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ON 1-ABSORBING FUZZY IDEALS OF COMMUTATIVE SEMIRINGS

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Abstract

In this study, the algebraic structure of 1-absorbing ideals is first examined and applied to fuzzy sets, along with an investigation into the relationships and algebraic properties between them. The contribution to this work's literature involves examining 1-absorbing fuzzy primary ideals. Features of 1-absorbing fuzzy primary ideals are explored, and it is demonstrated, for instance, that I is deemed a 1-absorbing fuzzy primary ideal of P if I is a fuzzy primary ideal of P. Additionally, I is considered a 2-absorbing fuzzy primary ideal of P if I is a 1-absorbing fuzzy primary ideal of P. Furthermore, these theorems are elucidated through specific examples.

Keywords: Fuzzy sets, Fuzzy ideals, 1-absorbing ideals, 1-absorbing fuzzy ideals, 1-absorbing fuzzy primary ideals

1. Introduction

American mathematician H. S. Vandiver established the notion of semirings in 1935, and numerous authors have subsequently examined it. Semirings are a natural generalization of rings with a wide area of applications in the mathematical foundations of computer science [1].

In 1965, Zadeh introduced the theory of a fuzzy set in [2], and many scientists worked on developing this theory. Earth science, life and behavioral sciences, medical science, logic, operations research, decision theory, and especially electrical engineering and computer science have all used fuzzy set theory [3, 4]. In 1968, the article “Fuzzy Topological Spaces” was written by Zadeh’s student Chang in [5]. Chang’s article served as a source of inspiration for Rosenfeld, leading him to believe that these operations had the potential to be utilized in algebraic structures. This led him to write ”Fuzzy Groups,” as documented in [6]. Fuzzy algebra is constructed upon this foundation, and various researchers have extensively examined this concept in a wide range of algebraic systems. The notion of a fuzzy ideal of a ring was introduced by Liu in [7]. Abou-Zaid created the concept of fuzzy subnear-rings and fuzzy ideals in [8].

Atanassov extended Zadeh’s fuzzy set notion to the intuitionistic fuzzy set in [9]. These are primarily intended to reflect that the degree of non-membership of an element in an intuitionistic fuzzy set may not always be equal to 1 minus the degree of membership, but there may be some degree of hesitation [10].

The growing interest in fuzzy logic is evident in academic research as well. While the phrase ‘fuzzy logic’ was present in the titles of scientific publications until 1990, it continued to be used in research paper titles starting from 2003, as noted in [11].

A. Badawi [12] proposed the 2-absorbing ideal of a commutative ring with identity as a generalization of the prime ideal in a commutative ring and investigated its features. Hashempoor et al. showed the view of a 2-absorbing fuzzy ideal of semiring P (2AFIS(P)) in [16]. A fuzzy ideal ϕ of P is called a 2AFIS(P) if $\phi(pqr) \geq \ell$ implies $\phi(pq) \geq \ell$ or $\phi(pr) \geq \ell$ or $\phi(qr) \geq \ell$ for all $p, q, r \in P$ and $\ell \in [0, 1]$. Researchers have conducted investigations into the concept of 2- absorbing, see ([18]-[25]).

Then, Badawi et al. were the first to introduce and study 1-absorbing primary ideals and investigated a few notable features of 1-absorbing primary ideals in [13]. After this article, they studied 1-absorbing prime ideals of commutative rings in [14]. Every prime ideal is a 1-absorbing prime ideal, and every 1-absorbing prime ideal is a 1-absorbing primary ideal. On the other hand, the inverse of this statement might not hold true as stated in [14]. Then, works on 1-absorbing have been studied by researchers, for more information see ([26]-[32]).

The main focus of this article is on proving theorems and propositions for 2AFIS(P) that are well-known in fuzzy theory, as well as 1-absorbing fuzzy ideals of semiring P (1AFIS(P)). The relationship between 2AFIS(P) and 1-absorbing fuzzy ideals of the semiring P (1AFIS(P)) is investigated. Examples are presented to illustrate the distinctions between these two concepts, emphasizing the importance of studying both.

2. Preliminaries

We acknowledge that all semirings are commutative in this section, with 1 = 0. To begin, we will go through the basics of fuzzy sets and 1-absorbing ideal of semiring P (1AIS(P)).

Definition 2.1 [17] A semiring is a non-empty set P on which operations addition and multiplication (denoted by + and ·, respectively) have been defined such that the following conditions are satisfied:

i. $(P,+)$ is a commutative monoid with identity *0*,

ii. $(P,\cdot)$ is a commutative semigroup,

iii. $p(q+r)=pq+pr$ *and* $(p+q)r=pr+qr$ *for all* $p,q,r\in P$,

iv. $0p=p0\ =\ 0$ *for all* $p\in P$.

Definition 2.2 [32] A proper ideal I of a semiring P is said to be a 1AIS(P) if $pqr\in I$ implies $pq\in I$ or $r\in I$ for all $p,q,r\in P$.

Definition 2.3 [32] Let P be a commutative semiring and I be a proper ideal of P . Then I is said to be a 1APIS(P) if whenever $p,q,r\in P$.

We recall some definitions and results.

Definition 2.4 [4] *A* fuzzy subset of a nonempty set P is defined as a function $\phi:P\ \to[0,1]$. For any fuzzy subset ϕ *of* P and $\ell\in[0,1]$, the level subset *of* ϕ is denoted by ϕ_ℓ and defined as $\phi_\ell=\{p\in P:\phi(p)\geq\ \ell\}$.

Definition 2.5 [4] *Let* ϕ *be a fuzzy subset of* P *and* $p\in P$. *Then the radical of* ϕ *is denoted by* $\sqrt{\phi}$ *and defined as* $\sqrt{\phi}=\sup\left\{\phi\left(p^n\right):n\in N\right\}$.

Definition 2.6 [15] Let ϕ be a non empty fuzzy subset of a semiring P (it means that anyone of $\phi(p)$ not equal to zero for some $p\in P$). Then ϕ is called a fuzzy left ideal of P if

i) $\phi(p+q)\geq\min\{\phi(p),\phi(q)\}$

ii) $\phi(pq)\geq\phi(q)$.

for all $p,q\in P$.

Likewise, we can define the fuzzy right ideal of *P*.

Definition 2.7 [15] Since $\xi\subseteq P$ the characteristic function ξ of I is defined by

$$\xi = \begin{cases} 1, & p \in I \\ 0, & p \notin I \end{cases}.$$

Now, we give the notion of morphism of semirings.

Definition 2.8 [17] If *R* and *S* are semirings then a function $\gamma : R \to S$ is a morphism of semirings if and only if :

i) $\gamma(0_R) = 0_S$,

ii) $\gamma(1_R) = 1_S$,

iii) $\gamma(r + r') = \gamma(r) + \gamma(r')$ and $\gamma(rr') = \gamma(r)\gamma(r')$ for all $r, r' \in R$.

Given any two semirings *R* and *S,* let μ be a fuzzy subset of *R* and let $\phi : R \to S$ be any function. We define a fuzzy subset v on *S* by

$$v(y) = \begin{cases} \sup\limits_{x \in \phi^{-1}(y)} \mu(x), & if\ \phi^{-1}(y) \neq \varnothing, y \in S \\ 0, & otherwise \end{cases}$$

and we call *v* the image of μ under ϕ, written $\phi(\mu)$. For any fuzzy subset v on $\phi(R)$, we define a fuzzy subset μ on R by

$$\phi^{-1}(v(x)) = v(\phi(x))$$

for all $x \in R$, and we call μ the preimage of v under ϕ [16].

3. On 1-absorbing fuzzy ideals on commutative semirings

In this section, we will concentrate on 1AFIS(*P*) and unless stated differently, *P* symbolizes a commutative semiring.

Definition 3.1 A fuzzy ideal ϕ of P is called a 1AFIS(*P*) if $\phi(pqr) \geq \ell$ implies $\phi(pq) \geq \ell$ or $\phi(r) \geq \ell$ for all $p, q, r \in P$ and $\ell \in [0, 1]$.

The following are some examples of this definition.

Example 3.2 Let P = {0, p, q} be a set with the operation addition and the multiplication defined as follows:

+	0	p	q
0	0	p	q
p	p	0	q
q	q	q	0

·	0	p	q
0	0	0	0
p	0	0	0
q	0	0	q

Then $(P,+,.)$ forms a commutative semiring Define fuzzy subset ϕ of P by $\phi(0)=1$, $\phi(p)=0.4$, $\phi(q)=1$. Then ϕ is a 1AFIS(P).

Example 3.3 Let $P=\{0,p,q,r\}$ be a set with the operation addition and the multiplication is defined as follows:

+	0	p	q	r
0	0	p	q	r
p	p	p	q	r
q	q	q	q	r
r	r	r	r	r

·	0	p	q	r
0	0	0	0	0
p	0	0	0	0
q	0	0	q	q
r	0	0	q	r

Then (P, +, ·) forms a commutative semiring. Define fuzzy subset ϕ of P by $\phi(0) = 1$, $\phi(p) = 0.3$, $\phi(q) = 1$ and $\phi(r) = 0.9$. Then ϕ is a 1AFIS(P).

Example 3.4 Let $P = Z$. Consider the ideal $I = 7Z$.

$$\phi(p)=\begin{cases}1, & p\in I\\ 0, & p\notin I\end{cases}$$

Since $\phi(7.1.1)\geq 1$, $\phi(1)=0\not\geq 1$ but $\phi(7.1)=1\geq 1$. Thus I is a 1AFIS(P).

Definition 3.5 A fuzzy ideal ϕ of P is called a 1-absorbing fuzzy primary ideal of semirin P (1AFIS(P)) if $\phi(pqr)\geq \ell$ implies $\phi(pq)\geq \ell$ *or* $\phi(r)\geq \ell$ for all $p,q,r\in P$ and $\ell\in[0,1]$.

Theorem 3.6 *Let ϕ be a fuzzy subset of P. Then ϕ is a 1AFIS(P) if and only if ϕ_ℓ is a 1AIS(P).*

Proof. Let ϕ be a 1AFIS(P) and $pqr\in\phi_\ell$ for $p,q,r\in P$ and $\ell\in[0,1]$. Since ϕ is a 1AFIS(P),

$$\begin{aligned} pqr\in\phi_\ell &\Rightarrow \phi(pqr)\geq \ell \\ &\Rightarrow \phi(pq)\geq \ell \text{ or } \phi(r)\geq \ell \\ &\Rightarrow pq\in\phi_\ell \text{ or } r\in\phi_\ell \end{aligned}$$

Hence ϕ_ℓ is a 1AIS(P). Conversely, suppose ϕ be a fuzzy subset of P such that ϕ_ℓ is a 1AIS(P) for all $\ell\in[0,1]$ but ϕ is not a 1AFIS(P). Therefore, $\phi(pqr)\geq \ell$ does not imply ϕ

$(pq) \geq \ell$ or $\phi(r) \geq \ell$ for $p,q,r \in P$ and $\ell \in [0,1]$ it means that $pqr \in \phi_\ell$ does not imply $pq \in \phi_\ell$ or $r \in \phi_\ell$ contradiction to the fact that ϕ_ℓ is a 1AIS(*P*). So ϕ is a 1AFIS(*P*).

Theorem 3.7 Let I be a 1AIS(*P*) and $\alpha \in [0,1]$. Then, a fuzzy subset ϕ of P given as

$$\phi(p) = \begin{Bmatrix} 1, & p \in I \\ \alpha, & p \notin I \end{Bmatrix}$$

Then ϕ is a 1AFIS(*P*).

Proof. Let I be a 1AIS(*P*) and $p,q,r \in P$. Suppose $\phi(pqr) \geq \ell$.

***C*ase 1)** Let $\ell = 1$, Let $pqr \in \phi_\ell \Rightarrow pqr \in I$. Now, if $\phi(pq) < \ell = 1$ or $\phi(r) < \ell = 1$, then $pq, r \notin I$ contradiction to the fact that I is a 1AIS(*P*).

***C*ase 2)** Let $\ell = \alpha$, Here, $\phi(pqr) = \alpha$. If anyone of $\phi(pq)$ or $\phi(r)$ is equal to α, we are done. Otherwise all of $\phi(pq)$ and $\phi(r)$ is equal to 1. In this case, $pq, r \in I$ but $pqr \notin I$, contradiction to the fact that I is a 1AIS(*P*). Therefore, the proof has been concluded.

Now, we will charactarize 1AFIS(*P*) with the radical.

Theorem 3.8 Let ϕ be a 1AFPIS(*P*). Then, $\sqrt{\phi}$ is a 1AFIS(*P*).

Proof. Let ϕ be a 1AFPIS(*P*). Suppose for $p,q,r \in P$ and $\ell \in [0,1]$, $\sqrt{\phi}(pqr) \geq \ell$. Then,

$$\begin{aligned} \sqrt{\phi}(pqr) \geq \ell &\Rightarrow \sup\left\{\phi(pqr)^n : n \in Z\right\} \geq \ell \\ &\Rightarrow \sup\left\{\phi\left(p^n q^n r^n\right) : n \in Z\right\} \geq \ell. \end{aligned}$$

Suppose that $r \notin \sqrt{\phi}$. So, $\phi(r^n) \ngeq \ell$, for $\ell \in [0,1]$ and $n \in Z$. Since ϕ is a 1AFPIS(*P*), $\phi\left(\left(p^n q^n\right)^k\right) \geq \ell$ for some $k \in Z$ and so $\sup\left\{\phi\left(p^m q^m\right) : m \in Z\right\} \geq \ell$ it follows that $pq \in \sqrt{\phi}$. Therefore, $\sqrt{\phi}$ is a 1AFIS(*P*).

Theorem 3.9 Let ϕ_1 and ϕ_2 be 1AFIS(*P*). Then $\phi_1 \cap \phi_2$ is also a 1AFIS(*P*).

Proof. The intersection of two fuzzy ideals of P is also a fuzzy ideal of P, as we know. Let ϕ_1 and ϕ_2 be 1AFIS(*P*) and $p,q,r \in P$. Then,

$$\begin{aligned} (\phi_1 \cap \phi_2)(pqr) \geq \ell &\Rightarrow \inf\left\{\phi_1(pqr), \phi_2(pqr)\right\} \geq \ell \\ &\Rightarrow \inf\left\{\phi_1(pq), \phi_2(pq)\right\} \geq \ell \;\; or \;\; \inf\left\{\phi_1(r), \phi_2(r)\right\} \geq \ell \\ &\Rightarrow (\phi_1 \cap \phi_2)(pq) \geq \ell \;\; or \;\; (\phi_1 \cap \phi_2)(r) \geq \ell. \end{aligned}$$

Hence, we reach $\phi_1 \cap \phi_2$ is a 1AFIS(*P*). Therefore, the result follows.

Corollary 3.10 Let ϕ_i be a non-empty family of 1AFIS(*P*) for $i = 1, 2, \ldots$, then $\bigcap \phi_i$ is also a 1AFIS(*P*).

Proof. We know that the intersection of a non-empty family of fuzzy ideals of P is also a fuzzy ideal of P. Now for 1AFIS(*P*), let $\{\phi_i : i = 1, 2, \ldots\}$ be a non-empty family of a1AFIS(*P*) and $p, q, r \in P$.

$$\begin{aligned}\left(\bigcap_{i\in I}\phi_i\right)(pqr) \geq \ell &\Rightarrow \inf\{\phi_i(pqr)\} \geq \ell \\ &\Rightarrow \inf\{\phi_i(pq)\} \geq \ell \ \ or\ \ \inf\{\phi_i(r)\} \geq \ell \\ &\Rightarrow \left(\bigcap_{i\in I}\phi_i\right)(pq) \geq \ell \ \ or\ \ \left(\bigcap_{i\in I}\phi_i\right)(r) \geq \ell.\end{aligned}$$

As a result, the proof has been completed.

Theorem 3.11 Let ϕ_1 and ϕ_2 be 1AFPIS(*P*). Then $\phi_1 \cap \phi_2$ is also a 1AFPIS(*P*).

Proof. Similar to Theorem 3.9.

Corollary 3.12 Let ϕ_i be a non-empty family of 1AFPIS(*P*) for $i = 1, 2, \ldots$, then $\bigcap \phi_i$ is also a 1AFPIS(*P*).

Proposition 3.13 Let $h : R \to P$ be a morphism of semirings and ϕ be a 1AFIS(*P*). Then, $h^{-1}(\phi)$ is also a 1AFIS(*R*).

Proof. Let $h : R \to P$ be a morphism of semirings and ϕ be a 1AFIS(*P*). Now, for $p, q, r \in P$ and $\ell \in [0,1]$, by the definition of pre-image we get $h^{-1}(\phi)(pqr) = \phi(h(pqr))$

$$\begin{aligned}h^{-1}(\phi)(pqr) \geq \ell &\Rightarrow \phi(h(pqr)) \geq \ell \\ &\Rightarrow \phi(h(p)h(q)h(r)) \geq \ell \\ &\Rightarrow \phi(h(p)h(q)) \geq \ell \quad or \quad \phi(h(r)) \geq \ell \\ &\Rightarrow h^{-1}(\phi)(pq) \geq \ell \quad or \ \ h^{-1}(\phi)(r) \geq \ell.\end{aligned}$$

Therefore, $h^{-1}(\phi)$ is a 1AFIS(*R*).

Proposition 3.14 Let ϕ_1, ϕ_2 be two fuzzy subsets of P such that ϕ_1 is a 1AFIS(*P*) and $\phi_1 \subseteq \phi_2$. If for $p, q, r \in P$ and $\ell \in [0,1]$, $\phi_2(pqr) \geq \phi_1(pqr) \geq \ell$, then ϕ_2 is also a 1AFIS(*P*).

Proof. Let ϕ_1, ϕ_2 be two fuzzy subsets of P such that $\phi_1 \subseteq \phi_2$ and ϕ_1 is a 1AFIS(*P*). Let $p, q, r \in P$ and $\ell \in [0,1]$ be such that $\phi_2(pqr) \geq \ell$. Since $\phi_2(pqr) \geq \phi_1(pqr) \geq \ell$ and ϕ_1 is a 1AFIS(*P*), $\phi_1(pq) \geq \ell$ or $\phi_1(r) \geq \ell$ which implies $\phi_2(pq) \geq \phi_1(pq) \geq \ell$ or $\phi_2(r) \geq \phi_1(r) \geq \ell$. Hence ϕ_2 is a 1AFIS(*P*).

Definition 3.15 Let ϕ_1 and ϕ_2 be two fuzzy subsets of P_1 and P_2, respectively. The cartesian product of ϕ_1 and ϕ_2 is defined by

$$(\phi_1 \times \phi_2)(p, q) = \min\{\phi_1(p), \phi_2(q)\}$$

for all $(p, q) \in P_1 \times P_2$.

Theorem 3.16 Let P_1, P_2 be two commutative semiring and ϕ be a 1AFIS(P_1). Then, $\phi \times \chi_{P_2}$ is a 1AFIS($P_1 \times P_2$), where χ_{P_2} is the characteristic function of P_2.

Proof. Let P_1, P_2 be two commutative semiring and ϕ be 1AFIS(P_1) and χ_{P_2} be 1AFIS(P_2) respectively. Now, for $p_1, q_1, r_1 \in P_1$, $p_2, q_2, r_2 \in P_2$ and $\ell \in [0,1]$

$$\begin{aligned}(\phi \times \chi_{P_2})(p_1q_1r_1, p_2q_2r_2) \geq \ell &\Rightarrow \min\{\phi(p_1q_1r_1), \chi_{P_2}(p_2q_2r_2)\} \geq \ell \\ &\Rightarrow \min\{\phi(p_1q_1), \chi_{P_2}(p_2q_2)\} \geq \ell \ \ or \ \ \min\{\phi(r_1), \chi_{P_2}(r_2)\} \geq \ell.\end{aligned}$$

For $p_2, q_2, r_2 \in P_2$, $\chi_{P_2}(p_2q_2r_2) = 1$ and so

$$\min\{\phi(p_1q_1), \chi_{P_2}(mn)\} \geq \ell \ \ or \ \min\{\phi(r_1), \chi_{P_2}(mn)\} \geq \ell$$

where $m, n \in \{p_2, q_2, r_2\}$. Therefore $\phi \times \chi_{P_2}$ is a 1AFIS($P_1 \times P_2$).

Theorem 3.17 Let ϕ be a 1AFPIS(P). Then $\sqrt{\phi}$ is a fuzzy prime ideal of P.

Proof. Let $\sqrt{\phi}(pq) \geq \ell$ for some $p, q \in P$ and $\ell \in [0,1]$. We may assume that p and q are non-unit elements of P. Let $n \geq 2$ be an even positive integer such that $\phi(pq)^n \geq \ell$. Then $n = 2m$ for some positive integer $m \geq 1$. Since $\phi(pq)^n = \phi(p^n q^n) = \phi(p^m p^m q^n) \geq \ell$ and ϕ is a 1AFPIS(P), we conclude that $\phi(p^m p^m) = \phi(p^n) \geq \ell$ *or* $\phi(q^n) \geq \ell$. Hence, $\sqrt{\phi}(p) \geq \ell$ or $\sqrt{\phi}(q) \geq \ell$. Thus $\sqrt{\phi}$ is a fuzzy prime ideal of P.

Theorem 3.18 Let ϕ be a fuzzy proper ideal of P. If ϕ is a fuzzy primary ideal of semiring P (FPIS(*P*)), then ϕ is a 1AFPIS(P).

Proof. Let ϕ be a fuzzy primary ideal of semiring and for all $p,q,r \in P$ and $\ell \in [0,1]$. Admit that $\phi(pqr) \geq \ell$. Since ϕ is a fuzzy primary ideal of semiring P, then

$$\phi(pq) \geq \ell \quad or \quad \sqrt{\phi}(r) \geq \ell .$$

It follows that ϕ is a 1AFPIS(P).

Theorem 3.19 Let ϕ be a fuzzy proper ideal of P. If ϕ is a 1AFPIS(P), then ϕ is a 2-absorbing fuzzy primary ideal of semiring P (2AFPIS(P)).

Proof. Let P be a 1-absorbing primary ideal of semiring R. It follows that the characteristic function χ_P of P is a fuzzy 1-absorbing primary ideal of R and admit that ϕ is a 1AFPIS(P). Since every 1-absorbing primary ideal is a 2-absorbing primary ideal, then we get P is a 2-absorbing primary ideal. It means that χ_P is a fuzzy 1-absorbing primary ideal of R and ϕ is a 2AFPIS(P).

Example 3.20 Let $P = Z$. Consider the ideal $I = 2Z$ and

$$\phi(p) = \begin{Bmatrix} 1, & p \in I \\ 0, & p \notin I \end{Bmatrix}$$

Then, ϕ is a 2AFPIS(P). When $\phi(2.1.1) \geq 1$, $\phi(1.1) = 0 \ngeq 1$ but $\sqrt{\phi}(2) = 1 \geq 1$. Thus, ϕ is a 1AFPIS(P). The following example shows that the converse of Theorem 3.19 may not be true.

Example 3.21 Let $P = Z$. Consider the ideal $I = 18Z$ and

$$\phi(p) = \begin{Bmatrix} 1, & p \in I \\ 0, & otherwise \end{Bmatrix}$$

Then, ϕ is a 2AFPIS(P). Since $\phi(2.3.3) \geq 1$, $\phi(2.3) = 0 \ngeq 1$ and $\sqrt{\phi}(3) = 0 \ngeq 1$. Therefore ϕ is not a 1AFPIS(P).The diagram below can be accessed from the algebraic structures obtained throughout the paper.

Remark 3.22 We can give the given theorems as a diagram:

$$\boxed{FPIS(P) \;\Rightarrow\; 1AFIS(P) \;\Rightarrow\; 1AFPIS(P) \;\Rightarrow\; 2AFPIS(P)}$$

4. Conclusion

In this study, the concept of 1AFIS(P) is investigated. Several theorems charac- terizing 1AFIS(P) are obtained with the concepts of level subset, radical, charac- teristic function, morphism, and Cartesian product. Finally, a diagram is provided that connects these definitions and theorems. To further expand upon this research, various algebraic structures can be examined, and additional research into their properties may be conducted. The authors employ this coherent approach to obtain various significant results regarding 1AFIS(P). Based on our work, we propose some research questions:

1) Exploration of 1-absorbing bipolar fuzzy ideals of commutative semiring.
2) Introduction of 1-absorbing bipolar valued intuitionistic fuzzy ideals of commutative semiring.
3) Identification of 1-absorbing vague ideals of commutative semiring.
4) Clarification of 1-absorbing neutrosophic ideals of commutative semiring.
5) Definition of 1-absorbing complex fuzzy ideals of commutative semiring.

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References

[1] Atani, R. E., and Atani, S. E., "Ideal theory in commutative semirings", Buletinul Academiei de Stiinte a Republicii Moldova Matematica 2 (2008) : 14-23.

[2] Zadeh, L.A., "Fuzzy sets", Inform. and Control 8 (1965) : 338-353.

[3] Roberts, D. W., "Ordination based on fuzzy set theory", Vegetation 66(3) (1986) : 123-131.

[4] Zimmermann, H. J., "Fuzzy set theory", Wiley interdisciplinary reviews: Computational statistics 2(3) (2010) : 317-332.

[5] Chang, C. L., "Fuzzy topological spaces", Journal of Mathematical Analysis and Applications 24(1) (1968) : 182-190.

[6] Rosenfeld, A., "Fuzzy groups", Journal of Mathematical Analysis and applications 35(3) (1971) : 512-517.

[7] Liu, W. J., "Fuzzy invariant subgroups and fuzzy ideals", Fuzzy sets and Systems 8(2) (1982) : 133-139.

[8] Abou-Zaid, S., "On fuzzy subnear-rings and ideals", Fuzzy sets and systems 44(1) (1991) : 139-146.

[9] Atanassov, K. T., "Intuitionistic fuzzy sets. In Intuitionistic fuzzy sets", Physica, Heidelberg (1999) : 1-137.

[10] Szmidt, E., and Kacprzyk, J., "Distances between intuitionistic fuzzy sets", Fuzzy sets and systems 114(3) (2000) : 505-518.

[11] Özdemir, O., & Kalınkara, Y., "Bulanık Mantık: 2000-2020 Yılları Arası Tez ve Makale Çalışmalarına Yönelik Bir İçerik Analizi", Acta Infologica 4(2) (2020) : 155-174.

[12] Badawi, A., "On 2-absorbing ideals of commutative rings", Bulletin of the Australian Mathematical Society 75(3) (2007) : 417-429.

[13] Badawi, A., & Celikel, E. Y., "On 1-absorbing primary ideals of commutative rings", Journal of Algebra and Its Applications 19(06) (2020) : 2050111.

[14] Yassine, A., Nikmehr, M. J., & Nikandish, R., "On 1-absorbing prime ideals of commutative rings", Journal of Algebra and its Applications 20(10) (2021): 2150175.

[15] Mandal, D., "On 2-absorbing fuzzy ideals of commutative semirings", TWMS Journal of Applied and Engineering Mathematics 11(2) (2021) : 368.
[16] Darani, A.Y., Hashempoor, A., "L-fuzzy 0-(1-or 2-or 3-) 2-absorbing ideals in semiring", Ann. Fuzzy Math. Inform. 7 (2014) : 303-311.
[17] Jonathan S. Golan, "Semirings and their applications", Springer (2013) : 105-120.
[18] Darani, Ahmad Yousefian, "On 2-absorbing and weakly 2-absorbing ideals of commutative semirings", Kyungpook Mathematical Journal 52 (1) (2012) : 91-97.
[19] Chaudhari, Jayprakash Ninu, "2-absorbing ideals in semirings", International Journal of Al- gebra 6 (6) (2012) : 265-270.
[20] Groenewald, Nico, "On weakly 2-absorbing ideals of non-commutative rings", Afrika Matematika 32(7-8) (2021) : 1669-1683.
[21] Soheilnia, Fatemeh, "On 2-absorbing and weakly 2-absorbing primary ideals of a commutative semiring", Kyungpook Mathematical Journal 56 (1) (2016).
[22] Behzadipour, Hussein, and Peyman Nasehpour, "On 2-absorbing ideals of commutative semir- ings", Journal of Algebra and Its Applications 19 (2)(2020) : 2050034.
[23] Jaber, Ameer., "Properties of weakly 2-absorbing primal ideals", Italian Journal of Pure and Applied Mathematics 7 (2022) : 609-619.
[24] Groenewald, N. J., "On 2-absorbing and weakly 2-absorbing principally right primary ideals", Journal of Algebra and Related Topics 9 (2) (2021) : 47-67.
[25] Sahoo, Taptee, Deepak Shetty, M., Groenewald, N. J., Harikrishnan, P. K., Kuncham, S. P., "On completely 2-absorbing ideals of N-groups", Journal of Discrete Mathematical Sciences and Cryptography 24 (2) (2021) : 541-556.
[26] Yassine, A., M. J. Nikmehr, and R. Nikandish., "On 1-absorbing prime ideals of commutative rings", Journal of Algebra and its Applications 20(10) (2021) : 2150175.
[27] Koc, Suat, Ünsal Tekir, and Eda Yıldız, "On weakly 1-absorbing prime ideals", Ricerche di Matematica 72(2) (2023) : 723-738.
[28] Groenewald, Nico., "1-absorbing prime ideals and weakly 1-absorbing prime ideals in non-commutative rings", S˜ao Paulo Journal of Mathematical Sciences (2022) : 1-17.
[29] Bouba, E. M., Tamekkante, M., Tekir, U¨ ., and Ko¸c, S., "Notes on 1-absorbing prime ideals",Proceedings of the Bulgarian Academy of Sciences 75(5) (2022) : 631-639.
[30] Abu-Dawwas, R., Yıldız, E., Tekir, U¨ ., Koc, S., "On graded 1-absorbing prime ideals." Sao Paulo Journal of Mathematical Sciences 15 (2021) : 450-462.
[31] Anbarloei, Mahdi., "On 1-absorbing prime hyperideal and some of its generalizations", Journal of Mathematics (2022) : 4947019.
[32] Saleh, Mohammad, and Ibaa M., "On weakly 1-absorbing primary ideals of commutative semirings", Communications in Advanced Mathematical Sciences 5(4) (2022) : 199-208.