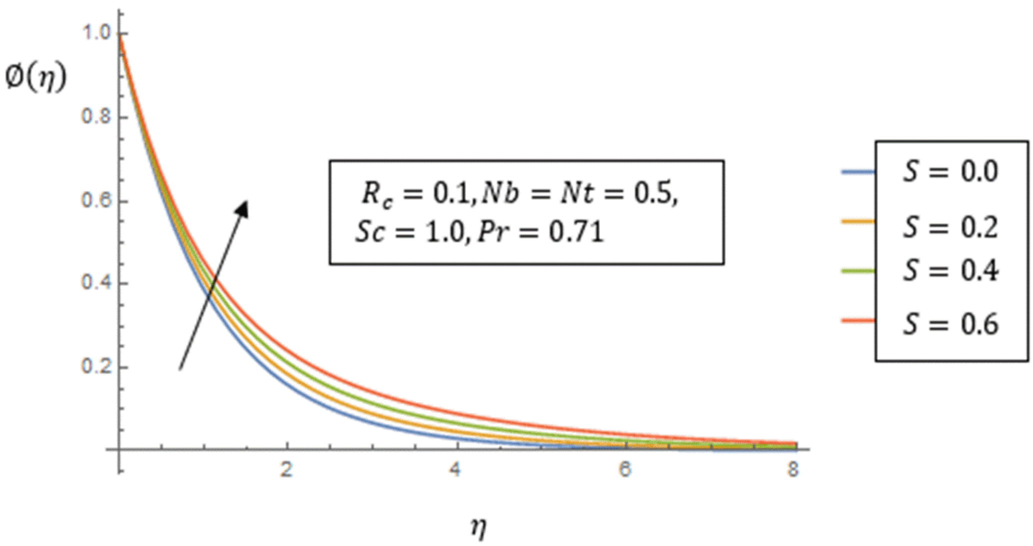

Fig. 5. Influence of radiation parameter on concentration

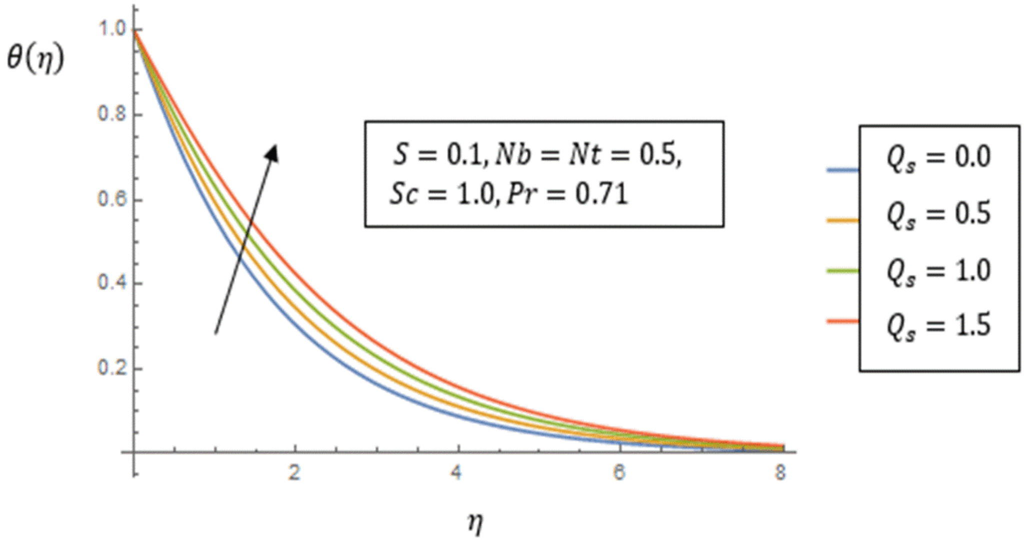

Fig 6. Influence of heat parameter on temperature

Figure 6 is indicative of the impact of heat source $(Q_s > 0)$ parameter on temperature. Heat is a type of energy which passes from a hotter body to a cooler one. Physically, the influx of heat $(Q_s > 0)$ to a system makes the kinetic energy of its molecules to increase, causing them to move quicker and hit with each other more frequently. As a result of this, the average kinetic energy of the particles increases and thus leads to a rise in temperature. The effect of Prandtl number Pr is depicted in **Figure 7**. As a dimensionless number which is expressed as the ratio of momentum diffusivity to thermal diffusivity of a fluid, it aids in the rate of cooling in conducting fluids. However, a surge in the Prandtl number implies that the fluid has higher momentum diffusivity relative to its thermal diffusivity. Thus, the fluid is more effective at transferring momentum, i.e. velocity, than it is at transporting heat, thereby resulting in a lower thermal transmission coefficient and smaller amount of heat removed from the surface. Therefore, heightening the Prandtl number leads to a fall in the thermal boundary layer which causes a decline in temperature of the fluid.

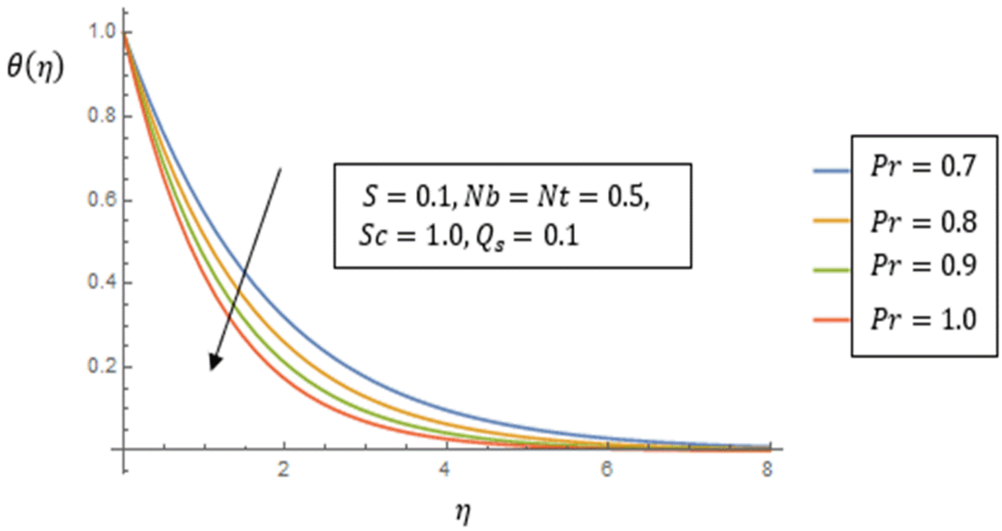

Fig 7. Influence of Prandtl factor on temperature

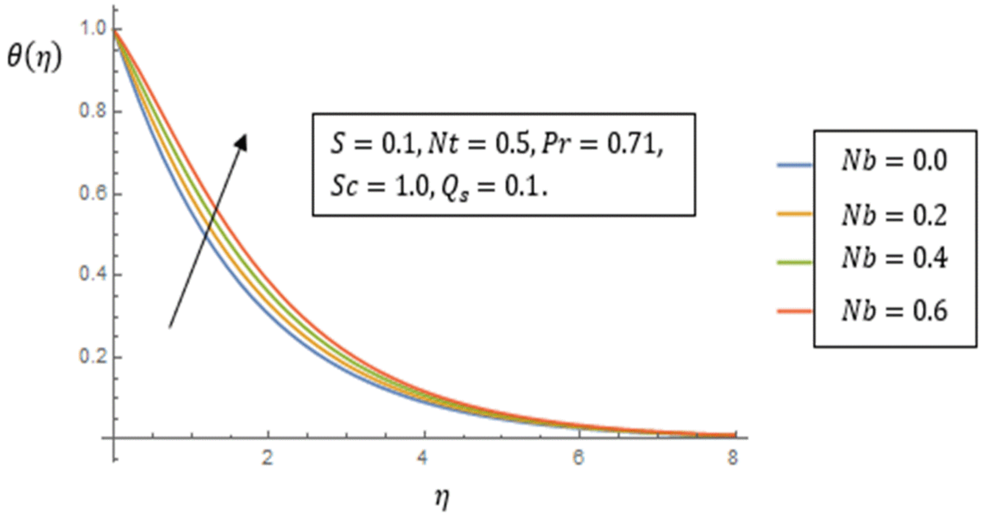

Fig 8. Influence of Brownian motion parameter on temperature

The impartation of Brownian motion, Nb, on temperature and nanoparticle concentration is shown in **Figures 8** and **9,** respectively. As the temperature of a fluid increases, the kinetic energy of its particles also rises, making them to travel faster and collide more often. Thus, this phenomenon between the particles brings about a growth in Brownian motion with an indirect impact of temperature increment. Conversely, an upsurge in this motion begets a decline in nanoparticle concentration. This is because Brownian motion causes nanoparticles to migrate haphazardly, striking each other in the fluid. As its value rises, the probability of nanoparticle hitting each other and accumulating also increases, thus breeding a decrease in the number of individual nanoparticles and an increase in the number of larger masses.

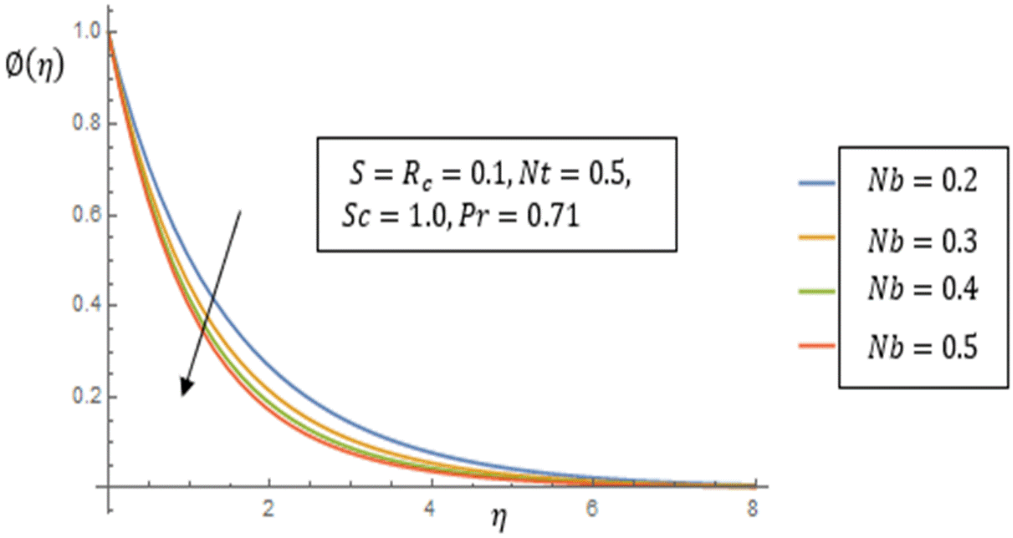

Fig 9. Influence of Brownian motion parameter on concentration

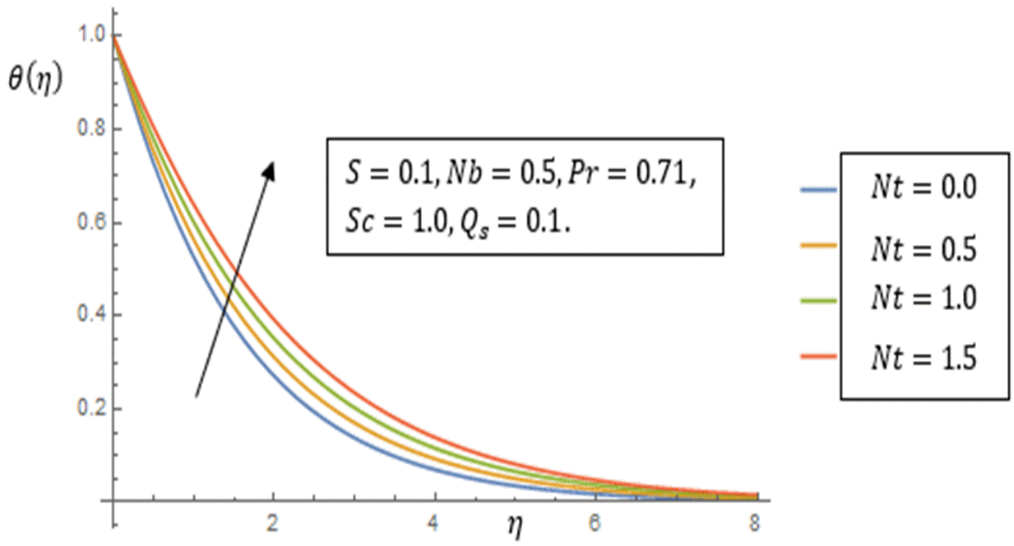

Figure. 10. Influence of thermomigration number on temperature

The impact of the thermo-migration parameter Nt on temperature and concentration is shown in **Figures 10** and **11.** Thermo-migration is the movement of particles in a fluid owing to temperature differences. When there is a change of temperature in a fluid with nanoparticles, such particles will possess a force that moves them toward areas of higher temperature. Hence, an augmentation in the value of this parameter produces the thickening of the thermal and mass boundary layers, which leads to a rise in the temperature and concentration.

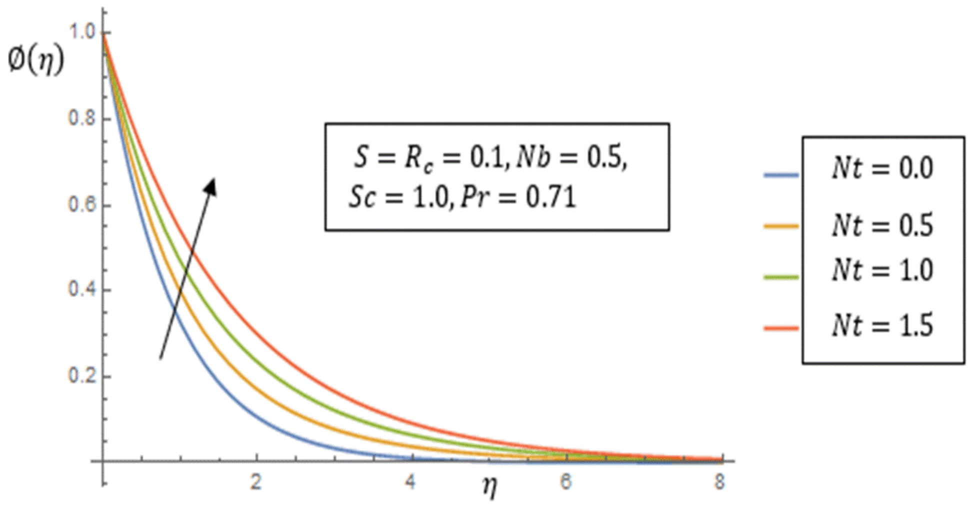

Figure. 11. Influence of thermomigration number on concentration

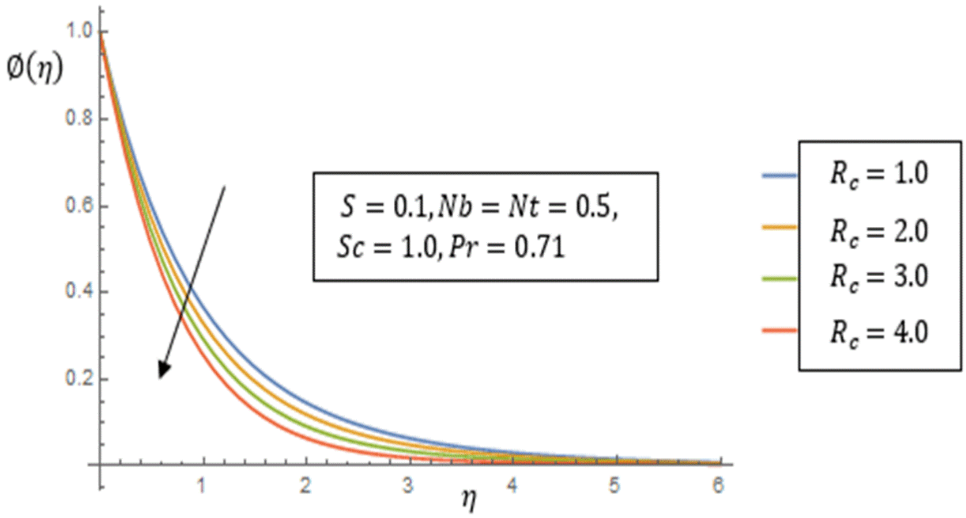

Figure. 12. Influence of chemical reaction parameter on concentration

The influence of chemical reaction R_c on concentration is depicted in **Figure 12.** However, a growth in temperature can quicken the reaction rate, which implies that the lessening of metal ions to nanoparticles happens quicker. Meanwhile, with a faster reaction, the nanoparticles may not possess sufficient time to develop to their required mass and may turn out to be unstable, thus leading to a reduction in concentration. **Figure 13** portrays the effect of Schmidt factor Sc on concentration. Due to the weaker nature of molecular diffusivity of the nano-particles caused by increasing values of the number, the diffusion rate reduces, hence there occurs a decrease in concentration.

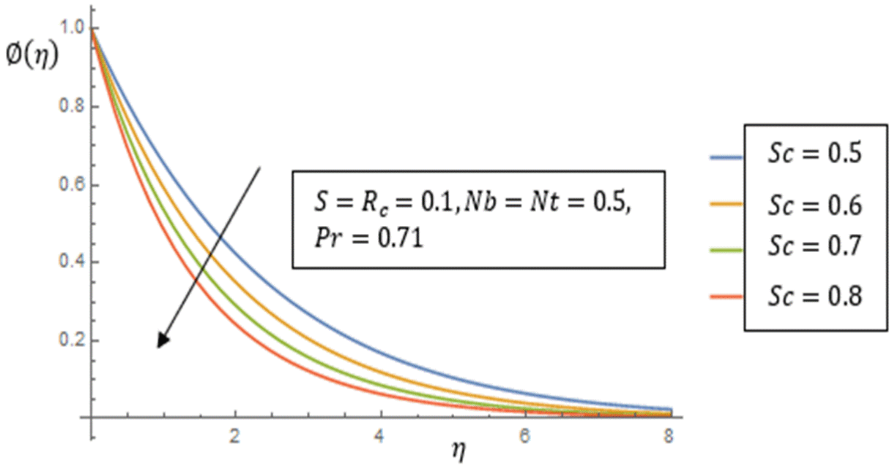

Figure. 13. Influence of Schmidt factor on concentration

Conclusion

The analysis of MHD nanofluid stream in the presence of enriched heat generation/absorption and thermo-migration have been examined. Thus, the conclusions regarding the scientific significance of the current study are stated below:

1. Improvement in the Hartman number H begets a fall in the fluid velocity while an increase in the local thermal Grashof number leads to the enhancement of rate of fluid flow.
2. Increasing the thermomigration parameters Nt causes a thicker, thinner thermal and mass boundary layer thickness with a corresponding rise in the temperature and concentration distribution.

3. As the heat Q_s, Brownian motion Nb and radiation S parameters surge, the thermal wall layer thickens followed by an upsurge in the temperature. Similarly, an increase in the radiation S parameter produces an enhancement in the concentration.
4. It is also observed that the concentration is decreased with rising chemical reaction R_c parameter.
5. **Improved understanding of MHD nanofluidic flow:** The research paper explores the flow behavior of a magnetohydrodynamic (MHD) nanofluid over a nonlinear exponentially stretched plate. This study contributes to the understanding of the behavior of nanofluids in the presence of a magnetic field and their reactivity to thermal source/sink and thermo-migration influences.
6. **Applications in energy engineering:** The results of this study can be applied in energy engineering, especially in the designing and optimization of MHD nanofluidic systems for heat transfer applications. Thus, the findings could aid to advance the proficiency of heat exchangers, boilers, and other energy systems that use nanofluids.
7. **Implications for materials science:** This study has significant implications for materials science since the application of nanoparticles into fluids can lead to improved thermal properties, making them useful for usage in heat transfer, energy storage, etc.
8. **Potential for biomedical applications:** The enriched thermal properties of nanofluids can also have implications for biomedical applications, such as hyperthermia treatments for cancer. The findings of this research paper could help in developing new and more effective hyperthermia treatment methods.
9. **Contribution to the development of numerical methods:** The numerical scheme used in this research paper can be applied to other fields of study such as computational fluid dynamics (CFD), and can aid the advancement of more accurate and efficient numerical methods for modelling fluid flow.

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Author Contribution

Planning the method that will be suitable for achieving results by Uchenna Awucha Uka 1; Designing, supervision and responsibility for the organization, writing an original frame work and course of study by Uchenna Awucha Uka 1, Digbo Idika Iku 2, Great Edwin Esekhaigbe 3; Taking responsibility for the rationale and presentation of the findings by Uchenna Awucha Uka 1, Digbo Idika Iku 2, Great Edwin Esekhaigbe 3; Taking responsibility of the analysis by Uchenna Awucha Uka 1, Digbo Idika Iku 2, Great Edwin Esekhaigbe 3; Re-checking of the article before submission, not only on for spelling and grammar but also for intellectual content by Uchenna Awucha Uka 1, Digbo Idika Iku 2, Great Edwin Esekhaigbe 3; Critical review in planning the technique that will be appropriate for achieving noble results by Uchenna Awucha Uka 1, Digbo Idika Iku 2; Detailed analysis, proof-reading, review and editing of the manuscript by Uchenna Awucha Uka 1, Digbo Idika Iku 2, Great Edwin Esekhaigbe 3. All authors have read and given their full approval on the manuscript.

Conflict of Interest

The authors did not report any form of conflict of interest.

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