Mathematical Sciences and Applications E-Notes
https://doi.org/10.36753/mathenot.1300609
12 (1) 12-19 (2024) - Research Article
ISSN: 2147-6268

Some Fixed Point Results for α-Admissible Mappings on Quasi Metric Space Via θ-Contractions

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Abstract
By implying α-admissible mapping, this study expands and investigates generalized contraction mappings in quasi-metric spaces, aiming to establish the existence of fixed points. Moreover, we show that the main outcomes of the paper encompass several previously reported results in the literature.

Keywords: *Fixed point, Right K-Cauchy sequence, Quasi metric space, α admissible*

AMS Subject Classification (2020): *54H25; 47H10*

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1. Introduction and preliminaries

The Banach contraction principle, also known as the Banach fixed point theorem, is a fundamental result in mathematics, specifically in the field of functional analysis. It is named after the Polish mathematician Stefan Banach, who first stated and proved the theorem in 1922. The theorem provides conditions under which a mapping using a complete metric space to itself has a unique fixed point. A fixed point of a mapping is a point in the space that remains unchanged after applying the mapping. The proof of the Banach contraction principle typically involves constructing a sequence of iterates using the contraction property and showing that it converges to the fixed point. The completeness of the metric space is crucial for guaranteeing the convergence of the sequence.(see [1–5]). But owing to the strict conditions of the metric space and the specific properties imposed, the necessity to consider topological structures that have more flexible conditions than the metric space has emerged.Therefore, many generalizations of the Banach fixed point theorem have been obtained in this space by defining the quasi metric space. Furthermore, quasi-metric spaces are useful in numerous topics of mathematics, like optimization, functional analysis and computer science. They provided a more general framework for studying approachs related to distances and convergence, allowing for more flexible and adaptable notions of proximity. (see [6–11]). Now, review the definitions and notations related to quasi-metric space:

$\Lambda \neq \emptyset$ and ρ be a function $\rho : \Lambda \times \Lambda \to \mathbb{R}$ such that for each $\omega, \gamma, \eta \in \Lambda$:

Received : *22-05-2023,* ***Accepted :*** *14-08-2023,* ***Available online :*** *02-11-2023*

*(**Cite as** "G. D. Güngör, İ. Altun, Some Fixed Point Results for α-Admissible mappings on Quasi Metric Space Via θ-Contractions, Math. Sci. Appl. E-Notes, 12(1) (2024), 12-19")*

i) $\rho(\omega,\omega)=0$ (Non-negativity),

ii) $\rho(\omega,\gamma)\leq\rho(\omega,\eta)+\rho(\eta,\gamma)$ (triangle inequality),

iii) $\rho(\omega,\gamma)=\rho(\gamma,\omega)=0\Rightarrow\omega=\gamma$ (asymmetry),

iv) $\rho(\omega,\gamma)=0\Rightarrow\omega=\gamma$.

If (i) and (ii) conditions are satisfied, then ρ it is called a quasi-pseudo metric(shortly q.-p.-m.), if (i), (ii) and (iii) conditions are satisfied, then ρ is called quasi metric(shortly q.-m.), in addition if a q.-m. ρ satisfies (iv), then ρ is called T_1-q.-m.. It is evident that

$$\begin{aligned}&\forall \text{ metric is a } T_1 \text{ quasi-metric,}\\&\forall\, T_1 \text{ quasi-metric is a quasi-metric,}\\&\forall \text{ quasi-metric is a quasi-pseudo metric.}\end{aligned}$$

Then, the pair (Λ,ρ) is also said to be a quasi pseudo metric space(shortly q.-p.-m. s.). Moreover, each q.-p. m. ρ on Λ generates a topology τ_ρ on Λ the family of open balls as a base defined as follows:

$$\{B_\rho(\omega,\varepsilon):\omega\in\Lambda \text{ and } \varepsilon>0\}$$

where $B_\rho(\omega_0,\varepsilon)=\{\gamma\in\Lambda:\rho(\omega_0,\gamma)<\varepsilon\}$.

If ρ is a q.-m. on Λ, then τ_ρ is a T_0 topology, and if ρ is a T_1-q.-m., then τ_ρ is a T_1 topology on Λ.

If ρ is a q.-m. and τ_ρ is T_1 topology, then ρ is T_1-q.-m.. In this case, the mappings, $\rho^{-1},\rho^s,\rho_+:\Lambda\times\Lambda\to[0,\infty)$ defines as

$$\begin{aligned}\rho^{-1}(\omega,\gamma)&=\rho(\gamma,\omega)\\\rho^{s}(\omega,\gamma)&=\max\{\rho(\omega,\gamma),\rho^{-1}(\omega,\gamma)\}\\\rho_{+}(\omega,\gamma)&=\rho(\omega,\gamma)+\rho^{-1}(\omega,\gamma)\end{aligned}$$

are also q.-p.-metrics on Λ. If ρ is a q.-m., then ρ^s and ρ_+ are (equivalent) metrics on Λ. To find the fixed point, the most important part is to use the completeness of the metric space. But since there is no symmetry conditions in a q.-m., there are many definitions of completeness in these spaces in the literature.(see [12–14])

Let (Λ,ρ) be a q.-m. and the convergence of a sequence $\{\omega_n\}$ to ω w. r. t.

$$\begin{aligned}\tau_\rho \text{ called } \rho-\text{convergence and is defined } \omega_n\overset{\rho}{\to}\omega\quad&\Leftrightarrow\quad\rho(\omega,\omega_n)\to0,\\\tau_{\rho^{-1}} \text{ called } \rho^{-1}-\text{convergence and is defined } \omega_n\overset{\rho^{-1}}{\to}\omega\quad&\Leftrightarrow\quad\rho(\omega_n,\omega)\to0,\\\tau_{\rho^{s}} \text{ called } \rho^{s}-\text{convergence and is defined } \omega_n\overset{\rho^{s}}{\to}\omega\quad&\Leftrightarrow\quad\rho(\omega_n,\omega)\to0\end{aligned}$$

for $\omega\in\Lambda$. A more detailed explanation of some essential metric properties can be found in [15]. Also, a sequence $\{\omega_n\}$ in Λ is called left(right) $K-$Cauchy if for every $\varepsilon>0$,there exists $n_0\in\mathbb{N}$ such that $\forall n,k,n\geq k\geq n_0(k\geq n\geq n_0),\rho(\omega_k,\omega_n)<\varepsilon$. The left K-Cauchy property under ρ implies the right K-Cauchy property under ρ^{-1}. Assuming

$$\sum_{n=1}^{\infty}\rho(\omega_n,\omega_{n+1})<\dot{\infty},$$

the sequence $\{\omega_n\}$ in the quasi-metric space (Λ,ρ) is left K-Cauchy.

In a metric space, every convergent sequence is indeed a Cauchy sequence, but since this may not hold true in q.-m., and so there have been several definitions of completeness. Let (Λ,ρ) be a q.-m.. Then (Λ,ρ) is said to be left(right) K (resp. $(M)(Smyth)$)- complete if every left(right) K -Cauchy sequence is ρ(resp. $(\rho^{-1})(\rho^s)$) -convergent.

Indeed, now explain the approach of α-admissibility as constructed by Samet et al. [16].

Let $\Lambda\neq\emptyset$, Υ be a self-mapping (a mapping from Λ to itself), and $\alpha:\Lambda\times\Lambda\to[0,\infty)$ be a function. In this context, Υ is said to be α-admissible if it satisfies the following condition:

$$\text{If } \alpha(\omega,\gamma)\geq1,\ \text{ then } \alpha(\Upsilon\omega,\Upsilon\gamma)\geq1.$$

By introducing the approach of α-admissibility, Samet et al. [16] were able to establish some general fixed point results that encompassed many well-known theorems of complete metric spaces. These fixed point results provide a framework for studying the existence and properties of fixed points for self-mappings on a complete metric space, using the approach of α-admissibility.(see [17–23])

In addition to these, in the study conducted by Jleli and Samet in [24], they they led to the introduction of a new type of contractive mapping known as a θ-contraction. This θ-contraction serves as an attractive generalization within the field. To better understand this approach, let's review some notions and related results concerning θ-contraction.

The family of $\theta : (0,\infty) \to (1,\infty)$ functions that satisfy the following conditions can be denoted by the set Θ.

(θ_1) θ is nondecreasing;

(θ_2) Considering every sequence $\{\varkappa_n\} \subset (0,\infty)$, $\lim_{n\to\infty} \varkappa_n = 0^+$ if only if $\lim_{n\to\infty} \theta(\varkappa_n) = 1$;

(θ_3) There exist $0 < p < 1$ and $\beta \in (0,\infty]$ such that $\lim_{\varkappa\to 0^+} \frac{\theta(\varkappa)-1}{\varkappa^p} = \beta$.

If we define $\theta(\varkappa) = e^{\sqrt{\varkappa}}$ for $\varkappa \leq 1$ and $\theta(\varkappa) = 9$ for $\varkappa > 1$, then $\theta \in \Theta$.

Let $\theta \in \Theta$ and (Λ, ρ) be a metric space. Then $\Upsilon : \Lambda \to \Lambda$ is said to be a θ-contraction if there exists $0 < \delta < 1$ such that

$$\theta(\rho(\Upsilon\omega, \Upsilon\gamma)) \leq [\theta(\rho(\omega,\gamma))]^{\delta} \tag{1.1}$$

for each $\omega, \gamma \in \Lambda$ with $\rho(\Upsilon\omega, \Upsilon\gamma) > 0$.

By choosing appropriate functions for θ, such as $\theta_1(\varkappa) = e^{\sqrt{\varkappa}}$ and $\theta_2(\varkappa) = e^{\sqrt{\varkappa e^{\varkappa}}}$, it is possible to obtain different types of nonequivalent contractions using (1.1).

Indeed, Jleli and Samet proved that every θ-contraction on a complete metric space possesses a unique fixed point. This result provides a valuable insight into the uniqueness and existence of fixed points for a wide range of contractive mappings. If you are interested in exploring more papers and literature related to θ-contractions, there are several resources available (see [25, 26]).

2. The results

Our basic results are based on a novel approach that we have developed.

Let (Λ, ρ) be a q.-m., $\Upsilon : \Lambda \to \Lambda$ be a given mapping and $\alpha : \Lambda \times \Lambda \to [0,\infty)$ be a function. We will consider the following set

$$\Upsilon_\alpha = \{(\omega,\gamma) \in \Lambda \times \Lambda : \alpha(\omega,\gamma) \geq 1 \text{ and } \rho(\Upsilon\omega, \Upsilon\gamma) > 0\}. \tag{2.1}$$

Let (Λ, ρ) be a q.-m. and $\Upsilon : \Lambda \to \Lambda$ be a mapping satisfying

$$\rho(\omega,\gamma) = 0 \Longrightarrow \rho(\Upsilon\omega, \Upsilon\gamma) = 0. \tag{2.2}$$

$\alpha : \Lambda \times \Lambda \to [0,\infty)$ and $\theta \in \Theta$ be two functions. Then we say that Υ is a generalized $(\alpha - \theta_\rho)$-contraction(shortly g. $(\alpha - \theta_\rho)$-c.) if there exists a constant $0 < \delta < 1$ such that

$$\theta(\rho(\Upsilon\omega, \Upsilon\gamma)) \leq [\theta(M(\omega,\gamma))]^{\delta}, \tag{2.3}$$

for each $\omega, \gamma \in \Upsilon_\alpha$, where

$$M(\omega,\gamma) = \max\left\{\rho(\omega,\gamma), \rho(\Upsilon\omega,\omega), \rho(\Upsilon\gamma,\gamma), \frac{1}{2}\left[\rho(\Upsilon\omega,\gamma) + \rho(\Upsilon\gamma,\omega)\right]\right\}.$$

Before presenting our main results, let us recall some important remarks:

- If (Λ, ρ) is a T_1-q.-m., then every mapping $\Upsilon : \Lambda \to \Lambda$ satisfies the condition (2.2).
- It is clear from (2.1), (2.2) and (2.3) that if Υ is an (α, θ_ρ)-contraction on a q.-m. (Λ, ρ), then

  $$\rho(\Upsilon\omega, \Upsilon\gamma) \leq \rho(\omega,\gamma),$$

  for each $\omega, \gamma \in \Lambda$ with $\alpha(\omega,\gamma) \geq 1$.

By utilizing the approach of g. $(\alpha - \theta_\rho)$-c., we will now present the following theorem.

Theorem 2.1. *Let (Λ, ρ) be a Hausdorff right K-complete T_1-q.-m., and let $\Upsilon : \Lambda \to \Lambda$ be a g. $(\alpha - \theta_\rho)$-c.. Presume that τ_ρ-continuous and Υ is α-admissible. If there exists $\omega_0 \in \Lambda$ such that $\alpha(\Upsilon\omega_0, \omega_0) \geq 1$, then Υ has a fixed point in Λ.*

Proof. Let $\omega_0 \in \Lambda$ be a such that $\alpha(\Upsilon\omega_0, \omega_0) \geq 1$. Define a sequence $\{\omega_n\}$ in Λ by $\omega_{n+1} = \Upsilon\omega_n$ for each n in $\mathbb{N}$. Since Υ is α-admissible then $\alpha(\omega_{n+1}, \omega_n) \geq 1$ for each n in $\mathbb{N}$. If there exist $k \in \mathbb{N}$ with $\rho(\omega_n, \Upsilon\omega_n) = 0$ then $\omega_n = \Upsilon\omega_n$, ssince ρ is T_1 q.-m.. Hence, ω_k is a fixed point of Υ. Presume $\rho(\omega_n, \Upsilon\omega_n) > 0$ for each n in $\mathbb{N}$. In this case the pair (ω_{n+1}, ω_n) for each n in $\mathbb{N}$ belongs to Υ_α. Since Υ is g. $(\alpha - \theta_\rho)$-c. and (θ_1), we obtain

$$\begin{aligned}\theta(\rho(\omega_{n+1}, \omega_n)) &\leq [\theta(M(\omega_n, \omega_{n-1}))]^\delta \\ &= \left[\theta(\max\left\{\begin{array}{c}\rho(\omega_n, \omega_{n-1}), \rho(\omega_{n+1}, \omega_n), \rho(\omega_n, \omega_{n-1}), \\ \frac{1}{2}\left[\rho(\omega_{n+1}, \omega_{n-1}) + \rho(\omega_n, \omega_n)\right]\end{array}\right\}\right]^\delta \\ &\leq [\theta(\max\{\rho(\omega_{n+1}, \omega_n), \rho(\omega_n, \omega_{n-1})\}]^\delta . \end{aligned} \tag{2.4}$$

If $\max\{\rho(\omega_{n+1}, \omega_n), \rho(\omega_n, \omega_{n-1})\} = \rho(\omega_{n+1}, \omega_n)$, using (2.4), we get

$$\theta(\rho(\omega_{n+1}, \omega_n)) \leq [\theta(\rho(\omega_{n+1}, \omega_n)]^\delta < \theta(\rho(\omega_{n+1}, \omega_n)),$$

which is a contradiction. Thus, $\max\{\rho(\omega_{n+1}, \omega_n), \rho(\omega_n, \omega_{n-1})\} = \rho(\omega_n, \omega_{n-1})$, and then we obtain

$$\theta(\rho(\omega_{n+1}, \omega_n)) \leq [\theta(\rho(\omega_n, \omega_{n-1}))]^\delta , \tag{2.5}$$

for each n in $\mathbb{N}$. Denote $f_n = \rho(\omega_{n+1}, \omega_n)$ for n in $\mathbb{N}$. Then $f_n > 0$ for each n in $\mathbb{N}$ and repeating this process with using (2.5), we have

$$\theta(f_n) \leq [\theta(f_0)]^{\delta^n} ,$$

i.e.

$$1 < \theta(f_n) \leq [\theta(f_0)]^{\delta^{n-1}} \tag{2.6}$$

for each n in $\mathbb{N}$. When taking the limit as $n \to \infty$ in (2.6), we obtain

$$\lim_{n\to\infty} \theta(f_n) = 1. \tag{2.7}$$

Using (θ_2), we can deduce that $\lim_{n\to\infty} f_n = 0^+$, thus using (θ_3), there exist $p \in (0, 1)$ and $\beta \in (0, \infty]$ such that

$$\lim_{n\to\infty} \frac{\theta(f_n) - 1}{(f_n)^p} = \beta.$$

Presume that $\beta < \infty$. In this case, let $F = \frac{\beta}{2} > 0$. Using the definition of the limit, there exists n_0 in $\mathbb{N}$ such that, for each $n_0 \leq n$,

$$\left|\frac{\theta(f_n) - 1}{(f_n)^p} - \beta\right| \leq F.$$

This implies that, for each $n_0 \leq n$,

$$\frac{\theta(f_n) - 1}{(f_n)^p} \geq \beta - F = F.$$

Then, for each $n_0 \leq n$,

$$n(f_n)^p \leq Dn\left[\theta(f_n) - 1\right],$$

where $D = 1/F$.

Presume now that $\beta = \infty$. Let $F > 0$ be an arbitrary positive number. Using the definition of the limit, there exists n_0 in $\mathbb{N}$ such that, for each $n_0 \leq n$,

$$\frac{\theta(f_n) - 1}{(f_n)^p} \geq F.$$

This implies that, for each $n_0 \leq n$,

$$n\left[f_n\right]^p \leq Dn\left[\theta(f_n) - 1\right],$$

where $D = 1/F$.

Thus, in all cases, there exist $D > 0$ and n_0 in $\mathbb{N}$ such that

$$n\left[f_n\right]^p \leq Dn\left[\theta(f_n) - 1\right],$$

for each $n_0 \leq n$. Using (2.6), we obtain

$$n\,[f_n]^p \leq Dn\left[[\theta(f_0)]^{\delta^{n-1}} - 1\right],$$

for each $n_0 \leq n$. Letting $n \to \infty$ from the given inequality, we have

$$\lim_{n\to\infty} n\,[f_n]^p = 0.$$

Thus, there exists n_1 in $\mathbb{N}$ such that $n\,[f_n]^p \leq 1$ for each $n \geq n_1$, so we have, for each $n \geq n_1$,

$$f_n \leq \frac{1}{n^{1/p}}. \tag{2.8}$$

In order to show that $\{\omega_n\}$ is a left K-Cauchy sequence, consider m, n in $\mathbb{N}$ such that $m > n \geq n_1$. Using the triangular inequality for ρ and using (2.8), we have

$$\begin{aligned} \rho(\omega_m, \omega_n) &\leq \rho(\omega_m, \omega_{m-1}) + \rho(\omega_{m-1}, \omega_{m-2}) + \cdots + \rho(\omega_{n+1}, \omega_n) \\ &= f_{m-1} + f_m + \cdots + f_n \\ &= \sum_{i=n}^{m-1} f_i \leq \sum_{i=n}^{\infty} f_i \leq \sum_{i=n}^{\infty} \frac{1}{i^{1/p}}. \end{aligned}$$

By the convergence of the series $\sum\limits_{i=1}^{\infty} \frac{1}{i^{1/p}}$, we get $\rho(\omega_m, \omega_n) \to 0$ as $n \to \infty$. This yields that $\{\omega_n\}$ is a right K-Cauchy sequence in the q.-m. (Λ, ρ). Since (Λ, ρ) is a right K-complete, there exists $\eta \in \Lambda$ such that the sequence $\{\omega_n\}$ is ρ-converges to $\eta \in \Lambda$; that is, $\rho(\eta, \omega_n) \to 0$ as $n \to \infty$. Since Υ is τ_ρ-continuous then $\rho(\Upsilon\eta, \Upsilon\omega_n) = \rho(\Upsilon\eta, \omega_{n+1}) \to 0$ as $n \to \infty$. Since Λ is Hausdorff, we get $\Upsilon\eta = \eta$. □

In Theorem 2.1, if we consider the approach of $\tau_{\rho^{-1}}$-continuity, we can derive the following theorem.

Theorem 2.2. *Let (Λ, ρ) be a right M-complete T_1-q.-m. such that $(\Lambda, \tau_{\rho^{-1}})$ is Hausdorff and $\Upsilon : \Lambda \to \Lambda$ be a g. $(\alpha - \theta_\rho)$-c.. Presume that Υ is $\tau_{\rho^{-1}}$-continuous and α-admissible. If there exists $\omega_0 \in \Lambda$ such that $\alpha(\Upsilon\omega_0, \omega_0) \geq 1$, then Υ has a fixed point in Λ.*

Proof. Similar to the proof of Theorem 2.1, we can take iterative sequence $\{\omega_n\}$ right K-Cauchy. Since (Λ, ρ) right M-complete, there exists $\eta \in \Lambda$ such that $\{\omega_n\}$ is ρ^{-1}-converges to η, that is, $\rho(\omega_n, \eta) \to 0$ as $n \to \infty$. Using $\tau_{\rho^{-1}}$-continuity of Υ, we get $\rho(\Upsilon\omega_n, \Upsilon\eta) = \rho(\omega_{n+1}, \Upsilon\eta) \to 0$ as $n \to \infty$. Since $(\Lambda, \tau_{\rho^{-1}})$ is Hausdorff, we get $\eta = \Upsilon\eta$. □

Theorem 2.3. *Let (Λ, ρ) be a right Smyth complete T_1 q.-m. and $\Upsilon : \Lambda \to \Lambda$ be a g. $(\alpha - \theta_\rho)$-c.. Presume that Υ is τ_ρ or $\tau_{\rho^{-1}}$-continuous and α-admissible. If there exists $\omega_0 \in \Lambda$ such that $\alpha(\Upsilon\omega_0, \omega_0) \geq 1$, then Υ has a fixed point in Λ.*

Proof. Similar to the proof of Theorem 2.1, we can take iterative sequence $\{\omega_n\}$ right K-Cauchy. Since (Λ, ρ) is right Smyth complete, there exists $\eta \in \Lambda$ such that $\{\omega_n\}$ is ρ^s-converges to $\eta \in \Lambda$; that is, $\rho^s(\omega_n, \eta) \to 0$ as $n \to \infty$. If Υ is τ_ρ-continuous, then

$$\rho(\Upsilon\eta, \Upsilon\omega_n) = \rho(\Upsilon\eta, \omega_{n+1}) \to 0 \text{ as } n \to \infty.$$

Therefore we get,

$$\rho(\Upsilon\eta, \eta) \leq \rho(\Upsilon\eta, \omega_{n+1}) + \rho(\omega_{n+1}, \eta) \to 0 \text{ as } n \to \infty.$$

If Υ is $\tau_{\rho^{-1}}$-continuous, then

$$\rho(\Upsilon\omega_n, \Upsilon\eta) = \rho(\omega_{n+1}, \Upsilon\eta) \to 0 \text{ as } n \to \infty.$$

Therefore we have,

$$\rho(\eta, \Upsilon\eta) \leq \rho(\eta, \omega_{n+1}) + \rho(\Upsilon\omega_{n+1}, \Upsilon\eta) \to 0 \text{ as } n \to \infty.$$

Since Υ is T_1-q.-m., we obtain $\Upsilon\eta = \eta$. □

Based on Theorem 2.1, we can derive the following corollaries.

Corollary 2.1. *Let (Λ, ρ) be a Hausdorff right K-complete T_1-q.-m. and $\Upsilon : \Lambda \to \Lambda$ be given a mapping that satisfies*

$$\theta(\rho(\Upsilon\omega, \Upsilon\gamma)) \leq \left[\theta(t_1\rho(\omega, \gamma) + t_2\rho(\Upsilon\omega, \omega) + t_3\rho(\Upsilon\gamma, \gamma), t_4\left[\rho(\Upsilon\omega, \gamma) + \rho(\Upsilon\gamma, \omega)\right])\right]^{\delta}, \tag{2.9}$$

for each $\omega, \gamma \in \Lambda$, where $0 < \delta < 1$, $t_1, t_2, t_3, t_4 \geq 0$, and $t_1 + t_2 + t_3 + 2t_4 < 1$. Presume that Υ is τ_ρ-continuous and α-admissible . If there exists $\omega_0 \in \Lambda$ such that $\alpha(\Upsilon\omega_0, \omega_0) \geq 1$, then Υ has a fixed point in Λ.

Proof. for each $\omega, \gamma \in \Lambda$, we have

$$\begin{aligned} & t_1\rho(\omega, \gamma) + t_2\rho(\Upsilon\omega, \omega) + t_3\rho(\Upsilon\gamma, \gamma), t_4\left[\rho(\Upsilon\omega, \gamma) + \rho(\Upsilon\gamma, \omega)\right] \\ \leq\ & (t_1 + t_2 + t_3 + 2t_4)\max\left\{\rho(\omega, \gamma), \rho(\Upsilon\omega, \omega), \rho(\Upsilon\gamma, \gamma), \frac{1}{2}\left[\rho(\Upsilon\omega, \gamma) + \rho(\Upsilon\gamma, \omega)\right]\right\} \\ \leq\ & M(\omega, \gamma). \end{aligned}$$

Then using (θ_1) we see that (2.3) is a consequence of (2.9). Therefore, the proof is concluded. □

Corollary 2.2. *Let (Λ, ρ) be a Hausdorff right K-complete T_1-q.-m. and $\Upsilon : \Lambda \to \Lambda$ be given a mapping that satisfies*

$$\rho(\Upsilon\omega, \Upsilon\gamma) \leq t_1\rho(\omega, \gamma) + t_2\rho(\Upsilon\omega, \omega) + t_3\rho(\Upsilon\gamma, \gamma),$$

for each $\omega, \gamma \in \Lambda$, where $t_1 + t_2 + t_3 \geq 0$ and $t_1 + t_2 + t_3 < 1$. Presume that Υ is τ_ρ-continuous or α-admissible. If there exists $\omega_0 \in \Lambda$ such that $\alpha(\Upsilon\omega_0, \omega_0) \geq 1$, then Υ has a fixed point in Λ.

Proof. If $\theta(\varkappa) = e^{\sqrt{\varkappa}}$ and $\delta = \sqrt{t_1 + t_2 + t_3}$, since $\rho(\Upsilon\omega, \Upsilon\gamma) \leq (t_1 + t_2 + t_3)M(\omega, \gamma)$, using Theorem 2.1, then the proof is concluded. □

Corollary 2.3. *Let (Λ, ρ) be a Hausdorff right K-complete T_1-q.-m. and $\Upsilon : \Lambda \to \Lambda$ be given a mapping that satisfies*

$$\rho(\Upsilon\omega, \Upsilon\gamma) \leq L\max\{\rho(\Upsilon\omega, \omega), \rho(\Upsilon\gamma, \gamma)\}$$

for each $\omega, \gamma \in \Lambda$, where $L \in [0, 1)$. Presume that Υ is τ_ρ-continuous or α-admissible. If there exists $\omega_0 \in \Lambda$ such that $\alpha(\Upsilon\omega_0, \omega_0) \geq 1$, then Υ has a fixed point in Λ.

Proof. If $\theta(\varkappa) = e^{\sqrt{\varkappa}}$ and $\delta = \sqrt{L}$, since $\rho(\Upsilon\omega, \Upsilon\gamma) \leq \lambda M(\omega, \gamma)$, using Theorem 2.1, then the proof is concluded. □

Remark 2.1. By considering the notion of left completeness in the sense of K, M and $Smyth$, we can extend similar fixed point results to the setting of q.- m. spaces.

Article Information

Acknowledgements: The authors are grateful to the referees for their careful reading of this manuscript and several valuable suggestions which improved the quality of the article.

Author's contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflict of Interest Disclosure: No potential conflict of interest was declared by the author.

Supporting/Supporting Organizations: No grants were received from any public, private or non-profit organizations for this research.

Ethical Approval and Participant Consent: It is declared that during the preparation process of this study, scientific and ethical principles were followed and all the studies benefited from are stated in the bibliography.

Plagiarism Statement: This article was scanned by the plagiarism program. No plagiarism detected.

Availability of data and materials: Not applicable.

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