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DETECTING UNKNOWN CHANGE POINTS FOR HETEROSKEDASTIC DATA

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ABSTRACT

In the realm of applied economics, models are created and predictions are derived based on available data. However, a crucial concern arises when structural changes are present within the dataset. Ignoring these changes during model development can lead to inaccurate estimates and predictions. Therefore, it is vital to identify these change points before building the model. In some cases, researchers may anticipate the presence of structural changes at specific points in time, such as during economic crises or significant events like the COVID-19 pandemic. When the change point is known, the widely accepted approach is to use the Chow test. However, if the change point is unknown, the detection of such points is typically achieved using tests like the Sup F test or CUSUM test. However, these tests assume homoskedasticity. In this paper, this assumption is relaxed and the presence of heteroskedasticity is considered as well. For this purpose, we employed the newly developed test of Sup MZ. In our application, we used weekly and daily returns data from Borsa İstanbul for the period 2003 – 2023. Our model consisted of a mean and a noise term, with occasional jumps in the level of mean or variance at unknown times. The main objective is to detect these jumps and adjust the model accordingly. We proposed a trading rule that utilized the forecasts from our procedure and compared its performance to the buy-and-hold strategy.

Keywords: Structural Change, Unknown Change Points, Sup MZ Test, Borsa İstanbul.
JEL Classification: C18, C58, G10.

HETEROSKEDASTİK VERİLERDE BİLİNMEYEN DEĞİŞİM NOKTALARININ TESPİT EDİLMESİ

ÖZ

Uygulamalı ekonomi alanında, modeller mevcut verilere dayalı olarak oluşturulur ve tahminler elde edilir. Ancak, veri içinde yapısal değişiklikler varsa önemli bir sorun ortaya çıkabilir. Modeli oluşturma sürecinde bu değişikliklerin göz ardı edilmesi, yanlış tahminlere ve kestirimlere yol açabilir. Bu nedenle, modele başlamadan önce bu değişim noktalarının tespit edilmesi önem arzetmektedir. Ekonomik krizler veya COVID-19 pandemisi gibi bazı durumlarda araştırmacılar yapısal değişikliklerin olabileceğini tahmin edebilirler. Değişim noktasının bilindiği durumlarda, geniş kabul gören yaklaşım Chow testini kullanmaktır. Ancak, değişim noktası bilinmiyorsa, genellikle Sup F testi veya CUSUM testi gibi testler kullanılarak değişim noktaları tespit edilebilir. Ne var ki, bu testler homoskedastisite varsayımı altında çalışmaktadırlar. Bu makalede verilerde heteroskedastisitenin olduğu durumu dikkate alınmaktadır. Bu amaçla, yeni geliştirilen Sup MZ testini kullanılmaktadır. Uygulamamızda, 2003-2023 dönemi için Borsa İstanbul haftalık ve günlük getiri verilerini kullanılmaktadır. Modelimiz ortalama ve gürültü teriminden oluşmaktadır ve bilinmeyen zamanlarda ortalamanın veya varyansın seviyesinde sıçramalar meydana gelmektedir. Hedef, bu sıçramaların yerlerini tespit etmek ve modeli uygun şekilde güncelleyerek bir al-sat kuralı önermektir. Ardından önerilen kuralın performansı al-tut stratejisiyle karşılaştırılmaktadır.

Anahtar Kelimeler: Yapısal Değişim, Bilinmeyen Değişim Noktaları, Sup MZ Testi, Borsa İstanbul.
JEL Sınıflandırması: C18, C58, G10.

INTRODUCTION

During the course of the sample, the data's structure may undergo changes. In the field of economics, these changes can happen due to various factors, such as shifts in economic policy, alterations in the economy's structure, or advancements in specific industries. Neglecting to account for these changes can result in misleading

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conclusions and inaccurate predictions. Thus, accurately identifying the points of change is crucial when conducting an econometric analysis

The literature consists of several theoretical papers that offer various techniques for detecting changes in data. These changes may be predictable in certain cases, such as economic crises or major events like the COVID-19 pandemic. However, there are instances where these change points occur unexpectedly at unknown times. The detection in such cases can be made by using unknown change point tests like Sup F or CUSUM. Many applied papers utilize these methods to address diverse economic challenges comprehensively. However, a notable limitation in this extensive body of work is the assumption of homoskedasticity, which implies that regression coefficients may change, but the variances remain constant in the data used. This assumption is puzzling because structural changes often involve alterations in variances, and in reality, heteroskedasticity is prevalent in many cases, particularly when dealing with financial data.

It is widely recognized that particularly stock exchange data often exhibits the characteristic of heteroskedasticity. Traders in the stock market must work with this data while formulating their trading rules. However, ignoring structural change points during this process can result in misleading predictions and potential financial losses for the traders. This paper introduces a novel technique Sup MZ test to identify unknown change points in the presence of heteroskedasticity. By doing so, it contributes to the econometric literature by acquainting econometricians and applied economists with this valuable technique.

Moreover, this paper applies the Sup MZ test to analyze the weekly and daily return data of Borsa Istanbul. The authors note that there is only one existing application of this technique, which is specifically for GDP data (Ahmed, Haider & Zaman, 2017, p. 6). The paper compares the performance of the Sup MZ test with the conventional Sup F test, which assumes homoskedasticity. Consequently, the paper makes a significant contribution to the econometric literature by demonstrating the practical application of considering heteroskedasticity, which leads to improved results.

This study employes the daily and Wednesday closing return values of the BIST 100 composite index denominated in US dollars, sourced from the Borsa İstanbul. The data is analyzed for the period spanning from April 30, 2003, to May 3, 2023. The research identifies estimated change points and their signaling times for a mean plus noise model. Additionally, the study proposes a buy and sell trading rule.

In the paper, we assess the effectiveness of the proposed trading strategy by contrasting it with the buy-and-hold strategy. The findings indicate that the proposed trading rule surpasses the buy-and-hold strategy solely when weekly data is considered, with statistical significance at the 10% level. This result arises from the fact that, in situations where the investor exits the stock market due to an anticipated negative return, no alternative investment is pursued, resulting in a return of zero percent.

The paper is structured as follows. Section 2 is the literature review. Section 3 provides a description of the model and the data used in the study. Section 4 outlines the method and test statistics employed in the analysis. Section 5 presents the results of the study, while Section 6 provides a conclusion.

LITERATURE REVIEW

Let's begin with the three theoretical papers that hold significance for this paper. Andrews (1993) examines tests for parameter instability and structural change with an unknown change point. The findings are applicable to a broad range of parametric models that are amenable to estimation through the generalized method of moments procedures. The paper explores various tests, including Wald, Lagrange multiplier, and likelihood ratio-like tests, all of which implicitly rely on an estimated change point. This change point could be entirely unknown or known to lie within a restricted interval. The paper establishes asymptotic null distributions for these tests and provides tables of critical values based on these asymptotic null distributions. As tests of parameter instability, the discussed tests exhibit nontrivial asymptotic local power against all alternative hypotheses. However, a crucial point to note is that all of the tests studied in the paper assume homoskedasticity.

Maasouimi et al. (2010) introduced a test named MZ that assesses changes in both regression coefficients and variance simultaneously. Building on this work, Ahmed et al. (2017) developed a similar approach based on Andrews (1993) and named it Sup MZ. The primary contribution of their paper to the literature, as opposed to the widely used Sup F test of Andrews (1993), is the consideration of heteroskedasticity as an underlying assumption. Through Monte Carlo simulations, they demonstrated that the Sup MZ test incurs only a minimal cost in cases of homoskedasticity while exhibiting significantly better performance in the presence of heteroskedasticity. Furthermore, in a real-world dataset of GDP, they observed that the Sup F test failed to detect structural changes and yielded misleading results, whereas the Sup MZ test performed well. As a result, they concluded that the Sup MZ test outperforms the current methodology for detecting structural changes.

Let's proceed by exploring relevant applied papers that are significant for this study. Initially, our attention will be on research endeavors aimed at modeling volatility. Following that, we will delve into papers that formulate portfolio strategies and asset pricing models, taking into account structural changes in the data-generating process, respectively.

The first paper that should be mentioned is Başçı et al. (2000) since the setting in that paper is almost the same as this paper. They proposed a procedure for updating a model based on the detection of structural changes at unknown change points using the SupF test of Andrews (1993). The model used in the study consisted of a mean plus noise, with occasional jumps in the mean level at unknown times. The aim was to identify these jumps and update the model accordingly. The authors applied the procedure to Borsa İstanbul weekly data and found that a trading rule utilizing the forecasts from the suggested procedure performed better than a buy-and-hold strategy.

Eizaguirre et al. (2002) conducted a comprehensive investigation into the factors influencing changes in stock market volatility. They employed diverse methodologies, including the use of Andrews' (1993) Sup LR test, to identify change points and examine whether the volatility of the Spanish stock market experienced significant shifts between 1941 and 2001. Their analysis was specifically centered on volatility and did not encompass both mean and volatility, which is the main focus of our paper.

Two research papers investigating fluctuations in stock market return volatility are Abdennadher and Hallara (2018) and Sethapramote and Prukumpai (2012). Both studies employed the Bai and Perron technique to analyze multiple structural changes in volatility. The former paper focused on emerging markets, while the latter centered on the Thai market. Both studies found compelling evidence of significant structural changes in volatility, highlighting the importance of detecting such changes during the model-building process. Furthermore, Abdennadher and Hallara (2018) demonstrated an improvement in their modeling by incorporating structural change. This underscored the significance of considering structural changes when developing models. Notably, while both papers concentrated solely on volatility changes, our study sets itself apart by investigating both mean and volatility fluctuations in stock markets.

Garcia & Ghysels (1998) emphasized the significance of testing for structural changes in emerging markets. Their study revealed that, while standard chi-square tests did not lead to the rejection of the asset pricing factor model for many countries, the application of the Sup LM test proposed by Andrews (1993) for structural change allowed them to reject the model. This paper serves as an excellent illustration of how the inclusion of structural change considerations in the analysis can potentially alter the results obtained.

Despite the differences in the techniques employed to identify structural changes, Mills' 1998 study holds significance for our research. This is because Mills' study compares various trading rules with the buy-and-hold strategy, analyzing daily data on the London Stock Exchange FT30 index from 1935 to 1994. The findings of the study revealed that the buy-and-hold strategy clearly outperformed the other trading rules.

Now, let's shift our focus to the concept of modern portfolio theory. Markowitz's groundbreaking paper (1952) introduced the notion that returns are desirable, while risk, measured through the variance of returns, is considered undesirable. More recently, Ureche-Rangau and Speeg (2011) highlighted that financial literature often views volatility as a reliable proxy for risk, making it a crucial parameter in numerous financial techniques and strategies. In light of these perspectives, the beauty of the Sup MZ test lies in its ability to capture both returns and the variance of returns simultaneously.

Our model encompasses a mean and a noise term, with occasional jumps in the level of mean or variance at unknown times, making it essential to investigate studies utilizing this setting. Early investigations in this special case, focused on detecting unknown change points, were conducted by Hawkins (1977), Worsley (1979), James et al. (1987), and Chu (1990). These studies primarily sought to obtain the asymptotic distributions of the Likelihood Ratio (LR) test statistic, with the Sup F test, introduced in Andrews (1993), being equivalent to the LR test. Chernoff and Zacks (1964) developed a Bayesian test for this problem by imposing a normal prior on the mean, while Hinkley (1970) examined the issue of inference about the location of the change point.

In more recent literature, numerous studies have referenced earlier works on this topic, including James et al. (1992), Bai (1993), Andrews et al. (1996), Kim and Ryu (2006), Li (2006), Hinich et al. (2010), Bautahar (2012), Lee and Wei (2017), Bautahar (2018), Wang et al. (2020), and Jewel et al. (2022). James et al. (1992) extended their earlier one-dimensional work to the multivariate case, Bai (1993) applied a simple least squares method, Bautahar (2012) and Wang et al. (2020) used the CUSUM test, Bautahar (2018) employed a Lagrange multiplier-type test while assuming heteroscedasticity similar to this paper. Lee and Wei (2017) applied the LR test, and finally, Jewel et al. (2022) adopted a newer technique known as LASSO (Least Absolute Shrinkage and Selection Operator).

THE MODEL AND THE DATA

For a homoscedastic regression model with a constant and a noise term, the following equation can be used:

$$R_t = \beta_0 + \varepsilon_t, t = 1, \cdots, T$$

Here, R_t represents the return at time t, β_0 is the mean and $\varepsilon_t \sim iidN(0, \sigma^2), t = 1, \dots, T$ represents the error term assumed to be identically and independently distributed normal with mean 0 and variance σ^2. However, if we relax the assumption about the error term and allow ε_t to follow a normal distribution with mean 0 and varying variances, i.e., $\varepsilon_t \sim iidN(0, \sigma_t^2), t = 1, \cdots, T$, then the model becomes heteroskedastic.

In this paper, the weekly and daily returns of the BIST 100 composite index are modeled for the period April 30, 2003, to May 3, 2023 using an independently and normally distributed error term, which accounts for the possibility of changes in both mean and variance. The dataset encompasses a total of 1012 and 5030 observations for weekly and daily data respectively, all of which are denominated in dollars to mitigate the impact of fluctuations in exchange rates. In terms of the weekly data examined in this research, it specifically involves the closing values of the BIST 100 composite index on Wednesdays. This choice of day aims to circumvent any potential biases stemming from the day of the week effect (Kasman & Kirkulak, 2007).

Figure 1 depicts the natural logarithm of the level, which reveals several time points where the mean rate appears to have shifted. However, it is possible that these changes may be illusions. Therefore, a rigorous statistical testing procedure should be employed to ascertain their validity.

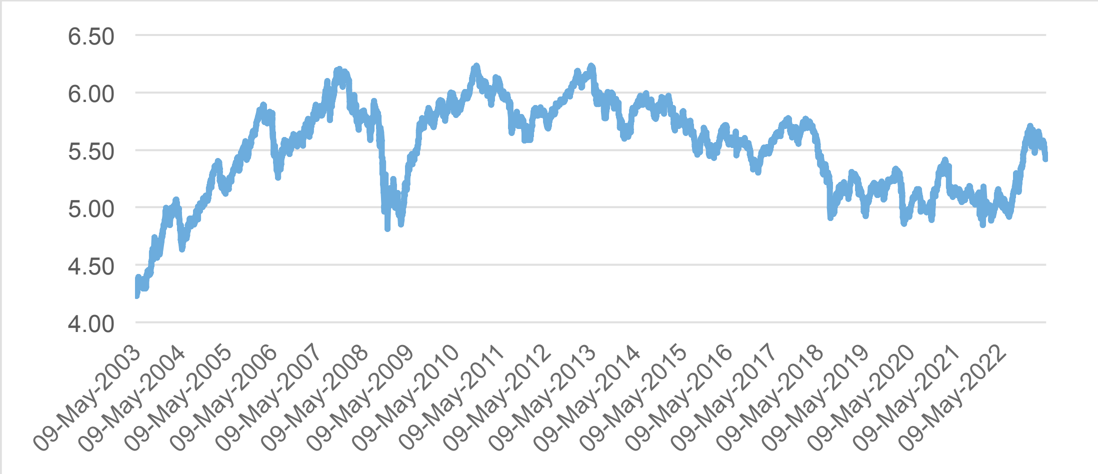

Figure 1: Natural logarithm of BIST 100 ($)
Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.

In Figure 2, the weekly continuously compounded rates of return, which are computed as the first differences of the natural logarithm of the index, are displayed. Unlike Figure 1, detecting mean changes in Figure 2 solely by visual inspection is not straightforward. However, it is possible to visually detect changes in volatility. Nonetheless, it is important to note once more that some of these changes may be illusions, and hence, a thorough statistical testing procedure should be employed to verify their authenticity.

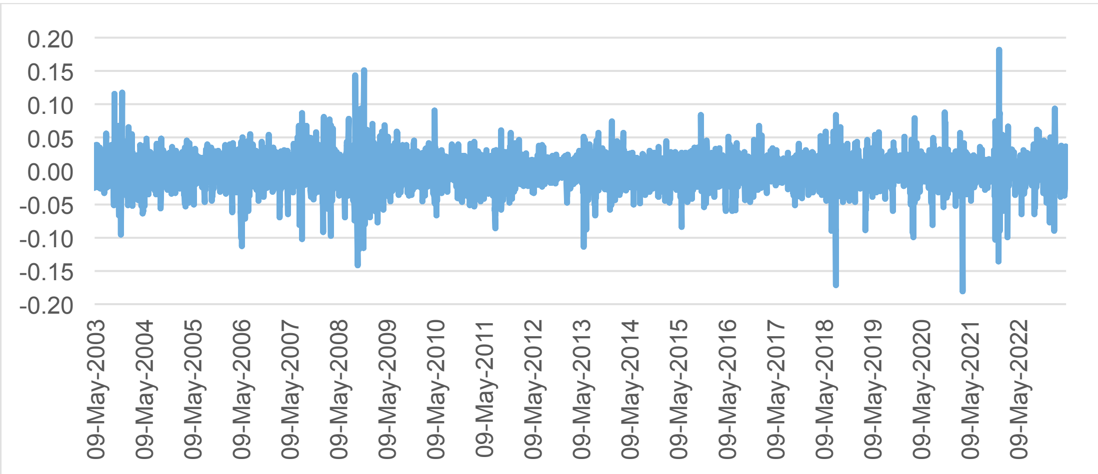

Figure 2: Weekly Continuously Compounded Rates of Return
Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.

THE METHOD AND TEST STATISTICS

The primary objective of this paper is to extend the existing model update procedure to include the detection of changes in variance, alongside changes in mean. However, the Sup F test, which assumes homoskedasticity, is not appropriate for this purpose. To overcome this limitation, Ahmed et al. (2017) introduced a novel test, called Sup MZ, which can detect jumps in both the regression coefficients and variance at unknown change points. This paper builds upon the work of Maasomi et al. (2010), where the MZ test was initially introduced. The approach for producing the Sup MZ test is analogous to the Sup F test.

In the present study, we employ the Sup MZ test rather than the Sup F test. Although the Sup MZ test is relatively new and has only been used once by Ahmed et al. (2017) to analyze GDP data, our paper can contribute to the literature by introducing this test to researchers. This will help to familiarize researchers with this novel approach.

Let $(\beta_i, \varepsilon_i^2)$ $i = 1, 2$ represent parameters in each subgroup. The regression model for each subgroup is

$$R_1 = \beta_1 + \varepsilon_1, \varepsilon_1 is\ i.i.d\ N(0, \sigma_1^2 I_{T_1})$$
$$R_2 = \beta_2 + \varepsilon_2, \varepsilon_2 is\ i.i.d\ N(0, \sigma_2^2 I_{T_2})$$

β_1 is the mean up to $T_1$and β_2 is the mean from $T_1 + 1$ to T_2, before and after the structural change. The null hypothesis of structural stability versus the alternative of simultaneous change in mean and variance is:

$$H_0: \beta_1 = \beta_2, \sigma_1^2 = \sigma_2^2\ versus\ \beta_1 \neq \beta_2\ or\ \sigma_1^2 \neq \sigma_2^2$$

The MZ test defined in Maasouimi et.al, (2010) is

$$MZ = (T - k) \log \hat{\sigma}_0^2 - \{(T_1 - k) \log \hat{\sigma}_1^2 + (T_2 - k) \log \hat{\sigma}_2^2\}$$

To adapt the test for use when the change point is unknown, Ahmed et.al (2017) propose the “sup” version of MZ and label it as sup MZ. When the change point is unknown, they calculate MZ for all potential change points j, k < a <= j <= b < T – k and take the maximum value of these MZj values. This maximum value is defined to be “supMZ”, calculated as:

$$SupMZ = \max_{a \leq j \leq b} MZ_j\ \ k < a \leq j \leq b < T - k$$

This method differs from the widely used sup F test in that it permits both regression coefficients and variance to change simultaneously at the change point. In contrast, the sup F test assumes that the variance of the regression error term remains constant before and after the structural change, which makes it optimal only if this assumption holds true. The null hypothesis is rejected if it is greater than the critical values for each sample size calculated by the bootstrap method.

The detection method involves the addition of new observations to the sample, in case the null of no change is not rejected. If there is a rejection, then the change point is estimated and the new starting point for the sample is set as the estimated change point. Then the regression model parameters to be used are estimated on this new and smaller sample. A Window Length (WL) of 52 is chosen for weekly data and a WL of 250 is chosen for daily data. The test is conducted if and only if there are at least 52 (250) sample observations from the most recent estimated change point to the current week (day). The algorithm can be described as follows:[1]

```
10   Let START=1;
20   Let T=1;
30   If T-START>=51(for weekly data), >=249 (for daily data), test for the null of no
     structural break on the most recent 52 (for weekly data), 250 (for daily data)
     return data;
40   If rejected set START=Estimated change-point;
50   MEAN=Average of returns from START to T;
60   Obtain the following week's return;
70   Let T=T+1;
80   Go to 30;
```

RESULTS AND DISCUSSION

The critical values obtained by the bootstrap method were utilized to apply the algorithm described in Section 3 to the stock index return data. Consequently, the algorithm generated a record of identified change points and their corresponding signalling time.

Weekly Wednesday Closing Prices

Table 1 lists the results for weekly Wednesday closing prices for significance levels of 5 % and 10 %. To illustrate, Table 1's left panel shows that the null hypothesis was first rejected in week 155, with an estimated change point in week 60 for significance level of 5%. In this scenario, a new mean would be used in the model for the period 60 to 153, the second estimated change point. The third column is the bootstrap critical values for the test.

Table 1: The Estimated Change Points and Their Signaling Times for Weekly Wednesday Closing Prices ($)

Estimated Change Point (5%)	**Signal Time (5%)**	**Critical Value (5%)**	**Estimated Change Point (10%)**	**Signal Time (10%)**	**Critical Value (10%)**
60	155	3.46310	60	155	0.83468
153	276	16.7055	153	205	12.3799
272	324	20.2173	158	274	7.67239
280	751	18.0180	272	324	15.0634
418	752	26.4590	280	492	11.1735
667	759	7.32797	441	513	23.2262
			506	560	5.49503
			542	722	-0.12193
			667	723	5.68655

[1] The analysis in this study was conducted using the software MATLAB. The codes are available upon request.

			692	759	-1.75879
			756	881	-2.40002
			851	999	11.6820

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023. Authors' Calculations.

Figure 3 and Figure 4 display the estimated change points for the natural logarithm of BIST 100 weekly Wednesday dollar closing prices at 5% and 10 % significance levels, respectively.

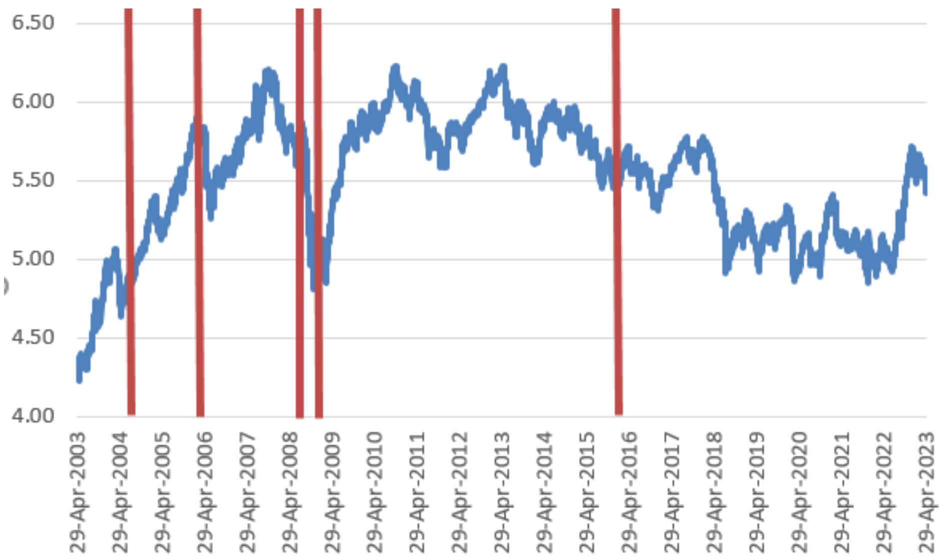

Figure 2: Natural logarithm of BIST 100 with estimated change points for weekly Wednesday closing prices located (5 % Significance Level) ($)

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.

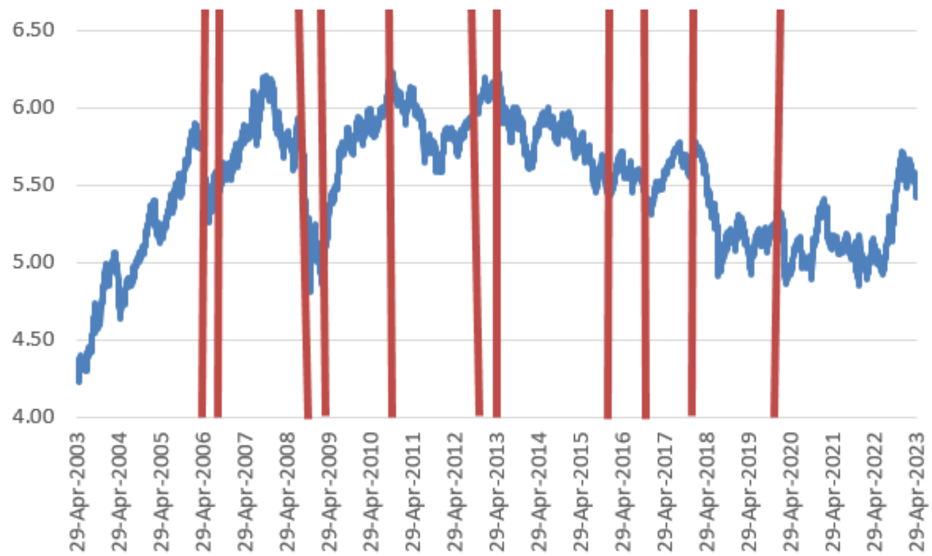

Figure 4: Natural logarithm of BIST 100 with estimated change points for weekly Wednesday closing prices located (10 % significance level) ($)

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.

We evaluated the performance of our algorithm by implementing a trading strategy on the sample dataset. The strategy involved entering the market and obtaining the index return for the following weeks/days if the algorithm generated a positive expected weekly/daily return. In contrast, if the expected weekly/daily return was negative, we exited the market and remained in US dollars during the following weeks/days, resulting in a return of 0%. We assume a 0,02% commission for each transaction.

Table 2 and Table 3 display the outcomes of the analysis conducted on the buy and sell strategy, utilizing weekly Wednesday Closing Prices. This analysis was performed at significance levels of 5% and 10%, respectively. To elaborate on how the results are structured in these tables, consider the first row of Table 2 as an example. In this instance, for the time frame spanning from period 1 to period 60, a positive expected return was observed, indicating engagement in the stock market. The achieved return for this period stood at 0.77%, and after accounting for transaction costs, the net return equated to 0.73%.

The buy and hold strategy yields an annual return of 5.95%. At a significance level of 5%, the trading strategy demonstrates an annual return of 3.59%. At a significance level of 10%, the return increases significantly to 16.30%. Consequently, the buy and hold strategy outperforms our trading strategy at the 5% significance level, but this superiority diminishes when considering the 10% significance level. This outcome is attributed to the trading strategy's tendency, at the 5% significance level, to recommend staying out of the market for the majority of instances, which leads to a resulting return of 0% when not invested.

Table 2: In and out Periods of the Buy and Sell Strategy and the Returns Obtained During the Periods for Weekly Wednesday Closing Prices ($) (5% Significance Level)

Period	Expected Return	In or Out	Return	Net Return
1 – 60	0.0097	In	0.7685	0.733128
61 - 153	0.0103	In	1.6435	1.590634
154 - 272	-0.0006	Out	0	0
273 - 280	-0.1141	Out	0	0
281 - 418	-0.0007	Out	0	0
419 - 667	-0.0004	Out	0	0
668 - 1012	-0.0005	Out	0	0

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023. Authors' Calculations.

Table 3: In and out Periods of the Buy and Sell Strategy and the Expected Returns of the Periods for Weekly Wednesday Closing Prices ($) (10% Significance Level)

Period	Expected Return	In or Out	Return	Net Return
1-153	0.0100	In	3.587792	3.4960
154-158	-0.0667	Out	0	0
159-273	0.0004	In	0.23495	0.2103
274-280	-0.0996	Out	0	0
281-441	0.0054	In	0.956971	0.9178
442-506	0.0067	In	0.39742	0.39742

507-542	-0.0127	Out	0	0
543-667	-0.0005	Out	0	0
668-692	-0.0133	Out	0	0
693-756	0.0043	In	0.217698	0.1933
757-851	-0.0051	Out	0	0
852-1012	0.0023	In	0.282814	0.2572

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023. Authors' Calculations.

Figures 5 and 6 illustrates the in and out periods for natural logarithm and continuously compounded rates returns of BIST 100 for weekly Wednesday closing prices, respectively. In both figures, the colour green corresponds to the in periods, while the colour orange corresponds to the out periods. The significance level is 5% for both of the figures.

The middle part of the figures, approximately from May 2010 to May 2017, can be considered as an interesting period. If one considers just Figure 5, that's just the logarithm of BIST 100, the decision to be out of the market is reasonable but if one just considers Figure 6, that's continuously compounded rates of returns of BIST 100, the decision of being out of market can be questionable since the period can be considered as stable period. Since the Sup MZ test considers both mean return and volatility, trading rule recommends that we exit the market.

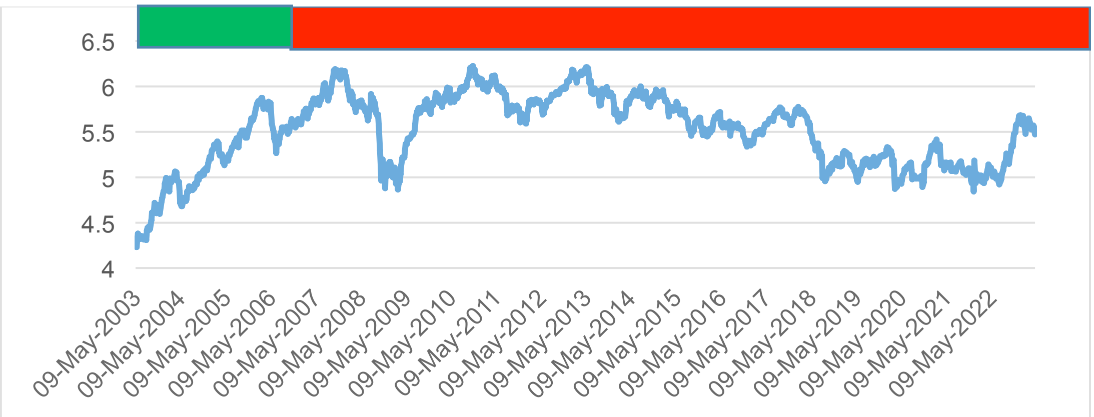

Figure 5: In and out Periods of the Trading Rule (Natural Logarithm of BIST 100) (5% Significance Level) ($)

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.
Green: Inside the stock market. Orange: Outside the stock market.

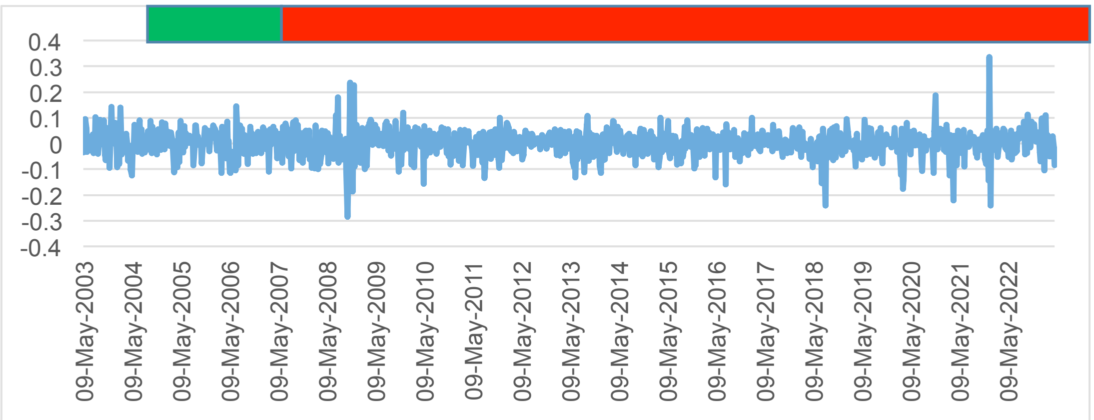

Figure 6: In and out Periods of the Trading Rule (Weekly Continuously Compounded Rates of Return, BIST 100) (5 % Significance Level) ($)

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.
Green: Inside the stock market. Orange: Outside the stock market.

Figures 7 and 8 illustrates the in and out periods for natural logarithm and continuously compounded rates returns of BIST 100 for weekly Wednesday closing prices, respectively. In both figures, the colour green corresponds to the in periods, while the colour orange corresponds to the out periods. This time, the significance level is 10% for both of the figures.

The last part of the figures, approximately from May 2019 to May 2022, can be considered as an interesting period. If one considers just Figure 7, that's continuously compounded rates of returns of BIST 100, the decision to be in the market can be questionable since the period is an unstable one although there is an upward trend. Since the Sup MZ test considers both mean return and volatility, trading rule recommends that we should stay in the market.

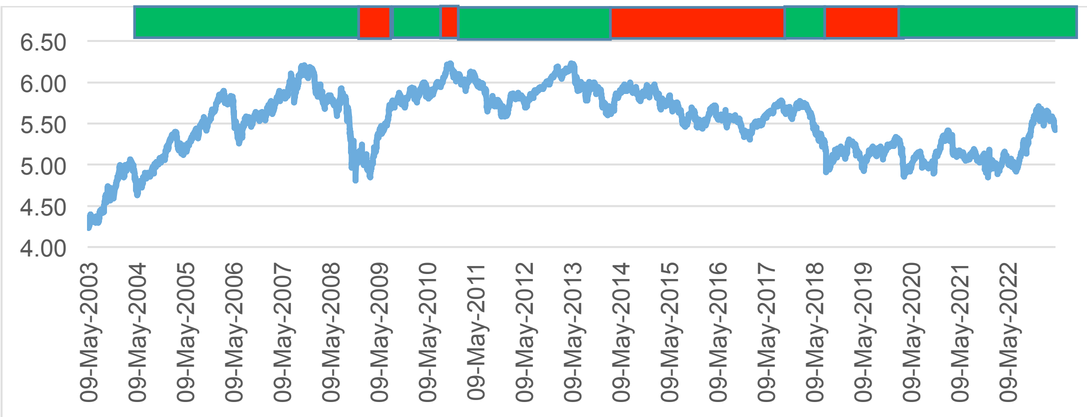

Figure 7: In and out Periods of the Trading Rule (Natural Logarithm of BIST 100) (10% Significance Level) ($)

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.
Green: Inside the stock market. Orange: Outside the stock market.

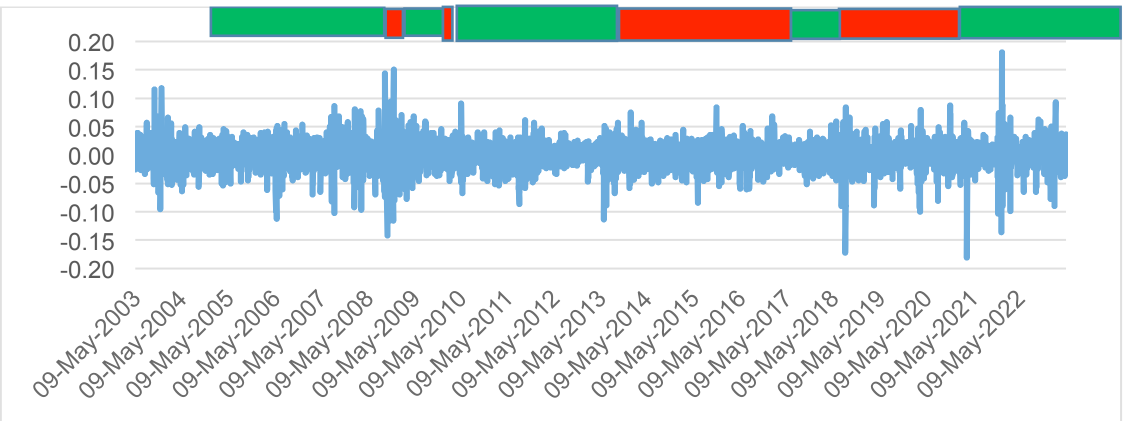

Figure 8: In and out Periods of the Trading Rule (Weekly Continuously Compounded Rates of Return, BIST 100) (5% Significance Level) ($)

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023.
Green: Inside the stock market. Orange: Outside the stock market.

Daily Closing Prices

Table 4 displays the outcomes pertaining to daily closing prices at significance levels of 5% and 10%. The format of this Table mirrors that of Table 1. As can be seen, there are numerous estimated change points for this case. Consequently, we have opted not to generate figures resembling Figures 3 through 8. Instead, we will solely compile a table presenting the periods of entry and exit, along with the expected returns associated with each of these periods.

Table 4: The Estimated Change Points and Their Signaling Times for Daily Closing Prices ($)

Estimated Change Point (5%)	Signal Time (5%)	Critical Value (5%)	Estimated Change Point (10%)	Signal Time (10%)	Critical Value (10%)
113	364	16.59266	113	364	14.04471
146	397	13.2698	146	397	10.59357
279	530	1.803255	279	530	-0.21283
466	717	0.015022	466	717	-1.37985
499	762	2.801036	499	761	1.511247
760	1011	23.93069	713	964	3.505311
809	1076	15.29275	809	1066	11.74772
1061	1312	10.94567	1061	1312	8.266763
1238	1489	8.094667	1238	1489	5.481924
1353	1604	15.9906	1353	1604	12.77886
1401	1652	15.1098	1401	1652	12.26269
1533	1882	4.058882	1533	1784	2.561381
1773	2026	12.83444	1762	2013	7.653622
1884	2135	2.292327	1777	2028	12.99698
2079	2330	8.404988	1884	2135	0.444876

2202	2453	10.19218	2079	2330	5.895106
2450	2701	3.427521	2202	2453	7.812342
2539	2793	18.72557	2450	2701	1.254757
2742	3578	6.37755	2539	2790	15.14274
3458	3709	15.89112	2552	2810	4.593116
3631	3882	1.8676	2742	3568	1.532625
3824	4077	52.3632	3458	3709	13.38219
3848	4182	45.70648	3631	3882	0.128633
4142	4395	11.77131	3824	4075	46.79911
4226	4477	29.71957	3848	4179	42.51055
4249	4682	22.23543	4142	4393	10.25721
4663	4914	83.23277	4226	4477	25.2629
4731	4982	50.05921	4249	4637	18.58397
			4498	4749	86.86229
			4663	4914	52.65874
			4731	4982	46.69486

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023. Authors' Calculations.

Table 5: In and out Periods of the Buy and Sell Strategy and the Expected Returns of the Periods for Daily Closing Prices ($) (5% Significance Level)

Period	Expected Return	In or Out	Return	Net Return
1 -113	0.004	In	0.38190616	0.354268
114 - 146	-0.001	Out	0	-0.02
147 - 279	0.000	Out	0	0
280 - 466	0.003	In	0.89986495	0.861868
467 - 499	-0.003	Out	0	-0.02
500 - 760	0.002	In	0.92497224	0.886473
761 - 809	-0.008	Out	0	-0.02
810 - 1061	0.002	In	0.75641859	0.72129
1062 - 1238	-0.002	Out	0	-0.02
1239 - 1353	0.001	In	-0.04983337	-0.06884
1354 - 1401	-0.018	Out	0	-0.02
1402 - 1533	0.004	In	0.41747245	0.389123
1534 - 1773	0.002	In	0.69015647	0.690156
1774 - 1884	0.003	In	0.45003848	0.450038
1885 - 2079	-0.002	Out	0	-0.02
2080 - 2202	-0.001	Out	0	0
2203 - 2450	0.002	In	0.45078974	0.421774
2451 - 2539	-0.001	Out	0	-0.02
2540 - 2742	-0.002	Out	0	0
2743 - 3458	-0.000	Out	0	0
3459 - 3631	0.002	In	0.2631672	0.237904
3632 - 3824	-0.002	Out	0	-0.02
3825 - 3848	-0.017	Out	0	0

3849 - 4142	0.001	In	0.15	0.123702
4143 - 4226	0.002	In	0.12572213	0.125722
4227 - 4249	-0.018	Out	0	-0.02
4250 - 4663	0.000	Out	0	0
4664 - 4731	-0.002	Out	0	0
4665 - 5029	0.001	In	0.56013742	0.528935

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023. Authors' Calculations.

Table 6: In and out Periods of the Buy and Sell Strategy and the Expected Returns of the Periods for Daily Closing prices ($) (10 % Significance Level)

Period	Expected Return	In or Out	Return	Net Return
1-113	0.004	In	0.381906	0.354268
114-146	-0.001	Out	0	-0.02
147-279	0.000	Out	0.056723	0
280-466	0.003	In	0.899865	0.899865
467-499	-0.014	Out	0	-0.02
500-713	0.003	In	1.120823	1.078407
714-809	-0.005	Out	0	-0.02
810-1061	0.002	In	0.756419	0.72129
1062-1238	-0.002	Out	0	-0.02
1239-1353	0.001	In	-0.04983	-0.06884
1354-1401	-0.018	Out	0	-0.02
1402-1533	0.004	In	0.464572	0.435281
1534-1762	0.002	In	0.810625	0.810625
1763-1777	-0.011	Out	0	-0.02
1778-1884	0.004	In	0.44897	0.419991
1885-2079	-0.002	Out	0	-0.02
2080-2202	-0.001	Out	0	0
2203-2450	0.002	In	0.45079	0.421774
2451-2539	-0.001	Out	0	-0.02
2540-2552	-0.008	Out	0	0
2553-2742	-0.002	Out	0	0
2743-3458	0.000	Out	0	0
3459-3631	0.001	In	0.263167	0.237904
3632-3824	-0.002	Out	0	-0.02
3825-3848	-0.017	Out	0	0
3849-4142	0.001	In	0.146635	0.123702
4143-4226	0.002	In	0.12619	0.12619
4227-4249	-0.018	Out	0	0
4250-4498	0.002	In	0.523576	0.493104
4499-4663	-0.002	Out	0	-0.02
4664-4731	-0.002	Out	0	0
4665 - 5029	0.000	Out	0	0

Source: BIST 100 Database. Period: April 30, 2003 – May 3, 2023. Authors' Calculations.

The buy and hold strategy yields an annual return of 48.57%. At a significance level of 5%, the trading strategy demonstrates an annual return of 20.40%. At a significance level of 10%, the return increases to 25.20%. Consequently, the buy and hold strategy outperforms our trading strategy at both 5% and 10% significance levels. This outcome is attributed to the trading strategy's tendency, to recommend staying out of the market which leads to a resulting return of 0% when not invested.

CONCLUSION

This paper presents a mean plus noise model, which includes occasional jumps in the mean level and variance at unknown times. The research proposes a technique for updating the model by detecting structural changes at unknown change points. The Sup MZ test is utilized for this purpose, which assumes heteroskedasticity. This assumption is not applicable for the widely used Sup F structural change test (Andrews, 1993). The goal is to identify these jumps and adjust the model accordingly. Additionally, the study employs a trading rule that depends on the estimated times of the jumps.

The analysis is made both for weekly and daily data of BIST 100 composite index in US dollars. The data is downloaded from Istanbul Stock Exchange for the period April 30, 2003 to May 3, 2023. For weekly data, in order to avoid the day of the week effect we used Wednesday closing values.

The buy and hold strategy generates an annual return of 5.95% when examining the weekly data. Our trading rule exhibits an annual return of 3.59% at a 5% significance level. However, when the significance level is raised to 10%, the return significantly improves to 16.30%. Consequently, the buy and hold strategy outperforms our trading strategy at the 5% significance level, but this advantage diminishes when we shift our focus to the 10% significance level.

When analysing the daily data, the buy and hold strategy produces an annual return of 48.57%. In contrast, when we apply a 5% significance level to the trading rule, it yields an annual return of 20.40%. At a 10% significance level, the return improves to 25.20%. As a result, it is evident that the buy and hold strategy outperforms our trading strategy at both the 5% and 10% significance levels.

The main factor contributing to these outcomes is that when the investor exits the stock market due to a negative expected return, no alternative investment is pursued, leading to a zero percent return. Therefore, for future research, incorporating an alternative investment strategy that could potentially influence the findings would be advantageous.

An intriguing feature of the proposed approach is that when it does offer suggestions to either stay in or exit the market, the market's direction frequently corresponds with the advice provided. This suggests the potential utility of the method in facilitating profitable trades.

While conducting the analysis, intriguing periods were noted. For instance, there were instances where a boost in returns is detected, yet the trading rule advised staying out of the market during those intervals. This phenomenon can be attributed to the high volatility characterizing those periods, and it's noteworthy that the Sup MZ test takes into account both shifts in the mean and the presence of volatility.

A potential area for future study could be to compare the performance of the Sup F test and the Sup MZ test. This could involve comparing the resulting trading strategies and returns. Such a comparison could help identify the strengths and weaknesses of each test and provide guidance on when each test may be more appropriate to use.

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