

Determination of Ionization Energy Loss of Electron-Positron Pair in Multi-GeV Region

Onur Buğra Kolcu¹ 

¹Istinye University, Department of Basic Sciences, Istanbul

Geliş Tarihi / Received Date: 06.05.2023

Kabul Tarihi / Accepted Date: 05.10.2023

Abstract

The suppression of ionization energy loss owing to the Chudakov effect is discussed using the opening angles of the electron-positron pairs from the GEANT4 simulation package. The expected Borsellino and Olsen angles for photon energies between 1-178 GeV were presented and compared with the simulated opening angles. The simulated opening angles of the electron-positron pair are mostly compatible with the Borsellino angle for energies below 30 GeV. The ionization-suppression effect was reproduced using known theoretical approaches and compared with the corresponding simulated results. The results showed that the GEANT4 simulation package is suitable for adopting the Chudakov effect in a simulation environment, with a dataset that provides theory-experiment consistency.

Keywords: pair production, ionization energy loss, Chudakov effect, King-Perkins-Chudakov effect

Elektron-Pozitron Çiftinin İyonlaşma Enerji Kaybının Çoklu-GeV Bölgesinde Belirlenmesi

Öz

Chudakov etkisi nedeniyle iyonlaşma enerji kaybının bastırılması, GEANT4 simülasyon paketinden elde edilen elektron-pozitron çiftlerinin açılma açıları kullanılarak tartışılmıştır. 1-178 GeV arasındaki foton enerjileri için beklenen Borsellino ve Olsen açıları gösterilmiş ve simüle edilen açılma açıları ile karşılaştırılmıştır. Elektron-pozitron çiftinin benzetimi yapılan açılma açıları, 30 GeV'in altındaki enerjiler için Borsellino açısı ile çoğunlukla uyumludur. İyonizasyon-bastırma etkisi bilinen teorik yaklaşımlar kullanılarak yeniden üretilmiş ve ilgili simülasyon sonuçları ile karşılaştırılmıştır. Elde edilen sonuçlar, GEANT4 simülasyon paketinin teori-deneysel tutarlılığı sağlayan bir veri seti ile Chudakov etkisini simülasyon ortamına uyarlamak için uygun olduğunu göstermiştir.

Anahtar Kelimeler: çift oluşumu, iyonizasyonla enerji kaybı, Chudakov etkisi, King-Perkins-Chudakov etkisi

Introduction

The ionization energy loss of the electron-positron produced by pair production is expected to reach twice the ionization loss of a single electron. However, when the electron-positron pair's transverse separation is too small around the production point, the ionization energy loss in a substance decreases, as the oppositely charged particles affect each other's electric field (Trofymenko & Shul'ga, 2015). That is, suppression of ionization occurs due to the interference of electromagnetic fields of electron-positron pairs (Iwadare, 1958). This is called the Chudakov or King-Perkins-Chudakov effect (Chudakov, 1955; Perkins, 1955) and has been extensively studied both theoretically and experimentally (Berestetskii, 1957; Burkhardt, 1958; Mito, 1957; Wolter, 1956; Yekutieli, 1957; Zieliński, 1985), with more detailed theoretical studies published over the last two decades (Shul'ga & Trofymenko, 2014; Trofymenko & Shul'ga 2017; Trofymenko, 2020; Thomsen & Uggerhøj, 2011; Trofymenko & Shul'ga, 2013). Cosmic ray experiments have been performed to observe the Chudakov effect, but these experiments offer limited statistics and are unable to control the beam energy (Iwadare, 1958; Perkins, 1955; Wolter, 1956). Also, an accelerator-based experiment was performed to directly measure the Chudakov effect with photon beam energy in the range of 1-178 GeV (Virkus, et al., 2008). In this experiment, 20 μm thick gold targets were placed at different distances from the CCD detector, and ionization suppression was measured with the most probable energy loss ratios (Virkus, et al., 2008). The results of this experiment show the effect is significantly stronger than expected such that the mechanism of suppression of ionization requires further investigation.

More recently, Trofymenko determined the struggling function for the most probable value of the pair ionization loss, which includes the Chudakov effect (Trofymenko, 2023). The theoretical results of this study can be used to adopt the Chudakov effect in the simulation packages used for high energy physics experiments such as GEANT4 (Agostinelli, et al., 2003).

In this study, a simplified version of the CERN NA63 experiment (Virkus, et al., 2008) was constructed using the GEANT4 simulation package to obtain the electron-positron pair opening angles, and the results were used to evaluate the ionization energy loss according to theoretical approaches. The simulations are used to reproduce the expected theoretical results and to show that the simulation package is suitable for adopting ionization loss suppression for future studies.

Material and Methods

Method for Calculating Ionization Suppression

The Chudakov effect is valid when the electron and positron are close to each other which are produced by highly energetic photons and the opening angle of the pair is too small. According to the Borsellino formula, the opening angle of the pair is $\vartheta \simeq 4m_e/E_\gamma$ (m_e is the electron mass and E_γ is the photon energy) where e^- and e^+ have approximately the same energies (Borsellino, 1953). When the distance is known between the detector and the target where the pair production takes place, the opening angle can be determined and the transverse distance (s) of the $e^- - e^+$ pair relative to each other can be found. Since the suppression of ionization occurs due to the interference of electromagnetic fields of the pair in the restricted transverse distance, the distance between the $e^- - e^+$ pair is the key point to determine the suppression ratio as can be seen in the equations 1, and 2.

Relative ionization loss of the pair (R) is defined by the Chudakov as

$$R = \frac{\ln(s/r_{min})}{\ln(r_{max}/r_{min})} \quad (1)$$

$r_{min} = \lambda_c$ Compton wavelength ($\lambda_c = \hbar/m_e c$), $r_{max} = c/\omega_p$, and the equation is valid for $s = \vartheta x \leq 0.6r_{max}$, where x is the distance from pair production vertex, ϑ is the opening angle of the pair and ω_p is the plasma frequency of substance (Chudakov, 1955; Thomsen & Uggerhøj, 2011).

The energy loss of the pair also defined by Berestetskii and Geshkenbain with

$$\frac{dE_{\pm}}{dt} = 2 \frac{\alpha \hbar \omega_p^2}{\beta} \left[\ln \left(\frac{\sqrt{2m_e c^2 T_{cut}}}{\hbar \omega_p} \right) - K_0 \left(\frac{s \omega_p}{\beta c} \right) \right] \quad (2)$$

where $T_{cut} = 10^5$ eV for $16 \mu\text{m}$ Si, the first term is responsible for restricted energy loss of the pair, and the second is the interference term (Berestetskii & Geshkenbain, 1957; Zieliński, 1985). Second kind with the order zero modified Bessel function is given with K_0 .

According to equations 1 and 2, the theoretical curves are determined using the Borsellino angles corresponding to the gamma energy. Also, the pair opening angle suggested by Olsen ($\vartheta \approx 3.2m_e/E_{\gamma}$) is taken into account; and intersection regions of the results for relative ionization loss are specified for Borsellino and Olsen angles (Borsellino, 1953; Olsen, 1963).

Monte Carlo Simulations

GEANT4 simulation toolkit allows for the simulation of the passage of particles through matter, including gamma interactions such as photoelectric effect, Compton scattering, and pair production (Geant4 Collaboration, 2023). The default "Physics List" constructor (G4EmStandardPhysics) was selected for the simulations. In the selected physics constructor, the polar angle is defined with respect to the incoming photon, the azimuthal angle is generated isotropically, and the momenta of the pair is coplanar with the photon (Geant4 Collaboration, 2023).

To determine the ionization suppression with equation 1 and 2, the transverse distance between the electron-positron pair must be calculated. The opening angle was used to determine the transverse distance for each photon energy and was obtained using the GEANT4 simulation package version 10.07.p03.

The experiment to measure the Chudakov effect in the accelerator environment used an electron beam; after generating the photons in the target, the electrons were deflected under a uniform magnetic field (Virkus, et al., 2008). The generated photons interact with the $20 \mu\text{m}$ Au targets and the ionization energy loss of the generated pairs is measured with the $16 \mu\text{m}$ CCD detector (for more details of the experimental setup see Virkus, et al., 2008). Au target is used in front of the CCD detector to increase the probability of pair production since the probability of pair production in Au is approximately 35 times higher than Si (Thomsen & Uggerhøj, 2011). To simplify this experimental setup in the simulation, the monoenergetic photon beam was used and its energy was adjusted in accordance with the energies in (Virkus, et al., 2008), thus simulating a simpler experimental setup. The setup includes gold targets (Au1, Au2) to which photons are directed and detectors (Det-1, Det-2) to determine the corresponding information of the generated $e^- - e^+$ pairs. The thickness of the targets (Au) was set to $20 \mu\text{m}$ and the thickness of the detectors (Si) was $16 \mu\text{m}$. A schematic representation of the simulation setup is shown in Figure 1.

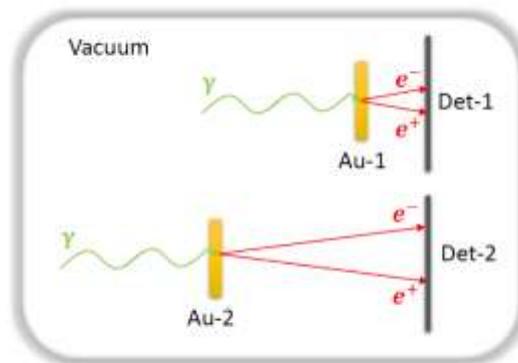


Figure 1. Schematic Representation of Simulation Setup

The drawing is not to scale.

The polar angle between the electron-positron pairs was obtained using the directions of the produced pairs. The photon beam moves along the Z-axis perpendicular to the X and Y-axes; the angles of the secondary particles with respect to the Z-axis are defined as

$$\theta_{(x,y)_i} = \frac{P_{(x,y)_i}}{\sqrt{P_{x_i}^2 + P_{y_i}^2 + P_{z_i}^2}} \quad (3)$$

where $P_{(x,y,z)_i}$ are the momentum directions of the electron-positron pairs.

The Borsellino angles were found numerically by assuming that the energies of the pairs were close to each other. In order to determine the angle between the produced pairs with a similar approach, 5% of half of the photon energy was chosen as the energy window and the opening angle was determined with events satisfying this condition. At least 10^6 events were produced for each selected photon energy and the average opening angle was determined for the pairs. The statistical error of the opening angle was found by repeating the simulations at least three times. The transverse distance of the e^- - e^+ pair to each other at the detectors was determined using the opening angle of the pairs and the known distances between the targets and detectors. The horizontal distance between the center of the Au-1 and Au-2 targets to the detectors Det-1 and Det-2 are $16 \mu\text{m}$ and $116 \mu\text{m}$ respectively.

Results and Discussions

The expected Borsellino and Olsen angles corresponding to the energies between 1-178 GeV are given in Figure 2. The energy points (1.21, 3.71, 11.8, 37.2, and 114 GeV) in the accelerator-based experiment (Virkus, et al., 2008) are considered to adjust the photon source energy in the simulation and the corresponding angles for these energies are given in Figure 2. The simulation results indicate that the opening angles are mostly compatible with the Borsellino angle for the energy region below 30 GeV. In the high energy region, the Borsellino and Olsen angles are approximately the same and the angles from the simulation are compatible with the expected results.

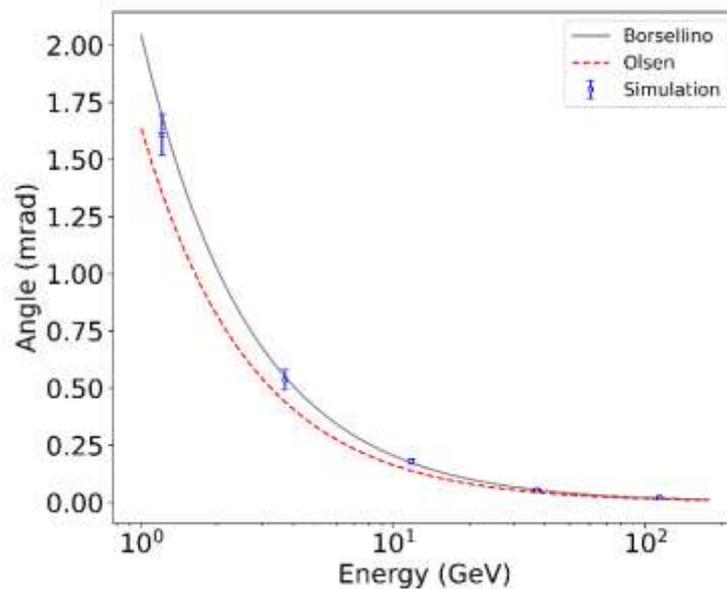


Figure 2. Pair Opening Angles for Gamma Energies between 1-178 GeV

The solid line represents the Borsellino angle, the dashed line Olsen angle, and the points are the simulation results.

To compute the reduction ratio of ionization energy loss in the detectors, the opening angles of the pairs corresponding to the photon beam energy were used in equations 1 and 2. Also, the expected Borsellino and Olsen angles were used to compare the results. The reduction ratio of the ionization

energy loss was defined as the expected ionization energy loss in the detector for the pairs originating from the close (Au1) and far (Au2) targets. The distances of Au1 and Au2 centers to the Det-1 and Det-2 are $16 \mu\text{m}$ and $116 \mu\text{m}$, respectively. Thus, the transverse distance s_1 between pairs produced in Au1 is smaller than the transverse distance of the pair s_2 generated in the second target when they arrived at the detectors. The ionization suppression effect is stronger for the pairs produced in Au1 while the ionization is almost unaffected for the pairs coming from the far ($116 \mu\text{m}$) target. Thus, the reduction of ionization energy loss (the relative ionization suppression (R)) can be calculated with the ratio of expected ionization energy loss of the pairs for s_1 and s_2 distances. The ratios of the ionization suppression are given in Figure 3.

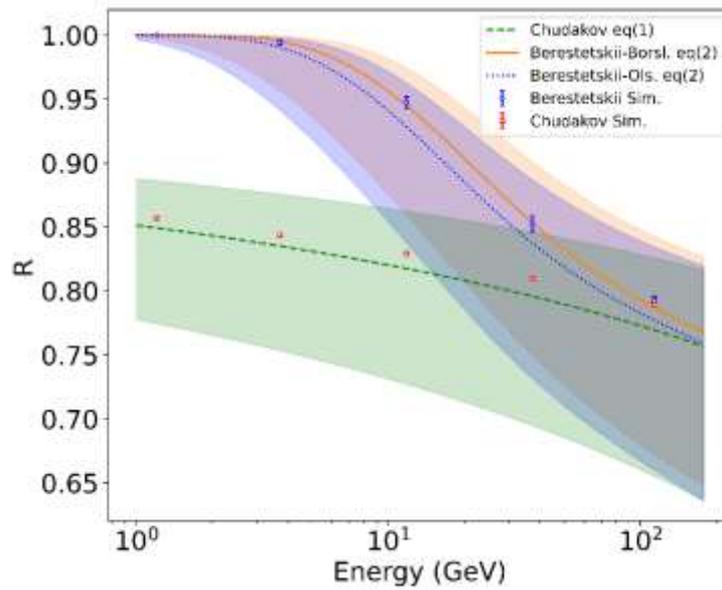


Figure 3. The Ratios of the Ionization Suppression

Relative ionization suppression (R) determined from the Chudakov (dashed) and Berestetskii-Geshkenbain (solid) equations with Borsellino angles, Berestetskii-Geshkenbain (dotted) equation with Olsen angles, Chudakov (open squares) and Berestetskii-Geshkenbain (open circles) equations with simulation angles. The color of the regions is the same as the color of the drawn equations.

In Figure 3, the colored plots represent regions determined with horizontal distances of $d_1 = 16 \pm 10 \mu\text{m}$ and $d_2 = 116 \pm 10 \mu\text{m}$, considering that gamma conversion occurs inside the $20 \mu\text{m}$ gold target. The ionization suppression effect derived from the simulation results are compatible with the results expected for the Berestetskii and Geshkenbain approach. The points derived from the equation 1 differ slightly from the Chudakov approach but are within the expected range. As can be seen in Figure 3, the theories of the Berestetskii-Geshkenbain and Chudakov are quite different below 10 GeV. Also, experimental results show a stronger ionization suppression below 10 GeV than theoretically expected, and the experimental results are mostly consistent with the theory of Chudakov (Thomsen & Uggerhøj, 2011). However, the theoretical expectations are almost the same for the high-energy regions above 100 GeV. The possible ionization suppression ratios determined by applying the simulation results to the theory are also compatible above 100 GeV.

In the accelerator-based experiment designed to directly measure the Chudakov effect, the most probable value of the pair ionization loss is used to determine the relative energy deposition of the pairs (Virkus, et al., 2008). GEANT4 simulation package is suitable for determining the energy loss of the particles in the medium without Chudakov effect. However, the expected relative energy deposition of the pairs can be used to derive the energy loss distribution of the pairs. Recently, Trofymenko proposed the struggling function for the most probable value of the pair ionization loss, which includes the effect of ionization suppression (Trofymenko, 2023). It is stated that the most probable value of the ionization loss of $e^- - e^+$ pairs can be written as

$$E_{MP}(s) = \eta \left[\ln \frac{2m_e \eta}{\left(\frac{\hbar \omega_p}{\beta C}\right)^2} - 2K_0 \left(\frac{\omega_p s}{\beta C}\right) + 0.2 \right] \quad (4)$$

like the equation 2, where $\eta/x \approx 0.0356 \text{ keV}/\mu\text{m}$ for Si detector (Trofymenko, 2023). This expression fit in the struggling function given in (Trofymenko, 2023) and can be used to determine the relative suppression. Thus, the ratio of the most probable energy loss (MPEL) of the pairs can be written as $E_{MP}(s_1)/E_{MP}(s_2)$. Equation 4 is used to numerically calculate the most probable energy deposition of the pairs in detectors and adopted to the simulated energy deposition to configure the suppressed ionization loss distribution indirectly. The relative suppression ratios for energies between 1-178 GeV are given in Figure 4.

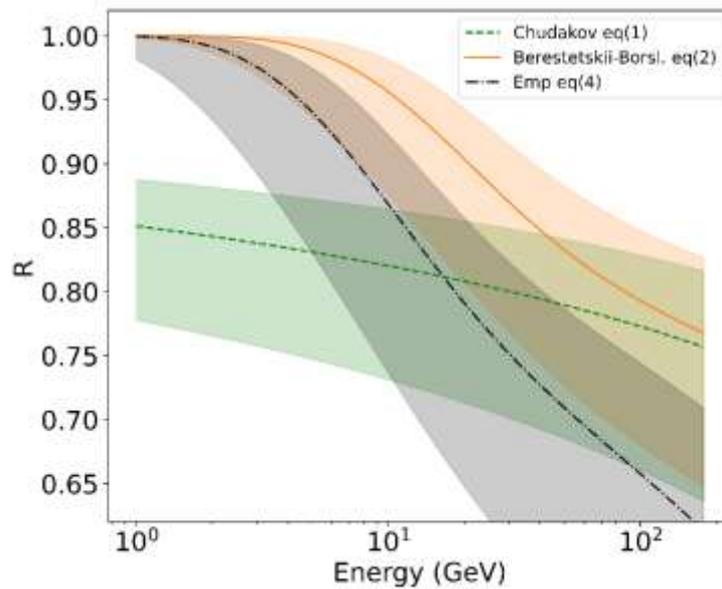


Figure 4. The Relative Suppression Ratios for Energies between 1-178 GeV

Relative ionization suppression (R) for equations (4) (dash-dot), (1) (dashed), and (2) (solid). The color of the regions is the same as the color of the drawn equations.

Although equation 4 mostly coincides with the experimental data (Virkus, et al., 2008) at the high energy region, the difference between curves below 10 GeV is still present. However, the R values can be used to simulate the expected energy deposition distributions for the energies below 40 GeV where $E_{MP}(s_2)$ is almost constant (Trofymenko, 2023). The expected distributions with ionization suppression were determined by applying the R values to the energy depositions of the pairs formed by photon sources with 11.8 and 37.2 GeV energies and are given in Figure 5.

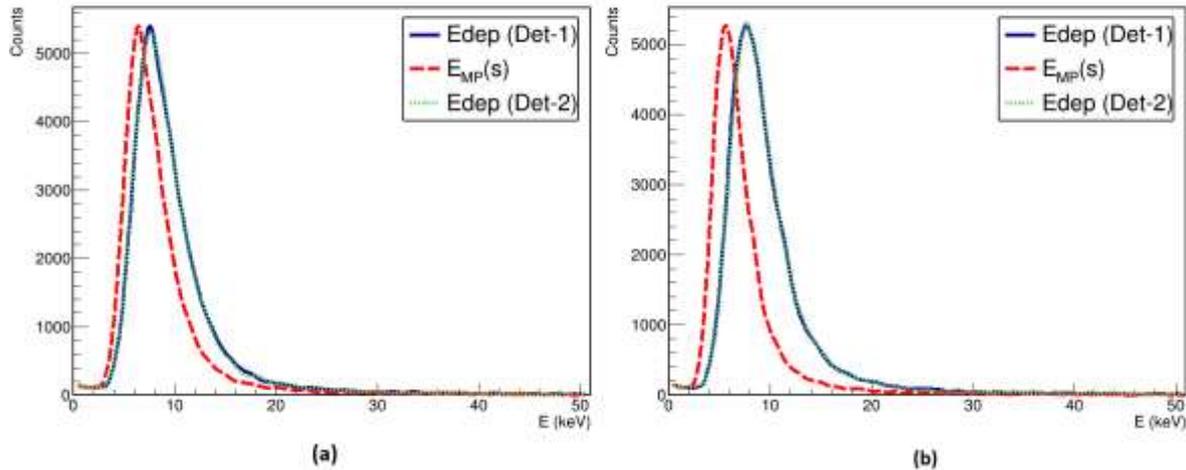


Figure 5. The Energy Depositions of the Pairs Formed by Photon Sources with 11.8 and 37.2 GeV Energies

Expected energy depositions the $e^- - e^+$ pairs for 11.8 GeV (a) and 37.2 GeV (b) photon beams. Red dashed lines represent the expected energy deposition with ionization suppression according to equation 4.

In Figure 5, the most probable value of the energy deposition with ionization suppression (red dashed lines) is specified by fitting the Landau distribution. The MPEL values are 6.58 ± 0.01 and 5.57 ± 0.02 for the given distributions (a) and (b) respectively. The MPEL for the pairs produced by 1.21 GeV photons is approximately the same with the energy deposition without ionization suppression as expected.

Conclusions

This is the first study to indirectly determine the Chudakov effect using the GEANT4 simulation package. The simplified demonstration of the accelerator-based experiment designed for observation of the Chudakov effect was simulated using the GEANT4 simulation package. Simulation results show that the opening angles are mostly in agreement with the Borsellino angle for the energy region below 30 GeV. Above 30 GeV, the Borsellino and Olsen angles are approximately the same and in agreement with the simulated opening angle. Also, the ionization suppression was derived using the simulated opening angles and the results were found to be in agreement with the reproduced literature results. These results show that the GEANT4 software package is suitable for adapting the Chudakov effect to the simulation environment. However, to complete adaptation of the ionization suppression mechanism to any of the simulation environments needs more experimental work, especially below the 10 GeV region. To eliminate the discrepancies between theoretical approaches, an experiment is needed in which the Chudakov effect is observed with high statistics and in a wide energy range. Finally, to adapt the Chudakov effect to the simulation environment, it is necessary to overcome the discrepancies between the theoretical approaches and thus provide consistent theory-experiment results.

Ethics

There are no ethical issues related to the publication of this article.

Conflict of Interest

The author state that there is no conflict of interest.

ORCID

Onur Buğra Kolcu  <https://orcid.org/0000-0002-9177-1286>

References

- Agostinelli, S., et al., (2003). Geant4—a simulation toolkit. *Nuclear Instruments & Methods in Physics Research. Section A, Accelerators, Spectrometers, Detectors and Associated Equipment*, 506(3), 250–303. [https://doi.org/10.1016/s0168-9002\(03\)01368-8](https://doi.org/10.1016/s0168-9002(03)01368-8)
- Berestetskii V. B., & Geshkenbain B. V. (1957). Ionizational slowing down of high-energy electron positron pairs. *Sov. Phys. JETP*, 4(609). <https://www.osti.gov/biblio/4350183>
- Borsellino, A. (1953). Momentum transfer and angle of divergence of pairs produced by photons. *The Physical Review*, 89(5), 1023–1025. <https://doi.org/10.1103/physrev.89.1023>
- Burkhardt, G. H. (1958). The suppression effect in ionization by fast electron pairs. *Il Nuovo Cimento*, 9(3), 375–399. <https://doi.org/10.1007/bf02725095>
- Chudakov, A. E. (1955). On an ionization effect associated with observation of electron-positron pairs at very high energy. *Izv. Akad. Nauk. USSR, Ser. Fiz.*, 19(651).
- Iwadare, J. (1958). Suppression effect of the ionization produced by electron-positron pairs of extremely high energy. *The Philosophical Magazine*, 3(31), 680–691. <https://doi.org/10.1080/14786435808237003>
- Geant4 Collaboration (2023). GEANT4 Physics Reference Manual (version: 10.07). Retrieved March 15, 2023 from <http://geant4.cern.ch>
- Mito, I., & Ezawa, H. (1957). Ionization loss near the origin of an electron pair of very high energy. *Progress of Theoretical Physics*, 18(4), 437–447. <https://doi.org/10.1143/ptp.18.437>
- Olsen, H. (1963). Opening angles of electron-positron pairs. *The Physical Review*, 131(1), 406–415. <https://doi.org/10.1103/physrev.131.406>
- Perkins, D. H. (1955). Ionization at the origin of electron Pairs, and the lifetime of the neutral pion. *The London Edinburgh and Dublin Philosophical Magazine and Journal of Science*, 46(381), 1146–1148. <https://doi.org/10.1080/14786441008521131>
- Shul'ga, N. F., & Trofymenko, S. V. (2014). Anti-Chudakov effect in high-energy electron–positron pair ionization loss in thin target. *Physics Letters. A*, 378(4), 315–318. <https://doi.org/10.1016/j.physleta.2013.11.026>
- Thomsen, H. D., & Uggerhøj, U. I. (2011). Measurements and theories of the King–Perkins–Chudakov effect. *Nuclear Instruments & Methods in Physics Research. Section B, Beam Interactions with Materials and Atoms*, 269(17), 1919–1924. <https://doi.org/10.1016/j.nimb.2011.05.019>
- Trofymenko, S. V., & Shul'ga, N. F. (2013). On ionization energy losses of high-energy electron–positron pair in thin targets. *Physics Letters. A*, 377(37), 2265–2269. <https://doi.org/10.1016/j.physleta.2013.06.044>
- Trofymenko, S. V., & Shul'ga, N. F. (2015). The influence of non-parallelism of electron and positron velocities upon high-energy e+e– pair ionization loss in thin plate. *Nuclear Instruments & Methods in Physics Research. Section B, Beam Interactions with Materials and Atoms*, 355, 140–143. <https://doi.org/10.1016/j.nimb.2015.02.041>
- Trofymenko, S. V., & Shul'ga, N. F. (2017). Anomalous ionization loss of high-energy e+e– pairs in thin targets. *Nuclear Instruments & Methods in Physics Research. Section B, Beam Interactions with Materials and Atoms*, 402, 44–48. <https://doi.org/10.1016/j.nimb.2017.03.017>
- Trofymenko, S. V., & Shul'ga, N. F. (2020). Energy loss by relativistic electron ensembles due to coherent excitation and ionization of atoms. *Physical Review Accelerators and Beams*, 23(8), 084501. <https://doi.org/10.1103/physrevaccelbeams.23.084501>

- Trofymenko, S. V. (2023). Chudakov effect for the most probable value of high-energy electron-positron pair ionization loss in thin targets. *The European Physical Journal. C, Particles and Fields*, 83(1). <https://doi.org/10.1140/epjc/s10052-023-11181-y>
- Virkus, T., Thomsen, H. D., Uggerhøj, E., Uggerhøj, U. I., Ballestrero, S., Sona, P., Mangiarotti, A., Ketel, T. J., Dizdar, A., Kartal, S., Pagliarone, C., & CERN NA63 Collaboration. (2008). Direct measurement of the Chudakov effect. *Physical Review Letters*, 100(16), 164802. <https://doi.org/10.1103/PhysRevLett.100.164802>
- Wolter, W. & Miesowich, M. (1956). Ionization at the origin of an electron pair of very high energy. *Nuovo Cimento*, 4(648). <https://link.springer.com/content/pdf/10.1007/BF02745390.pdf>
- Yekutieli, G. (1957). Ionization at the origin of high energy electron positron pairs. *Il Nuovo Cimento*, 5(6), 1381–1387. <https://doi.org/10.1007/bf02856030>
- Zieliński, I. P. (1985). On the possibility of the electronic measurement of the King-Perkins-Chudakov effect for electron pairs using a multilayer silicon detector. *Nuclear Instruments & Methods in Physics Research. Section A, Accelerators, Spectrometers, Detectors and Associated Equipment*, 238(2–3), 562–563. [https://doi.org/10.1016/0168-9002\(85\)90502-9](https://doi.org/10.1016/0168-9002(85)90502-9)