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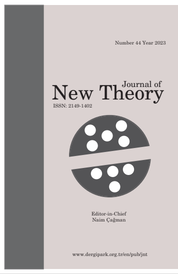

New Exact Solutions of the Drinfeld-Sokolov System by the Generalized Unified Method

Tuğba Aydemir[1]

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Abstract — In this study, we apply the generalized unified method (GUM), an enhanced version of the unified method, to find novel exact solutions of the Drinfeld-Sokolov System (DSS) that models the dispersive water waves in fluid dynamics. Moreover, 3D and 2D graphs of some of the obtained exact solutions are plotted to present how various characteristic forms they have. The results show that the presented method simplifies the computation process on the computer in a highly reliable and straightforward manner while providing the solutions in more general forms. In addition, the GUM has great potential to apply to a wide range of problems, including nonlinear partial differential equations (NPDEs) and fractional partial differential equations (FPDEs) for finding exact solutions.

Keywords *Drinfeld-Sokolov system, unified method, generalized unified method, exact solution, traveling wave solutions*

Mathematics Subject Classification (2020) 35B08, 76B15

1. Introduction

Nonlinear partial differential equations (NPDEs), usually represented by an equation that describes a relationship between an unknown function and its partial derivatives, are great tools for modeling these real-world phenomena in many scientific fields. Finding solutions for NPDEs is essential because it improves our understanding of the physical phenomena these equations model, accelerates technological advances, and optimizes the production process of these technological tools. Therefore, finding numerical and exact solutions to NPDEs corresponding to the physical problems in science and engineering becomes increasingly important.

In this study, the exact solutions of the nonlinear Drinfeld-Sokolov System (DSS), considered to model for dispersive water in fluid dynamics, are investigated by using the generalized unified method (GUM). The explicit mathematical form of the system is given as follows:

$$\begin{cases} u_t(x,t) + (v^2(x,t))_x = 0 \\ v_t(x,t) - \alpha v_{xxx}(x,t) + 3\beta u_x(x,t)v(x,t) + 3\gamma u(x,t)v_x(x,t) = 0 \end{cases}$$

where α, β, and γ are real constants. This system was first proposed by Drinfeld and Sokolov [1] as an extension of the Korteweg-de Vries (KdV), which possesses Lax pairs of a special form for affine Lie algebras. The solutions of the DSS exhibit different characteristics, such as static solitons

[1]tgb.aydemir@gmail.com (Corresponding Author)
[1]Department of International Trade and Finance, Faculty of Economics and Administrative Sciences, Yalova University, Yalova, Türkiye

that interact with moving solitons without deformations [2]. Morris and Kara [3] showed that the interaction between the underlying symmetries and conservation laws of a partial differential equation system results in double reductions of a class of Drinfeld–Sokolov–Wilson equations.

Due to its significant applications in fluid dynamics and optical fiber, many researchers have developed several methods to obtain exact and numerical solutions for the Drinfeld-Sokolov family. Wazwaz [4] used the sine–cosine and tanh methods to have exact solutions for the Drinfeld-Sokolov System. Arora and Kumar [5] applied the homotopy analysis method to obtain an approximation of the analytic solution for the coupled Drinfeld's–Sokolov–Wilson. He et al. [6] obtained exact solutions for the classic Drinfeld-Sokolov-Wilson equation using the F-expansion method combined with the exp-function method. Gómez [7] studied the generalized Drinfeld-Sokolov-Wilson equation to obtain exact solutions by applying the improved tanh-coth method. Düşünceli [8] employed the improved Bernoulli sub-equation function method (IBSEFM) to have exact solutions of the Drinfeld-Sokolov equation. Zhang [9] solved the Drinfeld-Sokolov-Wilson equation using a variational approach. Günay et al. [10] derived solitary wave solutions to the DSS using the generalized exponential rational function method. Salim et al. [11] solved the Drinfeld-Sokolov-Wilson system numerically using the modified Adomian decomposition method. Alam et al. [12] established some exact solutions of the Drinfeld-Sokolov-Wilson equation with $S(\xi)$-expansion method.

Recently, fractional partial differential equations (FPDEs) have also been studied by many authors because fractional order derivatives provide a powerful and enhanced model for expressing some real-world phenomena. In particular, these fractional equations are encountered in modeling considerable complex problems in networks, optics, and fluid dynamics. Jaradat et al. [13] investigated the analytical solution of the time-fractional Drinfeld–Sokolov–Wilson system through the residual power series method. Bhatter et al. [14] considered the fractionalized homotopy analysis transform method to solve the fractional Drinfeld–Sokolov–Wilson model numerically. Gao et al. [15] found the solutions of fractional Drinfeld–Sokolov–Wilson equation using amalgamations of Laplace transform technique with q-homotopy analysis scheme, called q-homotopy analysis transform method (q-HATM) with Atangana-Baleanu derivative. Taşbozan et al. [16] applied the Sine-Gordon expansion method to obtain exact solutions and the perturbation-iteration algorithm to obtain approximate solutions for the fractional Drinfeld-Sokolov-Wilson system. Wang and Wang [17] found the numerical solution for the time-space fractional nonlinear Drinfeld–Sokolov–Wilson system using He's variational method. Noor et al. [18] used the homotopy perturbation transform method and Sumudu transform decomposition method to solve approximately the time-fractional Drinfeld–Sokolov–Wilson system.

In this paper, we have applied the GUM [19] to derive exact solutions for the DSS. The GUM is an enhanced version of the unified method [20,21] that many authors [22–36] have successfully applied to solve different types of NPDEs. Compared with other methods, the GUM requires less computational work with high reliability to solve NPDEs.

This paper is structured as follows: After describing briefly the GUM in section 2, the exact solutions of the DSS are obtained by the GUM, and graphical illustrations of some selected solutions are displayed in section 3. In section 4, the derivation of the exact solution sets obtained by some methods from the solutions obtained by the GUM is discussed. Lastly, conclusive remarks are given in section 5.

2. The Generalized Unified Method

In this section, a brief explanation of the GUM for constructing wave solutions of NPDEs is provided. In the first step, when applying the GUM, the NPDE is converted to a nonlinear ordinary differential equation (NODE) by using wave transformation $u(x,t) = U(\eta)$ such that $\eta = x - \nu t + \eta_0$. The solution form of the obtained NODE is expressed by an ansatz statement as follows:

$$U(\eta) = a_0 + \sum_{m=1}^{M} \left[a_m \phi^m + b_m \phi^{-m}\right] \tag{1}$$

After finding the balance value M in the ansatz statement, this statement and its derivatives are substituted into NODE to obtain an algebraic polynomial system. Here, $\phi(\eta)$ is considered Riccati differential equation in the form $\phi'(\eta) = \phi^2(\eta) - \mu^2$ with $\phi' = \frac{d\phi}{d\eta}$ and $\mu = (c + id)$ where c and d are parameters. The polynomial system with powers ϕ provides to get the values of the coefficients in Equality 1. Finally, wave solutions of the NPDE are obtained in closed form with free parameters A, B, and C, using these coefficients and the solutions of the Riccati differential equation as given by:

$$\phi(\eta) = \begin{cases} \phi_1 = \frac{\mp(c+id)\sqrt{A^2+(B+iC)^2} - A(c+id)\cosh(2(c+id)(\eta+\eta_0))}{(B+iC)+A\sinh(2(c+id)(\eta+\eta_0))} \\ \phi_2 = \frac{\mp(c+id)\left(-A+e^{\mp 2(c+id)(\eta+\eta_0)}\right)}{\left(A+e^{\mp 2(c+id)(\eta+\eta_0)}\right)} \\ \phi_3 = -\frac{1}{\eta+\eta_0} \end{cases}$$

where $A \neq 0$, B, and C are real arbitrary parameters.

3. Application of The Generalized Unified Method to Drinfeld-Sokolov System

In this section, the GUM is applied to the DSS to obtain exact wave solutions by following the steps as explained in Section 2. The DSS is given by

$$\begin{cases} u_t(x,t) + (v^2(x,t))_x = 0 \\ v_t(x,t) - \alpha v_{xxx}(x,t) + 3\beta u_x(x,t)v(x,t) + 3\gamma u(x,t)v_x(x,t) = 0 \end{cases} \tag{2}$$

where α, β, and γ are real constants. Converting NPDE System 2 to ordinary differential equations (ODEs) by using the wave transformation $\eta = x - \nu t + \eta_0$ in $u(x,t)$ and $v(x,t)$ gives:

$$\begin{cases} -\nu U'(\eta) + 2V(\eta)V'(\eta) = 0 \\ -\nu V'(\eta) - \alpha V'''(\eta) + 3\beta U'(\eta)V(\eta) + 3\gamma U(\eta)V'(\eta) = 0 \end{cases} \tag{3}$$

Here, $U(\eta)$ and $V(\eta)$ denote the shape of the nonlinear wave with the wave variable $\eta = x - \nu t + \eta_0$. Integrating the first equation with respect to η yields the equality $U(\eta) = \dfrac{V^2(\eta)}{\nu}$ in ODE System 3. Substituting this equality into the second equation in ODE System 3, this second one can be rewritten in the form as follows:

$$\nu V(\eta) + \alpha V''(\eta) - \frac{6\beta + 3\gamma}{\nu} V^3(\eta) = 0 \tag{4}$$

The solutions of the DSS in Equation 2 can be written in the form with balance value $M = 1$, found from the highest order V'' and the nonlinear term V^3. Therefore, the solutions in closed form for Equation 2 are described as follows:

$$\begin{cases} V(\eta) = a_0 + a_1\phi + \frac{b_1}{\phi} \\ U(\eta) = \frac{V^2(\eta)}{\nu} \end{cases} \tag{5}$$

where a_0, a_1, and b_1 are coefficients of ϕ, determined later. Substituting the ansatz statement in Solution 5 and its derivatives into Equation 4 gives an algebraic polynomial equation system with a_0, a_1, b_1, and μ. Solving this polynomial equation system by using any symbolic computation program yields the following sets for the unknowns a_0, a_1, b_1, and μ. Substituting these values into Solution 5 with ϕ_1 and ϕ_2 gives rise to the following exact solutions to the DSS, respectively.

Case 1. For $a_0 = 0$, $a_1 = \mp\sqrt{\frac{2\alpha}{K}}$, $b_1 = 0$, and $\mu = \mp\sqrt{\frac{\nu}{2\alpha}}$ such that $K = \frac{6\beta+3\gamma}{\nu}$, the exact solutions are as follows:

$$\begin{cases} u_1(x,t) = \frac{v_1^2(x,t)}{\nu} \\ v_1(x,t) = \mp\frac{\sqrt{\frac{\nu^2}{6\beta+3\gamma}}\left(\sqrt{A^2+(B+iC)^2}\mp A\cosh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right)}{(B+iC)\mp A\sinh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)} \end{cases}$$

and

$$\begin{cases} u_2(x,t) = \frac{v_2^2(x,t)}{\nu} \\ v_2(x,y,t) = \frac{\sqrt{\frac{\nu^2}{6\beta+3\gamma}}\left(-A+e^{\mp\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)}\right)}{\left(A+e^{\mp\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)}\right)} \end{cases}$$

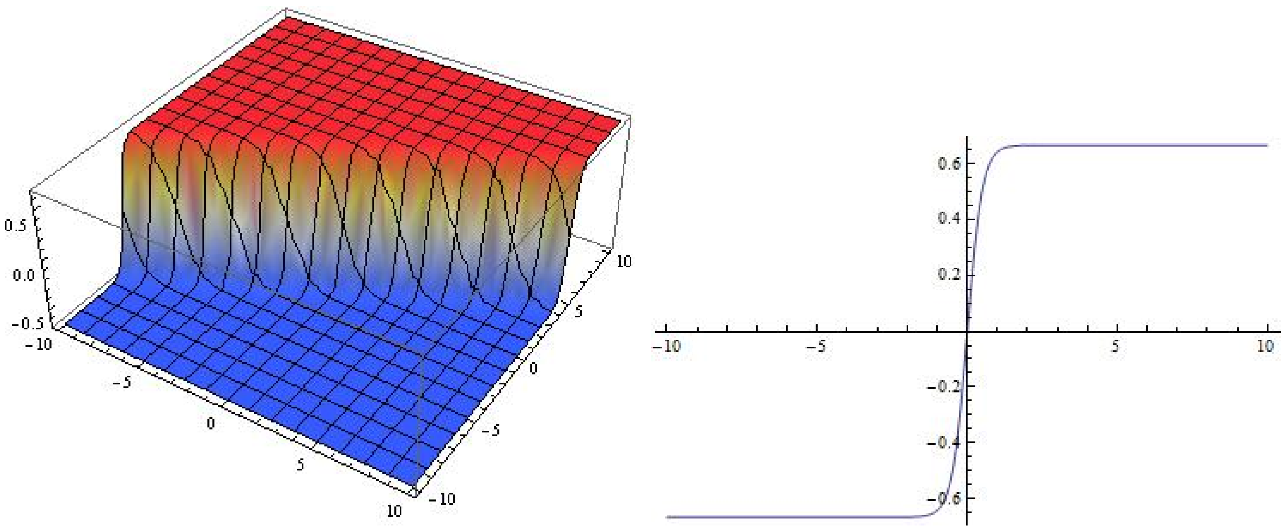

Figure 1. 3D and 2D graphs of the solution v_1 for the parameter $A = 1$, $B = 0$, and $C = 0$

Case 2. For $a_0 = 0$, $a_1 = 0$, $b_1 = \mp\nu\sqrt{\frac{1}{2K\alpha}}$, and $\mu = \mp\sqrt{\frac{\nu}{2\alpha}}$ such that $K = \frac{6\beta+3\gamma}{\nu}$,the exact solutions are as follows:

$$\begin{cases} u_3(x,t) = \frac{v_3^2(x,t)}{\nu} \\ v_3(x,t) = \mp\frac{\sqrt{\frac{\nu^2}{6\beta+3\gamma}}(B+iC)\mp A\sinh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)}{\left(\sqrt{A^2+(B+iC)^2}\mp A\cosh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right)} \end{cases}$$

and

$$\begin{cases} u_4(x,t) = \frac{v_4^2(x,t)}{\nu} \\ v_4(x,y,t) = \frac{\sqrt{\frac{\nu^2}{6\beta+3\gamma}}\left(A+e^{\mp\sqrt{\frac{2c}{\alpha}}(x-\nu t+\eta_0)}\right)}{\left(-A+e^{\mp\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)}\right)} \end{cases}$$

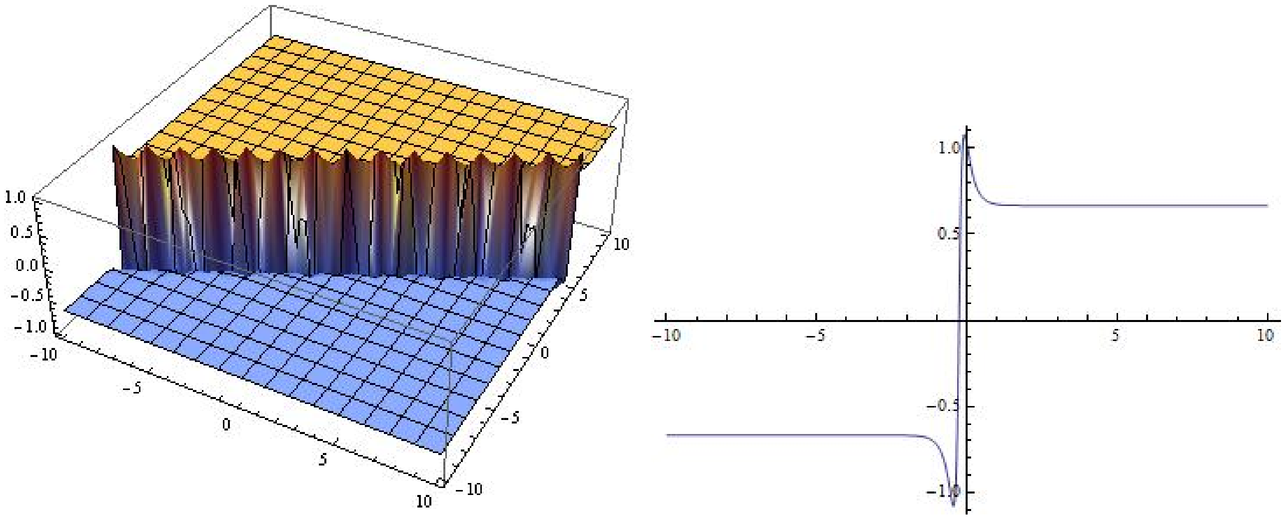

Figure 2. 3D and 2D graphs of real part for the solution v_3 for the parameter $A = 1$, $B = 1$, and $C = 1$

Case 3. For $a_0 = 0$, $a_1 = \frac{\nu}{2Kb_1}$, $b_1 = \mp\nu\sqrt{\frac{1}{8K\alpha}}$, and $\mu = \mp\frac{b_1}{\nu}\sqrt{-2\nu K}$ such that $K = \frac{6\beta+3\gamma}{\nu}$, the exact solutions are as follows:

$$\begin{cases} u_5\left(x,t\right) = \frac{v_5^2(x,t)}{\nu} \\ v_5(x,t) = \mp\sqrt{\frac{\nu^2}{-12\beta-6\gamma}}\left(\frac{(B+iC)\mp A\sinh\left(\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)\right)}{\left(\sqrt{A^2+(B+iC)^2}\mp A\cosh\left(\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right)} + \frac{\left(\sqrt{A^2+(B+iC)^2}\mp A\cosh\left(\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right)}{(B+iC)\mp A\sinh\left(\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)\right)}\right) \end{cases}$$

and

$$\begin{cases} u_6\left(x,t\right) = \frac{v_6^2(x,t)}{\nu} \\ v_6(x,t) = \mp\sqrt{\frac{\nu^2}{-12\beta-6\gamma}}\left(\frac{A+e^{\mp\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)}}{-A+e^{\mp\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)}} + \frac{-A+e^{\mp\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)}}{A+e^{\mp\sqrt{\frac{-\nu}{\alpha}}(x-\nu t+\eta_0)}})\right) \end{cases}$$

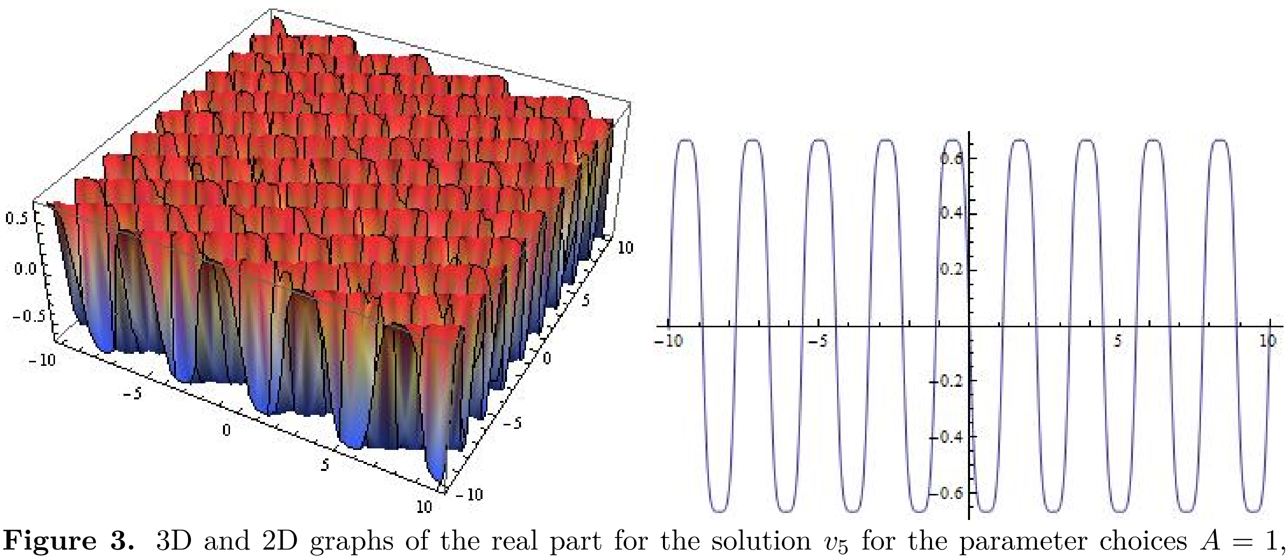

Figure 3. 3D and 2D graphs of the real part for the solution v_5 for the parameter choices $A = 1$, $B = 1$, and $C = 0$

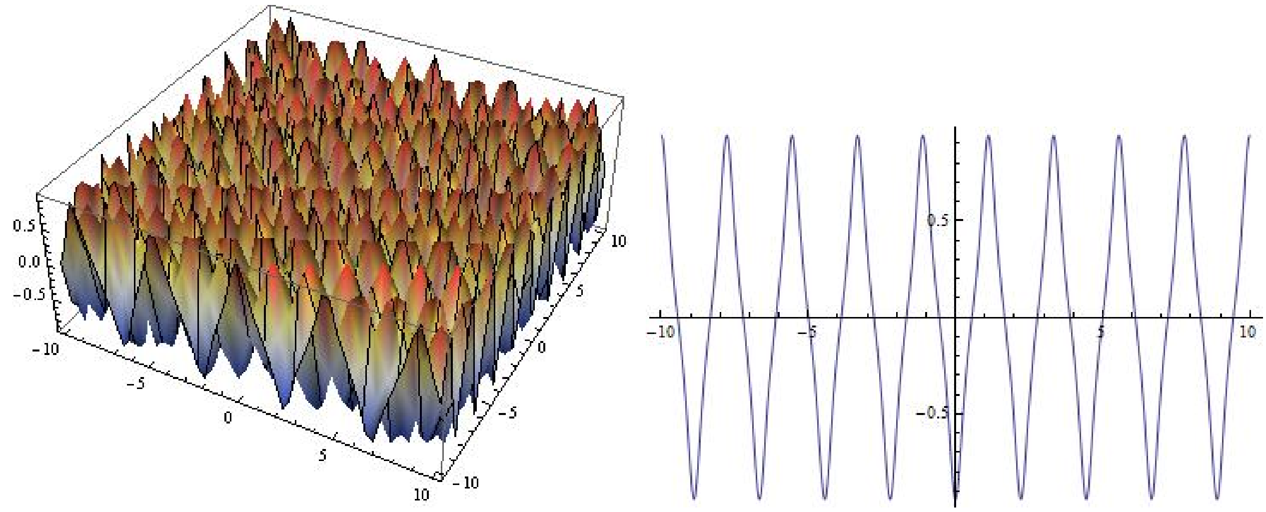

Figure 4. 3D and 2D graphs of the imaginary part for the solution v_5 for the parameter choices $A=1,\ \ B=1$, and $C=0$

Case 4. For $a_0=0$, $a_1=\frac{\nu}{4Kb_1}$, $b_1=\mp\nu\sqrt{\frac{1}{32K\alpha}}$, and $\mu=\mp\frac{2b_1}{\nu}\sqrt{\nu K}$ such that $K=\frac{6\beta+3\gamma}{\nu}$, the exact solutions are as follows:

$$\begin{cases} u_7\left(x,t\right)=\frac{v_7^2(x,t)}{\nu} \\ v_7(x,t)=\mp\frac{\nu}{2\sqrt{6\beta+3\gamma}}\left(\frac{(B+iC)\mp A\sinh\left(\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)\right)}{\left(\sqrt{A^2+(B+iC)^2}\mp A\cosh\left(\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)\right)\right)}+\frac{\left(\sqrt{A^2+(B+iC)^2}\mp A\cosh\left(\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)\right)\right)}{(B+iC)\mp A\sinh\left(\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)\right)}\right)\end{cases}$$

and

$$\begin{cases} u_8\left(x,t\right)=\frac{v_8^2(x,t)}{\nu} \\ v_8(x,y,t)=\mp\frac{\nu}{2\sqrt{6\beta+3\gamma}}\left(\frac{A+e^{\mp\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)}}{-A+e^{\mp\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)}}+\frac{-A+e^{\mp\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)}}{A+e^{\mp\sqrt{\frac{\nu}{2\alpha}}(x-\nu t+\eta_0)}})\right)\end{cases}$$

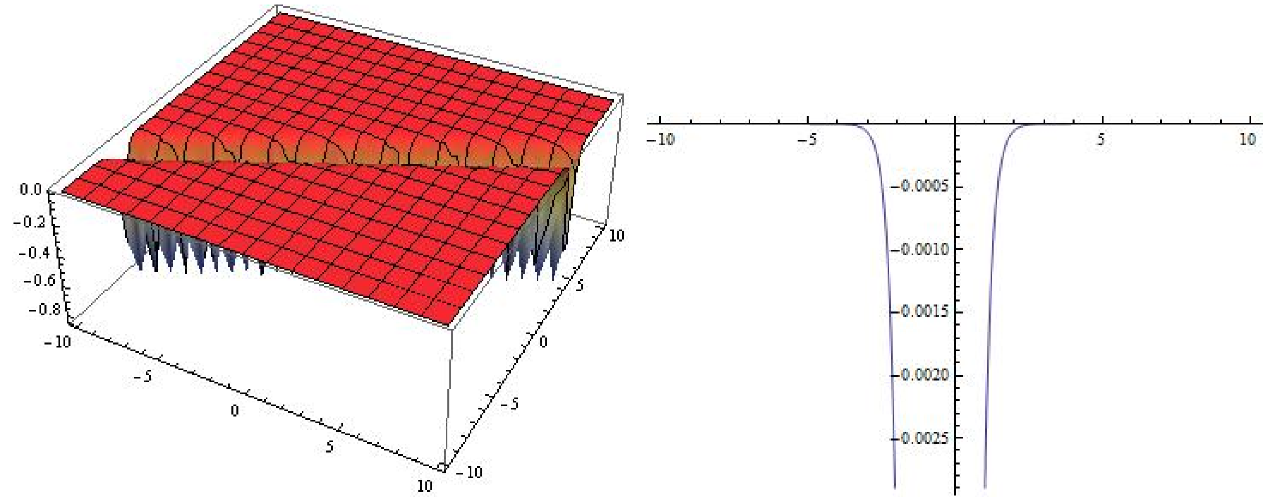

Figure 5. 3D and 2D graphs of imaginary part for the solution v_7 for the parameter choices $A=1$, $B=1$, and $C=0$

The 3D and 2D graphs of the solutions v_1, v_3, v_5, and v_7 are shown as in Figures 1-5 for the parameters $\nu=2$, $\alpha=1$, $\beta=1$, and $\gamma=1$ on the intervals $-10<x<10$ and $-10<t<10$. From these figures, it is observed that the solutions obtained by the GUM display a wide variety of characteristics, represented by hyperbolic and trigonometric functions. 2D graphs of the solutions are also plotted for $x=0$ to follow properly the characteristics of the solutions. The solutions in different shapes can be obtained by using free parameters for the corresponding physical problems.

4. Results and Discussion

We have obtained more general forms of exact solutions for the Drinfeld-Sokolov System by using the GUM in Section 3. In this section, we show more concretely how to derive solutions from some other methods from these solutions, considering some of the above solution sets.

The first group of hyperbolic and trigonometric solutions of the unified method can be obtained by taking $B = 0$ and $C = 0$, respectively, depending on the wave velocity in $\{u_1, v_1\}$ as follows:

$$\begin{cases} u_{1,1}(x,t) = \frac{v_1^2(x,t)}{\nu} \\ v_{1,1}(x,t) = \mp \frac{\sqrt{\frac{\nu^2}{6\beta+3\gamma}}\left(\sqrt{A^2+B^2} \mp A\cosh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right)}{\left(B \mp A\sinh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right.} \end{cases}$$

and

$$\begin{cases} u_{1,2}(x,t) = \frac{v_1^2(x,t)}{\nu} \\ v_{1,2}(x,t) = \mp \frac{\sqrt{\frac{\nu^2}{6\beta+3\gamma}}\left(\sqrt{A^2-C^2} \mp A\cosh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right)}{iC \mp A\sinh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)} \end{cases}$$

In addition, the solution sets obtained by the tanh method can be derived by taking $B = 0$ and $C = 0$ with the identities $\cosh(2x) = 2\cosh^2(x) - 1 = 2\sinh^2(x) + 1$ and $\sinh(2x) = 2\sinh(x)\cosh(x)$ as follows:

$$\begin{cases} u_{1,3}(x,t) = \frac{v_1^2(x,t)}{\nu} \\ v_{1,3}(x,t) = \mp \frac{\sqrt{\frac{\nu^2}{6\beta+3\gamma}}\left(A \mp A\cosh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)\right)}{A\sinh\left(\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)\right)} \end{cases}$$

Similarly, we can also obtain the second group of hyperbolic and trigonometric solutions of the unified method depending on the wave velocity after some simplification in $\{u_2, v_2\}$.

$$\begin{cases} u_{21}(x,t) = \frac{v_2^2(x,t)}{\nu} \\ v_{21}(x,y,t) = \sqrt{\frac{\nu^2}{6\beta+3\gamma}} - \frac{2A\sqrt{\frac{\nu^2}{6\beta+3\gamma}}}{\left(A + e^{\mp\sqrt{\frac{2\nu}{\alpha}}(x-\nu t+\eta_0)}\right)} \end{cases}$$

In the studies [20, 21], it is explained that the unified method gives many more solutions than the family of the tanh method, the $\left(\frac{G'}{G}\right)$-expansion method, and the extended homogeneous balance method. Therefore, it can be concluded that the GUM gives more general solution sets than these methods.

The GUM, an enhanced version of the unified method, provides a more general solution structure for the models in mathematical physics. Considering hyperbolic-trigonometric identities and the value of μ in the case of a real number or pure imaginary number, the solutions obtained by the unified method can be derived by GUM as above while setting $B = 0$ and $C = 0$, respectively. Further, when μ is a complex number, solutions combining trigonometric-hyperbolic solution sets can also be obtained. Moreover, performing the GUM as a new expansion method on a computer contributes to a very simple, straightforward, effective, and accurate way to solve a wide range of NPDEs. Since a more general closed solution form in a compact way is obtained by the GUM with free parameters, we applied this method to solve the Drinfeld-Sokolov System.

5. Conclusion

In this study, the GUM has been successfully applied to obtain solutions for the DSS. The main contributions of this study are:

i. More general forms of exact solutions are obtained for DSS using the GUM. Some of the obtained solutions have been visualized by plotting graphs to show how diverse characteristics the solutions have.

ii. These obtained solutions can be more functional in explaining the physical characteristics of various models arising in science and engineering.

iii. We have shown that the solution sets obtained by some other exact solution methods, including the unified method, can be derived from the GUM.

iv. It is obvious that the GUM is quite simple to perform on any symbolic computation program and gives reliable and straightforward results to find exact solutions for NPDEs.

v. Moreover, the results show that the GUM can be applied in future studies for NPDEs and fractional NPDEs.

Author Contributions

The author read and approved the final version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

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