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Analytical Rational Solitons of the Modified Lakshmanan-Porsezian-Daniel Equation

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Abstract

In this paper, the Lakshmanan-Porsezian-Daniel (LPD) equation is studied. New analytical rational solitons to the LPD equation are presented by an ansatz method. Wave solutions of three types, such as parabolic, trigonometric and hyperbolic function solutions have been retrieved. All solutions are plotted in 3D to enhance the understanding of their physical characteristics. These simulations, which represent the behavior of the resulting hyperbolic, parabolic and trigonometric solitons, are provided by choosing different appropriate values of the parameters.

1. Introduction

Many researchers in fields such as mathematics, physics, engineering and more are very interested in nonlinear partial equations, since most physical systems are not linear in nature. The Lakshmanan-Porsezian-Daniel (LPD) equation is a widely known nonlinear partial differential equation. It is a generalization of the nonlinear Schrödinger equation that includes higher order nonlinear and dispersed terms. In recent years, it has attracted great attention from mathematicians and physicists. The LPD equation has also been generalized and extended in many ways, including the addition of external forcing, the inclusion of damping effects, and the consideration of higher-dimensional versions of the equation. The LPD equation and its variants have been used to model a variety of physical systems in many areas of physics and engineering, including plasma physics, fluid dynamics, and nonlinear optics, and has been studied extensively from different perspectives such as integrability, symmetry analysis, solution methods and applications: Ricatti equation [1], $\tan(\frac{\psi(\eta)}{2})$-expansion technique [2], collective variable [3], modified simple equation method [4], method of undetermined coefficients [5], Darboux transformations [6, 7], Rogue wave equation [8], the modified auxiliary equation method [9] etc.

This paper investigates the Lakshmanan–Porsezian–Daniel equation [10, 11], a well known partial differential equation that describes the pulse propagation in an optical fiber which is in the form

$$iq_t + aq_{xx} + bq_{xt} + \zeta|q^2|q = \sigma q_{xxxx} + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \delta|q|^4 q \tag{1.1}$$

where the complex valued function $q(x,t)$ depends on space x and time t. The term iq_t denotes the temporal evolution of pulse. The group velocity dispersion and spatio-temporal dispersion are given by a and b, respectively. The fourth-order dispersion and two-photon absorption are represented by constants σ and δ, respectively. The parameters β and γ indicate the non-linear forms of dispersion.

In this paper, our aim is finding solutions in the form of parabolic, trigonometric and hyperbolic solitons of the LPD equation. First we start by using traveling wave variables to find a solution for Eq. (1.1). After analyzing the resulting system of equations to find the condition of its compatibility, it turned out that the structure for the system of equations. At the second stage, a special logarithmic transformation is applied. At the last stage, three different methods are applied to retrieved equation. The solutions obtained by appropriate selection of some parameters affecting the shape and velocity of the solitons are observed with three-dimensional plots.

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2. System of Differential Equations Corresponding to Eq. (1.1)

In order to find soliton solutions of Eq. (1.1), we use the traveling wave reduction in the form

$$q(x,t)=y(z)e^{i\theta}, \quad z=x-ct, \quad \theta=kx-wt \tag{2.1}$$

where $y(z)$ is a complex-valued function that represents the structure of the pulse, k, w, c, θ are parameters of solution. The phase component of the soliton is θ, k is represents the frequency of the soliton, c is the velocity of the soliton, while w is the wave number.
Substituting solution (2.1) into Eq. (1.1) and equating the real and imaginary parts of expression to zero, respectively the following equations are obtained

$$\sigma y_{zzzz}+(-a+bc-6\sigma k^2)y_{zz}+\gamma y^2 y_{zz}+\beta y y_z{}^2+(\beta k^2-\gamma k^2-\zeta)y^3+\delta y^5+(\sigma k^4-w+ak^2-bwk)y=0 \tag{2.2}$$

and

$$4\sigma k y_{zzz}+\left(-4\sigma k^3+c-2ak+b(ck+w)\right)y_z+2\gamma k y^2 y_z=0. \tag{2.3}$$

The $y(z)$ function must satify both of the above third and fourth order differential equations obtained, respectively (2.2) and (2.3).
For the purpose of evaluating the solution of the Eq. (1.1), we implement the following logaritmic transformation from [12] on y,

$$y=2(\ln f)_z. \tag{2.4}$$

If transformation (2.4) is substituted into in (2.2) and (2.3), then the resulting expressions obtained as:

$$\left(\begin{array}{c} -4\zeta f^2 f_z^3-60\sigma f f_{zz} f_z^3-12\gamma f f_{zz} f_z^3-8\beta f f_{zz} f_z^3+4\gamma f^2 f_z^2 f_{zzz}+20\sigma f^2 f_z^2 f_{zzz}+30\sigma f^2 f_{zzz}^2 f_z+4\beta f^2 f_{zz}^2 f_z \\ -10\sigma f^3 f_{zz} f_{zzz}+3af^3 f_{zz} f_z+bcf^4 f_{zzz}-5\sigma f^3 f_z f_{zzzz}-6\sigma k^2 f^4 f_{zzz}+8\gamma f_z^5+4\beta f_z^5+16\delta f_z^5+24\sigma f_z^5-wf^4 f_z \\ -2af^2 f_z^3-bkwf^4 f_z+k^4\sigma f^4 f_z+ak^2 f^4 f_z-4\gamma k^2 f^2 f_z^3+4\beta k^2 f^2 f_z^3-12\sigma k^2 f^2 f_z^3+2bcf^2 f_z^3 \\ -af^4 f_{zzz}+\sigma f_{zzzzz} f^4-3bcf^3 f_{zz} f_z+18\sigma k^2 f^3 f_{zz} f_z \end{array}\right)=0,$$

$$\left(\begin{array}{c} -4\sigma k^3 f^3 f_{zz}+4\sigma k^3 f^2 f_z^2+bckf^3 f_{zz}-bxkf^2 f_z^2-2akf^3 f_{zz}+bwf^3 f_{zz}+2akf^2 f_z^2-bwf^2 f_z^2+8\gamma k f f_z^2 f_{zz} \\ -8\gamma k f_z^4+4\sigma f_{zzz} f^3+cf^3 f_{zz}-16\sigma f^2 f_{zzz} f_z-cf^2 f_z^2-12\sigma f^2 f_{zz}^2+48\sigma f f_z^2 f_{zz}-24\sigma f_z^4 \end{array}\right)=0. \tag{2.5}$$

Now, we use this form to evaluate various rational solitons.

3. Hyperbolic Solitons of Differential Equations Corresponding to Eq. (1.1)

To get hyperbolic solitons of (2.5), we use the following transformation:

$$f=b_0\cosh z+b_1\sinh z \tag{3.1}$$

where b_0, b_1 are any constants to be determined. Substituting (3.1) into (2.5) and equating the coefficient terms that are containing independent combinations of cosh and sinh functions to zero, we obtain a system of algebraic equations:

$$-b_1(5b_0^4+10b_0^2b_1^2+b_1^4)(-k^4\sigma-ak^2+bkw-4\beta k^2+4\gamma k^2+4\zeta-16\delta+w)=0,$$

$$-b_0(b_0^4+10b_0^2b_1^2+5b_1^4)(-k^4\sigma-ak^2+bkw-4\beta k^2+4\gamma k^2+4\zeta-16\delta+w)=0,$$

$$2b_1\left(\begin{array}{c} \left(-2k^4\sigma-2ak^2+2bkw-14\beta k^2+14\gamma k^2+18k^2\sigma-3bc+14\zeta+3a-80\delta-12\gamma-12\sigma+2w\right)b_0^4 \\ +\left(-7k^4\sigma-7ak^2+7bkw-24\beta k^2+24\gamma k^2-12k^2\sigma+2bc+24\zeta-2a-80\delta+8\gamma+8\sigma+7w\right)b_1^2b_0^2 \\ +\left(-k^4\sigma-ak^2+bkw-2\beta k^2+2\gamma k^2-6k^2\sigma+bc+2\zeta-a+4\gamma+4\sigma+w\right)b_1^4 \end{array}\right)=0,$$

$$2b_0\left(\begin{array}{c} \left(-2\beta k^2+2\gamma k^2+6k^2\sigma-bc+2\zeta+a-16\delta-4\gamma-4\sigma\right)b_0^4 \\ +\left(-3k^4\sigma-3ak^2+3bkw-16\beta k^2+16\gamma k^2+12k^2\sigma-2bc+16\zeta+2a-80\delta-8\gamma-8\sigma+3w\right)b_0^2b_1^2 \\ +\left(-3k^4\sigma-3ak^2+3bkw-6\beta k^2+6\gamma k^2-18k^2\sigma+3bc+6\zeta-3a+12\gamma+12\sigma+3w\right)b_1^4 \end{array}\right)=0,$$

$$-b_1\left(\begin{array}{c} \left(-8\beta k^2+8\gamma k^2+24k^2\sigma-4bc+8\zeta+4a-4\beta-80\delta-24\gamma-40\sigma\right)b_0^4 \\ +\left(-4k^4\sigma-4ak^2+4bkw-12\beta k^2+12\gamma k^2-12k^2\sigma+2bc+12\zeta-2a+8\beta+24\gamma+56\sigma+4w\right)b_0^2b_1^2 \\ +\left(-k^4\sigma-ak^2+bkw-12k^2\sigma+2bc-2a-4\beta-16\sigma+w\right)b_1^4 \end{array}\right)=0,$$

$$b_0\left(\begin{array}{c} (8\gamma+4\beta+16\delta+24\sigma)b_0^4 \\ +\left(4\beta k^2-4\gamma k^2-12k^2\sigma+2bc-4\zeta-2a-8\beta-8\gamma-40\sigma\right)b_0^2b_1^2 \\ +\left(k^4\sigma+ak^2-bkw+12k^2\sigma-2bc+2a+4\beta+16\sigma-w\right)b_1^4 \end{array}\right)=0, \tag{3.2}$$

$$(b_0-b_1)(b_0+b_1)(b_0^2+b_1^2)(-4k^3\sigma+bck-2ak+bw+8\gamma k+c+16\sigma)=0,$$

$$2b_0b_1(b_0-b_1)(b_0+b_1)(-4k^3\sigma+bck-2ak+bw+8\gamma k+c+16\sigma)=0,$$

$$-(b_0-b_1)(b_0+b_1)(-4b_1^2k^3\sigma+bb_1^2ck-2ab_1^2k+bb_1^2w+8b_0^2\gamma k+24b_0^2\sigma+b_1^2c-8b_1^2\sigma)=0.$$

After solving the system (3.2) with the help of Maple software, three cases of parametric values are obtained as follows:

Case 1:

$$b_1 = b_0, \quad \sigma = \frac{-ak^2 + bkw - 4\beta k^2 + 4\gamma k^2 + 4\zeta - 16\delta + w}{k^4}. \tag{3.3}$$

Via the parametric values in (3.3), we have

$$f = b_0(\cosh z + \sinh z). \tag{3.4}$$

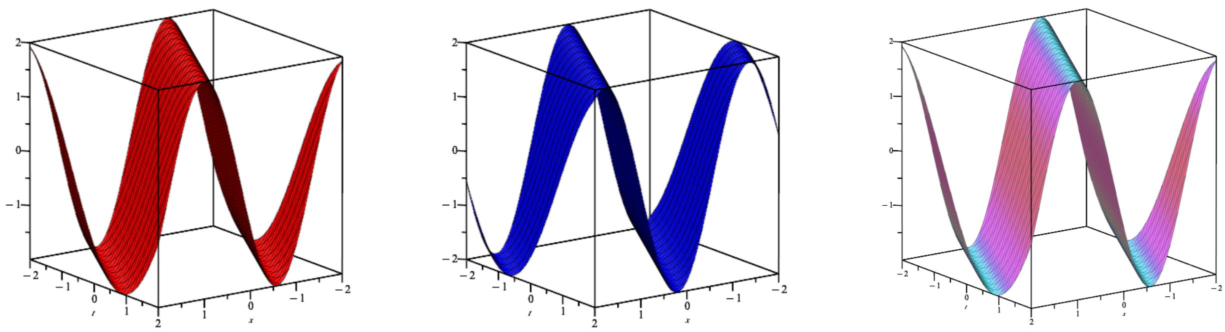
By using $y = 2(\ln f)_z$, we have

$$y = 2. \tag{3.5}$$

By using Eq. (3.5) into Eq. (2.1), we obtain a first type of rational hyperbolic solution of Eq. (1.1):

$$q(x,t) = 2e^{i(kx-wt)}. \tag{3.6}$$

(See Figure 3.1)



(a) **(b)** **(c)**

Figure 3.1: 3D plots of the rational solution (3.6) in Case 1 with the values of $k = 2$, $w = 1$, $c = 1$, (a) Real, (b) Imaginary and (c) Complex.

Case 2:

$$\begin{aligned}
\zeta &= \frac{-\left(\begin{array}{c} bck^8 + 2ak^8 - 3bk^7w + 2bck^6 + ck^7 - 20ak^6 + 18bk^5w - 4k^6w + 8ak^5 + 12bck^4 \\ -8bk^4w + 10ck^5 - 16ak^4 - 28bck^3 + 4bk^3w + 8k^4w + 24ak^3 + 8bck^2 + 4bk^2w + 4ck^3 \\ -8k^3w - 16ak^2 - 8bck + 8bkw - 12ck^2 + 32ak - 24bw + 8ck + 16kw - 24c \end{array}\right)}{16k(k^3+2)}, \\
\delta &= \frac{(bc+2a)k^6 + (-3bw+c)k^5 + 4(bc-4a-w)k^4 + 4(3bw+2a+3c)k^3 + 8(-(c+w)b+2a)k^2 + 8((c-w)b-4a-c-w)k + 24(bw+c)}{64k(k^3+2)}, \\
\gamma &= \frac{-3((bc-2a)k+bw+c)}{4k(k^3+2)}, \\
\beta &= \frac{-(bc+2a)k^5 + (3bw-c)k^4 + 4(-bc+4a+w)k^3 - 4(3bw+2a+3c)k^2 + 8((-2c+w)b+4a)k + 8(2(c-w)b-2(a+c)+w)}{16k^3+32}, \\
\sigma &= \frac{(bc-2a)k+bw+c}{4k^3+8}.
\end{aligned} \tag{3.7}$$

By using values in (3.7) into (3.1), we have

$$f = b_0 \cosh z + b_1 \sinh z. \tag{3.8}$$

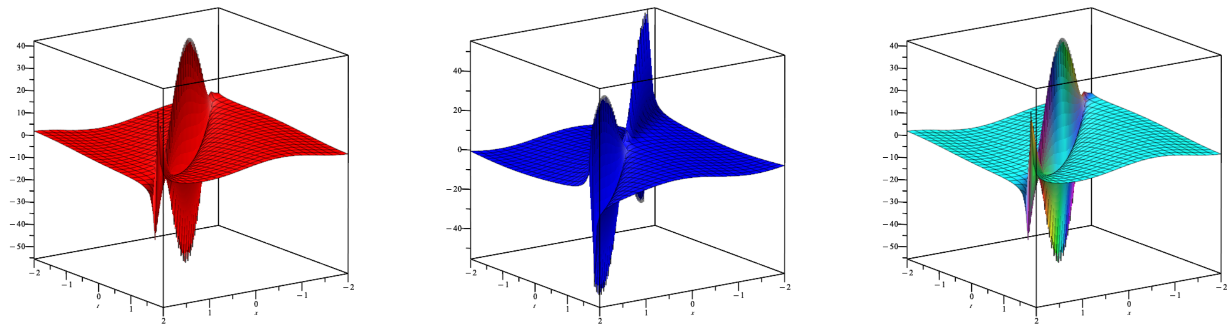
By using $y = 2(\ln f)_z$, we have

$$y = \frac{2(b_0 \sinh(z) + b_1 \cosh(z))}{b_0 \cosh(z) + b_1 \sinh(z)}. \tag{3.9}$$

By using Eq. (3.9) into Eq. (2.1), we obtain a second type of rational hyperbolic solution of Eq. (1.1):

$$q(x,t) = \frac{2(b_0 \sinh(z) + b_1 \cosh(z))e^{i(kx-wt)}}{b_0 \cosh(z) + b_1 \sinh(z)}. \tag{3.10}$$

(See Figure 3.2)



(a) (b) (c)

Figure 3.2: 3D plots of the rational solution (3.10) in Case 2 with the values of $k = 2$, $w = 1$, $c = 0.1$, $b_0 = -5$, $b_1 = 0.1$, (a) Real, (b) Imaginary and (c) Complex.

4. Parabolic Solutions of Differential Equations Corresponding to Eq. (1.1)

To get parabolic solution of (2.5), we choose f as following:

$$f = b_2 z^2 + b_1 z + b_0 \tag{4.1}$$

where b_0, b_1 and b_2 represent any constant parameters. By substituting (4.1) into (2.5) and equating the various coefficient terms of z, we then solve the following system of algebraic equations to find the values of the parameters:

$$2b_2^4(4k^3\sigma - bck + 2ak - bw - c) = 0$$

$$6b_1b_2^3(4k^3\sigma - bck + 2ak - bw - c) = 0,$$

$$b_2^2\left((8k^3\sigma - 2bck + 4ak - 2bw - 2c)b_2b0 + (28k^3\sigma - 7bck + 14ak - 7bw - 7c)b_1^2 + (-64\gamma k - 48\sigma)b_2^2\right) = 0,$$

$$4b_1b_2\left((4k^3\sigma - bck + 2ak - bw - c)b_2b0 + (4k^3\sigma - bck + 2ak - bw - c)b_1^2 + (-32\gamma k - 24\sigma)b_2^2\right) = 0,$$

$$\begin{bmatrix} \left((16k^3\sigma - 4bck + 8ak - 4bw - 4c)b_2b_1^2 + (64\gamma k + 288\sigma)b_2^3\right)b_0, \\ +(-8k^3\sigma + 2bck - 4ak + 2bw + 2c)b_2^2b0^2 + (4k^3\sigma - bck + 2ak - bw - c)b_1^4 + (-112\gamma k - 144\sigma)b_2^2b_1^2 \end{bmatrix} = 0,$$

$$-2b_1\left((4k^3\sigma - bck + 2ak - bw - c)b_2b0^2 + ((-4k^3\sigma + bck - 2ak + bw + c)b_1^2 + (-32\gamma k - 144\sigma)b_2^2)b0 + (24\gamma k + 48\sigma)b_2b_1^2\right) = 0,$$

$$\begin{bmatrix} (4k^3\sigma - bck + 2ak - bw - c)b_1^2 - 48b_2^2\sigma \\ (-8k^3\sigma + 2bck - 4ak + 2bw + 2c)b_2b_0^3 + (16\gamma k + 96\sigma)b_2b_1^2b0 + (-8\gamma k - 24\sigma)b_1^4 \end{bmatrix} = 0, \tag{4.2}$$

$$2b_2^5(k^4\sigma+ak^2-bkw-w)=0,$$

$$9b_1b_2^4(k^4\sigma+ak^2-bkw-w)=0,$$

$$-4b_2^3\left((-2k^4\sigma-2ak^2+2bkw+2w)b_2b_0+(-4k^4\sigma-4ak^2+4bkw+4w)b_1^2+(-8\beta k^2+8\gamma k^2+6k^2\sigma-bc+8\zeta+a)b_2^2\right)=0,$$

$$-14b_1b_2^2\left((-2k^4\sigma-2ak^2+2bkw+2w)b_2b_0+(-k^4\sigma-ak^2+bkw+w)b_1^2+(-8\beta k^2+8\gamma k^2+6k^2\sigma-bc+8\zeta+a)b_2^2\right)=0,$$

$$-2b_2\left[\begin{array}{c}(-18k^4\sigma-18ak^2+18bkw+18w)b_2b_1^2+(-32\beta k^2+32\gamma k^2-12k^2\sigma+2bc+32\zeta-2a)b_2^3\\(-6k^4\sigma-6ak^2+6bkw+6w)b_2^2b_0^2+(-3k^4\sigma-3ak^2+3bkw+3w)b_1^4\\+(-76\beta k^2+76\gamma k^2+66k^2\sigma-11bc+76\zeta+11a)b_2^2b_1^2+(-16\beta-256\delta-32\gamma-24\sigma)b_2^4\end{array}\right]=0,$$

$$-b_1\left[\begin{array}{c}((-20k^4\sigma-20ak^2+20bkw+20w)b_2b_1^2+(-160\beta k^2+160\gamma k^2-60k^2\sigma+10bc+160\zeta-10a)b_2^3)b_0\\+(-30k^4\sigma-30ak^2+30bkw+30w)b_2^2b_0^2+(-k^4\sigma-ak^2+bkw+w)b_1^4\\(-100\beta k^2+100\gamma k^2+120k^2\sigma-20bc+100\zeta+20a)b_2^2b_1^2+(-80\beta-1280\delta-160\gamma-120\sigma)b_2^4\end{array}\right]=0,$$

$$\left[\begin{array}{c}((24k^4\sigma+24ak^2-24bkw-24w)b_2b_1^2+(32\beta k^2-32\gamma k^2+120k^2\sigma-20bc-32\zeta+20a)b_2^3)b_0^2\\+(8k^4\sigma+8ak^2-8bkw-8w)b_2^2b_0^3+(32\beta k^2-32\gamma k^2-60k^2\sigma+10bc-32\zeta-10a)b_2b_1^4\\+(96\beta+1280\delta+208\gamma+240\sigma)b_2^3b_1^2\\+((4k^4\sigma+4ak^2-4bkw-4w)b_1^4+(144\beta k^2-144\gamma k^2-144\zeta)b_2^2b_1^2+(-64\beta-192\gamma-480\sigma)b_2^4)b_0\end{array}\right]=0,$$

$$-2b_1\left[\begin{array}{c}((-3k^4\sigma-3ak^2+3bkw+3w)b_1^2+(-24\beta k^2+24\gamma k^2-90k^2\sigma+15bc+24\zeta-15a)b_2^2)b_0^2\\+(-6k^4\sigma-6ak^2+6bkw+6w)b_2b_0^3+(-2\beta k^2+2\gamma k^2+6k^2\sigma-bc+2\zeta+a)b_1^4\\+(-32\beta-320\delta-76g-120\sigma)b_2^3b_1^2\\+((-28\beta k^2+28\gamma k^2+30k^2\sigma-5bc+28\zeta+5a)b_2b_1^2+(48\beta+144\gamma+360\sigma)b_2^3)b_0\end{array}\right]=0,$$

$$\left[\begin{array}{c}(2k^4\sigma+2ak^2-2bkw-2w)b_2b_0^4+((4k^4\sigma+4ak^2-4bkw-4w)b_1^2+(72k^2\sigma-12bc+12a)b_2^2)b_0^3\\+((24\beta k^2-24\gamma k^2+36k^2\sigma-6bc-24\zeta+6a)b_2b_1^2+(32\beta+240\sigma)b_2^3)b_0^2\\+((8\beta k^2-8\gamma k^2-24k^2\sigma+4bc-8\zeta-4a)b_1^4+(-64\beta-144\gamma-480\sigma)b_2^2b_1^2)b_0+(24\beta+160\delta+56\gamma+120\sigma)b_2b_1^4\end{array}\right]=0,$$

$$-b_1\left[\begin{array}{c}(-k^4\sigma-ak^2+bkw+w)b_0^4+(-36k^2\sigma+6bc-6a)b_2b_0^3+(-4\beta-16\delta-8\gamma-24\sigma)b_1^4\\+((-4\beta k^2+4\gamma k^2+12k^2\sigma-2bc+4\zeta+2a)b_1^2+(-16\beta-120\sigma)b_2^2)b_0^2+(16\beta+24\gamma+120\sigma)b_2b_1^2b_0\end{array}\right]=0.$$

Resolution of the system (4.2) with the help of Maple gives five cases of parametric values as follows:

Case 1:

$$a=\frac{4\gamma k^5+3bkw+3w}{3k^2},\quad b_0=b_1=0,\quad \beta=-\frac{12b\gamma k^6-24b\gamma k^5+20\gamma k^5-24\zeta bk^3-24\gamma k^4-24\zeta k^2-3w}{24k^4(bk+1)},$$
$$c=-\frac{8\gamma k^5-3bkw-6w}{3k(bk+1)},\quad \delta=\frac{60b\gamma k^6-72b\gamma k^5+68\gamma k^5-24\zeta bk^3-72\gamma k^4-24\zeta k^2-3w}{384k^4(bk+1)},\quad \sigma=-\frac{4\gamma k}{3}. \tag{4.3}$$

By using values in (4.3) into (4.1), we have

$$f=b_2z^2. \tag{4.4}$$

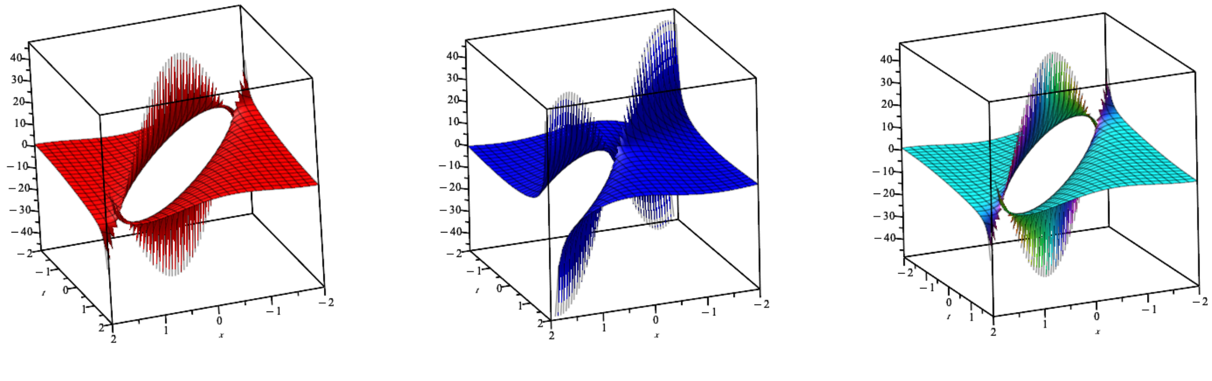
By using $y=2(\ln f)_z$, we have

$$y=\frac{4}{z}. \tag{4.5}$$

By replacing Eq. (2.1) with Eq. (4.5), we obtain the first type of rational parabolic solution of Eq. (1.1):

$$q(x,t)=\frac{4e^{i(kx-wt)}}{-ct+x}. \tag{4.6}$$

(See Figure 4.1)



(a) (b) (c)

Figure 4.1: 3D plots of the rational solution (4.6) in Case 1 with the values of $k = 2$, $w = 1$, $b = 128$, $\gamma = -3$, $b_1 = 0.1$, (a) Real, (b) Imaginary and (c) Complex.

Case 2:

$$a = \frac{4\gamma k^3}{3}, \quad b = -\frac{1}{k}, \quad b_0 = b_1 = 0, \quad \beta = \frac{-20\gamma k^4 + 24\gamma k^3 + 24\zeta k + 3c}{24k^3},$$
$$\delta = -\frac{-68\gamma k^4 + 72\gamma k^3 + 24\zeta k + 3c}{384k^3}, \quad \sigma = -\frac{4\gamma k}{3}, \quad w = \frac{8\gamma k^5}{3}. \tag{4.7}$$

By using values in (4.7) into (4.1), we have

$$f = b_2 z^2. \tag{4.8}$$

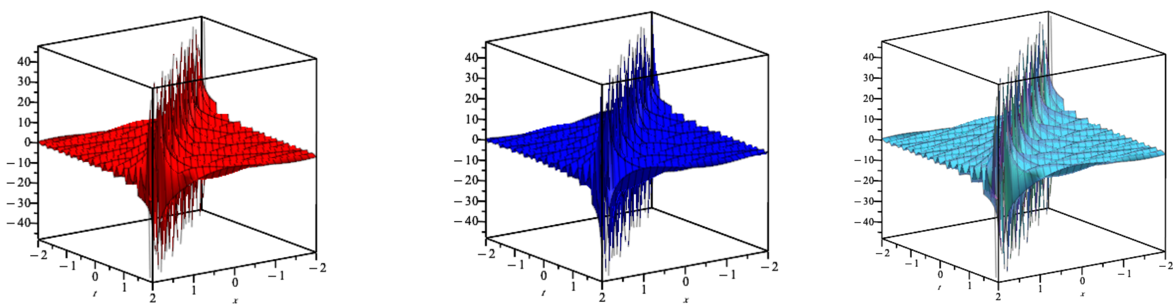
By using $y = 2(\ln f)_z$, we have

$$y = \frac{4}{z}. \tag{4.9}$$

By using Eq. (4.9) into Eq. (2.1), we obtain a second type of rational parabolic solution of Eq. (1.1):

$$q(x,t) = \frac{4e^{i\left(-\frac{8}{3}\gamma k^5 t + kx\right)}}{-ct + x}. \tag{4.10}$$

(See Figure 4.2)



(a) (b) (c)

Figure 4.2: 3D plots of the rational solution (4.10) in Case 2 with the values of $k = 2$, $c = 1$, $\gamma = 3$, (a) Real, (b) Imaginary and (c) Complex.

Case 3:

$$a = \frac{\gamma k^5 + 3bkw + 3w}{3k^2}, \quad b_2 = 0, \quad \beta = \frac{-3b\gamma k^6 + 6b\gamma k^5 - 5\gamma k^5 + 6\zeta bk^3 + 6\gamma k^4 + 6\zeta k^2 + 3w}{6k^4(bk+1)},$$
$$c = \frac{-2\gamma k^5 + 3bkw + 6w}{3k(bk+1)}, \quad \delta = -\frac{-15b\gamma k^6 + 18b\gamma k^5 - 17\gamma k^5 + 6\zeta bk^3 + 18\gamma k^4 + 6\zeta k^2 + 3w}{24k^4(bk+1)}, \quad \sigma = -\frac{\gamma k}{3}. \tag{4.11}$$

By using values in (4.11) into (4.1), we have

$$f = b_1 z + b_0. \tag{4.12}$$

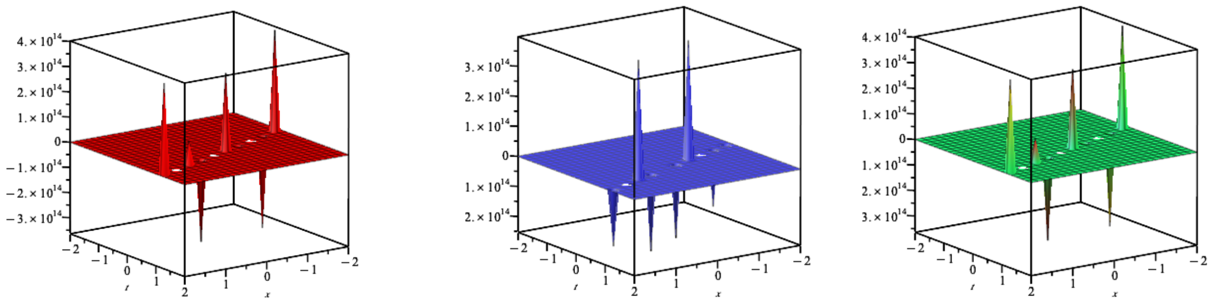
By using $y = 2(\ln f)_z$, we have

$$y = \frac{2b_1}{b_1 z + b_0}. \tag{4.13}$$

By replacing Eq. (2.1) with Eq. (4.13) , we obtain a third type of rational parabolic solution of Eq. (1.1):

$$q(x,t) = \frac{2b_1 e^{i(kx-wt)}}{b_1\left(-\frac{(-2\gamma k^5+3bkw+6w)t}{3k(bk+1)}+x\right)+b_0}. \tag{4.14}$$

(See Figure 4.3)



(a) **(b)** **(c)**

Figure 4.3: 3D plots of the rational solution (4.14) in Case 3 with the values of $b_1 = 1$, $b_0 = 1$, $k = 2$, $w = 1$, $b = \frac{-5}{12}$, $\gamma = \frac{1}{64}$, (a) Real, (b) Imaginary and (c) Complex.

Case 4:

$$a = \frac{k^3\gamma}{3}, \quad b = -\frac{1}{k}, \quad b_2 = 0, \quad \beta = \frac{-5\gamma k^4+6\gamma k^3+6\zeta k+3c}{6k^3},$$
$$\delta = -\frac{-17\gamma k^4+18\gamma k^3+6\zeta k+3c}{24k^3}, \quad \sigma = -\frac{k\gamma}{3}, \quad w = \frac{2k^5\gamma}{3}. \tag{4.15}$$

By using values in (4.15) into (4.1), we have

$$f = b_1 z + b_0. \tag{4.16}$$

By using $y = 2(\ln f)_z$, we have

$$y = \frac{2b_1}{b_1 z + b_0}. \tag{4.17}$$

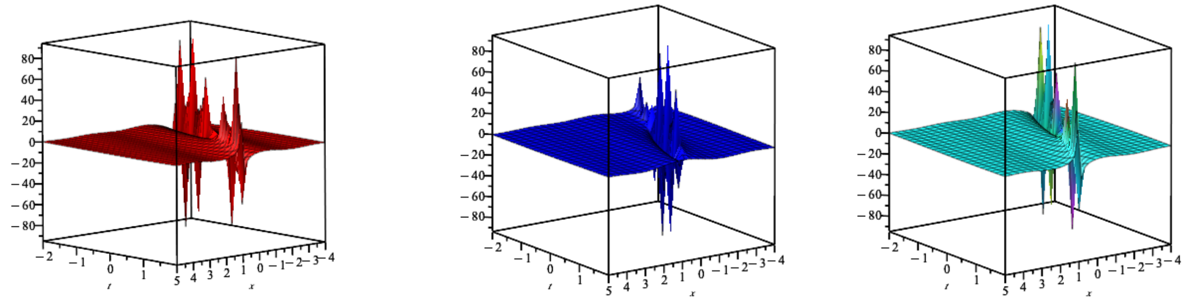
By using Eq. (4.17) into Eq. (2.1), we obtain a fourth type of rational parabolic solution of Eq. (1.1):

$$q(x,t) = \frac{2b_1 e^{i\left(\frac{-2}{3}\gamma k^5 t+kx\right)}}{b_1(-ct+x)+b_0}. \tag{4.18}$$

(See Figure 4.4)

Case 5:

$$a = \frac{4\gamma k^5+3bkw+3w}{3k^2}, \quad b_2 = \frac{b_1^2}{4b_0}, \quad \beta = \frac{-12b\gamma k^6+24b\gamma k^5-20\gamma k^5+24\zeta bk^3+24\gamma k^4+24\zeta k^2+3w}{24k^4(bk+1)},$$
$$c = \frac{-8\gamma k^5+3bkw+6w}{3k(bk+1)}, \quad \delta = -\frac{-60b\gamma k^6+72b\gamma k^5-68\gamma k^5+24\zeta bk^3+72\gamma k^4+24\zeta k^2+3w}{384k^4(bk+1)}, \quad \sigma = -\frac{4\gamma k}{3}. \tag{4.19}$$



(a) (b) (c)

Figure 4.4: 3D plots of the rational solution (4.18) in Case 4 with the values of $b_1 = 1$, $b_0 = 1$, $k = 2$, $w = 1$, $b = \frac{-5}{12}$, $\gamma = \frac{1}{64}$, $c = 1$, (a) Real, (b) Imaginary and (c) Complex.

By using values in (4.19) into (4.1), we have

$$f = \frac{b_1^2}{4b_0} z^2 + b_1 z + b_0. \tag{4.20}$$

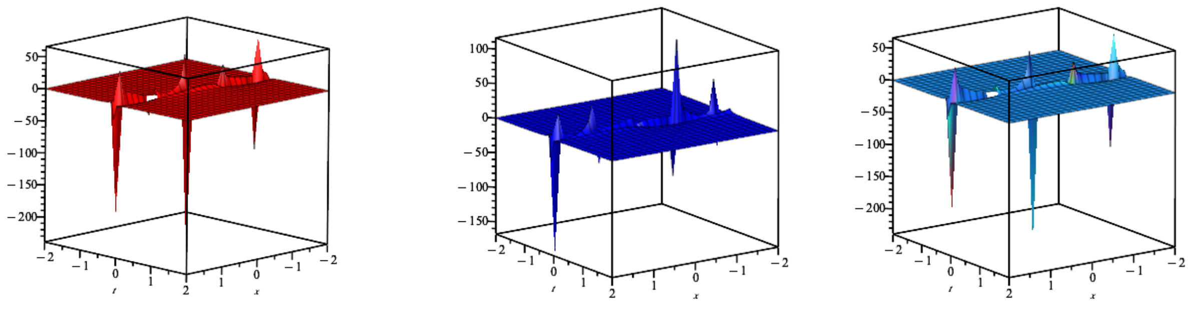
By using $y = 2(\ln f)_z$, we have

$$y = \frac{4b_1}{b_1 z + 2b_0}. \tag{4.21}$$

By using Eq. (4.21) into Eq. (2.1), we obtain a fifth type of rational parabolic solution of Eq. (1.1):

$$q(x,t) = \frac{4b_1 e^{i(kx-wt)}}{b_1\left(-\frac{(-8\gamma k^5 + 3bkw + 6w)t}{3k(bk+1)} + x\right) + 2b_0}. \tag{4.22}$$

(See Figure 4.5)



(a) (b) (c)

Figure 4.5: 3D plots of the rational solution (4.22) in Case 5 with the values of $b_1 = 2$, $b_0 = -1$, $k = 2$, $w = 1$, $b = \frac{-1}{3}$, $\gamma = 0.1$, (a) Real, (b) Imaginary and (c) Complex.

Case 6:

$$a = \frac{4k^3\gamma}{3}, \quad b = \frac{-1}{k}, \quad b_2 = \frac{b_1^2}{4b_0}, \quad \beta = \frac{-20\gamma k^4 + 24\gamma k^3 + 24\zeta k + 3c}{24k^3},$$
$$\delta = -\frac{-68\gamma k^4 + 72\gamma k^3 + 24\zeta k + 3c}{384k^3}, \quad \sigma = -\frac{4\gamma k}{3}, \quad w = \frac{8k^5\gamma}{3}. \tag{4.23}$$

By using values in (4.23) into (4.1), we have

$$f = \frac{b_1^2}{4b_0} z^2 + b_1 z + b_0. \tag{4.24}$$

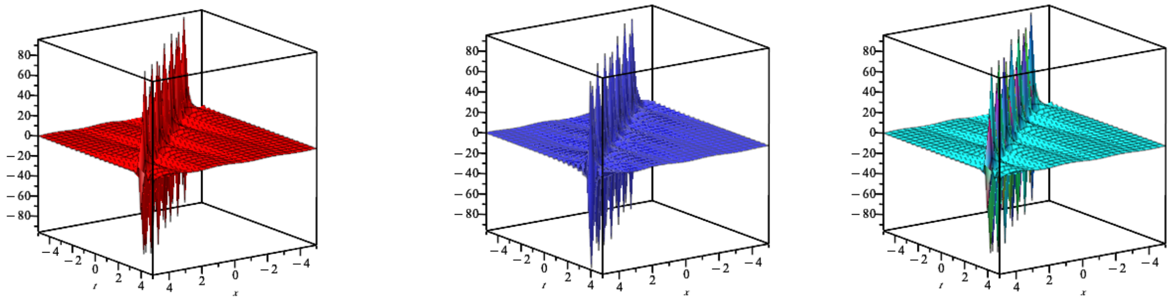
By using $y = 2(\ln f)_z$, we have

$$y = \frac{4b_1}{b_1 z + 2b_0}. \tag{4.25}$$

By using Eq. (4.25) into Eq. (2.1), we obtain a sixth type of rational parabolic solution of Eq. (1.1):

$$q(x,t) = \frac{4b_1 e^{i\left(kx - \frac{8k^5\gamma}{3}t\right)}}{b_1(x-ct)+2b_0}. \tag{4.26}$$

(See Figure 4.6)



(a) (b) (c)

Figure 4.6: 3D plots of the rational solution (4.26) in Case 5 with the values of $b_1 = 2$, $b_0 = -1$, $k = 2$, $c = 1$, $\gamma = 3$, (a) Real, (b) Imaginary and (c) Complex.

5. Trigonometric Solutions of Differential Equations Corresponding to Eq. (1.1)

To get trigonometric solution, we suppose a solution of (2.5) in the following form:

$$f = b_0 \cos z + b_1 \sin z \tag{5.1}$$

where b_0, b_1 are all constants. Inserting (5.1) into (2.5) and setting the coefficient terms that are containing independent combinations of cos and sin functions to zero, we get a system of algebraic equations:

$$\begin{aligned}
&4b_1^4(4k^3\sigma - bck + 2ak - bw + 8\gamma k - c + 16\sigma) = 0,\\
&2b_1^4(4k^3\sigma - bck + 2ak - bw - 8\gamma k - c - 32\sigma) = 0,\\
&4b_1^5(k^4\sigma + ak^2 - bkw - 4\beta k^2 + 4\gamma k^2 + 4\zeta + 16\delta - w) = 0,\\
&8b_1^5(k^4\sigma + ak^2 - bkw - 2\beta k^2 + 2\gamma k^2 - 6k^2\sigma + bc + 2\zeta - a - 4\gamma - 4\sigma - w) = 0,\\
&-8b_1^5(-2\beta k^2 + 2\gamma k^2 + 6k^2\sigma - bc + 2\zeta + a + 16\delta + 4\gamma + 4\sigma) = 0,\\
&-b_1^5(3k^4\sigma + 3ak^2 - 3bkw - 4\beta k^2 + 4\gamma k^2 - 24k^2\sigma + 4bc + 4\zeta - 4a - 16\beta - 80\delta - 48\gamma - 112\sigma - 3w) = 0,\\
&b_1^5(-k^4\sigma - ak^2 + bkw - 4\beta k^2 + 4\gamma k^2 + 24k^2\sigma - 4bc + 4\zeta + 4a - 16\beta - 16\delta - 16\gamma - 80\sigma + w) = 0.
\end{aligned} \tag{5.2}$$

After solving the system (5.2) with the help of Maple, we obtain three cases of parametric values as follows:

Case 1:

$$
\begin{aligned}
a &= \frac{\begin{pmatrix} 3b\gamma k^7+3b^2ck^4+18b\gamma k^5+4\gamma k^6-8b\gamma k^4+12b^2ck^2 \\ +6bck^3-4b\gamma k^3+8\gamma k^4-4b\gamma k^2-8\gamma k^3+6b^2c \\ +12bck+8b\gamma k+3ck^2+12\zeta b-24b\gamma-16\gamma k+6c \end{pmatrix}}{3(bk^4+4bk^2+2k^3+2b+4k)}, \\
\beta &= -\frac{\begin{pmatrix} 3b\gamma k^5-6b\gamma k^4-20b\gamma k^3+8\gamma k^4-6\zeta bk^2 \\ +8b\gamma k^2-12\gamma k^3-24b\gamma k-40\gamma k^2-12\zeta b \\ -12\zeta k+24b\gamma+32\gamma k-3c \end{pmatrix}}{6(bk^4+4bk^2+2k^3+2b+4k)}, \\
\delta &= \frac{\begin{pmatrix} 15b\gamma k^5-18b\gamma k^4+28b\gamma k^3+32\gamma k^4 \\ -6\zeta bk^2-40\gamma k^2-36\gamma k^3+8b\gamma k^2 \\ -12\zeta b-12\zeta k-16\gamma k-3c \end{pmatrix}}{24(bk^4+4bk^2+2k^3+2b+4k)}, \\
w &= \frac{\begin{pmatrix} 2\gamma k^8+3bck^5+20\gamma k^6-8\gamma k^5+12bck^3 \\ +3ck^4-16\gamma k^4+24\gamma k^3+6bck+16\gamma k^2 \\ +24\zeta k-32\gamma k-6c \end{pmatrix}}{3(bk^4+4bk^2+2k^3+2b+4k)}, \\
\sigma &= -\frac{1}{3}\gamma k.
\end{aligned}
\tag{5.3}
$$

By utilizing values in (5.3) into (5.1), we have

$$f = b_1\cos z + b_1\sin z. \tag{5.4}$$

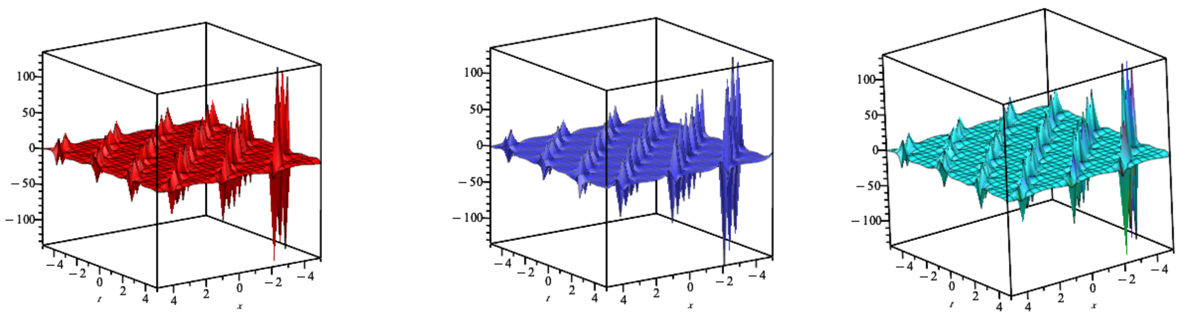
Inserting Eq. (5.4) into (2.4) yields

$$y = -\frac{2(\sin(z)-\cos(z))}{\sin(z)+\cos(z)}. \tag{5.5}$$

By using Eq. (5.5) into Eq. (2.1), we obtain a first type of trigonometric solution of Eq. (1.1):

$$q(x,t) = -\frac{2(\sin(x-ct)-\cos(x-ct))e^{i(kx-wt)}}{\sin(x-ct)+\cos(x-ct)}. \tag{5.6}$$

(See Figure 5.1)



(a) **(b)** **(c)**

Figure 5.1: 3D plots of the rational solution (5.6) in Case 1 with the values of $k=2$, $c=1$, $\gamma=3$, $b=1$, $\zeta=-1$ (a) Real, (b) Imaginary and (c) Complex.

Case 2:

$$a=-b^2w+2\zeta-4\gamma-w,\quad \beta=\zeta-2\gamma-\frac{1}{4}w,\quad c=-bw,\quad \delta=-\frac{1}{4}\zeta+\frac{1}{16}w,\quad k=0,\quad \sigma=0, \tag{5.7}$$

Using the obtained values in (5.1) gives

$$f = b_1\cos z + b_1\sin z. \tag{5.8}$$

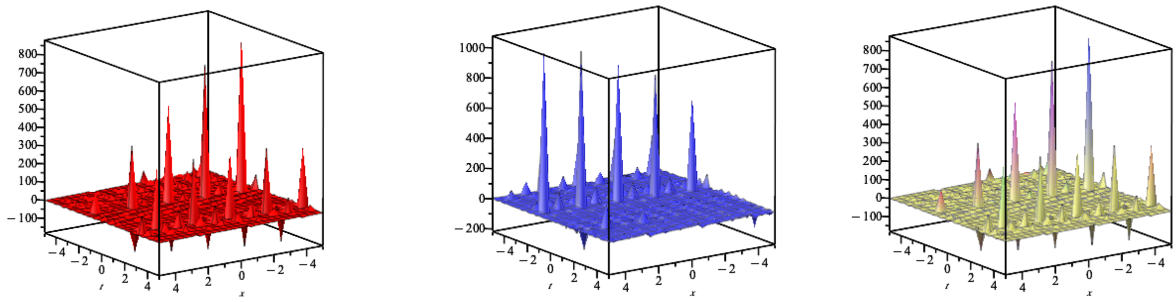
Inserting Eq. (5.8) into (2.4) yields

$$y = -\frac{2(\sin(z)-\cos(z))}{\sin(z)+\cos(z)}. \tag{5.9}$$

By using Eq. (5.9) into Eq. (2.1), we obtain a second type of trigonometric solution of Eq. (1.1):

$$q(x,t) = -\frac{2(\sin(x-ct)-\cos(x-ct))e^{i(kx-wt)}}{\sin(x-ct)+\cos(x-ct)}. \tag{5.10}$$

(See Figure 5.2)



(a) (b) (c)

Figure 5.2: 3D plots of the rational solution (5.10) in Case 2 with the values of $b=10$, $w=1$ (a) Real, (b) Imaginary and (c) Complex.

Case 3:

$$\begin{aligned}
a &= -\frac{\left(\begin{array}{c} -\gamma k^{11} - 6\gamma k^9 - 50\gamma k^7 + 44\gamma k^6 + 3k^6 w - 44\gamma k^5 + 12\zeta k^4 + 72\gamma k^4 \\ +18k^4 w + 8\gamma k^3 + 40\gamma k^2 + 30k^2 w - 16\gamma k + 48\gamma + 12w - 24\zeta \end{array}\right)}{3(k^4+4k^2+2)^2}, \\
\beta &= \frac{\left(\begin{array}{c} -\gamma k^9 + 6\gamma k^8 + 28\gamma k^7 + 8\gamma k^6 + 6k^6\zeta + 82\gamma k^5 - 20\gamma k^4 - 3k^4 w + 36k^4\zeta \\ +152\gamma k^3 - 144\gamma k^2 - 12k^2 w + 84k^2\zeta + 48\gamma k - 48\gamma - 6w + 24\zeta \end{array}\right)}{6(k^4+4k^2+2)^2}, \\
\delta &= -\frac{\left(\begin{array}{c} -13\gamma k^9 + 18\gamma k^8 - 68\gamma k^7 + 104\gamma k^6 + 6k^6\zeta - 158\gamma k^5 + 220\gamma k^4 \\ -3k^4 w + 36k^4\zeta - 40\gamma k^3 + 48\gamma k^2 - 12k^2 w + 84k^2\zeta + 24\zeta - 6w \end{array}\right)}{24(k^4+4k^2+2)^2}, \\
c &= \frac{2k\left(\gamma k^7 + 10\gamma k^5 - 4\gamma k^4 - 8\gamma k^3 + 12\gamma k^2 + 8\gamma k - 16\gamma + 12\zeta\right)}{3(k^4+4k^2+2)}, \quad b = -\frac{2(k^2+2)k}{k^4+4k^2+2}, \quad \sigma = -\frac{1}{3}\gamma k.
\end{aligned} \tag{5.11}$$

By utilizing values in (5.11) into (5.1), we have

$$f = b_1\cos z + b_1 \sin z. \tag{5.12}$$

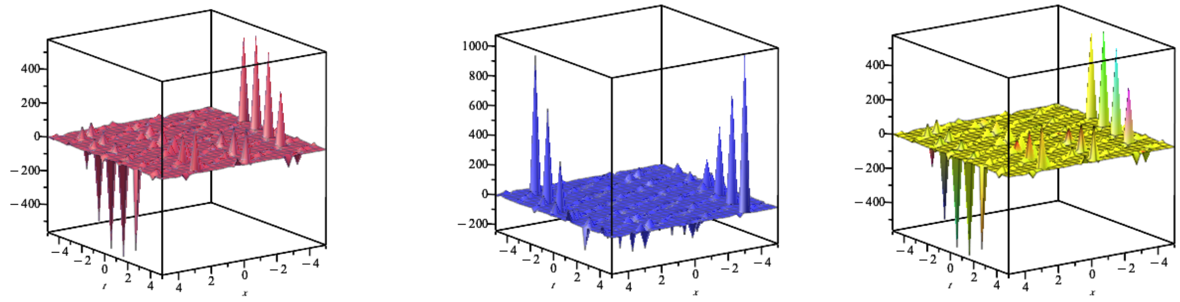
Inserting Eq. (5.12) into (2.4) yields

$$y = -\frac{2(\sin(z)-\cos(z))}{\sin(z)+\cos(z)}. \tag{5.13}$$

By using Eq. (5.13) into Eq. (2.1), we obtain a first type of trigonometric solution of Eq. (1.1):

$$q(x,t) = -\frac{2(\sin(x-ct)-\cos(x-ct))e^{i(kx-wt)}}{\sin(x-ct)+\cos(x-ct)}. \tag{5.14}$$

(See Figure 5.3)



(a) **(b)** **(c)**

Figure 5.3: 3D plots of the rational solution (5.14) in Case 3 with the values of $k=5$, $b=10$, $\gamma=0.5$, $w=1$, $\zeta=-1$ (a) Real, (b) Imaginary and (c) Complex.

6. Conclusion

New soliton solutions of the LPD equation were obtained with three different current, systematic and powerful methods. In order to understand how the obtained solutions change under different conditions, the solutions obtained by appropriate selection of some parameters affecting the shape and velocity of the solitons are observed. In this context, to understand the mechanism of the original equation (1.1) real, imaginary and complex three-dimensional plots have been drawn for each case of solitons. This paper presents novel solutions of LPD equation that have not been reported in the literature before. Also, comparing with the existing literature, our result is complete and our method is simple and direct. By providing novel solutions, this study contributes to the knowledge of the dynamical aspects of various physical phenomena that are modeled by the LPD equation.

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