Sinopfbd, 2023; 8(1): 65-74 https://doi.org/10.33484/sinopfbd.1292996

Sinop Üniversitesi Fen Bilimleri Dergisi
Sinop Uni J Nat Sci

ISSN: 2536-4383
E-ISSN: 2564-7873

https://dergipark.org.tr/tr/pub/sinopfbd

Examination of Dark and Bright Solitons of (2+1)-Dimensional Kundu-Mukherjee-Naskar Equation Via Unified Solver Technique

Volkan ALA[1]

How to cite: Ala, V. (2023). Examination of Dark and Bright Solitons of (2+1)-Dimensional Kundu-Mukherjee-Naskar Equation Via Unified Solver Technique. *Sinop Üniversitesi Fen Bilimleri Dergisi*, *8*(1), 65-74. https://doi.org/10.33484/sinopfbd.1292996

Research Article

Corresponding Author
Volkan ALA
volkanala@mersin.edu.tr

ORCID of the Authors
V.A: 0000-0002-8499-9979

Received: 05.05.2023
Accepted: 18.07.2023

Abstract

In this study, dark and bright solitons of the (2+1)-dimensional Kundu-Mukherjee-Naskar equation are constructed with unified solver in terms of He's variations method. In accordance with basic properties of proposed technique, some dark and bright solitons are obtained. Moreover, giving specific values to the achieved solutions, 2D and 3D graphics are plotted with the help of software package.The unified solver technique extract vital solutions in explicit way. It is an easy-to-use method applied to obtain various exact solutions of nonlinear partial differential equations arising in fluid mechanics, nuclear, plasma and particle physics.

Keywords: Wave transformation, unified solver technique, solitons

(2+1)-Boyutlu Kundu-Mukherjee-Naskar Denkleminin Birleşik Çözücü Teknik Yoluyla Dark ve Bright Solitonlarının Incelenmesi

[1]Mersin University, Science Faculty, Mathematics Department, Mersin, Türkiye

Öz

Bu çalışmada, (2+1)-boyutlu Kundu-Mukherjee-Naskar denkleminin dark ve bright soliton çözümleri varyasyonel metot aracılığıyla birleştirilmiş çözücü teknikle inşa edilmiştir. tekniğin temel özelliklerine uygun olarak dark ve bright soliton çözümleri elde edilmiştir. Ayrıca elde edilen çözümlere spesifik değerler verilerek, çözümlerin iki ve üç boyutlu grafikleri paket program aracılığıyla çizilmiştir. Birleşik çözüm tekniği, akışkanlar mekaniği, nükleer, plazma ve parçacık fiziğindeki kısmi diferansiyel denklemlerin çeşitli tam çözümlerini elde etmek için uygulanabilen kullanımı kolay bir yöntemdir.

Anahtar Kelimeler: Dalga dönüşümü, birleştirilmiş çözücü teknik, solitonlar

Introduction

Solitons are the solutions to a broad variety of weakly nonlinear dispersive partial differential equations that describe physical systems. The transfer of information in optical communication lines is based on soliton propagation in the study of Doran and Blow [1], Hausand and Wong [2]. For more than a decade the dynamic of solitons has received considerable interest in fibers [3–5]. Kundu-Mukherjee-Naskar

(KMN) is a significant equation that explains solitons in optical fibers and contains mixed forms of nonlinear effects. It is provided as follows:

$$i\psi_t + \alpha\psi_{xy} + i\beta\psi(\psi\psi_x^* - \psi^*\psi_x) = 0, \ \ i = \sqrt{-1}, \tag{1}$$

here $\psi(x, y, t)$ is wave function in nonlinear sense for optical solitons with the independent variables x, y and t, asterisk sign shows complex conjugation. While t denotes a temporal variable, x and y symbolize spatial variables. The parameter α represents the dispersion term while β ensures the presence of the distinct cases of nonlinearity media that does not fall into any of the forms non-Kerr and Kerr medias. This model can be used to explain the movement of oceanic rogue waves, optical fiber waves and ion-acoustic waves in a magnetic plasma [7–9]. First time, Kundu and Mukherjee [6] presented this equation in 2013.

In literarute Biswas,Vega-Guzman and et al. [10], Yıldırım and Mirzazadeh [11] used Sine–Gordon equation method; Yıldırım [12] used trial equation; Rizvi, Afzal and et al. [13] used tanh–coth; Al-Ghafri [14] used the ansatz approach; Mamedov, Demirbilek and et al. [15] used improved Bernoulli sub-equation function; Önder, Seçer and et al. [16] used Sardar sub-equation; Kumar, Paul and et al. [17] used new auxiliary equation; Günerhan, Khodadad and et al. [18] used extended direct algebraic; Kudryashov [19] and Petrovic [20] used the Weierstrass and Jacobi elliptic functions methods; Ekici, Sönmezoğlu and et al. [21] used extended trial function approach; Rezazadeh, Kurt and et al. [22] used functional variable technique; Çakıcıoğlu, Çınar and et al. [23] used modified extended tanh method; Mohammed, Al-Askar and et al. [24] used G'/G-expansion to get optical solitons of KMN equation.

In this work, optical soliton solutions of (1) are given via unified solver technique. Considered equation has still infant stage, therefore future research on it can focus on its potential applications in a variety of physical fields.

In the organization of this paper, in first section, basic structure of the unified solver technique is expressed. In second section, considered method is successfully applied to the governing model and graphical simulations of the solutions are plotted. Finally, some important conclusions and physical meanings of solutions are given in last section.

Essential Framework of Unified Solver Method

This section contains description of unified solver technique. Consider the nonlinear partial differential equation (NPDE) of the following form:

$$F\left(\phi, \phi_t, \phi_x, \phi_{tt}, \phi_{xx}, ...\right). \tag{2}$$

Applying the wave transformation $\phi\left(x, y, t\right) = \phi\left(\xi\right), \ \xi = k_1x + k_2y + k_3t$, (where $k_i = \overline{1, 3}$ are velocity of the wave) into (2) the following equation is obtained:

$$N\left(\phi, \phi^2, \phi', \phi'', ...\right) = 0, \tag{3}$$

here N is a nonlinear ordinary differential equation (NODE) that has partial derivatives of ϕ dependent on ξ. Based on He's semi-inverse method [25–27], the variational model for (3) can be obtained by the

ISSN: 2536-4383

semi-inverse method [27] which reads:

$$I(\phi) = \int \mathcal{L} d\xi, \tag{4}$$

where $\mathcal{L}$ is Lagrangian function connected with the derivative of ϕ given in the form:

$$\mathcal{L} = \frac{1}{2}\left(\phi'\right)^2 - Q, \tag{5}$$

here Q is the potential function. We look for a solitary wave solution in the form

$$\phi(x,y,t) = \lambda \text{sech}(\mu\xi), \phi(x,y,t) = \lambda \text{sech}^2(\mu\xi), \phi(x,y,t) = \lambda \tanh(\mu\xi), \tag{6}$$

where λ and μ are constants to be determined later. Assume that systems of equations can be reduced to the form:

$$\Omega_1\phi'' + \Omega_2\phi^3 + \Omega_3\phi = 0, \tag{7}$$

in which Ω_i, $i = \overline{1,3}$ are real coefficients. Multiplying (7) by ϕ' and taking integral with respect to ξ:

$$\frac{1}{2}\left(\phi'\right)^2 + \gamma_2\frac{\Omega_2}{4\Omega_1}\phi^4 + \frac{\Omega_3}{2\Omega_1}\phi^2 + \Omega_0 = 0, \tag{8}$$

where Ω_0 is a constant of integration. Thus (7) can be written in the form:

$$\phi'' = -\frac{\partial Q}{\partial \phi},\ Q = -\left(\gamma_2\phi^4 + \gamma_1\phi^2 + \gamma_0\right), \tag{9}$$

where

$$\gamma_2 = -\frac{\Omega_2}{4\Omega_1},\ \gamma_1 = -\frac{\Omega_3}{2\Omega_1},\ \gamma_0 = -\Omega_0. \tag{10}$$

Implementing the semi-inverse method [25–27] to solve (7) that constructs the following variational formulation from (8):

$$I = \int \left[\frac{1}{2}\left(\phi'\right)^2 + \gamma_2\phi^4 + \gamma_1\phi^2 + \ \gamma_0\right] d\xi. \tag{11}$$

Substituting (6) into (11) then making I stationary according to λ and μ:

$$\frac{\partial I}{\partial \lambda} = 0;\ \frac{\partial I}{\partial \mu} = 0. \tag{12}$$

Solving (12), we get λ and μ. Thus the solitary wave solution given by (6) is well determined.

The first family

The first family of solution is as follows:

$$\phi(\xi) = \lambda \text{sech}\theta,\ \theta = \mu\xi, \tag{13}$$

Substituting (13) into (11), we get

$$I = \frac{1}{\mu}\int_{0}^{\infty}\left[\frac{1}{2}\lambda^2\mu^2\text{sech}^2\theta\tanh^2\theta + \gamma_2\lambda^4\text{sech}^4\theta + \gamma_1\lambda^2\text{sech}^2\theta + \gamma_0\right]d\theta.$$

Taking $\gamma_0 = 0$ as an integration constant, it is obtained:

$$I = \frac{\lambda}{12\mu}\left[2\lambda\mu^2 + 8\gamma_2\lambda^3 + 12\gamma_1\lambda\right]. \tag{14}$$

Making I stationary in relation to λ and μ results in

$$\frac{\partial I}{\partial\lambda} = \frac{1}{12\mu}\left[32\gamma_2\lambda^3 + 24\gamma_1\lambda + 4\lambda\mu^2\right] = 0, \tag{15}$$

$$\frac{\partial I}{\partial\mu} = -\frac{\lambda}{12\mu^2}\left[8\gamma_2\lambda^3 + 12\gamma_1\lambda - 2\lambda\mu^2\right] = 0. \tag{16}$$

Solving these equations and using (13), the solution of (9) takes the form:

$$\phi(\xi) = \pm\sqrt{\frac{-\gamma_1}{\gamma_2}}\text{sech}\left(\pm\sqrt{2\gamma_1}\xi\right). \tag{17}$$

Using (10), the first family of solution can be written as:

$$\phi(\xi) = \pm\sqrt{\frac{-2\Omega_3}{\Omega_2}}\text{sech}\left(\pm\sqrt{\frac{-\Omega_3}{\Omega_1}}\xi\right). \tag{18}$$

The second family

The second family of solution is as follows:

$$\phi(\xi) = \lambda\text{sech}^2\theta,\ \theta = \mu\xi. \tag{19}$$

The substitution of (19) into (11) leads to

$$I = \frac{\lambda}{\mu}\int_{0}^{\infty}\left[2\lambda\mu^2\text{sech}^4\theta\tanh^2\theta + \gamma_2\lambda^3\text{sech}^8\theta + \gamma_1\lambda\text{sech}^4\theta + \frac{\gamma_0}{\lambda}\right]d\theta. \tag{20}$$

Suppose that $\gamma_0 = 0$ then

$$I = \frac{2\lambda^2}{105\mu}\left[14\lambda\mu^2 + 24\gamma_2\lambda^3 + 35\gamma_1\lambda\right]. \tag{21}$$

Making I stationary in relation to λ and μ results in

$$14\lambda\mu^2 + 48\gamma_2\lambda^3 + 35\gamma_1\lambda = 0, \tag{22}$$

ISSN: 2536-4383

$$14\lambda\mu^2 - 24\gamma_2\lambda^3 - 35\gamma_1\lambda = 0. \tag{23}$$

Solving (22)-(23) and using (19), solutions of (9) and (7) have following format:

$$\phi(\xi) = \pm\sqrt{\frac{-35\gamma_1}{36\gamma_2}}\operatorname{sech}^2\left(\pm\sqrt{\frac{5}{6}\gamma_1}\xi\right) \tag{24}$$

Using (10), the second family of solution can be written as:

$$\phi(\xi) = \pm\sqrt{\frac{-35\Omega_3}{18\Omega_2}}\operatorname{sech}^2\left(\pm\sqrt{-\frac{5\Omega_3}{12\Omega_1}}\xi\right). \tag{25}$$

The third family

Third family of solution is as follows:

$$\phi(\xi) = \lambda\tanh(\theta), \theta = \mu\xi. \tag{26}$$

Substituting (26) into (11), we have

$$\begin{aligned} I &= \frac{1}{\mu}\int_0^\infty \left[\frac{1}{2}\lambda^2\mu^2\operatorname{sech}^4\theta + \gamma_2\lambda^4\tanh^4\theta + \gamma_1\lambda^2\tanh^2\theta + \gamma_0\right]d\theta \\ &= \frac{1}{\mu}\int_0^\infty \left[\lambda^2\left(\gamma_2\lambda^2 + \frac{1}{2}\mu^2\right)\operatorname{sech}^4\theta - \lambda^2\left(2\gamma_2\lambda^2 + \gamma_1\right)\operatorname{sech}^2\theta\right]d\theta \\ &\quad + \frac{1}{\mu}\left(\gamma_2\lambda^4 + \gamma_1\lambda^2 + \gamma_0\right)\int_0^\infty d\theta. \end{aligned}$$

Under the condition

$$\gamma_0 = -\lambda^2\left(\gamma_2\lambda^2 + \gamma_1\right), \tag{27}$$

we find that

$$I = \frac{-\lambda^2}{3\mu}\left[4\gamma_2\lambda^2 + 3\gamma_1 - \mu^2\right]. \tag{28}$$

By resolving the two requirements, the values of λ and μ that makes I to be stationary with respect to λ and μ are found:

$$\frac{\partial I}{\partial\lambda} = \frac{-\lambda}{3\mu}\left[16\gamma_2\lambda^2 + 6\gamma_1 - 2\mu^2\right] = 0, \tag{29}$$

$$\frac{\partial I}{\partial \mu} = -\frac{\lambda^2}{3\mu^2}\left[4\gamma_2\lambda^2 + 3\gamma_1 + \mu^2\right] = 0. \tag{30}$$

Solving (29)-(30) and using (26), solution of (9) take the form:

$$\phi(\xi) = \pm\sqrt{\frac{-\gamma_1}{2\gamma_2}}\tanh\left(\pm\sqrt{-\gamma_1}\xi\right). \tag{31}$$

Resultantly, the third family of solution in (7) has the following structure:

$$\phi(\xi) = \pm\sqrt{\frac{-\Omega_3}{\Omega_2}}\tanh\left(\pm\sqrt{\frac{\Omega_3}{2\Omega_1}}\xi\right). \tag{32}$$

Application of The Proposed Method

Let us take the following stance to obtain the precise exact solutions of (1):

$$\psi(x,y,t) = u(\xi)\,e^{i\Phi(x,y,t)}, \tag{33}$$

$u(\xi)$, $\Phi(x,y,t)$ represent the amplitude and phase portion in the order given,

$$\xi = \delta_1 x + \delta_2 y - \delta_3 t, \Phi(x,y,t) = -\eta_1 x - \eta_2 y + \eta_3 t, \tag{34}$$

where δ_i, η_i $(i = 1, 2, 3)$ are real parameters different than zero. Also δ_1 and δ_2 are the width of the soliton along $x-$ and $y-$ directions respectively, whereas δ_3 is the velocity of the soliton. The parameters η_1 and η_2 indicate the soliton frequencies in the x- and y- directions respectively, η_3 represents the soliton wave number. Taking (34) and (33) into (1) produces imaginary and real parts as:

$$\delta_3 = -\alpha(\eta_2\delta_1 + \eta_1\delta_2), \tag{35}$$

$$\alpha\delta_1\delta_2\psi'' - (\eta_3 + \alpha\eta_1\eta_2)\psi - 2\beta\eta_1\psi^3 = 0. \tag{36}$$

If we compare (36) with (7) it can be easily seen that

$$\Omega_1 = \alpha\delta_1\delta_2,\ \Omega_2 = -(\eta_3 + \alpha\eta_1\eta_2),\ \Omega_3 = -2\beta\eta_1. \tag{37}$$

Having described the unified solver in previous section, below the formulae can be used to provide solutions to (1):

The first family solution

Taking into account (18), (32) and (33) the first family solution of (1) is get as:

$$\psi_{1,2}(x,y,t)=\pm i\sqrt{\frac{4\beta\eta_1}{(\eta_3+\alpha\eta_1\eta_2)}}\operatorname{sech}\left(\pm\sqrt{\frac{2\beta\eta_1}{\alpha\delta_1\delta_2}}(\delta_1 x+\delta_2 y-\delta_3 t)\right)e^{-i(\eta_1 x-\eta_2 y+\eta_3 t)}. \tag{38}$$

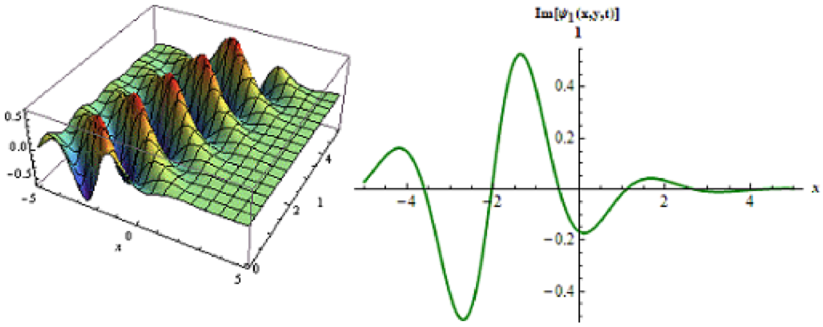

Figure 1. *3D graphs of* $\psi_1(x,y,t)$ *for* $\alpha=1;\ \beta=0.5;\ \eta_1=2;\ \eta_2=2.5;\ \eta_3=5;\ y=1;\ \delta_1=1;\ \delta_2=2.5;\ \delta_3=0.5;\ -5<x<5, -5<t<5$*, 2D plot for* $t=1$.

The second family solution

Considering (25), (33) and (34) the second family solution of (1) is obtained as:

$$\psi_{3,4}(x,y,t)=\pm i\sqrt{\frac{4\beta\eta_1}{18(\eta_3+\alpha\eta_1\eta_2)}}\operatorname{sech}^2\left(\pm\sqrt{\frac{5\beta\eta_1}{6\alpha\delta_1\delta_2}}(\delta_1 x+\delta_2 y-\delta_3 t)\right)e^{-i(\eta_1 x-\eta_2 y+\eta_3 t)}. \tag{39}$$

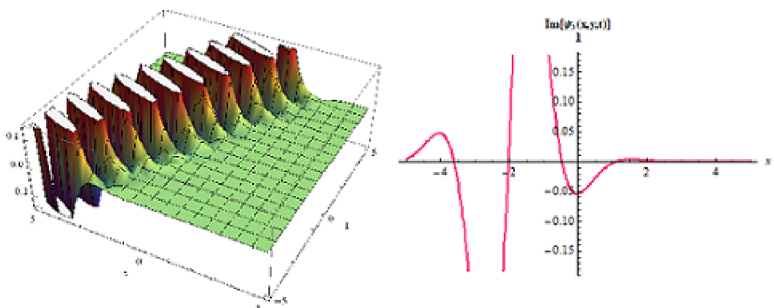

Figure 2. *3D graphs of* $\psi_3(x,y,t)$ *for* $\alpha=1;\ \beta=0.5;\ \eta_1=2;\ \eta_2=2.5;\ \eta_3=5;\ y=1;\ \delta_1=1;\ \delta_2=2.5;\ \delta_3=0.5;\ -5<x<5, 0<t<5$*, 2D plot for* $t=1$.

The third family solution

Considering (32), (33) and (34) the third family solution of (1) is obtained as:

$$\psi_{5,6}(x,y,t)=\pm\sqrt{\frac{2\beta\eta_1}{(\eta_3+\alpha\eta_1\eta_2)}}\tanh\left(\pm i\sqrt{\frac{\beta\eta_1}{\alpha\delta_1\delta_2}}(\delta_1 x+\delta_2 y-\delta_3 t)\right)e^{-i(\eta_1 x-\eta_2 y+\eta_3 t)}. \tag{40}$$

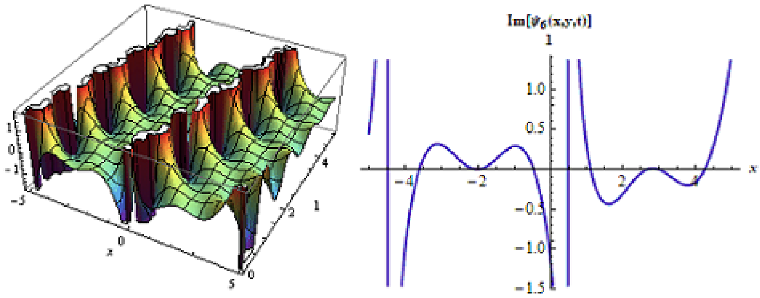

Figure 3. *3D graphs of* $\psi_5\left(x,y,t\right)$ *for* $\alpha = 1$; $\beta = 0.5$; $\eta_1 = 2$; $\eta_2 = 2.5$; $\eta_3 = 5$; $y = 1$; $\delta_1 = 1$; $\delta_2 = 2.5$; $\delta_3 = 0.5$;, $-5 < x < 5, 0 < t < 5$, *2D plot for* $t = 1$.

Conclusion

This paper finds entirely soliton solutions for governing model with the help of unified solver technique. The acquired solutions are hyperbolic function solutions. These solutions explain some interesting physical phenomena in applied science and physics.

The hyperbolic secant (bright soliton) arises in the profile of a laminar jet, the hyperbolic tangent (dark soliton) arises in the calculation of magnetic moment. Indeed, the options given comprised bright and dark as well as soliton solutions. In this sense, $\psi_{1,2}\left(x,y,t\right)$ and $\psi_{3,4}\left(x,y,t\right)$ are bright solitons, $\psi_{5,6}\left(x,y,t\right)$ are dark solitons of the considered model respectively.

By the selection of suitable values for the model's parameters, structures of solitons are clearly depicted. Graphs are presented to prescribe the dynamical behavior of selected solutions. Also the obtained solitons with special parameters in the figures satisfy the KMN equation.

In the light of this results, it seems that the unified solver method has been influential for the analytical solutions of nonlinear partial differential equations emerging in natural science.

Acknowledgments -

Funding/Financial Disclosure The authors have no received any financial support for the research, authorship, or publication of this study.

Ethics Committee Approval and Permissions The work does not require ethics committee approval and any private permission.

Conflict of Interests The authors stated that there are no conflict of interest in this article.

Authors Contribution Authors contributed equally to the study.

References

[1] Doran, N., & Blow, K. (1983). Solitons in optical communications. *IEEE Journal of Quantum Electronics*, *19*(12), 1883-1888. https://doi.org/10.1109/JQE.1983.1071806

[2] Haus, H. A., & Wong, W. S. (1996). Solitons in optical communications. *Reviews of Modern Physics*, *68*(2), 423-444. https://doi.org/10.1103/RevModPhys.68.423

[3] Ataş, S. S., Ali, K. K., Sulaiman, T.A., & Bulut, H. (2022). Optical solitons to the Fokas system equation in monomode optical fibers. *Optical and Quantum Electronics*, *54*(11), 1-13. https://doi.org/10.1007/s11082-022-04120-w

[4] Başkonuş, H. M., Sulaiman, T. A., & Bulut, H. (2018). Dark, bright and other optical solitons to the decoupled nonlinear Schrödinger equation arising in dual-core optical fibers. *Optical and Quantum Electronics*, *50*, Article 165. https://doi.org/10.1007/s11082-018-1433-0

[5] Al-Ghafri, K., Krishnan, E., & Biswas, A. (2021). W-shaped and other solitons in optical nanofibers. *Results in Physics*, *23*, Article 103973. https://doi.org/10.1016/j.rinp.2021.103973

[6] Kundu, A., & Mukherjee A. (2013). Novel integrable higher-dimensional nonlinear Schroedinger equation: properties, solutions, applications. arXiv:1305.4023v1 [nlinSI] https://doi.org/10.48550/arXiv.1305.4023

[7] Kundu A, Mukherjee A, & Naskar T. (2014). Modelling rogue waves through exact dynamical lump soliton controlled by ocean currents. *Proceedings of the Royal Society A-Math Phy.*, *470*(2164), 1-20. https://doi.org/10.1098/rspa.2013.0576

[8] Mukherjee, A., & Kundu, A. (2019). Novel nonlinear wave equation: Regulated rogue waves and accelerated soliton solutions. *Physics Letters A*, *383*(10), 985-990. https://doi.org/10.1016/j.physleta.2018.12.023

[9] Mukherjee, A., Janaki, M., & Kundu, A. (2019). A new (2+1)-dimensional integrable evolution equation for an ion acoustic wave in a magnetized plasma. *Physics of Plasmas*, *22*, Article 072302. https://doi.org/10.1063/1.4923296

[10] Biswas, A., Vega-Guzman, J., Bansal, A., Kara, A. H., Alzahrani, A. K., Zhou, Q., & Belic, M. R. (2020). Optical dromions, domain walls and conservation laws with Kundu-Mukherjee-Naskar equation via traveling waves and Lie symmetry. *Results in Physics*, *16*, 102850. https://doi.org/10.1016/j.rinp.2019.102850

[11] Yıldırım, Y., & Mirzazadeh, M. (2019). Optical pulses with Kundu-Mukherjee-Naskar model in fiber communication systems. *Chinese Journal of Physics*, *64*, 183-193. https://doi.org/10.1016/j.cjph.2019.10.025

[12] Yıldırım, Y. (2019). Optical solitons to Kundu–Mukherjee–Naskar model with trial equation approach. *Optik*, *183*, 1061–1065. https://doi.org/10.1016/j.ijleo.2019.02.117

[13] Rizvi, S. T. R., Afzal, I., & Ali, K. (2020). Dark and singular optical solitons for Kundu-Mukherjee-Naskar model. *Modern Physics Letters B*, *34*(6), 1-9. https://doi.org/10.1142/S0217984920500748

[14] Al-Ghafri, K. S. (2021). Soliton structures in optical fiber communications with Kundu–Mukherjee–Naskar model. *Open Physics*, *19*, 679-682. https://doi.org/10.1515/phys-2021-0074

[15] Mamedov, K. R., Demirbilek, U., & Ala, V. (2022). Exact solutions of the (2+1)-dimensional Kundu–Mukherjee–Naskar Model via IBSEFM. *Bulletin of the South Ural State University Series, Mathematical Modelling, Programming, Computer Software*, *15*(2), 17–26. https://doi.org/10.14529/mmp220202

[16] Önder, I., Seçer, A., Özışık, M., & Bayram, M. (2022). On the optical soliton solutions of Kundu–Mukherjee–Naskar equation via two different analytical methods. *Optik*, *257*, Article 168761. https://doi.org/10.1016/j.ijleo.2022.168761

[17] Kumar, D., Paul, G. C., Biswas, T., Seadawy, A. R., Baowali, R., Kamal, M., & Rezazadeh, H. (2020). Optical solutions to the Kundu-Mukherjee-Naskar equation: mathematical and graphical analysis with oblique wave propagation. *Physica Scripta*, *96*(2), Article 025218. https://doi.org/10.1088/1402-4896/abd201

[18] Günerhan H., Khodadad F. S., Rezazadeh H., & Khate, M. M. A. (2020). Exact optical solutions of the (2+1) dimensions Kundu–Mukherjee–Naskar model via the new extended direct algebraic method. *Modern Physics Letters B*, *34*(22), Article 2050225. https://doi.org/10.1142/S0217984920502255

[19] Kudryashov, N. A. (2019). General solution of traveling wave reduction for the Kundu–Mukherjee–Naskar model. *Optik*, *186*, 22-27. https://doi.org/10.1016/j.ijleo.2019.04.072

[20] Petrovic, N. (2022). Chirped solitary and traveling wave solutions for the Kundu–Mukherjee–Naskar equation using the Jacobi elliptic function expansion method. *Optical and Quantum Electronics*, *54*(10), 1–8. https://doi.org/10.1007/s11082-022-04024-9

[21] Ekici M., Sönmezoğlu A., Biswas A., & Belic, M. R. (2019). Optical Solitons in (2+1)-Dimensions with Kundu–Mukherjee–Naskar Equation by Extended Trial Function Scheme. *Chinese Journal of Physics*, *57*, 72-77. https://doi.org/10.1016/j.cjph.2018.12.011

[22] Rezazadeh, H., Kurt A., Tozar A., Tasbozan, O., & Mirhosseini-Alizamini, S. M. (2021). Wave behaviors of Kundu–Mukherjee–Naskar model arising in optical fiber communication systems with complex structure. *Optical and Quantum Electronics*, *53*, Article 317. https://doi.org/10.1007/s11082-021-02926-8

[23] Çakıcıoğlu, H., Çınar M., Seçer A., & Bayram, M. (2023). Optical solitons for Kundu–Mukherjee–Naskar equation via enhanced modified extended tanh method. *Optical and Quantum Electronics*, *55*, Article 400. http://doi.org/10.1007/s11082-023-04686-z

[24] Mohammed, W. W., Al-Askar, F. M., Cesarano, C., & El-Morshedy, M. (2022). The optical solutions of the stochastic fractional Kundu–Mukherjee–Naskar Model by Two Different Methods. *Mathematics*, *10*(9), Article 1465. https://doi.org/10.3390/math10091465

[25] He, J. (1997). Semi-inverse method of establishing generalized variational principles for fluid mechanics with emphasis on turbo machinery aerodynamics. *International Journal of Turbo and Jet Engines*, *14*, 23-28. https://doi.org/10.1515/TJJ.1997.14.1.23

[26] He, J. (2006). Some asymptotic methods for strongly nonlinear equations. *International Journal of Modern Physics B*, *20*, 1141-1199. https://doi.org/10.1142/S0217979206033796

[27] Kudryashov, N. (2009). Seven common errors in finding exact solutions of nonlinear differential equations. *Communications in Nonlinear Science and Numerical Simulation*, *14*(9-10), 3507-3529. https://doi.org/10.1016/j.cnsns.2009.01.023