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Mathematical Model for Multi Depot Simultaneously Pick Up and Delivery Vehicle Routing Problem with Stochastic Pick Up Demand

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Highlights

- This paper focuses on multi depot simultaneously pick up and delivery vehicle routing problem.
- Pick up demands are assumed to be stochastic.
- Chance constrained programming approach was used in the creation of the stochastic model.

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Keywords

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Abstract

In a classical vehicle routing problem (VRP), customer demands are known with certainty. On the other hand, in real-life problems, customer demands may change over time. Therefore, in the classical VRP, the assumption that customer demands are stochastic should be taken into account. To expedite consumer demands and minimize fuel use and carbon emissions, organizations must concurrently address client distribution and collection requirements. Customers' distribution requirements can be predicted, but it is impossible to predict in advance the product requirements they will send for recycling. Hence, in this study, a mathematical programming model is developed for the multi-depot simultaneous pick-up and delivery vehicle routing problem under the assumption that customers' picking demands are stochastic. However, there are non-linear constraints in the developed model. Thereby, firstly, the stochastic model is linearized, and then the effectiveness of the model is analyzed. The efficacy of the linearized model is ascertained by generating test problems. The study investigated the impact of varying reliability levels and the number of depots on the model. As a result of the sensitivity analysis, it was determined that by decreasing the reliability level, the solution time of the problems decreased and the number of problems reaching the best solution increased. In the study, 135 test problems were solved by changing the reliability level, and the best result was achieved in 105 of these problems within 7200 s. The increase in the number of depots both reduced the solution time of the problems and was effective in reaching the best solution for all solved test problems.

1. INTRODUCTION

In the globalizing world, companies need to reduce their costs in order to take advantage of the market and compete with rival companies. One of the most important cost items for companies that want to meet customer demands in a timely and complete manner is logistics costs. In recent years, companies have attached importance to the creation of suitable routes for product distribution in order to reduce both logistics costs and fuel consumption and carbon emissions.

Vehicle Routing Problem (VRP) is a substantial operational obstacle faced by companies in the domains of logistics, transportation, distribution, and supply chain management. VRP involves finding the most efficient routes for a group of vehicles to serve a specific group of consumers, taking into account specific operational limitations. To effectively respond to growing competition, organizations must strategically plan vehicle routes to optimize consumer satisfaction. The main objective of the VRP is to optimize the choice of routes in order to minimize the total cost, time, or fuel consumption [1]. As a result, VRP and its different variations have attracted considerable interest from researchers in recent years. It has been observed that researchers have recently focused on new versions of VRP, taking into account the assumptions encountered in real-life problems.

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The depletion of natural resources, efforts by non-governmental groups, and growing consumer awareness of recycling have all contributed to the increased importance of reverse logistics activities in today's world [2]. Contemporary enterprises are increasingly prioritizing reverse logistics operations due to concerns such as the scarcity and diminishing availability of resources on our planet, as well as the growing consciousness among customers about environmental pollution and recycling. Enterprises utilize a single vehicle in this transportation mode to handle both consumer collection requests and customer distribution requirements. The objective is to save costs and promote recycling. The topic being addressed is commonly referred to as the simultaneous pick-up and delivery vehicle routing problem (SPD-VRP) in academic literature [3]. This situation has turned it into an interesting subject for researchers as it both increases customer satisfaction and reduces the transportation costs of companies.

Companies can enhance customer satisfaction by establishing several depots to expedite customer requests and provide prompt service. The type of problem in which the most suitable routes are determined for the vehicles by meeting the customer demands from more than one depot is defined as the multi-depot vehicle routing problem (MDVRP). MDVRP is a type of VRP where customer demands are met by many depots located in different areas, with the objective of minimizing the total cost [4]. MDVRP has a more complex structure than classical VRP.

In classical VRP, customer demands are deterministic. Therefore, in most studies in the literature, it is assumed that customer demands are fixed and known. However, in practical situations, it is observed that client requirements can vary based on evolving circumstances. Customer demand is uncertain until the truck reaches the customer's location, and changes may occur after the route has been determined for the consumers to be served. Today, by taking advantage of the developments in information and communication technology, stochastic and dynamic VRPs, which include variable situations in order to provide faster service to customers under competition and to ensure customer satisfaction, have become a subject of attention by researchers [5]. In the literature, these problems are called stochastic vehicle routing problems (SVRP). SVRP emerges when one or more variables in deterministic VRP are not known exactly. Variables that are not known with certainty may be service time, customer requests, travel time, or customers [6]. Recent studies on SVRP reveal that the researchers mostly concentrate on the Vehicle Routing Problem with Stochastic Demand (VRPSD). It is interesting to notice that there aren't many studies in the literature that include time window in VRPSD, multi-depot problem type serving customers from more than one depot, and collect/distribute and simultaneously collect/distribute constraints. On the other hand, there is no study examining the problem type that includes these constraints together.

Real-world issues show that not only are client demands unpredictable, but so are product demands conveyed to factories or depots for purposes of recycling, maintenance, etc. Therefore, it is assumed that the collection demands sent by customers for recycling are stochastic [5]. When the studies in the literature in recent years are examined, it is seen that researchers have conducted studies that deal with more than one VRP type together [4]. In the literature review, it was determined that there was no study examining VRPSD, MDVRP, and SPD-VRP simultaneously. Therefore, in this study, considering the assumption that customers' picking demands are stochastic, the multi-depot simultaneous pick-up and delivery vehicle routing problem with stochastic pick-up demand (MD-SPDVRP-SPD) is investigated. In the study, firstly, a model has been proposed for the multi-depot simultaneous pick-up and delivery vehicle routing problem (MD-SPD-VRP). Afterwards, the collection demands were taken as stochastic, the relevant constraints were rearranged, and a stochastic programming model was created for MD-SPDVRP-SPD. The created stochastic model was converted to linear, and the efficiency of the model was analyzed.

The flow of the sections in the article can be summarized as follows: In the study, firstly, VRPSD and literature review are discussed. The assumptions for the MD-SPD-VRP model and the proposed model are then outlined. Afterwards, MD-SPD-VRP was examined, and the collecting demands of the customers were taken as stochastic in the model. Therefore, the deterministic mathematical model was transformed into the MD-SPDVRP-SPD model. The stochastic programming model was first made linear since the newly constructed model had nonlinear restrictions. The CPLEX Solver was used in the GAMS software to solve the linearized model, and the model's efficiency was examined. In order to assess the efficacy of the developed model, a series of test issues were generated and the outcomes of addressing these problems were assessed. In the last part of the study, a general evaluation of the investigated problem and a detailed analysis of the results were made. Sensitivity analysis was conducted in this part to assess the impact of changes in dependability level and the number of depots on the model. The acquired results were then presented. The conclusion section presents the findings of the study together with recommendations for further research.

2. LITERATURE REVIEW

VRPSD refers to a situation when the exact client needs are not known at the beginning of the vehicle's route. The vehicle only learns about the customer's demand when it arrives at the customer's location. VRPSD assumes that customer needs are derived from a specific probability distribution [6].

In real life problems, it is seen that the parameters in the problem are not deterministic, but mostly consist of random variables. In this case, the linear programming approach cannot be used in modeling the problem, instead the problems are modeled with stochastic programming approaches [7]. In the literature, it is seen that Chance Constrained Programming (CCP) and Stochastic Programming with Recourse approaches are used in modeling VRPSD. CCP approach was first developed by Charnes and Cooper and is now widely used in the creation of stochastic models [8, 9]. In the constraints and objective function of variables containing stochastic information, the CCP approach aims to convert variables with known probability distributions into deterministic ones as much as is feasible. In studies on VRPSD, it has been stated that the CCP approach is easier to apply to models and its solution is easier, and therefore it is preferred by researchers [6]. Hence, the CCP approach was employed in the construction of the stochastic model in this investigation. Desticioğlu and Özyörük (2019) examined in detail the studies on the VRPSD in their literature review [7].

VRPSD has garnered significant attention from researchers in recent years. Markovic et al. (2020) employed a VRPSD model to simulate the garbage collection in the city of Niš. They utilized local search and simulated annealing techniques to solve the problem [10]. Florio et al. (2020) employed the branch-price-and-cut method to effectively solve the VRPSD issue and obtained an optimal solution [11]. Omori and Shiina (2020) devised the integer L-Shaped algorithm specifically for solving the VRPSD problem [12]. Gaur et al. (2020) developed the approximation algorithm to solve the cumulative VRPSD problem [13]. Ma et al. (2021) performed an extensive literature assessment on dynamic and stochastic VRP, encompassing current studies [14]. Niu et al. (2021) examined green VRPSD in their study. They solved the model using a multi-objective evolutionary algorithm [15]. Xia et al. (2021) employed a hybrid system that combined a discrete spider monkey optimization technique with a genetic algorithm to solve the VRPSD model [16]. Komatsu et al. (2021) considered the premise that extra expenses occur when there are subtours in the VRPSD model. They employed the decomposition approach to solve the model [17]. In their study, Bernardo et al. (2021) presented a robust simulation-based method to address the capacitated stochastic VRP model, which involves uncertain demand [18]. In their study, Zarouk et al. (2022) investigated the green VRP, which considers the random nature of demand, supply, and service time. They addressed this problem by employing a metaheuristic algorithm that combines a genetic algorithm with annealing simulation [19]. Desticioğlu et al. (2022) devised a mathematical model to address reverse logistics issues, considering the stochastic nature of product returns to e-commerce enterprises. The study [2] conducted sensitivity studies to evaluate the efficacy of the model. Ledvina et al. (2022), on the other hand, solved VRPSD using an algorithm that combines process flexibility and route assignment [20]. Florio et al. (2022) did another study on VRPSD and used the branch-cut-and-price algorithm to solve the model. They used the switch policy, which is one of the recourse policy approaches. It was stated in the study that optimal results were achieved for up to 50 customers with this algorithm [21]. Niu et al. (2022) used a multi-objective evolutionary algorithm based on artificial neural networks to solve the multi-objective VRPSD [22]. Singh et al. (2022) created a mathematical model for VRP that incorporates uncertain demand and journey time, and using the branch and bound technique to solve the model. The study obtained an optimal solution by examining a minuscule problem [23]. Hoogendoorn and Spliet (2023) used the L-shaped method to solve the VRPSD [24]. De La Vega et al. (2023) conducted a study on the truck routing problem that involved time windows and uncertain demand. They developed a stochastic programming model with recourse to address this challenge [25]. In their study, Che and Zhang (2023) developed a stochastic programming model with recourse for SPD-VRP with stochastic pick-up demand [26]. Zhang et al. (2023) employed tabu search metaheuristics to solve the two-dimensional loading-constrained stochastic customer VRP [27]. Marinaki et al. (2023) used the Dragonfly algorithm to solve the VRPSD model [28]. Sluijk et al. (2023) developed a CCP model for two-stage VRPSD and solved this model using the novel labeling algorithm [29]. In their work, Fukasawa and Gunter (2023) employed the brunch and pricing algorithms to address the stochastic demand capacity vehicle routing problem [30].

Based on the literature assessment, VRPSD has emerged as a topic of interest among researchers in recent years. The literature study revealed that researchers focused on two main areas: designing algorithms to solve the VRPSD model, and exploring novel variations of the VRPSD problem by considering alternate assumptions. In real life, variations of VRP occur in many areas. For example, a cargo company that sends its cargo to its customers from different depots knows the amount of cargo to be distributed to its customers before the route, but when the customers go to the customer's location, they learn the amount of product they will send to the depots to be shipped with the same vehicle when they receive the cargo from the customer. In such a case, the amount of cargo that customers will send can be estimated with various probability distributions, taking into account their past demands, and the routes of the vehicles can be created. In another example, let's take a food company that distributes perishable products in glass containers to supermarkets from different depots in a province. In this problem, the quantities of products to be sent to supermarkets are determined in advance, and the routes of the vehicles are determined with the aim of minimizing the total distance traveled in distributing the products. Empty glass containers are collected simultaneously with the vehicle that distributes the products to the markets. In this case, the amount of glass containers to be collected from markets is not known in advance and is learned when the vehicle goes to drop off products at the market. As a result, the problem type in which both distribution and collection demands from customers are met simultaneously from more than one depot and where the distribution demands of the customers are known before the route starts and the collection requests are learned when the vehicle goes to the customer is modeled as a MD-SPDVRP-SPD. Upon reviewing the existing literature, it was discovered that no previous research had been conducted on MD-SPD-VRP-SPD. Therefore, in this study, a mathematical model for MD-SPD-VRP-SPD was developed for such problems encountered in real life. However, due to the NP-hard feature of VRPSD, it is noteworthy that there are still not many studies in the literature. In the literature research, it has been determined that the MD-SPD-VRP, in which stochastic demand is taken into account, has not been examined before. Therefore, in this study, taking into account the assumption that the picking demands are stochastic.

3. MATHEMATICAL MODEL FOR MD-SPDVRP-SPD

In order to create the model for MD-SPDVRP-SPD, a mathematical model created for MD-SPD-VRP, where the demands are deterministic, is first needed. In this study, the mathematical model developed for MD-SPD-VRP by Karaoğlan et al. (2012) was used in the creation of the stochastic model [31]. The assumptions considered in the creation of the MD-SPD-VRP model are:

1. All vehicles commence their movement from a single depot and conclude their path at the same depot.
2. Each customer is served only once with one vehicle.
3. At no point during the movement of the vehicles, the sum of the distribution and collection demands cannot exceed the capacity of the vehicle.
4. Vehicles serving customers have the same features and capacities.
5. The distribution and collection demands of customers are met simultaneously with the same vehicle.
6. The vehicle that starts the route from one depot is not allowed to go to another depot at the end of the route.
7. Each depot must have enough capacity to accommodate the complete distribution demands of the consumers assigned to it.
8. Vehicles serving customers are allocated to only one depot.

9. It is assumed that the products returned from the customers will be separated as waste, and the warehouse capacity is not taken into account for the collection demands of the customers.

Considering the above assumptions, Karaoğlan et al.'s mathematical model for MD-SPD-VRP is given below [31]:

Sets:

NS: Set of depots Ns = {0,1,…,n}
NC: Set of customers Nc= {n+1, n+2, …,n+m}
N: Set of all customers and depots {Ns U Nc }

Parameters:

$$Min\ Z = \sum_{\substack{i \in N \\ i \neq j}} \sum_{j \in N} c_{ij} x_{ij} \ , \quad (1)$$

$$\sum_{j \in N} x_{ij} = 1 \quad i \in N_c, i \neq j \ , \quad (2)$$

$$\sum_{j \in N} x_{ji} = \sum_{j \in N} x_{ij} \quad i \in N, i \neq j \ , \quad (3)$$

$$\sum_{j \in N_c} x_{kj} = M_k \quad \forall k \in N_s \ , \quad (4)$$

$$\sum_{i \in N_c} x_{ik} = M_k \quad \forall k \in N_s \ , \quad (5)$$

$$\sum_{k \in N_s} y_{ik} = 1 \quad \forall i \in N_c \ , \quad (6)$$

$$x_{ik} \leq y_{ik} \quad \forall i \in N_c, k \in N_s \ , \quad (7)$$

$$x_{ki} \leq y_{ik} \quad \forall i \in N_c, k \in N_s \ , \quad (8)$$

$$x_{ij} + y_{ik} + \sum_{\substack{m \in N_s \\ m \neq k}} y_{jm} \leq 2 \quad \forall i, j \in N_c,\ k \in N_s, \quad (9)$$

$$U_j - U_i + Q * x_{ij} + (Q - d_i - d_j) * x_{ji} \leq Q - d_i \ , \quad \forall i \in N_c, i \neq j \quad (10)$$

$$V_i - V_j + Q * x_{ij} + (Q - q_i - q_j) * x_{ji} \leq Q - q_j \ , \quad \forall i \in N_c, i \neq j \quad (11)$$

$$U_i + V_i - d_i \leq Q \quad \forall i \in N_c, i \neq j \ , \quad (12)$$

$$U_i \geq d_i + \sum_{j \in N_c} d_j x_{ij} \quad \forall i \in N_c, i \neq j \ , \quad (13)$$

$$V_i \geq q_i + \sum_{j \in N_c} q_j x_{ji} \quad \forall i \in N_c, i \neq j \ , \quad (14)$$

$$V_i \leq Q - (Q - q_i)\left(\sum_{k \in N_s} x_{ki}\right), \ \forall i \in N_c \ , \quad (15)$$

$$U_i \leq Q - (Q - d_i)\left(\sum_{k \in N_s} x_{jk}\right), \ \forall i \in N_c \ , \quad (16)$$

$$\sum_{i \in N_c} d_i y_{ik} \leq B_k, \ \forall k \in N_s \ , \quad (17)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in N \ , \quad (18)$$

$$y_{ik} \in \{0, 1\} \quad \forall i \in N_c, \forall k \in N_s \ , \quad (19)$$

$$U_i \geq 0, \ \forall i \in N_c \ , \quad (20)$$

$$V_i \geq 0,\ \forall i \in N_c\ , \tag{21}$$

$$M_k \geq 0,\ \forall k \in N_s\ , \tag{22}$$

Q: Vehicle capacity
c_{ij} : Distance between nodes, i, j Є N, i≠j
d_i : Amount of product to be distributed to customers, i Є N_C
q_i : Amount of product to be collected from customers, i Є N_C
M_k: Number of vehicles moving from k depot, k Є N_S
B_k: Maximum capacity of the k depot, k Є N_S

Decision Variables

$$x_{ij}:\begin{cases} 1, & \text{If the } (i,j) \text{edge is on any route } \forall i,j \in N \\ 0, & \text{Otherwise} \end{cases}$$

$$y_{ik}:\begin{cases} 1, & \text{If customer } i \text{ is served from depot } k\ \forall i \in N_C, \forall k \in N_s \\ 0, & \text{Otherwise} \end{cases}$$

Additional Decision Variables:
U_i: Auxiliary decision variable showing the amount of product to be distributed in the vehicle just before entering the node i, ∀i∈N_C
V_i: Auxiliary decision variable showing the amount of product collected in the vehicle at the exit of the i node, i∈N_C

Mathematical Model

The objective function presented in Equation (1) aims to reduce the total cost, distance, or trip time. Equation (2) guarantees that every node is visited exactly once. Equation (3) establishes an equality between the number of lines that enter and exit all nodes. Equation (4) ensures that at most M_k vehicles leave from k depot, and Equation (5) ensures that M_k vehicles departing from k depot return to k depot at the end of the route. Equation (6) guarantees that each customer is allocated to a single depot. Equations (7) to (9) impose limits that guarantee a vehicle's trip concludes at the same depot from where it originated. Equation (7) ensures that the last customer on the route will be assigned to the depot. Equation (8) ensures that the first customer on the route is assigned to the depot from which the first customer leaves. Equation (9) prevents two customers connected to each other on the route from being assigned to different depots. Equations (10) and (11) guarantee that the combined delivery and pick-up demand on a route does not surpass the vehicle's capacity. Additionally, these equations preclude the occurrence of sub-tours. Equation (12) guarantees that the quantity of product being transported along the route is less than the maximum capacity of the vehicle. Equations (12) to (16) constraints enable the determination of the lower and upper limits of the decision variables U_i and V_i. Equation (17) represents the constraint on depot capacity, which guarantees that the overall demand of customers assigned to a depot does not surpass the depot's capacity. Equations (18) and (19) show the sign constraints; they ensure that the x_{ij} and y_{ik} variables take the value "0" or "1." Equations (20) – (22) show the positivity constraints.

In real-life problems, customer demands are not always deterministic. Therefore, in recent years, VRPSD, where customer demands are stochastic, has become an interesting subject for researchers. VRPSD refers to an issue when the exact consumer requests are not known in advance when the vehicle begins its route. The demand is only discovered when the vehicle arrives at the customer's location.

In recent years, recycling activities have started to gain importance due to the depletion of resources, the deformation of the environment, legal obligations, the effects of non-governmental organizations, etc. Collecting the products sent for recycling from the customers simultaneously with the vehicles that meet the distribution demands of the customers both reduces the transportation costs of the companies and reduces environmental damage by reducing carbon emissions. In addition, by delivering customer demands from more than one depot, it both increases customer satisfaction by providing faster delivery of products to customers and is effective in reducing the total transportation cost by meeting the demands of customers from the nearest depot. In real-life problems, the product demands of the customers can be known in advance, while the product demands of the customers for recycling cannot be known. Customers' recycling demands can only be learned when the vehicle reaches the customer's location. Therefore, it is assumed that the collection demands of customers that are sent for recycling are stochastic. In the model given above for MD-SPD-VRP, the constraints considering the picking demand are reconstructed using the chance-constrained programming approach, and the model is transformed into stochastic. In the studies in the literature, it has been stated that chance-constrained programming adapts to the normal distribution [2, 32, 33]. Therefore, it is assumed that stochastic variables provide a normal distribution.

The aim of the mathematical model, which was developed under the assumption that the picking demands fit the normal probability distribution, is to ensure that the sum of the pickup demands. The distribution requirements of customers along the routes do not beyond the assigned vehicle's capacity, denoted as Q. Additionally, the collected load carried by the vehicle does not exceed both the lower and upper limits. It was tried to obtain the minimum route length (P≤α) within the determined limits (probability level) [5, 8, 9]. Here, it is assumed that the customers' distribution requirements are deterministic, that is, they are known with certainty in advance, whereas the customers' collection requirements are made up of random variables that follow a normal distribution. Assuming that the collection demands are stochastic, the probability constraints with stochastic variables created instead of Equations (11), (14), and (15), respectively, are as follows:

$$\mu_i + \sum_{j\epsilon N_c} \mu_j x_{ij} + Z_{1-\alpha}\sqrt{\sigma_i^2 + \sum_{j\in N_c} \sigma_j^2 x_{ij}} \leq V_i \,, \quad \forall i \in N_c, i \neq j \tag{23}$$

$$\mu_j - x_{ji}(\mu_j + \mu_i) + Z_{1-\alpha}\sqrt{\sigma_j^2 - x_{ji}(\sigma_j^2 + \sigma_i^2)} \leq Q(1 - x_{ij} - x_{ji}) - V_i + V_j \quad \forall i \in N_c \;,\; i \neq j \tag{24}$$

$$Q - Q\sum_{k\in N_s} x_{ki} + \mu_i \sum_{k\in N_s} x_{ki} + Z_{1-\alpha}\sqrt{\sigma_i^2 \sum_{k\epsilon N_s} x_{ki}} \geq V_i \,. \quad \forall i \in N_c \tag{25}$$

Equation (23) is the upper bound constraint of the chance-constrained Vi variable, which allows the stochastic picking demands to be exceeded by a certain percentage of the determined variable V_i. Similarly, Equation (24) shows the lower bound constraint of the V_i variable created by the chance constraint approach. Equation (25), on the other hand, shows the capacity constraint that allows the stochastic collection demands on the vehicle to exceed the capacity to a certain extent while the vehicle continues on its route. In addition, Equation (25) prevents the formation of sub-routes.

When Equation (23), Equation (24) and Equation (25) are examined, it is seen that the constraints are not linear. Since the solution of nonlinear constraints will be difficult and suitable solutions cannot be found for these constraints with the software used in the solution of mathematical models, the constraints must be converted to linear. Various techniques are employed in the literature to convert nonlinear constraints into linear ones. One of the techniques employed in the process of linearizing constraints is the linear approximation approach, which allows for the rapid attainment of the optimal solution. Hence, in this work, the linear approximation technique was employed to convert the constraints into linear form. In the linear approximation method, the constraints are converted to linear by using the equation given in Equation (26) [34]

$$\sqrt{\sum_{i=1}^{n} a_i^2} \le \sum_{i=1}^{n} a_i \qquad a_i \in R^+. \tag{26}$$

In the mathematical model developed for MD-SPD-VRP, constraints 11, 14 and 15 can be replaced with Equations (23), (24) and (25), and the model can be adapted for MD-SPDVRP-SPD. When the constraints Equation (23), Equation (24) and Equation (25) are examined, it is noteworthy that these constraints $\sqrt{\sigma_i^2 + \sum_{j \in N_C} \sigma_j^2 x_{ji}}$, $\sqrt{\sigma_i^2 \sum_{k \in N_S} x_{ki}}$, $\sqrt{\sigma_j^2 - \sigma_j^2 x_{ji} - \sigma_i^2 x_{ji}}$ contain non-linear expressions. This part utilizes the linear approximation approach to convert these formulae into linear forms. Linearized constraints with linear approach are given in Equation (27), Equation (28) and Equation (29), respectively:

$$\mu_i + \sum_{j \epsilon N_c} \mu_j x_{ij} + Z_{1-\alpha}\left[\sigma_i + \sum_{j \epsilon N_c} \sigma_j x_{ji}\right] \le V_i \quad \forall i \in N_c, i \neq j, \tag{27}$$

$$\mu_i \sum_{k \epsilon N_s} x_{ki} + Z_{1-\alpha}\left[\sigma_i \sum_{k \epsilon N_s} x_{ki}\right] \ge V_i - Q \sum_{k \epsilon N_s} x_{ki} - Q \quad \forall i \in N_c, \tag{28}$$

$$\mu_j - x_{ji}(\mu_j + \mu_i) + Z_{1-\alpha}\left[\sigma_j - x_{ji}(\sigma_j - \sigma_i)\right] \le Q(1 - x_{ij} - x_{ji}) - V_i + V_j \quad \forall i, j \in N_c, i \neq j. \tag{29}$$

As a result of the necessary operations, Equation (23), (24) and (25) constraints are replaced with Equation (27), (28) and (29) constraints, and the model developed for MD-SPD-VRP-SPD becomes linear. After the mathematical model developed for MD-SPD-VRP-SPD is converted to linear, the effectiveness of the model can be tested by coding it into programs that can solve linear models. Consequently, the linearized model was implemented in the GAMS program and its efficacy was assessed by applying it to the test problems found in the literature. In the next section, the results obtained in solving the test problems are evaluated.

4. RESULTS

Test problems are used to analyze the effectiveness of the model developed in this section. Due to the absence of test problems for MD-SPDVRP-SPD in existing literature, attempts have been made to generate test problems by assuming that the product demand sent by customers for recycling follows a stochastic pattern. Here, the test problems developed by Christofides and Eilon (1969) and Augerat et al. (1995) were used [35, 36]. In order to transform these problems into SPD-VRP, the method developed by Salhi and Nagy (1999) to determine collection and distribution demands was applied [37]. The distribution demands obtained by this method are accepted as the distribution demands of the stochastic problem [38]. In the determination of the collection demands, the method of forming the demand in similar problems involving stochastic demand was used. Accordingly, the range of the aggregation demands of the deterministic problem with the formulation proposed by Salhi and Nagy (1999) was taken into account, and the collection demands were randomly generated within this range with a uniform distribution [37].

The standard deviation also needs to be determined in the model developed for the stochastic problem. While creating test problems for MD-SPD-VRP-SPD, the method used for similar problems in the literature was used [14, 32]. Accordingly, in order to test the effectiveness of the model, three different standard deviations at low, medium, and high levels were taken into account. 10% of the demand averages are determined as low standard deviation (σ1), 15% of the demand average is determined as medium standard deviation (σ2), and in the high standard deviation, 20% of the demand averages are determined as standard deviation (σ3). In solving the problem, it is also necessary to determine at what level of reliability the customer demands will be met. When it is aimed at meeting customer demands with a high reliability level, the reliability level in test problems is primarily taken as 97.5%. However, in the studies in the literature, it was determined that test problems were created with different reliability levels. Thus, this study involved the creation of test problems utilizing reliability levels of 95% and 90% to investigate the impact of reliability on the model [2, 26, 27, 33, 34]. In the studies conducted in the literature, it has been determined that half of the total demand is taken into account in determining the vehicle capacities of the test problems [5, 26]. Therefore, in this study, half of the sum of the collection demands is taken as the vehicle capacity in determining the vehicle capacity.

In order to show the model works correctly, firstly, Christofides and Eilon's (1995) E-n51-k5 problem is solved by transforming it into an example with 15 customers and 2 depots [35]. The vehicle capacity in the problem is taken as 85, the standard deviation is determined as 0.1, and the confidence interval is assumed to be 97.5%. In this study, the distribution network created with the results obtained by solving this problem is shown in Figure 1.

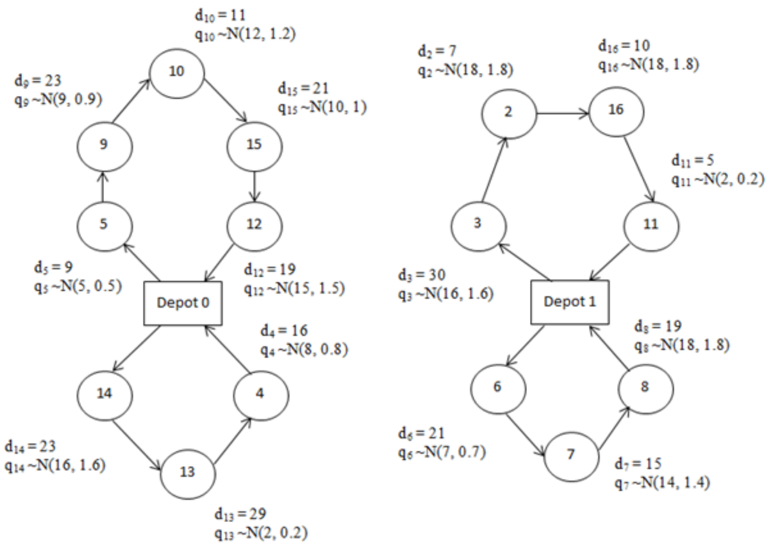

Figure 1. *Showing the results obtained from solving the test problem with linearized model on the network*

Since there are non-linear constraints in the mathematical model developed for MD-SPDVRP-SPD, the model was first linearized with the linear approximation method. The linearized model Intel Core i5-8265U CPU @ 1.60 GHz processor, 8 GB buffer, and "Windows Home Single Language" features are coded in the licensed GAMS 24.1.3 program. Different sizes of the test problems that are created for MD-SPDVRP-SPD are solved by using the CPLEX 12.5.1.0 solver. The success of the linearized model is assessed using three performance criteria: average solution time (AST), number of problems with the optimal solution (NPOS), average number of vehicles (ANV). Test problems proposed by Christofides and Eilon (1969) and Augerat et al. (1995) were used to create test problems [35, 36]. To adapt these problems to SPD-VRP, the formulation proposed by Salhi and Nagy was used, and the distribution demands of the customers were determined with this formulation [37]. Calculations were made by taking the standard deviation value of 0.1 in the creation of the test problems.

Today, companies need to meet customer expectations in the best way possible in order to compete with each other. This is possible by responding quickly to customer demands and meeting customer demands at a high rate, even with variable demands. In order to analyze the effectiveness of the mathematical model, 5 different samples from each of them were solved by changing the number of customers between 10 and 50 at a reliability level of 97.5%. In the solved test problems, it was assumed that customers were served from two depots. In this direction, the reliability level was accepted as 97.5%, the model was run using different test problems, and Table 1 was created with the results obtained. The first three columns in the table show the number of customers, the reliability level of the test problem, and the number of problems solved, respectively. The fourth column in the chart shows the NPOS among the 5 problems solved in each customer group and the average solution time value considering the average of the solution times of the 5 problems.

***Table 1.** Results obtained by solving the model with 97.5% reliability and 2 depots*

Number of Customer	*Reliability Level*	*Number of Problem*	*NPOS*	*ANV*	*AST (s)*
10	97.5%	5	5	1.4	1.08
15	97.5%	5	5	1.6	42.57
20	97.5%	5	5	1.8	381.22
25	97.5%	5	5	2.4	302.96
30	97.5%	5	5	2.6	1433.56
35	97.5%	5	4	3.2	1438.04
40	97.5%	5	3	3.2	2918.75
45	97.5%	5	1	3.2	5175.50
50	97.5%	5	1	3.4	4062.04

Upon examination of the table, it becomes apparent that NPOS exhibits a negative correlation with the quantity of customers. When the problems solved with 97.5% reliability are examined, it is seen that the AST of the problems varies between 1.08 and 5175.50 s. As anticipated, the average resolution time rises proportionally with the number of consumers. Since VRP is an operational decision, the solution must be obtained in a reasonable time. Therefore, 7200.00 s is determined as the upper limit for the solution times of the problems. However, due to the capacity error in the program for large-size problems, the program terminated before 7200.00 s. In this case, the solution time is taken as the moment when the capacity error is encountered. Especially as the number of customers increases, there is a possibility of encountering a capacity error, so the AST value of the problems with 50 customers is lower than the AST value of the problems with 45 customers. When the table is examined, it is noteworthy that the best solution was reached in all test problems up to 30 customers, and as the problem size increases, the complexity increases, so there is a decrease in the number of problems with the best solution in large-size problems. In the analysis, the number of vehicles obtained from the solution of each problem was also taken into account and the average of these vehicle numbers was taken for the solved problems. As a result of the calculations, it is seen that the number of customers generally increases in the number of vehicles serving.

When the studies were examined, it was determined that the problems were solved by using different reliability levels to specify the effect of the reliability change on [2, 32, 33, 39, 40]. Therefore, the test problems were solved by taking a reliability level of 95%. Table 2 was created by using the results obtained by taking the reliability level as 95%.

***Table 2.** Results obtained by solving the model with 95% reliability and 2 depots*

Number of Customer	*Reliability Level*	*Number of Problem*	*NPOS*	*ANV*	*AST (s)*
10	95%	5	5	1.2	1.06
15	95%	5	5	1.4	30.43
20	95%	5	5	1.6	209.54
25	95%	5	5	2.2	290.73
30	95%	5	5	2.6	821.62
35	95%	5	4	3.2	993.64
40	95%	5	3	3.2	2471.08
45	95%	5	2	3.2	2796.70

50	95%	5	1	3.4	3017.90

Upon examining Table 2, it is evident that the optimal solution is achieved for all cases with a customer range of 10–30. The average solution times for the problems vary between 1.06 and 3052.39 s. It is worth mentioning that as the number of clients increases, the AST normally increases. However, as the number of customers increases, when a capacity error is encountered in the program, the program terminates before 7200.00 seconds, and this value was taken as the solution time for the problem. Capacity errors were encountered, especially in problems involving 45 and 50 customers. With the test problems solved with a 95% reliability level, it was determined that ANV increased as the number of customers increased.

When the reliability level was taken at 90%, Table 3 was created with the results obtained from the test problems. Upon examining Table 3, it becomes evident that the optimal solution is achieved for all test cases with up to 35 customer. The AST of the test problems ranged from 0.78 to 3834.36 seconds. As expected, the larger the problem, the longer the solution time. A capacity error was encountered in the test problems, which 50 customers solved using this reliability level. Also it was concluded that ANV increased as the number of customers increased in solving test problems that were solved with a 90% reliability level.

Table 3. *Results obtained by solving the model with 90% reliability and 2 depots*

Number of Customer	*Reliability Level*	*Number of Problem*	*NPOS*	*ANV*	*AST (s)*
10	90%	5	5	1.2	0.78
15	90%	5	5	1.4	22.19
20	90%	5	5	1.4	205.04
25	90%	5	5	2.2	238.23
30	90%	5	5	2.4	420.58
35	90%	5	4	3.2	1220.79
40	90%	5	4	3.2	2414.73
45	90%	5	2	3.2	2223.05
50	90%	5	1	3.4	3700.29

Considering the assumption that customers' picking demands are stochastic, it is thought that the change in the number of depots in the mathematical model developed for MD-SPDVRP-SPD will also have an impact on the effectiveness of the model. Therefore, test problems were created by changing the number of depots in the study. In the test problems created, the reliability level was taken as 95% and the standard deviation was 0.10. To assess the impact of varying the number of depots on the model, the test problems from the previous section were re-solved. The initial assumption of customers being served from 2 depots was reconsidered, and the problems were solved again assuming customers are served from 3 and 4 depots. In order to examine the change in the number of depots at a 95% reliability level, 135 test problems were solved and the results were compared, taking into account the assumption that between 10 and 50 customers are served from 2 depots, 3 depots, and 4 depots.

In Table 2, the results obtained in solving test problems with two depots, where the standard deviation was determined to be 0.10 and the 95% reliability level established to analyze the effectiveness of the model, are given. For the same test problems, the number of depots was set to 3, and the problems were solved once more. Table 4 was created with the results obtained in solving the test problems with 3 depots at a 95% reliability level.

When Table 4 is examined, it is noted that the best result was achieved within 7200.00 s in all 45 problems solved to determine the effect of the number of depot changes on the model. The average solution time of the test problems varies between 0.13 and 1019.95 s. Table 4 shows that AST values increase as the number of customers increases. In the test problems solved under the assumption that customers are served from 3 depots, it was determined that ANV increases as the number of customers increases. There also generally appears to be a decrease in ANV compared to 2 depots test problems.

Table 4. *Results obtained by solving the model with 95% reliability and 3 depots*

Number of Customer	*Reliability Level*	*Number of Problem*	*NPOS*	*ANV*	*AST (s)*
10	95%	5	5	1	0.13
15	95%	5	5	1.4	0.29
20	95%	5	5	1.4	1.78
25	95%	5	5	2.2	8.18
30	95%	5	5	2.4	25.25
35	95%	5	5	2.4	50.25
40	95%	5	5	2.4	64.34
45	95%	5	5	2.6	641.90
50	95%	5	5	3	1019.95

A total of 45 test problems were solved by considering 4 depots, and the findings were used to generate Table 5. Upon analysis of Table 5, it is evident that the optimal outcomes were obtained for all 45 test problems encountered during the assessment of 4 depot scenarios. In addition, it was determined that the AST values of the test problems varied between 0.08 and 730.58 s and that the AST value increased as the number of customers increased. In the solved test problems, it is seen that ANV varies between 1.2 and 3.6 and as the number of customers increases, ANV also increases.

Table 5. *Results obtained by solving the model with 95% reliability and 4 depots*

Number of Customer	*Reliability Level*	*Number of Problem*	*NPOS*	*ANV*	*AST (s)*
10	95%	5	5	1.2	0.08
15	95%	5	5	1.4	0.17
20	95%	5	5	1.4	0.43
25	95%	5	5	2.4	2.20
30	95%	5	5	2.4	23.35
35	95%	5	5	2.6	26.73
40	95%	5	5	2.6	38.77
45	95%	5	5	3	477.30
50	95%	5	5	3.6	730.58

To evaluate the model's effectiveness, a total of 135 test problems of varying sizes were solved, with 45 test problems solved at each reliability level. To determine the impact of the dependability level on the established model for MD-SPDVRP-SPD, it is necessary to analyze Table 1, Table 2, and Table 3 together. It has been determined that the best solution has been achieved in 34 of 45 test problems solved at a 97.5% reliability level. While the best solution was obtained in 34 of 45 test problems solved at a 95% reliability level, the best solution was found in 36 of 45 problems solved at a 90% reliability level. This shows that as the level of reliability decreases, the number of problems for which the best solution is found generally increases. Similarly, the average solution time generally decreases as the reliability level decreases, but this decrease cannot be seen clearly because capacity error is encountered in large-sized problems. When the problems involving 10 to 40 customers are examined in all three charts, it can be deduced that the AST decreases as the level of security decreases. When the problems involving 10 to 40 customers are examined in all three charts, it can be deduced that the AST decreases as the level of security decreases. To evaluate the efficacy of the model, it was found that the optimal solution was achieved in 105 out of the 135 problems solved within the designated timeframe. As anticipated, the rise in client volume resulted in a decrease in the occurrence of issues that required the optimal resolution. As the quantity of clients experiencing issues rises, the typical duration for resolving these issues tends to climb. In addition, when the size of the problem increased, it was observed that in some problems, the appropriate solution could not be reached within 7200.00 seconds. As expected, again, as the number of customers increases, it has been determined that the best solution cannot be reached in a reasonable time. It was determined that 27 of the solved test problems could not be found due to capacity error, and only 3 of them could not reach the best solution within 7200.00 seconds due to time constraints. These problems were solved again by removing the time constraint of 7200.00 s. When these problems were solved again, it was determined that there was an increase in the average solution times, as expected. On the other hand, it was determined that the best solution was achieved in the results of these problems.

The objective of the second stage of this section is to analyze the impact of altering the number of depots on the mathematical model created for MD-SPDVRP-SPD. In order to achieve this objective, a total of 45 problems were addressed. These difficulties involved different numbers of customers, ranging from 10 to 50. The assumption made was that the customers would be supplied from either 2, 3, or 4 depots. As a result, three tables were developed, namely Table 2, Table 4, and Table 5, to present the outcomes achieved from solving these problems. When Table 2, Table 4 and Table 5 are examined together, it is seen that AST decreases as the number of depots serving customers increases. As in the test problems solved at 90% reliability level and 97.5% reliability level, it is seen that the solution time increases as the number of customers increases in the change in the number of depots. It is seen that optimal results were achieved in only 35 of the 45 test problems solved under the assumption that customers are served from 2 depots at a 95% reliability level. When the same test problems were solved under the assumption that customers were served from 3 depots and 4 depots, it was determined that the optimal result was achieved in all of them. As a result, the increase in the number of depots reduces the complexity of the problem. It has been determined that the optimal solution is reached faster when the number of depots is increased. It has been determined that in the number of depots does not have a direct effect on ANV, and the number of vehicles may increase or decrease in order to reduce the distance travelled. As a result of the solved problems, it is noteworthy that as the number of customers increases, the ANV also increases. As the reliability level increases, ANV also increases.

5. CONCLUSIONS

When the studies in the literature are examined, it is seen that many studies have been carried out separately on the VRPSD, MDVRP, and SPD-VRP types. However, in the literature search, it has been determined that the combination of these three VRPs, MD-SPDVRP-SPD has not been examined before. Therefore, in this study, MD-SPDVRP-SPD has been examined and a model has been proposed for this problem.

First, the mathematical model with the assumptions taken into account in MD-SPD-VRP is included. Meeting the recycling demands of customers with the same vehicle in real-life situations will both reduce the total distance traveled and fuel consumption, as well as reduce carbon emissions. Meeting the recycling demands of customers with the same vehicle will both reduce the total distance traveled and fuel consumption, as well as reduce carbon emissions. However, in real-life problems, product requests sent by customers for recycling cannot be learned until the vehicle reaches the customer's location. For this reason, in this study, it is assumed that customers' picking demands are stochastic. The CCP approach was used in the development of the mathematical model, in which stochastic demand is taken into account. Researchers who have looked at previous research on VRPSD have found that those that use the CCP method to build the random model assume that customer demands follow a normal distribution. Thus, this study assumes that the collection needs are stochastically distributed according to a normal distribution.

The model developed for MD-SPDVRP-SPD contains a nonlinear structure since it contains stochastic information. The non-linearity of the model also prevents the GAMS program from solving with the CPLEX solver. Therefore, the developed mathematical model has been converted to linear with the linear approximation method. To evaluate the efficiency of the linearized model, a total of 135 test cases were generated with varying reliability levels, ranging from 10 to 50 customersWhile the best solution was achieved in 34 of 45 test problems solved at 97.5%. The optimal solution was obtained in 35 out of 45 test issues, resulting in a reliability level of 95%, on the other hand it was determined that the best solution was reached in 36 of 45 test problems with the same feature, which were solved at 90% reliability level. Based on the data gained from solving these test issues, it has been established that when the reliability level improves, the average solution time of the problem also increases. Furthermore, it is observed that the solution time and the number of problems attaining the optimal solution reduce as the customer count rises. It has been determined that the best solution could not be reached within the specified time, especially in the problems involving 40 to 50 customers. This shows that the best solution cannot be obtained in a reasonable time for large-sized problems. The study also investigated the impact of altering the number of depots on the established model. To assess the impact of altering the number of depots on the model, a total of 135 test problems were solved with a 95% reliability level. The assumption made was that clients are supplied from 2, 3, and 4 depots. In the test problems solved with different numbers of depots, it is seen that the solution time of the test problems in the examples where customers are served from 2 depots is between 1.06 and 3017.40, while the solution time of the test problems where customers are served from 4 depots varies between 0.08 and 730.58. This shows that the increase in the number of depots reduces the average solution time for problems. It has been determined that as the number of depots increases, the solution time for the problems decreases, and the best results are achieved with more problems. As a result, the increase in the number of depots shortens the solution time of the problem and reduces the complexity of the model.

In this study, MD-SPDVRP-SPD has been examined, and a mathematical model has been proposed for this problem. In the study, the mathematical model developed for MD-SPDVRP-SPD was linearized, and the effectiveness of the model was analyzed. In future studies, MD-SPD-VRP can be examined where the distribution demands of the customers are stochastic as well as the collection demands. In addition, in the analysis of the developed mathematical model, it was determined that the problems involving 50 customers could not be solved in a reasonable time. Hence, in future research, it is possible to devise solution algorithms employing heuristic and meta-heuristic techniques to efficiently identify appropriate solutions for issues of medium and large scale within a tolerable timeframe. This study assessed the efficacy of the mathematical model created for MD-SPDVRP-SPD by testing it with various challenges. The mathematical model that has been built can be applied to real-life problems in future studies, and heuristic methods can be employed to solve the model. In this study, a mathematical model was developed by assuming that the vehicles are homogeneous, that is, their capacities and speeds are equal to each other. In future studies, a mathematical model can be created in which heterogeneous vehicle fleets are taken into account, and solution algorithms can be proposed by using heuristic or meta-heuristic methods for its solution. In future studies, researchers also can focus on a model that will minimize the number of vehicles.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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