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A Modified Water Cycle Algorithm: An Opposition Based Meta-Heuristic Optimization to Solve Real World Engineering Problems

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Highlights

- This paper focuses introducing opposition learning on optimization technique.
- 15 benchmark problems are considered for the proposed method verification.
- ANOVA test is done and the data are shown for statistical verification of the data received.

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Abstract

This paper proposes the Opposition based learning on a latest recent population based Water Cycle Algorithm on different benchmark constraint optimization techniques. Water cycle is a Hydrological based technique which works on better search location of the stream and river that flows to the sea which works on certain control parameters that will be defined initially and obtain the population matrix. With the help of the application of the opposition learning opposite search will be made to receive the better search location to find the better fitness value and avoid the premature convergence and get best convergence rate. This Proposed Opposition based Water Cycle Algorithm is implemented and tested on fifteen benchmark problems mentioning the fitness value as well as the constraints value. The convergence plot using a comparative study between Water Cycle Algorithm and Opposition based Water Cycle Algorithm, the proposed method had proved to obtain the best result and superior for the problems on to which it had implemented. The ANOVA test result is shown for the statistical analysis of the data obtained.

1. INTRODUCTION

Going back to the past 20 years, the pattern of the swarm based behaviour had been encouraging to evolve many renowned optimization techniques. Optimization had been expanded the attention over the years passed by for its omnipresent nature, various present problems faced or detected from application on engineering fields to decision making. The most practical way to approach to the optimization problems is considered as meta-heuristics. All those problems which cannot be dealt with the normalised or generalised techniques can be handled or taken care of are known as the meta-heuristics optimization techniques. There had been various optimization techniques that had been applied to various such problems like Ant Colony optimization [1] which had the concern on the relation between ant and the environment. Genetic Algorithm [2, 3] was the generation in terms of chromosomes had been broadly discussed. Particle Swarm Optimization [4] which is concerned on the convergence to get the optimal solution. Whale optimizations were used for finding a better location of placing SVC and TVSS from [5]. Eskander from [6] had seen to have discussed about water cycle algorithm and its natural cycle which had also played a huge role on optimization. Optimization had also discussed on the field of the overcurrent relay [7] as well. This topic optimization had been widely used in the field of optimal bidding strategically using various optimizations leading to cost minimization or for its profit maximization from [8-12]. Rather optimization is an elementary task where algorithm learns from the functions previous data, thereby optimizing it to some best

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solution estimated for assigned boundaries to search through a large specified areas or space to obtain a best and optimal value for the individual function of each problem solution. Each and every proposed optimization available in the market follows the similar procedure with some of its specific parameters for individual techniques for its execution along with the specialization of the techniques and finds out the best fitness function within minimum computation time and better convergence rate.

Opposition Based Learning (OBL) which is quite an interesting field of research that had already grabbed an attention of a huge researchers over the decades. Many of the optimizing techniques have already accepted the learning and have applied to the existing one to enhance the original techniques. This learning had been made known to all by Tizhoosh [13]. Opposition learning falls under a type of machine learning which had proved successful, constructive and a productive method that generates a better random data to obtain a better fitness for the given function. Several existing methods using this learning like OPSO [14] utilized for day ahead scheduling in distribution system which had proved better than PSO. Multi-verse optimizer [15] which with the help of two steps had been used to speed up the output with better accuracy was used. Also an Improved Equilibrium Optimization [16] using OBL for the area of medical dataset had been utilised. Whale optimization [17] also used the OBL learning for the problem of shop scheduling. OBL concept was even applied to Evaporation rate Water cycle optimization [18] on optimal coordination for overcurrent relay. In order to obtain better global optima OBL concept [19] was introduced with Grey Wolf Optimization. For utilization of the optimization for all the continuous problems Ant Colony optimization [20] too up the concept of OBL into the actual optimization which received to provide a better response as well. For the application of exploitation and exploration phase improvement OBL was used on Crow search Algorithm [21] as well.

In this study an OBL-WCA optimization method have been proposed. Both the WCA and OBL-WCA had been applied on some benchmark problems for the better comparison between the two methods along with the convergence curve also have been provided under the result section for better understanding and find the improvement made proposing the new technique. The OBL-WCA had been proved to provide the best value for all the functions that have been used for providing the better performance, random value detection and best fitness data with superior computation time. [22-25] had utilised the concept of OBL on various problem statement and considered to provide a better response.

Contribution and motivation of the paper:

Going through several papers regarding optimization concept a new meta-heuristic optimization which is a population based algorithm had been chosen which itself is best one due to its search and better convergence. Hence Opposition based learning; a new learning had been introduced along with the WCA algorithm for the better global search, avoiding premature and fast response with better convergence time. The proposed method Opposition based Water Cycle Algorithm (OWCA) had been tested on 15 benchmark problem and had been compared with the Water Cycle Algorithm (WCA) and had obtained a better response regarding the objective function with fast convergence time, better search space for the minimized as well as maximized function. Except few for all the cases we have taken 50 populations size with 100 iterations for better comparison for both OWCA and WCA.

The paper section had been made as follows: section 2 describes the proposed method with the following equation and a flowchart. Section 3 explains briefly the various functions that had been taken up mentioning their constraint boundaries and parameters used. Section 4 explains about the result obtained using the proposed method and its improvement on the functions used. Section 5 explains the conclusion part of the article.

2. OPPOSITION BASED WATER CYCLE ALGORITHM

Water Cycle algorithm (WCA) is a meta-heuristic population based algorithm which was first introduced by Eskander [6] that applied and coded for both unconstraint and constraints problems. It uses the concept of getting the best solution of the optimal location of the stream and river flowing to the sea. Though it is very much similar to various other available optimization in the market but due to the exploration and evaporation rate calculation it gives better global search opportunity and optimal data within best convergence rate. Opposition learning is one of the types of machine learning which is added to the WCA in order to get the best search of the optimal value by increasing the search rate in a reverse manner and to avoid the premature convergence. Figure 1 shows the flowchart of the proposed method stepwise for the better understanding.

2.1. Initial Population Calculation

At first the control parameters belonging to WCA has to be declared $M_p * M_v$, where M_v denotes the variable, M_p the population size.

$$M_S = No.\ of\ rivers + 1(Sea) \tag{1}$$

$$M_{streams} = M_p - M_S \tag{2}$$

$$Populations\ of\ Streams = \begin{bmatrix} Stream_1 \\ Stream_2 \\ Stream_3 \\ \vdots \\ Stream_{M_{stream}} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_M^1 \\ x_1^2 & x_2^2 & \cdots & x_M^2 \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{M_{Stream}} & x_2^{M_{Stream}} & \cdots & x_M^{M_{Stream}} \end{bmatrix} \tag{3}$$

$$Total\ \ Population = \begin{bmatrix} Sea \\ River_1 \\ River_2 \\ \vdots \\ Stream_{M_S+1} \\ Stream_{M_S+2} \\ Stream_{M_S+3} \\ \vdots \\ Stream_{M_p} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_D^1 \\ x_1^2 & x_2^2 & \cdots & x_D^2 \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{M_p} & x_2^{M_p} & \cdots & x_D^{M_p} \end{bmatrix} \tag{4}$$

Firstly the stream of M_p has to be created with M_S that is equal to rivers involve and one sea. The overall method to return the minimum value for the stream for minimum objective function and maximum value for maximized objective function that flows to the sea. The rest will flow from river and river to sea or directly to sea. Rivers will be absorbing water from streams, Hence there is a variation as to no of stream move to river and sea.

The next step is to calculate the cost calculation using the below equation,

$$C_i = f\left(x_1^i, x_2^i, x_3^i \dots \dots x_{M_v}^i\right)\ \ where\ i = 1,2,3, \dots, M_p \tag{5}$$

$$C_m = cost_m - cost_{M_S+1} \qquad where\ m = 1,2,3, \dots, M_S \tag{6}$$

$$MS_m = round\left\{\left|\frac{cost_M - cost_{M_S+1}}{\sum_{m=1}^{M_S} C_m}\right| * M_{streams}\right\} \quad where\ m = 1,2,3, \dots, M_S \tag{7}$$

where MS_m are the streams which particularly flow into some rivers and a sea.

2.2. Development of a New Streams Flow to River or River Flow to Sea Using Below Equation

Calculation of the exploitation phase of WCA,

Newly positioned streams and rivers are written below

$$\vec{K}_{stream}(t+1) = \vec{K}_{stream}(t) + rand * B * (\vec{K}_{sea}(t) - \vec{K}_{stream}(t)) \tag{8}$$

$$\vec{K}_{stream}(t+1) = \vec{K}_{stream}(t) + rand * B * (\vec{K}_{river}(t) - \vec{K}_{stream}(t)) \tag{9}$$

$$\vec{K}_{river}(t+1) = \vec{K}_{river}(t) + rand * B * (\vec{K}_{sea}(t) - \vec{K}_{river}(t)). \tag{10}$$

‘t’ is an iteration index, $1 < B < 2$, and the random variables denoted as *rand* lies within zero and one. Equations (8-10) are streams flow towards the rivers and sea. If the solution of stream found to give the better optimal solution than river, then it has the ability to exchanges between sea and river and vice versa. The evaporation rate solution is also added to stay away from premature meeting at the exploitation phase. Evaporation causes the water of the sea to evaporate as the rivers or stream flowing towards the sea leading to precipitation.

2.3. The Evaporation Condition

The criteria for such purposes are:

$$if \ \|\vec{K}_{sea}^{j} - \vec{K}_{\mathrm{River}}^{j}\| < d_{max} \quad or\ rand < 0.1 \quad j = 1,2,3 \ldots, M_S - 1 \tag{11}$$

Performance of raining process with uniform random search

End

‘d_{max}‘ a number nearest to null.

Following the evaporation process and then the raining procedure had been accepted and implemented by forming new streams to some other location. If $d_{max}$is greater enough then the further additional process will not be done. So d_{max}increase the search intensity near the sea.

$$d_{max}(t+1) = d_{max} - \frac{d_{max}(t)}{Max\_iteration} \quad where\ t = 1,2,3, \ldots \ldots, Max\_iteration \tag{12}$$

$$\vec{K}_{new\_stream}^{t} = LB + rand(UB - LB) \tag{13}$$

LB and UB are denoted as Lower and Upper Boundary of the constraints handling parameters for the various functions that are taken to and search the better location for better value in the problem or objective function.

To improve the rate of performance and convergence concerning on problems of constraints for the cases where sea will receive the water directly from stream. The proceeding equation belongs to the exploration phase of the optimization giving a better optimum search solution

$$K_{stream}^{new} = K_{sea} + \sqrt{\mu} * rand(1, nvar) \tag{14}$$

μ is the variance to for to the optimum solution of the various applied problems.

2.4. End of Loop

The loop gets repeated until a best convergence is received.

2.5. Opposition Learning

The Opposition based learning is a current concept to improve the performance of various population based techniques. It is the concept that had been implemented as a simultaneous search for the present estimate with its opposite search as well in order to get a better global search within iterative procedure maintaining the limited boundaries. Having used this concept on several benchmark problems used in the paper discussion to get an understanding of the performance level between the method and OBL on the method which had result to get a better search solution to get a better fitness value for the function used.

Considering Z as a number whose reverse number needs to have a random search, then

$$\hat{Z} = L + U - Z \tag{15}$$

in which $Z \in (L, U)$. Now this above equation can also be applied for more than a number as well thereby creating an opposite matrix for the algorithm to find out the random with a reverse search with

$$\hat{Z}_n = L_n + U_n - Z_n \tag{16}$$

where n=1,2,…d in which d denotes the number of variables involved for the specific objective function of the operating system. Below Figure shows the flowchart of the proposed method for calculation of various benchmark problems used.

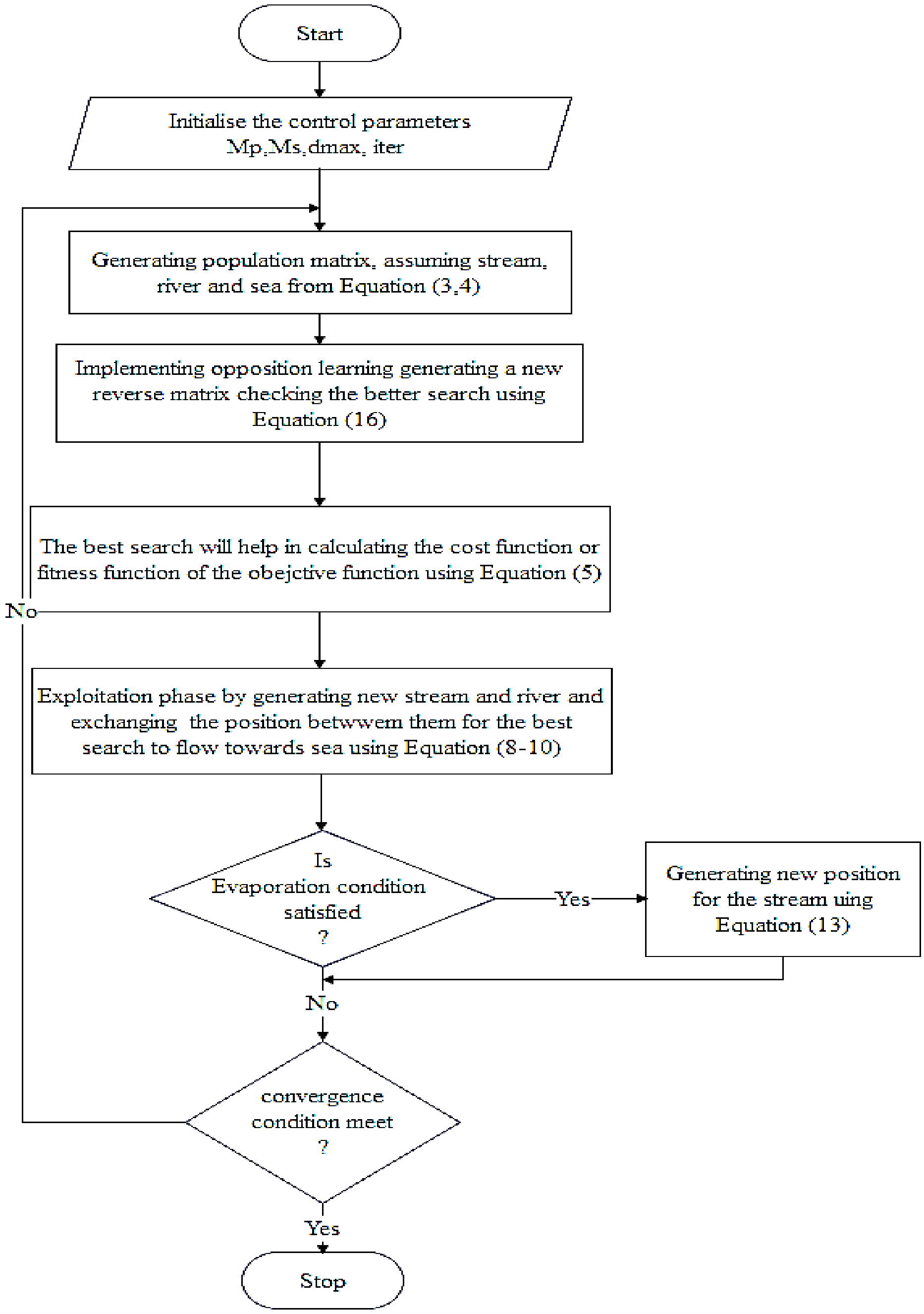


Figure 1. *The flowchart of the proposed method*

3. PROBLEMS CONSIDERED TO TEST THE PROPOSED METHOD

In these section different problems have been taken up that are used to verify the proposed method is suitable enough and provides the better response or not are expressed with their objective function, their different equal and in-equal constraints, linear and non-linear constraints. This benchmark problem can be some generalised problem and some mechanical problems used. Problem 1 is used as a minimized problem using seven parameters with four inequality constraints. Problem 2 is a maximized problem using ten parameters and single nonlinear constraint. Problem 3 is a minimized problem with three parameters and nonlinear constraints. Problem 4 is a minimized problem with eight parameters and three linear and three nonlinear constraints. Problem 5 is a process synthesis problem with two parameters and two inequality constraints. Problem 6 is a process design problem. Problem 7 used four parameters and three inequality constraints in its function. Problem 8 is process sheeting problem having two parameters and three inequality constraint. Problem 9 is a welded beam [26] minimized problem design using four continuous design parameters with population size of 50. Problem 10 is a pressure vessel [27] minimized design cost reduction problem with three linear and one nonlinear constraint. Problem 11 discusses about tension or spring minimized design [28] with one linear and three nonlinear constraints with three parameters. Problem 12 is weight minimization of speed reducer design [29] with a discrete and six continuous parameters along with four liners and seven nonlinear constraints. Problem 11, 12 are the minimized multi disk clutch design [30] and maximized rolling element design [31] and with its parameters and the constraints shown below. Problem 15 is the three bar Thrush problem [32, 33] with three inequality constraints with parameters mentioned below.

Problem-1:

$$\min f(u) = (u_1-10)^2 + 5(u_2-12)^2 + u_3^4 + 3(u_4-11)^2 + 10u_5^6 + 7u_6^2 + u_7^4 - 4u_6u_7 - 10u_6 - 8u_7 \tag{17}$$

$$Subject\ to:\ h_1(u) = -127 + 2u_1^2 + 3u_2^4 + u_3 + 4u_4^2 + 5u_5 \le 0 \tag{18}$$

$$h_2(u) = -282 + 7u_1 + 3u_2 + 10u_3^2 + u_4 - u_5 \le 0 \tag{19}$$

$$h_3(u) = -196 + 23u_1 + u_2^2 + 6u_6^2 - 8u_7 \le 0 \tag{20}$$

$$h_4(u) = 4u_1^2 + u_2^2 - 3u_1u_2 + 2u_3^2 + 5u_6 - 11u_7 \le 0 \tag{21}$$

$where - 10 \le u_j \le 10, j = 1,2,3,\dots,7.$

Problem-2

$$\max f(u) = \left(\sqrt{n}\right)^n \prod_{j=1}^{n} u_j \tag{22}$$

$$Subject\ to:\ g(u) = \sum_{j=1}^{n} u_1^2 - 1 = 0 \tag{23}$$

$where\ n = 10\ and\ 0 \le u_j \le 10, \quad j = 1,2,3,\dots,n.$

Problem-3

$$\max f(u) = \frac{100-(u_1-5)^2-(u_2-5)^2-(u_3-5)^2}{100} \tag{24}$$

$$Subject\ to: (u_1-p)^2 - (u_2-q)^2 - (u_3-r)^2 \le 0 \tag{25}$$

$where\ 0 \le u_j \le 10, \quad j = 1,2,3, p,q,r = 1,2,3,\dots,9$

Problem-4

$$\min f(u) = u_1 + u_2 + u_3 \tag{26}$$

$$Subject\ to: h_1(u) = -1 + 0.0025(u_4 + u_6) \le 0 \tag{27}$$

$$h_2(u) = -1 + 0.0025(u_5 + u_7 - u_4) \le 0 \tag{28}$$

$$h_3(u) = -1 + 0.01(u_8 - u_5) \le 0 \tag{29}$$

$$h_4(u) = -u_1u_6 + 833.3325u_4 + 100u_1 - 83333.333 \le 0 \tag{30}$$

$$h_5(u) = -u_2u_7 + 1250u_5 + u_2u_4 - 1250u_4 \le 0 \tag{31}$$

$$h_6(u) = -u_3u_8 + 1250000 + u_3u_5 - 2500u_5 \le 0 \tag{32}$$

$where\ 100 \le u_1 \le 10{,}000$

$1000 \le u_j \le 10{,}000, \quad j = 2,3$

$100 \le u_j \le 10{,}000, \quad j = 4,5,6,7,8$

Problem-5: Process synthesis problem

$$\min f = 2u + v \tag{33}$$

$$Subject\ to: 1.25 - u^2 - v \le 0 \tag{34}$$

$$u + v \le 1.6 \tag{35}$$

$where\ u \in [0,1.6]; \ v \in [0,1]$

Problem-6: Process design problem

$$\min f(u) = 5.357854u_1^2 + 0.835689v_1u_3 + 37.29329v_1 - 40792.141 \tag{36}$$

$$Subject\ to: 85.334407 + 0.0056858v_2u_3 + 0.0006262v_1u_2 - 0.0022053u_1u_3 \le 92 \tag{37}$$

$$80.51249 + 0.0071317v_2u_3 + 0.0029955v_1v_2 + 0.0021813u_1^2 - 90 \le 20 \tag{38}$$

$$9.300961 + 0.0047026u_1u_3 + 0.0012547v_1u_1 + 0.0019085u_1u_2 - 20 \le 5 \tag{39}$$

$where\ u_1, u_2, u_3 \ \in [27,45];\ v_1 \in [78,\dots,102], v_2 \in [33,\dots,45]$

Problem-7

$$\min f(u,v) = 2u_1 + 3u_2 + 1.5v_1 + 2v_2 - 0.5v_3 \quad (40)$$
$$Subject\ to: \ (u_1)^2 + v_1 = 1.25 \quad (41)$$
$$(u_2)^{1.5} + 1.5y^2 = 3 \quad (42)$$
$$u_1 + v_1 \leq 1.6 \quad (43)$$
$$1.333u_2 + v_2 \leq 3 \quad (44)$$
$$-v_1 - v_2 + v_3 \leq 0 \quad (45)$$
$$where\ u_1, u_2 \in [0,20]\ ;\ v_1, v_2 \in \{0,1\}$$

Problem-8:

$$\min f(u,v) = 7.5v_1 + 6.4u_1 + 5.5v_2 + 6.0u_2 \quad (46)$$
$$Subject\ to: 0.8u_1 + 0.67u_2 = 10 \quad (47)$$
$$u_1 - 20v_1 \leq 0 \quad (48)$$
$$u_2 - 20v_2 \leq 0 \quad (49)$$
$$where\ u_1, u_2 \in [0,20], v_1, v_2 \in [0,1]$$

Problem-9: Welded Beam design

$$\min f(x) = 1.10471u_1^2u_2 + 0.04811u_3u_4(14.0 + u_2) \quad (50)$$
$$Subject\ to: \quad h_1(u) = \tau(u) - \tau_{max} \leq 0 \quad (51)$$
$$h_2(u) = \sigma(u) - \sigma_{max} \leq 0 \quad (52)$$
$$h_3(u) = u_1 - u_4 \leq 0 \quad (53)$$
$$h_4(u) = 0.10471u_1^2 + 0.04811u_3u_4(14.0 + u_2) - 5.0 \leq 0 \quad (54)$$
$$h_5(u) = 0.125 - u_1 \leq 0 \quad (55)$$
$$h_6(u) = \delta(u) - \delta_{max} \leq 0 \quad (56)$$
$$h_7(u) = P - P_c(u) \leq 0 \quad (57)$$

where

$$\tau(u) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x^2}{2R} + (\tau')^2} \quad (58)$$
$$(\tau') = \frac{P}{2^{0.5}u_1u_2} \quad (59)$$
$$(\tau'') = \frac{MR}{J} \quad (60)$$
$$M = P\left(L + \frac{u_2}{2}\right) \quad (61)$$
$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{u_1+u_2}{2}\right)^2} \quad (62)$$
$$J = 2\left\{2^{0.5}u_1u_2\left[\frac{u_2^2}{12} + \left(\frac{u_1+u_2}{2}\right)\left(\frac{u_1+u_2}{2}\right)\right]\right\} \quad (63)$$
$$\sigma(u) = \frac{6PL^3}{u_4u_3^2} \quad (64)$$
$$\delta(x) = \frac{4PL^3}{Eu_3^3u_4} \quad (65)$$
$$P_c(x) = \frac{4.013E\sqrt{u_3^2u_4^6/36}}{L^2} \quad (66)$$

$where\ P = 6000lb, L = 14in,. E = 30 * 10^6 psi, G = 12 * 10^6 psi, \tau_{max} = 13{,}600\ psi, \sigma_{max} = 30{,}000\ psi, \quad \delta_{max} = 0.25\ in.,$

$0.1 \leq u_1 \leq 2,$
$0.1 \leq u_2 \leq 10$
$0.1 \leq u_3 \leq 10$
$0.1 \leq u_4 \leq 2$

Problem-10: Pressure vessel design

$$\min f(x) = 0.6224u_1u_3u_4 + 1.7781u_2u_3^2 + 3.1661u_1^2u_4 + 19.84u_1^2u_3 \quad (67)$$
$$Subject\ to: \quad h_1(u) = 0.0193u_3 - u_1 \leq 0 \quad (68)$$

$$h_2(u) = 0.0095u_3 - u_2 \leq 0 \tag{69}$$
$$h_3(u) = -\pi u_3^2 u_4 - \frac{4}{3}\pi u_3^3 + 1296000 \leq 0 \tag{70}$$
$$h_4(u) = u_4 - 240 \leq 0 \tag{71}$$

$where\ 0 \leq u_1 \leq 99$
$0 \leq u_2 \leq 99$
$10 \leq u_3 \leq 200$
$10 \leq u_4 \leq 200$

Problem-11: Tension/Compression spring design

$$\min f(u) = (u_3 + 2)u_2 u_1^2 \tag{72}$$
$$Subject\ to: \quad h_1(u) = 1 - \frac{u_2^3 u_3}{71785u_1^4} \leq 0 \tag{73}$$
$$h_2(u) = \frac{4u_2^2 - u_1 u_2}{12566(u_2 u_1^3 - u_1^4)} + \frac{1}{5108u_1^2} - 1 \leq 0 \tag{74}$$
$$h_3(u) = 1 - \frac{140.45u_1}{u_2^2 u_3} \leq 0 \tag{75}$$
$$h_4(u) = \frac{u_1 + u_2}{1.5} - 1 \leq 0 \tag{76}$$

$where\ \ 0.05 \leq u_1 \leq 2, \quad 0.25 \leq u_2 \leq 1.3, \quad 2 \leq u_3 \leq 15$

Problem-12: Speed reducer design

$$\min f(u) = 0.7854u_1 u_2^2(3.3333u_3^2 + 14.9334u_3 - 43.0934) - 1.508(u_6^2 + u_7^2) + 7.4777(u_6^3 + u_7^3) \tag{77}$$
$$Subject\ to: \ h_1(u) = \frac{27}{u_1 u_2^2 u_3} - 1 \leq 0 \tag{78}$$
$$h_2(u) = \frac{397.5}{u_1 u_2^2 u_3^2} - 1 \leq 0 \tag{79}$$
$$h_3(u) = \frac{1.93u_4^3}{u_2 u_3 u_6^4} - 1 \leq 0 \tag{80}$$
$$h_4(u) = \frac{1.93u_5^3}{u_2 u_3 u_7^4} - 1 \leq 0 \tag{81}$$
$$h_5(u) = \frac{\sqrt{\left(745u_4/u_2 u_3\right)^2 + 16.9\times10^6}}{119.0u_6^3} - 1 \leq 0 \tag{82}$$
$$h_6(u) = \frac{\sqrt{\left(745u_4/u_2 u_3\right)^2 + 157.5\times10^6}}{85.0u_6^3} - 1 \leq 0 \tag{83}$$
$$h_7(u) = \frac{u_2 u_3}{40} - 1 \leq 0 \tag{84}$$
$$h_8(u) = \frac{5u_2}{u_1} - 1 \leq 0 \tag{85}$$
$$h_9(u) = \frac{u_1}{12u_2} - 1 \leq 0 \tag{86}$$
$$h_{10}(u) = \frac{1.5u_6 + 1.9}{u_4} - 1 \leq 0 \tag{87}$$
$$h_{11}(u) = \frac{1.51u_7 + 1.9}{u_5} - 1 \leq 0 \tag{88}$$

$where\ 2.6 \leq u_1 \leq 3.6, \quad 0.7 \leq u_2 \leq 0.8, \quad 17 \leq u_3 \leq 28, \quad 7.3 \leq u_4 \leq 8.3,$
$7.8 \leq u_5 \leq 8.3, \ \ 2.9 \leq u_6 \leq 3.9, \quad 25.0 \leq u_7 \leq 5.5$

Problem-13: Multiple Disk Clutch Brake design

$$\min f(x) = \pi(r_o^2 - r_i^2)t(Z + 1)\rho \tag{89}$$
$$h_1(u) = r_o - r_i - \Delta r \geq 0 \tag{90}$$
$$h_2(u) = l_{max} - (Z + 1)(t + \delta) \geq 0 \tag{91}$$
$$h_3(u) = p_{max} - p_{rz} \geq 0 \tag{92}$$
$$h_4(u) = p_{max} v_{srmax} - p_{rz} v_{sr} \geq 0 \tag{93}$$
$$h_5(u) = v_{srmax} - v_{sr} \geq 0 \tag{94}$$
$$h_6(u) = T_{max} - T \geq 0 \tag{95}$$
$$h_7(u) = M_h - sM_s \geq 0 \tag{96}$$

$$h_8(u) = T \geq 0 \quad (97)$$

$$where \ \ M = \frac{2}{3}\mu FZ \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \quad (98)$$

$$p_{rz} = \frac{F}{\pi(r_o^2 - r_i^2)} \quad (99)$$

$$v_{sr} = \frac{2\pi n(r_o^3 - r_i^3)}{90(r_o^2 - r_i^2)} \quad (100)$$

$$T = \frac{I_z \pi n}{30(M_h + M_f)} \quad (101)$$

$where \ \ \Delta r = 20mm, \quad t_{max} = 3mm, t_{min} = 1.5mm, l_{max} = 30mm,$ $Z_{max} = 10, v_{srmax} = \frac{10m}{s}, \mu = 0.5, s = 1.5, M_s = 40Nm, M_f = 3Nm, n = 250rpm,$ $p_{max} = 1\ MPa, \ I_z = 55kg\ mm^2, T_{max} = 15s, F_{maxx} = 1000N, r_{imin} = 55mm,$ $r_{omax} = 110mm.$

Problem-14: Rolling element bearing

$$Max\ C_d = f_c z^{\frac{2}{3}} D_b^{1.8} \quad if D_b \leq 25.4\ mm \quad (102)$$

$$C_d = f_c z^{\frac{2}{3}} D_b^{1.8} \quad if D_b \leq 25.4\ mm \quad (103)$$

$$Subject\ to\!: \ h_1(u) = \frac{\phi_0}{2sin^{-1}\left({D_b}/{D_m}\right)} - Z + 1 \geq 0 \quad (104)$$

$$h_2(u) = 2D_b - K_{Dmaxx}(D - d) \geq 0 \quad (105)$$

$$h_3(u) = K_{Dmaxx}(D - d) - 2D_b \geq 0 \quad (106)$$

$$h_4(u) = \zeta B_\omega - D_b \leq 0 \quad (107)$$

$$h_5(u) = D_m - 0.5(D + d) \geq 0 \quad (108)$$

$$h_6(u) = (0.5 + e)(D + d) - D_m \geq 0 \quad (109)$$

$$h_7(u) = 0.5(D - D_m - D_b) - \varepsilon D_b \geq 0 \quad (110)$$

$$h_8(u) = f_1 \geq 0.515 \quad (111)$$

$$h_9(u) = f_0 \geq 0.515 \quad (112)$$

$where$

$$f_c = 37.91\left[1 + \left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_1(2f_0 - 1)}{f_0(2f_1 - 1)}\right)^{0.41}\right\}^{\frac{10}{3}}\right]^{-0.3} \quad (113)$$

$$\gamma = \frac{D_b cos\alpha}{D_m} \quad (114)$$

$$f_1 = \frac{r_1}{D_b} \quad (115)$$

$$f_c = 2\pi - 20cos^{-1}\frac{\left[\left\{\frac{D-d}{2} - 3\left(\frac{T}{4}\right)^2 + \left\{\frac{D}{2} - \left(\frac{T}{4} - D_b\right)\right\}^2 - \left\{\frac{d}{2} + \left(\frac{T}{4}\right)\right\}^2\right\}\right]}{2\left\{D - \frac{d}{2} - 3\left(\frac{T}{4}\right)\right\}\left\{\frac{D}{2} - \left(\frac{T}{4}\right) - D_b\right\}} \quad (116)$$

$$T = D - d - 2D_b \quad (117)$$

$where \ \ D = 160, d = 90, B_w = 30$

$$0.5(D + d) \leq D_m \leq 0.6(D + d) \quad (118)$$

$$0.15(D - d) \leq D_b \leq 0.45(D - d) \quad (119)$$

$4 \leq Z \leq 50$
$0.515 \leq f_1 \leq 0.6$
$0.515 \leq f_0 \leq 0.6$
$0.4 \leq K_{Dmin} \leq 0.5$
$0.6 \leq K_{Dmax} \leq 0.7$
$0.3 \leq \epsilon \leq 0.4$
$0.02 \leq e \leq 0.1$
$0.6 \leq \xi \leq 0.85$

Problem-15: Three Bar thrush problem

$$\min f(u_1,u_2) = \left(2\sqrt{2}u_1 + u_2\right)l \quad (120)$$

$$s.t.\, g_1(u) = \frac{(\sqrt{2}u_1+u_2)}{(\sqrt{2}u_1^2+2u_1u_2)}P - \sigma \leq 0 \quad (121)$$

$$s.t.\, g_2(u) = \frac{u_2}{(\sqrt{2}u_1^2+2u_1u_2)}P - \sigma \leq 0 \quad (122)$$

$$g_3(u) = \frac{1}{(\sqrt{2}u_2+u_1)}P - \sigma \leq 0 \quad (123)$$

$where\ 0 \leq u_1, u_2 \leq 1$

$L = 100cm, P = \frac{2KN}{cm^2}, \sigma = 2KN/cm^2$

4. SIMULATION RESULT AND DISCUSSION

In this work OWCA method had been proposed and implemented and result analysis had been discussed that are applied and investigated to all well-known benchmark problems to signify the improvement from the existing WCA algorithm and compared to some other conventional techniques as well. All the problems had been operated on MATLAB software. WCA was already an available optimizing technique on which a new opposition leaning had been implemented to get a better fitness value to receive a better good global data and better convergence at a faster rate. Table 1 includes all the conventional techniques applied on mentioned benchmark problems discussed along with the proposed method used obtained data. Table 2 includes the constraint parameters data obtained with executing the proposed method for securing the better fitness for each of the benchmark problem with the best, worst value including the computational time mentioned in second. Figure 2 shows various problem convergence curves comparing with the conventional WCA along with the proposed OWCA method. The first eight figures below indicate the convergence plot of generalised benchmark problem and the rest of the plot are of the mechanical problems which have been widely discussed under section 3. OWCA which proved for better understanding of the stability condition and time of the plot on the system for better comparison which had proved to give the best result without any premature convergence and good fitness data. Tables 1 and 2 includes the proposed and the conventional methods for all eight general maximum and minimum function benchmark problem. Table 3 and 4 includes all the seven mechanical design benchmark problems considered for implementing and testing the proposed method. Table 5 includes the proposed method constraint value along with its best, worst data and the computational time. Table 6 includes the ANOVA test data for the understanding of the statistical data for significant difference between the proposed and all the conventional techniques. Figure 2 shows the convergence plot of all the problems that had been considered for the article.

Table 1. *Comparing Problem 1-4 using the proposed and conventional methods*

Sl No.		OWCA	Jaya [34]	Elitist TLBO [35]	DETPS [22]	TLBO [23]	ABC [24]	CoDE [25]
1.	Problem-1	**680.420**	680.630	680.630	680.630	680.630	680.634	680.771
	NFE	**5000**	30,000	30,019	32,586	100,000	100,000	240,000
2.	Problem-2	**1.000**	1.000	1.000	1.001	1.000	1.000	---
	NFE	**5000**	25,000	69,996	90,790	100,000	100,000	---
3.	Problem-3	**1.000**	1.000	1.000	1.000	1.000	1.000	1.000
	NFE	**5000**	5000	5011	6540	50,000	100,000	240,000
4.	Problem-4	**5264.5183**	7049.248	7049.248	7049.257	7049.248	7053.904	---
	NFE	**5000**	99,000	99,987	100,000	100,000	240,000	---

Table 2. *Comparing Problem 5-8 with proposed and some other conventional methods*

Sl No.		OWCA	Jaya [34]	Elitist TBLO [35]	DETPS [22]	MDE [36]	MA-MDE [36]	MDE-HIS [36]
1	Problem-5	**1.3671**	2.0000	2.0000	2.0000	2.0093	2.0000	2.0000
	NFE	**5000**	5000	1,019	1720	1075	1430	3297
2	Problem-6	**-30658.273**	-32,21743	-32,217430	-32,217407	-32,21742	-32,217427	-32,2174277
	NFE	**5000**	300	312	3242	1240	1955	493
3	Problem-7	**7.6671**	7.6671	7.8156	7.9311	7.91861	7.88384	7.84889
	NFE	**5000**	79,000	79,994	100,000	96,718	93,524	83,442
4	Problem-8	**84.7404**	87.500012	87.50002	87.500012	89.879034	88.230145	87.49755
	NFE	**5000**	4000	4033	14360	7777	4436	5359

Table 3. *Comparing the best fitness value of the mechanical problem using OWCA with other applied conventional methods*

Sl No,		OWCA	Jaya [34]	Elitist TLBO [35]	DETPS [22]	MBA [37]	TLBO [23]	ABC [24]
5.	Welded Beam	**1.728**	1.7248	1.72485	1.72485	1.72485	1.72485	1.72485
	NFE	**5000**	10,000	9991	10,000	47340	10,000	30,000
6.	Pressure Vessel	**5884.024**	5885.333	5885.33	5885.33	5889.3216	6059.71	6059.71
	NFE	**5000**	10,000	4992	10,000	70,650	10,000	30,000
7.	Spring problem	**0.012144**	0.012665	0.01266	0.01266	0.012665	0.012665	0.012665
	NFE	**5000**	10,000	7022	10,000	7650	10,000	30,000
8.	Speed reducer	**2994.474**	2996.348	2996.348	2996.348	2994.744	2996.34	2997.06
	NFE	**5000**	10,000	9988	10,000	6300	10,000	30,000
5	Multi Disk	**0.25977**	0.31365	NA	**NA**	NA	0.31365	0.31365
	NFE	**5000**	600	NA	NA	NA		
6	Rolling Element	**234703.4705**	81,859.7	NA	**NA**	NA	81,859.7	81,859.7
	NFE	**5000**	600	NA	NA	NA		

Table 4. *Best fitness value of the Three bar thrush problem using both OWCA and the conventional methods*

	OWCA	Kalman filter [33]	AAL [32]	CA [38]	MBA [37]	BA [39]	CSA [40]	SOA [41]
Three Bar	**263.236**	263.8958	263,8958	263.8958	263.895	263.8962	263.9716	264.300

Table 5. *Comparative study on the constraint data, best, worst value and c.p.u. time between WCA and OWCA method*

Sl. No	Problems Used	Method Applied	Constraint Value	Best Value	Worst Value	Computational Time (sec)
1	Problem-1	WCA	$u_1 = 2.178644$ $u_2 = 1.88667$ $u_3 = -0.51167$ $u_4 = 4.557710$ $u_5 = -0.616597$	681.6432	21410	0.60204

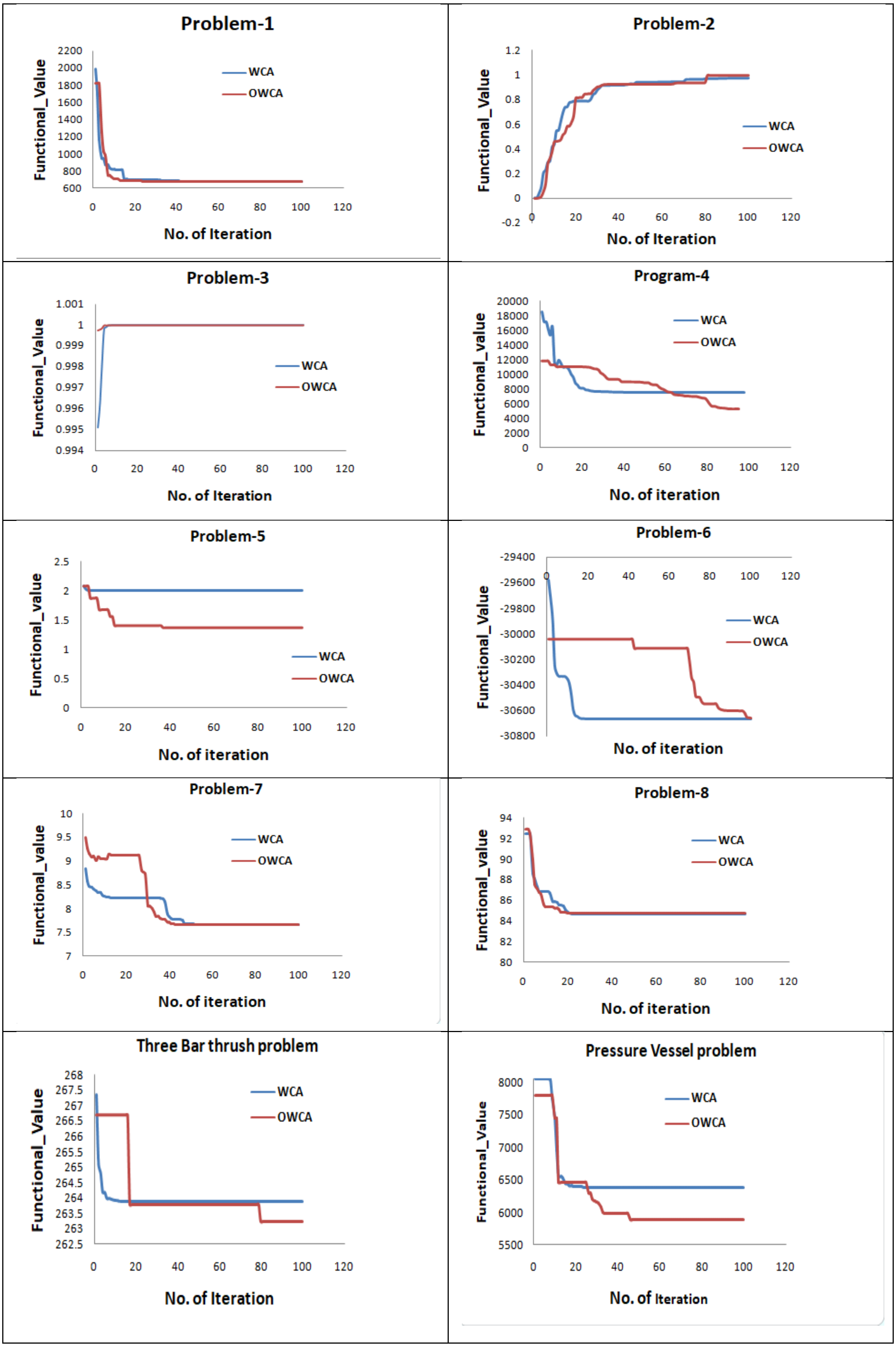
			$u_6 = 1{,}183889$ $u_7 = 1.51431$			
		OWCA	$u_1 = 2.30186$ $u_2 = 1.94857$ $u_3 = -0.37461$ $u_4 = 4.37647$ $u_5 = -0.61741$ $u_6 = 1.138756$ $u_7 = 1{,}183889$	680.420	11150.6776	0.49221
2	Problem-2	WCA	$u_1 = 0.337582$ $u_2 = 0.301929$ $u_3 = 0.328431$ $u_4 = 0.330691$ $u_5 = 0.323132$ $u_6 = 0.313910$ $u_7 = 0.308629$ $u_8 = 0.318755$ $u_9 = 0.32467$ $u_{10} = 0.3071345$	1.000	0.97375	0.523977
		OWCA	$u_1 = 0.337582$ $u_2 = 0.301929$ $u_3 = 0.328431$ $u_4 = 0.330691$ $u_5 = 0.323132$ $u_6 = 0.313910$ $u_7 = 0.287117$ $u_8 = 0.318822$ $u_9 = 0.337945$ $u_{10} = 0.314056$	1.000	0.95451	0.322104
3	Problem-3	WCA	$u_1 = 5.0000$ $u_2 = 5.0058$ $u_3 = 4.9999$	1	0.99513	0.49849
		OWCA	$u_1 = 4.9999$ $u_2 = 4.9999$ $u_3 = 4.9999$	1	0.99976	0.62494
4	Problem-4	WCA	$u_1 = 100$ $u_2 = 2112.368$ $u_3 = 5289.27$ $u_4 = 100$ $u_5 = 2884.289$ $u_6 = 100$ $u_7 = 211.5648$ $u_8 = 388.4289$	7501.6437	17166.6698	0.5970
		OWCA	$u_1 = 100$ $u_2 = 1000$ $u_3 = 4164.518$ $u_4 = 335.8958$ $u_5 = 333.5317$ $u_6 = 2073.8264$ $u_7 = 365.5282$ $u_8 = 433.4689$	5264.5183	11767.67	0.6442
5	Problem-5	WCA	$u = 0.500$ $v = 1.0000$	2	2.0813	0.4378
		OWCA	$u = 1.37335$ $v = 0.07720$	1.3671	2.079.9	0.46133
6	Problem-6	WCA	$v_1 = 78$ $v_2 = 33$ $u_1 = 30.035$ $u_2 = 45$	-30665.537	-2957.1712	0.39526

			$u_3 = 36.6905$			
		OWCA	$v_1 = 78$ $v_2 = 33$ $u_1 = 29.995$ $u_2 = 44.999$ $u_3 = 37.7758$	-30658.273	-30045.825	0.51574
7	Problem-7	WCA	$u_1 = 1.118034$ $u_2 = 1.31037$ $v_1 = 0$ $v_2 = 1$ $v_3 = 1$	7.6672	8.836	0.367757
		OWCA	$u_1 = 1.118033$ $u_2 = 1.31037$ $v_1 = 0$ $v_2 = 1$ $v_3 = 1$	7.6671	9.4988	0.41942
8	Problem-8	WCA	$u_1 = 12.500$ $u_2 = 0$ $v_1 = 0.62500$ $v_2 = 0$	84.6975	92.4059	0.509113
		OWCA	$u_1 = 12.42951$ $u_2 = 0.084156$ $v_1 = 0.52147$ $v_2 = 0.0046254$	84.7404	92.84288	0.371458
9	Welded Beam Design	WCA	$u_1 = 0.280834$ $u_2 = 2.73517$ $u_3 = 7.73436$ $u_4 = 0.280840$	1.98	4.3914	0.55481
		OWCA	$u_1 = 0.205520$ $u_2 = 3.46077$ $u_3 = 9.07371$ $u_4 = 0.20554$	1.728	3.093	0.523536
10	Pressure Vessel design	WCA	$u_1 = 0.99665$ $u_2 = 0.49264$ $u_3 = 51.64030$ $u_4 = 85.84320$	63735315	63108.95	0.59937
		OWCA	$u_1 = 0.008279$ $u_2 = 0.0040926$ $u_3 = 0.428998$ $u_4 = 1.66953$	5884.0430	175954.4354	0.356106
11	Spring Design	WCA	$u_1 = 0.068990$ $u_2 = 0.933266$ $u_3 = 2.000627$	0.017772	0.092446	0.69825
		OWCA	$u_1 = 0.061065$ $u_2 = 0.627064$ $u_3 = 4.028385$	0.012144	0.10108	0.61102
12	Speed reducer Design	WCA	$u_1 = 3.500$ $u_2 = 0.7000$ $u_3 = 17.00034$ $u_4 = 7.3345$ $u_5 = 7.715433$ $u_6 = 3.353433$	2995.456	3303.562	1.741
		OWCA	$u_1 = 3.500$ $u_2 = 0.7000$ $u_3 = 17.00034$ $u_4 = 7.3678$ $u_5 = 7.715433$ $u_6 = 3.35021$	2994.474	3967.942	0.53151
13	Multiple Disk	WCA	$u_1 = 0.07999$	0.26977	0.31612	0.49328

			$u_2 = 0.0900$ $u_3 = 0.01000$ $u_4 = 1.0000$ $u_5 = 0.002312$			
		OWCA	$u_1 = 0.06999$ $u_2 = 0.0900$ $u_3 = 0.001000$ $u_4 = 1.0000$ $u_5 = 0.002312$	0.25977	0.51363	1.0191
14	Rolling Element	WCA	$D_m = 125.04560$ $D_b = 21.4233$ $Z = 50.0908$ $f_1 = 0.515003$ $f_0 = 0.5150900$ $K_{Dmin} = 0.40285$ $K_{Dmax} = 0.69246$ $\epsilon = 0.305175$ $e = 0.09414$ $\xi = 0.60098$	234656.9639	72451.8564	2.3714
		OWCA	$D_m = 125.72771$ $D_b = 21.4233$ $Z = 50.0128$ $f_1 = 0.515050$ $f_0 = 0.5150905$ $K_{Dmin} = 0.46599$ $K_{Dmax} = 0.617028$ $\epsilon = 0.302330$ $e = 0.020098$ $\xi = 0.600023$	234703.4705	72878.098	0.779123
15	Three Bar thrush	WCA	$u_1 = 0.79102$ $u_2 = 0.400260$	263.9017	267.363	0.51560
		OWCA	$u_1 = 0.78452$ $u_2 = 0.40003$	263.2368	267.942	0.50345

Table 6. *ANOVA test*

Sl No.	**Comparison**	***p-value***
1.	OWCA and JAYA	0.311665
2.	OWCA and Elitist TLBO	0.328683
3.	OWCA and DEPTS	0.328684
4.	OWCA and TBLO	0.548378
5.	OWCA an ABC	0.548392
6.	OWCA and CoDE	0.87018
7.	OWCA and MBA	0.99941
8.	OWCA and MDE	0.360211
9.	OWCA and MA-MDE	0.35634
10.	OWCA and MDE-HIS	0.422686


Problem-1
Functional_Value
No. of Iteration
WCA
OWCA
Problem-2
Functional_Value
No. of Iteration
WCA
OWCA
Problem-3
Functional_Value
No. of Iteration
WCA
OWCA
Program-4
Functional_value
No. of iteration
WCA
OWCA
Problem-5
Functional_value
No. of iteration
WCA
OWCA
Problem-6
Functional_Value
No. of iteration
WCA
OWCA
Problem-7
Functional_value
No. of iteration
WCA
OWCA
Problem-8
Functional_value
No. of iteration
WCA
OWCA
Three Bar thrush problem
Functional_Value
No. of Iteration
WCA
OWCA
Pressure Vessel problem
Functional_Value
No. of Iteration
WCA
OWCA

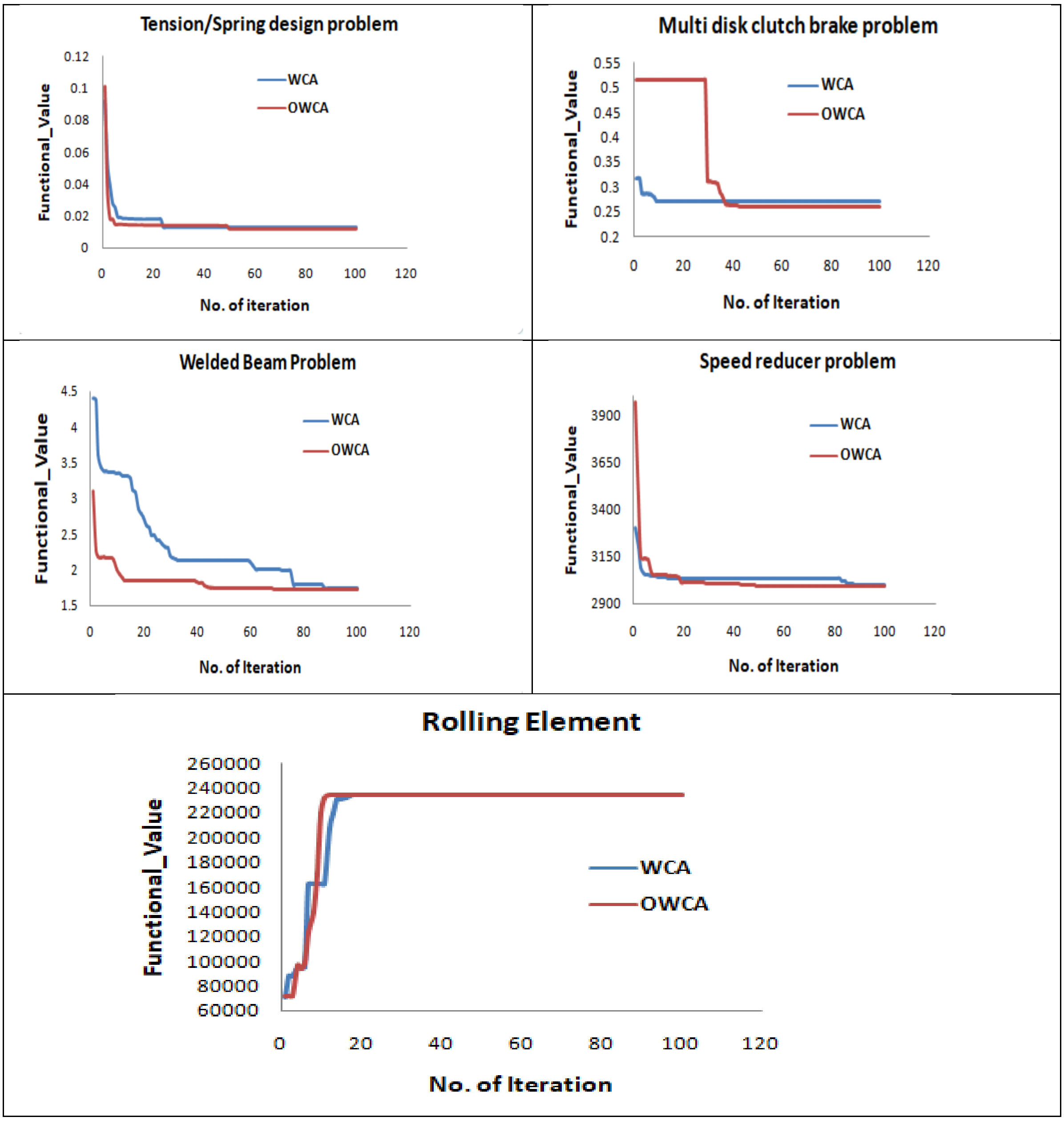


Figure 2. *Convergence Curve of various benchmark problems using WCA and OWCA*

5. CONCLUSION

Opposition learning is a new perception in the machine learning that had introduced looking into reverse relationship amongst entities that helps in a better global search on the opposite search as well. Though the search time might be longer than any other conventional search since it conducted in a two way process but it provides a better search random number followed with all the equality and inequality constraints and specific boundaries of the parameters to get a better functional value. This learning had been used on WCA technique and has seen to give a better fitness value, avoid any kind of premature result, better timing of settlement of the plot and good convergence rate than conventional WCA. This article had considered 15 such benchmark problems and have performed and implemented the OWCA(Opposition+ WCA) concept maintaining all the available constraints and boundaries with running for 100 iterations and taking up 50 populations for each and the result data received had been shown in tabular form individually for each function and compared to the other conventional algorithms from literature and had proved to be superior to the other algorithms for the problems giving a successful run and a better outcome. For every functions utilized to test the proposed method, with the constraint data along with the computational time had been added in the table and the convergence plot against each functions had been shown between the WCA and

OWCA showing the execution of the problems for 100 iterations. Many more complex problems would also take up for future work. The ANOVA test had been performed between each of the conventional methods with the proposed one to check the statistical analysis of the data and check whether the methods between them are significantly different from each other or not.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Dorigo, M., and Stutzle, T., “Ant Colony Optimization”, The MIT Press Cambridge, London, UK, (2004).

[2] Haupt, R., and Haupt, S., “Practical Genetic Algorithms”, John Wiley & Sons, Hoboken, NJ, USA, (2004).

[3] Kramer, O., “Genetic Algorithm Essentials”, Springer International Publishing AG, Berlin, Germany, (2017).

[4] Lazinica, A., “Particle swarm optimization”, In-Tech, Rijeka, Croatia, (2009).

[5] Raj, A., Bhattacharya, B., “Optimal Placement of TCSC and SVC for reactive power planning using whale optimization algorithm”, Swarm and Evolutionary Computation BASE DATA, (2018).

[6] Esander, H., Sadollah, A., Bahreininejad, A., Hamdi, M., “Water cycle Algorithm-a novel meta-heuristic optimization method for solving constrained engineering optimization problems”, Elsevier Computer and Structures, 110: 151-166, (2012).

[7] Kudkelwar, S., Sarkar, D., “Online Implementation of time augmentation of over-current relay coordination using water cycle algorithm”, SN Applied Science Springer Nature Journal, (2019).

[8] Ramapriya, B., “Profit maximization and optimal bidding strategies of gencos in Electricity market using self-Adaptive Differential Solution”, International Journal on Electrical and Electronics and Informatics, 8(4): 753-761, (2016).

[9] He, S., Wu, Q. H., and Saunders, J. R., “Group search optimizer: an optimization algorithm inspired by animal searching behaviour”, In IEEE Transactions on Evolutionary Computation, 13: 973–990, (2009).

[10] Reddy, S., Panwar, L. K., Kumar, R., Panigrahi, B. K., “Binary fireworks algorithm for profit based unit commitment (PBUC) problem”, International Journal of Electrical Power & Energy Systems, 83: 270–282, (2016).

[11] Manisha, H. Y. K. A., Thakur, N., Siddiqui, A. S., “Advanced hybrid intelligent model and algorithm for de-regulated electricity market”, Australian Journal of Electrical and Electronics Engineering, Taylor & Francis, 1-11, (2020).

[12] Aggarwal, R., Mallick, R. K., and Choudhury, A. R., “Bidding strategies for GENCOS in pool based DAEM Using GWO Method”, International Journal of Scientific & Technology Research, 9(1): 3229-3235, (2020).

[13] Tizhoosh, H. R., “Opposition-based learning: a new scheme for machine intelligence. In International conference on computational intelligence for modelling”, control and automation and international conference on intelligent agents, web technologies and internet commerce (CIMCA-IAWTIC'06), 695-701, (2005). DOI: 10.1109/CIMCA.2005.1631345

[14] Jabeen, H., Jalil, Z., Baig, A. R., “Opposition based initialization in particle swarm optimization”, (O-PSO) In Proceedings of the 11th Annual Conference Companion on Genetic and Evolutionary Computation Conference: Late Breaking Papers, Jul 8, 2047-2052, (2009).

[15] Shehab, M., Abualigah, L., “Opposition-based learning multi-verse optimizer with disruption operator for optimization problems”, Soft Computing, 26(21): 11669-11693, (2022).

[16] Elgamal, Z. M., Yasin, N. M., Sabri, A. Q., Sihwail, R., Tubishat, M., Jarrah, H., “Improved equilibrium optimization algorithm using elite opposition-based learning and new local search strategy for feature selection in medical datasets”, Computation, 9(6): 68, (2021).

[17] Alamri, H.S., Alsariera, Y.A., Zamli, K.Z., “Opposition-based whale optimization algorithm”, Advanced Science Letters, 24(10): 7461-7464, (2018).

[18] Korashy, A., Kamel, S., Houssein, E.H., Jurado, F., Hashim, F.A., “Development and application of evaporation rate water cycle algorithm for optimal coordination of directional overcurrent relays”, Expert Systems with Applications, 185: 115538, (2021).

[19] Dhargupta, S., Ghosh, M., Mirjalili, S., Sarkar, R., “Selective opposition based grey wolf optimization”, Expert Systems with Applications, 151: 113389, (2020).

[20] Zhang, Z., Xu, Z., Luan, S., Li, X., Sun, Y., “Opposition-based ant colony optimization algorithm for the traveling salesman problem”, Mathematics, 8(10): 1650, (2020).

[21] Al-Fakih A.M., Algamal, Z.Y., Qasim, M.K., “An improved opposition-based crow search algorithm for biodegradable material classification”, SAR and QSAR in Environmental Research, 33(5): 403-415, (2022).

[22] Zhang, G., Cheng, J., Gheorghe, M., and Meng, Q., “A hybrid approach based on differential evolution and tissue membrane systems for solving constrained manufacturing parameter optimization problems”, Applied Soft Computing, 13(3): 1528-1542, (2013).

[23] Rao, R.V., Savsani, V. J., and Vakharia, D. P., “Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems”, Computer-Aided Design, 43(3): 303-315, (2011).

[24] Akay, B., and Karaboga, D., “Artificial bee colony algorithm for large-scale problems and engineering design optimization”, Journal of Intelligent Manufacturing, 23: 1001-1014, (2012).

[25] Huang, F. Z., Wang, L., and He, Q., “An effective co-evolutionary differential evolution for constrained optimization”, Applied Mathematics and Computation, 186(1): 340-356, (2007).

[26] David, D. C. N., and Stephen, C. E. A., “Cost Minimization of Welded Beam Design Problem using Non-traditional optimization through Matlab and validation through analyses simulation”, International Journal of Mechanical Engineering and Technology, 9(8): 180-192, (2018).

[27] Hassan, S., Kumar, K., Raj, C.D., and Sridhar, K., “Design and optimisation of pressure vessel using metaheuristic approach”, In Applied Mechanics and Materials, 401-406, Trans Tech Publications Ltd, (2014).

[28] Satyanarayana, M., Charyulu, T.N., Ramanamurthy Naidu, S.C.V., and Appala Naidu, G., “Optimization of spring weight using genetic algorithm”, International Journal of Engineering Research & Technology (IJERT), 1(9): 1-10, (2012).

[29] Lin, M. H., Tsai, J. F., Hu, N. Z., and Chang, S. C., “Design optimization of a speed reducer using deterministic techniques”, Mathematical Problems in Engineering, (2013).

[30] Malcolm, J.D., Roth, A., Radic, M., Martin-Ramiro, P., Oillarburu, J., Orus, R., and Mugel, S., “Multi-disk clutch optimization using quantum annealing”, arXiv preprint arXiv:2208.05916 (2022).

[31] Dandagwhal, R.D., and Kalyankar, V.D., “Design optimization of rolling element bearings using advanced optimization technique”, Arabian Journal for Science and Engineering, 44(9): 7407-7422, (2019).

[32] Erdoğan Yıldırım, A., and Karci, A., “Application of three bar truss problem among engineering design optimization problems using artificial atom algorithm”, In 2018 International Conference on Artificial Intelligence and Data Processing (IDAP) 1-5, IEEE (2018).

[33] Fauzi, H., and Batool, U., “A three-bar truss design using single-solution simulated Kalman filter optimizer”, Mekatronika, 1(2): 98-102, (2019).

[34] Rao, R.V., and Waghmare, G.G., “A new optimization algorithm for solving complex constrained design optimization problems”, Engineering Optimization, 49(1): 60-83, (2017).

[35] Rao, R.V., and Waghmare, G.G., “Complex constrained design optimisation using an elitist teaching-learning-based optimisation algorithm”, International Journal of Metaheuristics, 3(1): 81-102, (2014).

[36] Liao, T.W., “Two hybrid differential evolution algorithms for engineering design optimization”, Applied Soft Computing, 10(4): 1188-1199, (2010).

[37] Sadollah, A., Bahreininejad, A., Eskandar, H., and Hamdi, M., “Mine blast algorithm: A new population-based algorithm for solving constrained engineering optimization problems”, Applied Soft Computing, 13(5): 2592-2612, (2013).

[38] Canayaz, M., and Karci, A., “Cricket behaviour-based evolutionary computation technique in solving engineering optimization problems”, Applied Intelligence, 44: 362-376, (2016).

[39] Wang, Y., Wang, P., Zhang, J., Cui, Z., Cai, X., Zhang, W., and Chen, J., “A novel bat algorithm with multiple strategies coupling for numerical optimization”, Mathematics, 7(2): 135, (2019).

[40] Gandomi, A.H., Yang, X.S., and Alavi, A.H., “Cuckoo search algorithm: a meta-heuristic approach to solve structural optimization problems”, Engineering with Computers, 29: 17-35, (2013).

[41] Ray, T., and Saini, P., “Engineering design optimization using a swarm with an intelligent information sharing among individuals”, Engineering Optimization, 33(6): 735-748, (2001).