JAMAME Journal of Advanced Mathematics and Mathematics Education
2023, VOL. 06, NO. 3, 1-10

The Complex plane field

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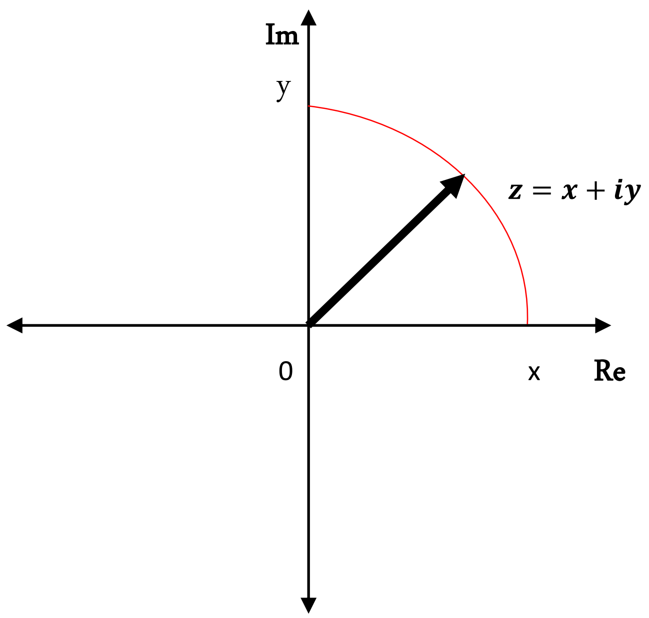
Abstract

Definition, and formulation of the complex plane, as the basis for deriving Euler's number along with Euler's identity. From which complex squares, and primes, are derived; and the value of pi approximated.

Introduction

Complex numbers are a simpler way of locating points of reference on the Cartesian coordinate system. Complex numbers can be manipulated like simple arithmetic, and be simplified to an arithmetic representation of simple expressions such as a set of whole numbers. This is because, every point on the plane has a complex representation, and thus, complex numbers are as important as any set of numbers known to number theory. Without algebra, the plane is an obscure field, and as dense as impenetrable space. However, with it; the plane is easier to define. And any object can be argued against, or in favour for; without deforming logic. The simplest way to generalise is to prove that the theorem is one-to-one with any known constant in mathematics. Often much of the simplicity of mathematics is based on Euler's approach to numbers. While obscurity doesn't justify the intricate nature of the plane itself, it is the nature of numbers to be simple. That's their inherent property from abstract thought. What follows is proof of the fundamentality of the mathematical constants in proving the validity of theorems, and justifying their existence. In general; to advance is to simplify. Had Isaac Newton's formulae fell on abstractionism; the world of mathematics wouldn't be so beautiful. But because there was intuitionism; that is why it is not ugly. And written simply; as follows:

a)



If $\boldsymbol{z = x + iy}$ holds; then $\boldsymbol{z = \sqrt{x^2 + y^2}}$, such that $\boldsymbol{\sqrt{x^2 + y^2} = x + iy}$

$$\boldsymbol{x^2 + y^2 = (x + iy)^2}$$

$$\boldsymbol{x^2 + y^2 = (x + iy)(x + iy)}$$

$$\boldsymbol{x^2 + y^2 = x^2 + 2(ixy) + i^2y^2}$$

$$\boldsymbol{y^2 = 2ixy + i^2y^2}$$

$$\boldsymbol{y^2 - i^2y^2 = 2ixy}$$

$$\boldsymbol{\frac{y^2(1-i^2)}{2} = \frac{2ixy}{2}}$$

$$\boldsymbol{\frac{\frac{y^2(1-i^2)}{2}}{xy} = \frac{ixy}{xy}}$$

$$\boldsymbol{\frac{y^2(1-i^2)}{2xy} = i}$$

$$\boldsymbol{\therefore i = \frac{y}{x} \; or \; \therefore ix = y \; or \; \therefore x = \frac{y}{i}}$$

Since a circle is defined as $\boldsymbol{x^2 + y^2 = 1}$, then it implies that the theorem of Pythagoras holds; and logically follows that: $\boldsymbol{y^2 = -x^2 + 1}$. And $\therefore \boldsymbol{y = \sqrt{-x^2 + 1}}$, or $\therefore \boldsymbol{x = \sqrt{-y^2 + 1}}$ defines half a unit-circle (that is, a semi-circle). Now, since $\boldsymbol{i^2 = -1}$ then $\boldsymbol{i^2 = e^{i\pi}}$; so that $\boldsymbol{i = \sqrt{e^{i\pi}}}$ holds. Such that $\boldsymbol{i = \frac{y}{x}}$

$$\boldsymbol{\sqrt{e^{i\pi}} = \frac{y}{x}}$$

$$\boldsymbol{e^{i\pi} = \left(\frac{y}{x}\right)^2}$$

$$\boldsymbol{e^{i\pi} = \frac{y^2}{x^2}}$$

$$\boldsymbol{\therefore e^{i\pi} = -1}$$

Or $\boldsymbol{\frac{e^{i\pi}}{-1} = \frac{-1}{-1}}$

$$\boldsymbol{\therefore -e^{i\pi} = 1}$$

When $\boldsymbol{z = 0}$, it cannot be false that the above holds, and thus, never seizes to exist. Already the theorem proves to be the simple basis of the Cartesian coordinate system. And it is justified as follows: $\boldsymbol{x^2 + y^2 = z^2}$

$$\boldsymbol{x^2 + y^2 = 0}$$

$$\boldsymbol{x^2 = -y^2}$$

$$\boldsymbol{\frac{x^2}{x^2} = \frac{-y^2}{x^2}}$$

$$\boldsymbol{1 = -\frac{y^2}{x^2}}$$

$$\therefore \frac{y^2}{x^2} = -1$$

Which can be further verified by the fact that $e^{i\pi} = -1$

$$e^{i\pi} = \frac{y^2}{x^2}$$

$$e = \sqrt[i\pi]{\frac{y^2}{x^2}}$$

$$e = \sqrt[i\pi]{-1}$$

$$e = \sqrt[i\pi]{i^2}$$

$$e = i^{\frac{2}{i\pi}}$$

$$e = e$$

$$e - e = 0$$

$$\therefore 0 = 0$$

So that $e^{i\pi} = \frac{y^2}{x^2}$

$$\log_e \frac{y^2}{x^2} = i\pi$$

$$\frac{\log_e \frac{y^2}{x^2}}{i} = \frac{i\pi}{i}$$

$$\therefore \pi = \frac{\log_e \frac{y^2}{x^2}}{i}$$

Or if the above holds, such that it is true that $e^{i\pi} = \frac{y^2}{x^2}$

$$e^{i\pi} x^2 = y^2$$

$$\sqrt{e^{i\pi} x^2} = y$$

$$\therefore y = e^{\frac{i\pi}{2}} x \; or \; \therefore x = \frac{y}{e^{\frac{i\pi}{2}}}$$

Then $x = \sqrt{-y^2 + 1}$

$$\left(\frac{y}{e^{\frac{i\pi}{2}}}\right)^2 = -y^2 + 1$$

$$\frac{y^2}{e^{\frac{i\pi}{2}+\frac{i\pi}{2}}} = -y^2 + 1$$

$$\frac{y^2}{e^{i\pi}} = -y^2 + 1$$

$$\frac{y^2}{e^{i\pi}} = x^2$$

$$\frac{y^2}{i^2} = x^2$$

$$y^2 = i^2x^2$$

$$\therefore y = ix \ or \ \therefore x = \frac{y}{x}$$

So that when $x = x$, the following holds: $\frac{y}{e^{\frac{i\pi}{2}}} = \frac{y}{i}$

$$\frac{iy}{y} = \frac{e^{\frac{i\pi}{2}}y}{y}$$

$$\therefore i = e^{\frac{i\pi}{2}}$$

Or such that $y = y$

$$e^{\frac{i\pi}{2}}x = ix$$

$$\frac{e^{\frac{i\pi}{2}}x}{x} = \frac{ix}{x}$$

$$\therefore i = e^{\frac{i\pi}{2}}$$

Thus, the following cannot be proven false, since it is true that $y = y$

$$e^{\frac{i\pi}{2}} = \sqrt{-x^2 + 1}$$

$$\left(e^{\frac{i\pi}{2}}x\right)^2 = -x^2 + 1$$

$$e^{i\pi}x^2 = -x^2 + 1$$

$$e^{i\pi}x^2 = y^2$$

$$i^2x^2 = y^2$$

$$\sqrt{i^2x^2} = y$$

$$\therefore y = ix \ or \ \therefore x = \frac{y}{i}$$

Then, $y = e^{i\pi}x$

$$\frac{y}{y} = \frac{e^{i\pi}x}{y}$$

$$1 = e^{i\pi}\left(\frac{x}{y}\right)$$

$$\therefore \frac{e^{i\pi}x}{y} = 1$$

So that for $-e^{i\pi} = e^{i\pi}i^2$, it follows that $e^{i\pi}i^2 = x^2 + y^2$

$$e^{i\pi}i^2 - x^2 = y^2$$

$$\sqrt{e^{i\pi}i^2 - x^2} = y$$

$$\therefore y = \sqrt{-x^2 + 1}$$

While it follows that $e^{i\pi}i^2 = x^2 + y^2$

$$e^{i\pi}i^2 - y^2 = x^2$$

$$\sqrt{e^{i\pi}i^2 - y^2} = x$$

$$\therefore x = \sqrt{-y^2 + 1}$$

From there then, it holds as follows: $e^{i\pi} = e^{-i\pi}$

$$e^{i\pi} = \frac{1}{e^{i\pi}}$$

$$e^{i\pi}(e^{i\pi}) = 1$$

$$\therefore e^{2i\pi} = 1$$

Whereas it is true that $i = -e^{i\pi}i$, while $-i = e^{i\pi}i$. Or $-e^{2i\pi}i^{-1} = i$, while $e^{i\pi}i^{-4} = -1$. For $-e^{-i\pi} = 1$, and such- can be proven as follows: $e^{i\pi} = \frac{y^2}{x^2}$

$$e^{i\pi}x^2 = y^2$$

$$\frac{e^{i\pi}x}{e^{i\pi}} = \frac{y^2}{e^{i\pi}}$$

$$x^2 = \frac{y^2}{\frac{y^2}{x^2}}$$

$$x^2 = \frac{x^2y^2}{y^2}$$

$$x^2 = x^2$$

$$x^2 - x^2 = 0$$

$$\therefore 0 = 0$$

What this therefore suggests is that $r = z$ for $0 \leq z \leq 1$. Rotating the radius of a circle result into derivatives which are retained through transforming the function at $\theta = 45°$. It is a continuous function on restriction, while it rotates- it therefore; cannot be greater than 2, since $-1 \leq 2r \leq 1$ is within an interval of a unit circle. Hence, $e^{i\pi} = \frac{y^2}{x^2}$

$$y^2 = e^{i\pi}x^2$$

$$y = \sqrt{e^{i\pi}x^2}$$

$$y = e^{\frac{i\pi}{2}}x$$

$$\frac{y}{e^{\frac{i\pi}{2}}x} = \frac{e^{\frac{i\pi}{2}}x}{e^{\frac{i\pi}{2}}x}$$

$$\frac{1}{e^{\frac{i\pi}{2}}}\left(\frac{y}{x}\right) = 1$$

$$\frac{1}{e^{\frac{i\pi}{2}}}(i) = 1$$

$$\frac{i}{i} = 1$$

$$1 - 1 = 0$$

$$\therefore 0 = 0$$

Holding that $-1 = e^{i\pi}$

$$i^2 = e^{i\pi}$$

$$\log_e i^2 = i\pi$$

$$\frac{\log_e i^2}{i} = \frac{i\pi}{i}$$

$$\therefore \pi = \frac{\log_e i^2}{i}$$

Pi has a simple definition on the Complex Cartesian coordinate system, much simpler than its definition which holds on the Cartesian coordinate system. Its geometry is clearer, as it follows by substitution from a unit-circle that: $x^2 + y^2 = -i^2$

$$x^2 = -i^2 - y^2$$

$$x^2 = -1(i^2 + y^2)$$

$$x^2 = i^2(i^2 + y^2)$$

$$x^2 = i^4 + i^2y^2$$

$$x = \sqrt{i^4 + i^2y^2}$$

$$\therefore x = \sqrt{-y^2 + 1}$$

Or $x^2 + y^2 = -i^2$

$$y^2 = -i^2 - x^2$$

$$y^2 = -1(i^2 + x^2)$$

$$y^2 = i^2(i^2 + x^2)$$

$$y = \sqrt{i^4 + i^2x^2}$$

$$\therefore \boldsymbol{y} = \sqrt{-\boldsymbol{x}^2 + \mathbf{1}}$$

Now, considering that an ellipse is a circle. And only, different from a circle with respect to its eccentricity; as an ellipse is defined by its eccentricity. Therefore, $\boldsymbol{y} = \boldsymbol{a}\sqrt{-\boldsymbol{x}^2 + \mathbf{1}}$ holds, and cannot be false, only and only if, $\boldsymbol{a} \neq \mathbf{1}$. Such that it is also true, even if $\boldsymbol{a} < \mathbf{0}$: as its eccentricity does not cease to exist.

Say then, if $\boldsymbol{i}^{\mathbf{2}} = \boldsymbol{e}^{\boldsymbol{i\pi}}$

$$\left(\boldsymbol{e}^{\frac{\boldsymbol{i\pi}}{\mathbf{2}}}\right)^{\mathbf{2}} = \boldsymbol{e}^{\boldsymbol{i\pi}}$$

$$\boldsymbol{e}^{\frac{\mathbf{2}\boldsymbol{i\pi}}{\mathbf{2}}} = \boldsymbol{e}^{\boldsymbol{i\pi}}$$

$$\boldsymbol{e}^{\boldsymbol{i\pi}} = \boldsymbol{e}^{\boldsymbol{i\pi}}$$

$$\boldsymbol{e}^{\boldsymbol{i\pi}} - \boldsymbol{e}^{\boldsymbol{i\pi}} = \mathbf{0}$$

$$\therefore \mathbf{0} = \mathbf{0}$$

So that $\boldsymbol{i}^{\mathbf{2}} = \boldsymbol{e}^{\boldsymbol{i\pi}}$

$$\boldsymbol{i}^{\mathbf{2}} = -\mathbf{1}$$

$$\therefore \boldsymbol{i} = \sqrt{-\mathbf{1}}$$

Or it can be simplified as follows: $\boldsymbol{i} = \sqrt{-\mathbf{1}}$

$$\boldsymbol{i} = \sqrt{\boldsymbol{i}^{\mathbf{2}}}$$

$$\boldsymbol{i} = \boldsymbol{i}^{\mathbf{2}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)}$$

$$\boldsymbol{i} - \boldsymbol{i} = \mathbf{0}$$

$$\therefore \mathbf{0} = \mathbf{0}$$

Thus, all being referred to- is within, and on the circumference of a unit circle whose radius is equal to one. It is true then that Euler's identity holds, and can also be justified and argued for its existence as follows: $\boldsymbol{e}^{\mathbf{2}\boldsymbol{i\pi}} = \boldsymbol{e}^{\boldsymbol{i\pi}}\boldsymbol{i}^{\mathbf{2}}$

$$\frac{\boldsymbol{e}^{\mathbf{2}\boldsymbol{i\pi}}}{\boldsymbol{e}^{\boldsymbol{i\pi}}} = \frac{\boldsymbol{e}^{\boldsymbol{i\pi}}\boldsymbol{i}^{\mathbf{2}}}{\boldsymbol{e}^{\boldsymbol{i\pi}}}$$

$$\frac{\boldsymbol{e}^{\mathbf{2}\boldsymbol{i\pi}}}{\boldsymbol{e}^{\boldsymbol{i\pi}}} = \boldsymbol{i}^{\mathbf{2}}$$

$$\sqrt{\frac{\boldsymbol{e}^{\mathbf{2}\boldsymbol{i\pi}}}{\boldsymbol{e}^{\boldsymbol{i\pi}}}} = \boldsymbol{i}$$

$$\frac{\sqrt{\boldsymbol{e}^{\mathbf{2}\boldsymbol{i\pi}}}}{\sqrt{\boldsymbol{e}^{\boldsymbol{i\pi}}}} = \boldsymbol{i}$$

$$\frac{\sqrt{\boldsymbol{e}^{\mathbf{2}\boldsymbol{i\pi}}}}{\boldsymbol{i}} = \boldsymbol{i}$$

$$\sqrt{e^{2i\pi}} = i^2$$

$$e^{i\pi} = i^2$$

$\therefore e^{i\pi} = -1$ And if true, thus $i = i^{\frac{i\pi}{2}}$

$$\frac{i\pi}{2} = \log_e i$$

$$i\pi = 2\log_e i$$

$$\frac{i\pi}{i} = \frac{2\log_e i}{i}$$

$$\therefore \pi = \frac{2\log_e i}{i}$$

However, since $e = i^{\frac{2}{i\pi}}$, then $\log_i e = \frac{2}{i\pi}$

$$i\pi \log_i e = 2$$

$$\frac{i\pi \log_i e}{\log_i e} = \frac{2}{\log_i e}$$

$$i\pi = \frac{2}{\log_i e}$$

$$\frac{i\pi}{i} = \frac{\frac{2}{\log_i e}}{i}$$

$$\pi = \frac{2}{\log_i e}(i)$$

$$\pi = \frac{2i}{\log_i e}$$

$\therefore \pi \neq -\pi$ And $\therefore \pi = -\frac{2i}{\log_i e}$

Given such then $\therefore i = -\frac{\pi \log_i e}{2}$. And it follows naturally, that $\pi = \frac{2}{\log_i e^i}$

$$\pi \log_i e^i = 2$$

$$\frac{\pi \log_i e^i}{\pi} = \frac{2}{\pi}$$

$$\log_i e^i = \frac{2}{\pi}$$

$$e^i = i^{\frac{2}{\pi}}$$

$$\therefore e = \sqrt[i]{i^{\frac{2}{\pi}}}$$

Let $\frac{2}{\log_i e^{i\pi}} = x^2 + y^2$

$$2 = \log_i e^{i\pi}(x^2 + y^2)$$

$$\frac{2}{(x^2+y^2)} = \frac{\log_i e^{i\pi}(x^2+y^2)}{(x^2+y^2)}$$

$$\frac{2}{(x^2+y^2)} = \log_i e^{i\pi}$$

$$e^{i\pi} = i^{\frac{2}{(x^2+y^2)}}$$

$$-1 = i^{\frac{2}{1}}$$

$$\therefore i^2 = -1$$

All this leads to a mere simple fact that $z^2 = x^2 - y^2 + 2ixy$, and $\left(\frac{z-x}{y}\right)^2 = -1$; when $z = x + iy$. Which translate as follows:

$$\left(\frac{z-x}{y}\right)^2 = \frac{y^2}{x^2}$$

$$\frac{z^2-2xz+x^2}{y} = \frac{y^2}{x^2}$$

$$x^2(z^2 - 2xz + x^2) = y^3$$

$$y = \sqrt[3]{x^2(z^2 - 2xz + x^2)}$$

$$y = \sqrt[3]{x^2(-y^2)}$$

$$\therefore y = \sqrt[3]{-x^2y^2}$$

Or the value of x can be located in the following way: $x^2(z^2 - 2xz + x^2) = y^3$

$$\frac{x^2(z^2-2xz+x^2)}{(z^2-2xz+x^2)} = \frac{y^3}{(z^2-2xz+x^2)}$$

$$x = \sqrt{\frac{y^3}{(z^2-2xz+x^2)}}$$

$$x = \sqrt{\frac{y^3}{-y^2}}$$

$$\therefore x = \sqrt{-y}$$

Conclusion

At $r = 1$ the area of a circle is equal to π. And transforming **r** does not change the value of its area, nor that of its circumference. That is, if we're only reflecting it around the plane. Therefore, as that happens: only our view of mathematics changed and not with its regard- to the nature of objects themselves. However, it only changes the location of **r**. That is, **z**: since it is equal to the radius. Thus, by transformation- functions such as curves: can be straightened, resulting into linear functions. And vice versa... As a result, simple functions are so, because of the changes they have undergone on the plane. Such that given specific conditions, or restrictions; objects either expands, contracts, or are reflected. Such is simple calculus, and if one where to utilise above; one would realise that did not affect the nature of existence of the object, but only, changed the view towards that particular object. These are the foundations of mathematics, and hence it is crucial for calculus to base its entire philosophy on the nature of algebra. Meaning therefore, that objects can be spatially distorted; and alter states, as they change dimensions on the plane. And such that with all the above being stated and proven: then there exist a set of complex primes and complex squares; on the complex Cartesian plane.

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