DOI: 10.38061/idunas.1296023

IDUNAS | NATURAL & APPLIED SCIENCES JOURNAL | 2023 Vol. 6 No. 1 (9-20)

Stability of Solution of Quasilinear Parabolic Two-Dimensional with Inverse Coefficient by Fourier Method

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DOI: 10.38061/idunas.1296023

Received:11.05.2023; Accepted: 19.06.2023

Abstract

In this article, the heat inverse two-dimensional quasilinear parabolic problem is examined. The stability and numerical solution for the problem are discussed. Since the problem is not linear, Picard's successive approximations theorem is used. In the numerical part, the solution is made with the finite difference and linearization method.
Keywords: Inverse problem, Fourier method, Periodic boundary conditions, Picard Method, Two-dimension parabolic problem, Fourier coefficient.

1. INTRODUCTION

Inverse problems are used to find an unknown character of a matter or a place. Especially inverse problems are important for many calculations used in aircraft, missiles and submarines. In geophysics, the inverse problem is finding subsurface inhomogeneities. When measuring the frequencies of a material, the inverse problem is finding whether there is a defect (a hole in a metal) in that material. There may be a tumor or some abnormalities in the human body in medicine. The inverse problem is examined [4], [5],[6]. It is used in many fields such as population, electrochemistry, engineering, chemistry. The problems with nonlocal boundary conditions discussed in this article are not easy to study. Various boundary conditions have been studied in this area [1],[2],[3].

2. MATERIAL AND METHODS

For the solution of this problem, the Fourier Method and Picard successive approximations method were used, while the linearization method was used for the numeric solution. It was used implicit finite-difference method for numeric problem.

3. STABILITY OF SOLUTIONS

$$\frac{\partial v}{\partial t} = b(t)\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + h(\alpha, \beta, \tau, v), \tag{1}$$

$$v(\alpha, \beta, 0) = \varphi(\alpha, \beta), \alpha \in [0, \pi], \beta \in [0, \pi] \tag{2}$$

$$\begin{aligned} &v(0, \beta, \tau) = v(\pi, \beta, \tau), \beta \in [0, \pi], \tau \in [0, T] \\ &v(\alpha, 0, \tau) = v(\alpha, \pi, \tau), \alpha \in [0, \pi], \tau \in [0, T] \\ &v_\alpha(0, \beta, \tau) = v_\alpha(\pi, \beta, \tau), \beta \in [0, \pi], \tau \in [0, T] \\ &v_\beta(\alpha, 0, \tau) = v_\beta(\alpha, \pi, \tau), \alpha \in [0, \pi], \tau \in [0, T] \end{aligned} \tag{3}$$

$$k(t) = \int_0^\pi \int_0^\pi \alpha\beta v(\alpha, \beta, \tau) d\alpha dy, \tau \in [0, T], \tag{4}$$

where (1) is the inverse coefficient problem, (2) is the initial condition, (3) are the periodic boundary conditions [8] and (4) is the integral overdetermination data (k(t) is heat diffusions) [7].
As known, in Fourier Method, we get the following structure:

$$\begin{aligned} v(\alpha, \beta, \tau) = &\frac{v_0(\tau)}{4} \\ &+ \sum_{m,n=1}^{\infty} v_{scmn}(\tau) \sin(2m\alpha) \cos(2n\beta) \\ &+ \sum_{m,n=1}^{\infty} v_{smn}(\tau) \sin(2m\alpha) \sin(2n\beta). \end{aligned}$$

$$v_0(\tau) = v_0(0) + \frac{4}{\pi^2} \int_0^t \int_0^\pi \int_0^\pi h(\alpha, \beta, \tau, v) d\alpha d\beta d\tau$$

$$\begin{aligned} v_{cmn}(\tau) = &v_{cmn}(0) e^{-\int_0^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} \\ &+ \frac{4}{\pi^2} \int_0^t \int_0^\pi \int_0^\pi e^{-\int_\tau^t \left[b(s)(2m)^2 + (2n)^2 \right] ds} h(\alpha, \beta, \tau, v) \cos(2m\alpha) \cos(2n\beta) d\alpha d\beta d\tau \end{aligned}$$

$$v_{csmn}(\tau) = v_{csmn}(0) e^{-\int_0^t \left[b(r)(2m)^2 + (2n)^2 \right] dr}$$

$$+ \frac{4}{\pi^2} \int_0^t \int_0^\pi \int_0^\pi e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} h(\alpha, \beta, \tau, v) \cos(2m\alpha) \sin(2n\beta) d\alpha d\beta d\tau$$

$$v_{scmn}(\tau) = v_{scmn}(0)e^{-\int_0^t \left[b(r)(2m)^2 + (2n)^2 \right] dr}$$

$$+ \frac{4}{\pi^2} \int_0^t \int_0^\pi \int_0^\pi e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} h(\alpha, \beta, \tau, v) \sin(2m\alpha) \cos(2n\beta) d\alpha d\beta d\tau$$

$$v_{smn}(\tau) = v_{smn}(0)e^{-\int_0^t \left[b(r)(2m)^2 + (2n)^2 \right] dr}$$

$$+ \frac{4}{\pi^2} \int_0^t \int_0^\pi \int_0^\pi e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} h(\alpha, \beta, \tau, v) \sin(2m\alpha) \sin(2n\beta) d\alpha d\beta d\tau$$

Then we obtain the solution:

$$v(\alpha, \beta, \tau) = \frac{1}{4} \left(\varphi_0 + \frac{4}{\pi^2} \int_0^t h_0(\tau, v) d\tau \right)$$

$$+ \sum_{m,n=1}^{\infty} \left(\varphi_{cmn} + \frac{4}{\pi^2} \int_0^t e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} h_{cmn}(\tau, v) d\tau \right)$$

$$\cos(2m\alpha) \cos(2n\beta)$$

$$+ \sum_{m,n=1}^{\infty} \left(\varphi_{csmn} + \frac{4}{\pi^2} \int_0^t e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} h_{csmn}(\tau, v) d\tau \right)$$

$$\cos(2m\alpha) \sin(2n\beta)$$

$$+ \sum_{m,n=1}^{\infty} \left(\varphi_{scmn} + \frac{4}{\pi^2} \int_0^t e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} h_{scmn}(\tau, v) d\tau \right)$$

$$\sin(2m\alpha) \cos(2n\beta)$$

$$+ \sum_{m,n=1}^{\infty} \left(\varphi_{smn} + \frac{4}{\pi^2} \int_0^t e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2 \right] dr} h_{smn}(\tau, v) d\tau \right)$$

$$\sin(2m\alpha) \sin(2n\beta)$$

Here,

$\varphi_0 = v_0(0),$

$\varphi_{cmn} = v_{cmn}(0)e^{-\int_0^t \left[b(r)(2m)^2+(2n)^2\right]dr},$

$\varphi_{csmn} = v_{csmn}(0)e^{-\int_0^t \left[b(r)(2m)^2+(2n)^2\right]dr},$

$\varphi_{scmn} = v_{scmn}(0)e^{-\int_0^t \left[b(r)(2m)^2+(2n)^2\right]dr},$

$\varphi_{smn} = v_{smn}(0)e^{-\int_0^t \left[b(r)(2m)^2+(2n)^2\right]dr}.$

Let's assume the following rules for the functions used in the problem:

(A1) $k(t) \in C^1[0,T]$

$\varphi(\alpha,\beta)\varepsilon C^{1,1}([0,\pi]\times[0,\pi]),$

(A2) $\varphi(0,\beta) = \varphi(\pi,\beta), \varphi_x(0,\beta) = \varphi_x(\pi,\beta), \quad \int_0^\pi\int_0^\pi \alpha\beta\varphi(\alpha,\beta)d\alpha d\beta = k(0),$

$\varphi(\alpha,0) = \varphi(\alpha,\pi), \varphi_y(\alpha,0) = \varphi_y(\alpha,\pi),$

(A3) Let $h(\alpha,\beta,\tau,v)$ have the following properties:

$$\left|\frac{\partial h(\alpha,\beta,\tau,v)}{\partial\alpha} - \frac{\partial h(\alpha,\beta,\tau,\bar{v})}{\partial\alpha}\right| \le l(\alpha,\beta,\tau)\left|v-\bar{v}\right|,$$

$$\left|\frac{\partial h(\alpha,\beta,\tau,v)}{\partial\beta} - \frac{\partial h(\alpha,\beta,\tau,\bar{v})}{\partial\beta}\right| \le l(\alpha,\beta,\tau)\left|v-\bar{v}\right|,$$

$$\left|\frac{\partial h(\alpha,\beta,\tau,v)}{\partial\alpha\partial\beta} - \frac{\partial h(\alpha,\beta,\tau,\bar{v})}{\partial\alpha\partial\beta}\right| \le l(\alpha,\beta,\tau)\left|v-\bar{v}\right|, \text{ where } l(\alpha,\beta,\tau) \in L_2(D), l(\alpha,\beta,\tau) \ge 0,$$

(2) $h(\alpha,\beta,\tau,v) \in C^{2,2,0}[0,\pi], \quad \tau \in 0,T],$

(3) $h(\alpha,\beta,\tau,v)\big|_{\alpha=0} = h(\alpha,\beta,\tau,v)\big|_{\alpha=\pi},$

$h_\alpha(\alpha,\beta,\tau,v)\big|_{\alpha=0} = h_\alpha(\alpha,\beta,\tau,v)\big|_{\alpha=\pi},$

$h_\beta(\alpha,\beta,\tau,v)\big|_{\beta=0} = h_\beta(\alpha,\beta,\tau,v)\big|_{\beta=\pi},$

$h_{\alpha\beta}(\alpha,\beta,\tau,v)\big|_{\alpha=0} = h_{\alpha\beta}(\alpha,\beta,\tau,v)\big|_{\alpha=\pi},$

$h_{\alpha\beta}(\alpha,\beta,\tau,v)\big|_{\beta=0} = h_{\alpha\beta}(\alpha,\beta,\tau,v)\big|_{\beta=\pi}.$

$$\int_0^\pi\int_0^\pi \alpha\beta v_t(\alpha,\beta,\tau)d\alpha d\beta = k'(\tau), 0 \le t \le T.$$

$$b(\tau) = \frac{k'(\tau) - \int_0^\pi\int_0^\pi xyh(\alpha,\beta,\tau,v)d\alpha d\beta - \frac{\pi^3}{2}v_\beta(\pi,\tau)}{\frac{\pi^3}{2}v_\alpha(\pi,t)}.$$

Definition 1.
Show the set $\{v(\tau)\} = \{v_0(\tau), v_{cmn}(\tau), v_{csmn}(\tau), v_{scmn}(\tau), v_{smn}(\tau)\}$ of continuous functions on [0, T] which satisfy the condition.

$$\max_{0\le t\le T}\frac{|v_0(\tau)|}{4} + \sum_{m,n=1}^{\infty}\begin{pmatrix}\max\limits_{0\le t\le T}|v_{cmn}(\tau)| + \max\limits_{0\le t\le T}|v_{csmn}(\tau)| \\ + \max\limits_{0\le t\le T}|v_{scmn}(\tau)| + \max\limits_{0\le t\le T}|v_{smn}(\tau)|\end{pmatrix} < \infty \, .$$

$$\|v(\tau)\| = \max_{0\le t\le T}\frac{|v_0(\tau)|}{4} + \sum_{m,n=1}^{\infty}\begin{pmatrix}\max\limits_{0\le t\le T}|v_{cmn}(\tau)| + \max\limits_{0\le t\le T}|v_{csmn}(\tau)| \\ + \max\limits_{0\le t\le T}|v_{scmn}(\tau)| + \max\limits_{0\le t\le T}|v_{smn}(\tau)|\end{pmatrix}$$

is called Banach norm.

Theorem 1: According to (A1)-(A3), the solution of (1)-(4) is constantly dependent on the data.

Proof:
Let $\theta = \{\varphi, k, h\}, \bar{\theta} = \{\bar{\varphi}, \bar{k}, \bar{h}\}$ and M, L_i, $i = 1,2$ positive constant.

$$\|h\|_{C^{1,1,0}[\Gamma]} \le M, \|\bar{h}\|_{C^{1,1,0}[\Gamma]} \le M,$$

$$\|\varphi\|_{C^3[0,\pi]} \le L_1, \|\bar{\varphi}\|_{C^3[0,\pi]} \le L_1,$$

$$\|k\|_{C^1[0,T]} \le L_2, \|\bar{k}\|_{C^1[0,T]} \le L_2,$$

$$\|\theta\| = (\|k\|_{C^1[0,T]} + \|\varphi\|_{C^{1,1}[0,\pi]} + h_{C^{1,1,0}[\bar{\Gamma}]}).$$

$$v - \bar{v} = \frac{(\varphi_0 - \overline{\varphi_0})}{4} + \sum_{m,n=1}^{\infty}\varphi_{cmn} e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2\right]dr}\cos(2m\alpha)\cos(2n\beta)$$

$$\sum_{m,n=1}^{\infty}\overline{\varphi_{cmn}} e^{-\int_\tau^t \left[\bar{b}(r)(2m)^2 + (2n)^2\right]dr}\cos(2m\alpha)\cos(2n\beta)$$

$$+ \sum_{m,n=1}^{\infty}\varphi_{csmn} e^{-\int_\tau^t \left[b(r)(2m)^2 + (2n)^2\right]dr}\cos(2m\alpha)\sin(2n\beta)$$

$$+\sum_{m,n=1}^{\infty}\overline{\varphi_{csmn}}\,e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\cos(2m\alpha)\sin(2n\beta)$$

$$+\sum_{m,n=1}^{\infty}\varphi_{scmn}\,e^{-\int_{\tau}^{t}\left[b(r)(2m)^2+(2n)^2\right]dr}\sin(2m\alpha)\cos(2n\beta)$$

$$+\sum_{m,n=1}^{\infty}\overline{\varphi_{scmn}}\,e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\sin(2m\alpha)\cos(2n\beta)$$

$$+\sum_{m,n=1}^{\infty}\varphi_{smn}\,e^{-\int_{\tau}^{t}\left[b(r)(2m)^2+(2n)^2\right]dr}\sin(2m\alpha)\sin(2n\beta)$$

$$+\sum_{m,n=1}^{\infty}\overline{\varphi_{smn}}\,e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\sin(2m\alpha)\sin(2n\beta)$$

$$+\frac{1}{4}\left(\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}\left[h(\alpha,\beta,\tau,v)-h(\alpha,\beta,\tau,\bar{v})\right] e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\,d\alpha\, d\beta\, d\tau\right)$$

$$+\frac{1}{4}\left(\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}h(\alpha,\beta,\tau,\bar{v})\left(e^{-\int_{\tau}^{t}\left[b(r)(2m)^2+(2n)^2\right]dr}-e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\right)d\alpha\, d\beta\, d\tau\right)$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}\left[h(\alpha,\beta,\tau,v)-h(\alpha,\beta,\tau,\bar{v})\right] e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\cos(2m\alpha)\cos(2n\beta)\,d\alpha\, d\beta\, d\tau$$

$$+\sum_{k=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}h(\alpha,\beta,\tau,\bar{v})\left(e^{-\int_{\tau}^{t}\left[b(r)(2m)^2+(2n)^2\right]dr}-e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\right)$$

$$\cos(2m\alpha)\cos(2n\beta)d\alpha d\beta d\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}\left[h(\alpha,\beta,\tau,v)-h(\alpha,\beta,\tau,\bar{v})\right]$$

$$e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\cos(2m\alpha)\sin(2n\beta)d\alpha d\beta d\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}h(\alpha,\beta,\tau,\bar{v})\left(e^{-\int_{\tau}^{t}\left[b(r)(2m)^2+(2n)^2\right]dr}-e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\right)$$

$$\cos(2m\alpha)\sin(2n\beta)d\alpha d\beta d\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}\left[h(\alpha,\beta,\tau,v)-h(\alpha,\beta,\tau,\bar{v})\right]$$

$$e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}$$

$$\sin(2m\alpha)\cos(2n\beta)d\alpha d\beta d\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^{\pi}\int_0^{\pi}h(\alpha,\beta,\tau,\bar{v})\left(e^{-\int_{\tau}^{t}\left[b(r)(2m)^2+(2n)^2\right]dr}-e^{-\int_{\tau}^{t}\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\right)$$

$$\sin(2m\alpha)\cos(2n\beta)d\alpha d\beta d\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi\left[h(\alpha,\beta,\tau,v)-h(\alpha,\beta,\tau,\bar{v})\right]$$

$$e^{-\int_\tau^t\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\sin(2m\alpha)\sin(2n\beta)d\alpha d\beta d\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi h(\alpha,\beta,\tau,\bar{v})$$

$$\left(e^{-\int_\tau^t\left[b(r)(2m)^2+(2n)^2\right]dr}-e^{-\int_\tau^t\left[\bar{b}(r)(2m)^2+(2n)^2\right]dr}\right)$$

$$\sin(2m\alpha)\sin(2n\beta)d\alpha d\beta d\tau$$

$$\left\|v-\bar{v}\right\|_B\le\frac{\left\|\varphi_0-\overline{\varphi_0}\right\|}{4}$$

$$+\sum_{m,n=1}^{\infty}\left\|\varphi_{cmn}-\overline{\varphi_{cmn}}\right\|+\left\|\varphi_{csmn}-\overline{\varphi_{csmn}}\right\|$$

$$+\left\|\varphi_{scmn}-\overline{\varphi_{scmn}}\right\|+\left\|\varphi_{smn}-\overline{\varphi_{smn}}\right\|$$

$$+\sqrt{T}\left(\frac{3\sqrt{\pi}+16}{3\pi}\right)\left\|l(\alpha,\beta,\tau)\right\|_{L_2(\Gamma)}\left\|v(\tau)-\bar{v}(\tau)\right\|_B+\sqrt{T}\left(\frac{3\sqrt{\pi}+16}{3\pi}\right)\left\|l(\alpha,\beta,\tau)\right\|_{L_2(\Gamma)}M\left\|b(\tau)-\bar{b}(\tau\right\|\text{ where}$$

$$\left\|\theta-\bar{\theta}\right\|=\frac{\left\|\varphi_0-\overline{\varphi_0}\right\|}{4}$$

$$+\sum_{m,n=1}^{\infty}\left\|\varphi_{cmn}-\overline{\varphi_{cmn}}\right\|+\left\|\varphi_{csmn}-\overline{\varphi_{csmn}}\right\|$$

$$+\left\|\varphi_{scmn}-\overline{\varphi_{scmn}}\right\|+\left\|\varphi_{smn}-\overline{\varphi_{smn}}\right\|.$$

$$\left\|b(\tau)-\bar{b}(\tau)\right\|_{C[0,T]}\le C\left\|v(\tau)-\bar{v}(\tau)\right\|_B$$

where $C=\left(\frac{\pi M}{2\left\|u(\tau)\right\|_B\left\|\bar{u}(\tau)\right\|_B}+\frac{\pi\left\|l(\alpha,\beta,\tau)\right\|_{L_2(\Gamma)}}{2\left\|u(\tau)\right\|_B}\right).$

$$\left\|v-\bar{v}\right\|_B^2\le 2D^2\left\|\theta-\bar{\theta}\right\|^2 x0$$

$$D=\frac{1}{1-\left(\sqrt{T}\left(\frac{3\sqrt{\pi}+16}{3\pi}\right)\|l(x,y,t)\|_{L_2(\Gamma)}+MC\right)}.$$

For $\theta\to\bar{\theta}$ then $v\to\bar{v}$. Hence $b\to\bar{b}$.

4. NUMERIC METHOD FOR THE PROBLEM

Let's linearize it for the nonlinear data in the problem:

$$u_t^{(n)}=b(\tau)u_{xx}^{(n)}+u_{yy}^{(n)}+h(\alpha,\beta,\tau,u^{(n-1)}),\quad(\alpha,\beta,\tau)\in\Gamma$$

$$u^{(n)}(\alpha,\beta,0)=\varphi(\alpha,\beta),\alpha\in[0,\pi],\beta\in[0,\pi],$$

$$u^{(n)}(0,\beta,\tau)=u^{(n)}(\pi,\beta,\tau),\beta\in[0,\pi],\tau\in[0,T],$$
$$u^{(n)}(\alpha,0,\tau)=u^{(n)}(\alpha,\pi,\tau),\alpha\in[0,\pi],\tau\in[0,T],$$

$$u_\alpha^{(n)}(0,\beta,\tau)=u_\alpha^{(n)}(\pi,y,t),\beta\in[0,\pi],\tau\in[0,T],$$
$$u_\beta^{(n)}(\alpha,0,\tau)=u_\beta^{(n)}(\alpha,\pi,\tau),\alpha\in[0,\pi],\tau\in[0,T],$$

Let us $u^{(n)}(\alpha,\beta,\tau)=v(\alpha,\beta,\tau)$ ve $h(\alpha,\beta,\tau,u^{(n-1)})=\bar{h}(\alpha,\beta,\tau)$.

$$v_t=b(\tau)v_{xx}+v_{yy}+\bar{f}(\alpha,\beta,\tau),\quad(\alpha,\beta,\tau)\in\Gamma$$

$$v(\alpha,\beta,0)=\varphi(\alpha,\beta),\alpha\in[0,\pi],\beta\in[0,\pi]$$

$$v(0,\beta,\tau)=v(\pi,\beta,\tau),\beta\in[0,\pi],\tau\in[0,T]$$
$$v(\alpha,0,\tau)=v(\alpha,\pi,\tau),\alpha\in[0,\pi],\tau\in[0,T]$$

$$v_\alpha(0,\beta,\tau)=v_\alpha(\pi,\beta,\tau),\beta\in[0,\pi],\tau\in[0,T]$$
$$v_\beta(\alpha,0,\tau)=v_\beta(\alpha,\pi,\tau),\beta\in[0,\pi],\tau\in[0,T]$$

$[0,\pi]^2\times[0,T]$ is divided to an $M^2\times N$ mesh with the step sizes $h=\pi/M$, $\tau=T/N$.

Let's take $v_{i,j}^k$, $f_{i,j}^k$, φ_i and b^k that instead of $v(\alpha_i,\beta_j,\tau_k)$, $h(\alpha_i,\beta_j,\tau_k)$, $\varphi(\alpha_i,\beta_j)$ and $b(\tau_k)$, respectively.

Then we examine implicit finite-difference method for the last problem :

$$\frac{1}{\tau}\left(v_{i,j}^{k+1}-v_{i,j}^{k}\right)$$
$$=\frac{1}{h^2}\begin{bmatrix} b^k\left(v_{i-1,j}^{k+1}-2v_{i,j}^{k+1}+v_{i+1,j}^{k+1}\right) \\ +\left(v_{i,j-1}^{k+1}-2v_{i,j}^{k+1}+v_{i,j+1}^{k+1}\right)\end{bmatrix}$$
$$+h_{i,j}^{k+1},$$

$$v_{i,j}^{0}=\varphi_i,$$

$$v_{0,j}^{k}=v_{M+1,j}^{k},v_{M+1,j}^{k}=\frac{v_{1,j}^{k}-v_{M,j}^{k}}{2}$$

$$v_{i,0}^{k}=v_{i,M+1}^{k},v_{i,M+1}^{k}=\frac{v_{i,1}^{k}-v_{i,M}^{k}}{2}$$

If we take integral to x and y from 0 to π , we get,

$$\mathrm{b}(\tau)=\frac{\mathrm{k}'(\tau)-\int_0^\pi\int_0^\pi \mathrm{xy}\,\mathrm{h}(\alpha,\beta,\tau)\mathrm{d}\alpha\mathrm{d}\beta-\frac{\pi^3}{2}\mathrm{v}_\mathrm{y}(\pi,\tau)}{\frac{\pi^3}{2}\mathrm{v}_\mathrm{x}(\pi,\tau)}.$$

$$\mathrm{b}^{\mathrm{k}+1}=-\frac{\left(\left(\mathrm{k}^{\mathrm{k}+2}-\mathrm{k}^{\mathrm{k}}\right)/\tau\right)}{\left(\frac{\pi^3}{2}\mathrm{v}_\mathrm{x}(\pi,\tau)\right)^{\mathrm{k}}}$$
$$\frac{-\left(\int_0^\pi\int_0^\pi \mathrm{xyh}(\alpha,\beta,\tau)\mathrm{d}\alpha\mathrm{d}\beta\right)^{\mathrm{k}}}{\left(\frac{\pi^3}{2}\mathrm{v}_\mathrm{x}(\pi,\tau)\right)^{\mathrm{k}}}$$
$$-\left(\frac{\pi^3}{2}\mathrm{v}_\mathrm{y}(\pi,\tau)\right)^{\mathrm{k}}$$

where $k^k=k(t_k),\ k=0,1,...,N.$

Here, Simpson's central difference scheme is applied. $b^{k(s)},\ v_{i,j}^{k(s)}$ of $b^k,\ v_{i,j}^{k}$ at the s-th iteration step.

$$\frac{1}{\tau}\left(v_{i,j}^{k+1(s+1)} - v_{i,j}^{k+1(s)}\right)$$
$$= \frac{1}{h^2}\begin{bmatrix} b^{k(s+1)}\left(v_{i-1,j}^{k+1(s+1)} - 2v_{i,j}^{k+1(s+1)} + v_{i+1,j}^{k+1(s+1)}\right) \\ +\left(v_{i,j-1}^{k+1(s+1)} - 2v_{i,j}^{k+1(s+1)} + v_{i,j+1}^{k+1(s+1)}\right) \end{bmatrix}$$
$$+ h_{i,j}^{k+1},$$

$$v_{i,j}^{0} = \varphi_i,$$
$$v_{0,j}^{k+1(s)} = v_{M+1,j}^{k+1(s)}, v_{M+1,j}^{k+1(s)} = \frac{v_{1,j}^{k+1(s)} - v_{M,j}^{k+1(s)}}{2}$$
$$v_{i,0}^{k+1(s)} = v_{i,M+1}^{k+1(s)}, v_{i,M+1}^{k+1(s)} = \frac{v_{i,1}^{k+1(s)} - v_{i,M}^{k+1(s)}}{2}$$

$v_{i,j}^{k+1(s+1)}$ is found.

5. CONCLUSION

The inverse problem of defining thermal dissipation and the heat in the semi-linear two-dimensional parabolic equation with periodic boundary and integral conditions is investigated. The problem has been studied both theoretically and numerically. In this article, periodic boundary conditions are studied.
Nonlocal Periodic boundary conditions for heat inverse coefficient problems are more difficult than local boundary conditions. In this study, the results were obtained by using the Fourier method and the finite difference method.

6. ACKNOWLEDGMENTS

We would like to thank the reviewers.

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