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Time-Varying Fractal Analysis of Exchange Rates

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ABSTRACT The foreign exchange (forex) market is a dynamic and complex financial arena where the exchange rates of various currency pairs fluctuate continuously. Among these currency pairs, EUR/TRY and USD/TRY hold significant economic relevance due to their roles in international trade and finance. In this study, we analyze the multifractality of hourly EUR/TRY and USD/TRY exchange rate data for the whole period, as well as its time-varying individual and cross correlations, spanning from May 31, 2018, to March 21, 2022. We employ multifractal detrended cross-correlation analysis (MF-DCCA) and multifractal detrended fluctuation analysis (MF-DFA) methodologies. The aim of studying multifractality in exchange rates is to comprehend and model the complex and intricate nature of price movements and dynamics of the EUR/TRY and USD/TRY exchange rates. In the analysis of the whole period, multifractality is detected in individual exchange rates and cross correlations. In the rolling window analysis, we demonstrated how multifractality and cross correlation multifractality change over time. Additionally, contributions of the sources of the multifractality are investigated in a time-varying framework. Multifractal nature of these exchange rates indicate that they exhibit complex and scale-dependent behaviors, which go beyond the traditional linear models. The existence of multifractality in EUR/TRY and USD/TRY exchange rates has significant implications for financial modeling, risk management, and trading strategies. It implies that standard linear models may not capture the full complexity of these markets, necessitating the development of more sophisticated models that account for multifractal properties.

KEYWORDS
Exchange rates
Multifractality
MF-DFA
MF-DCCA
Fractal theory

INTRODUCTION

Fractal theory is originated from (Mandelbrot 1982) and used to provide an explanation for economic and financial data where traditional efficient market hypothesis (EMH) failed. Fractal geometry is applied in the analysis of systems which are irregular and self-similar at all scales. One of the key characteristics of these systems are non-integer dimensions. Fractal systems can be categorized as monofractal or multifractal. Monofractal systems can be defined by a single scaling exponent and different regions of these systems have same scaling properties. However, multifractal systems display varying scaling properties in different regions, requiring multiple scaling exponents to describe the system.

Firstly, in the field of hydrology (Hurst 1951, 1957) suggested rescaled range (R/S) methodology for studying systems exhibiting fractal properties. However, Lo (1991) demonstrated the shortcomings of Hurst methodology such as sensitivity to short-term autocorrelation. To address this deficiency (Peng *et al.* 1994) proposed a methodology called Detrended Fluctuation Analysis (DFA). DFA methodology is successfully applied to noisy and non-stationary time series which exhibiting long-range correlations and fractal scaling properties. Numerous data sets have been successfully analyzed using this method, including geological, economic, financial, weather and earthquake data (Liu *et al.* 1999; Buldyrev *et al.* 1998; Blesić *et al.* 1999; Bunde *et al.* 2000; Ashkenazy *et al.* 2001; Talkner and Weber 2000). However, studies in this field have revealed that some data from various fields such as medicine, geophysics, economy and finance do not exhibit monofractal scaling behavior. Consequently, a single scaling exponent cannot adequately represent these multifractal systems (Kantelhardt *et al.* 2001; Hu *et al.* 2001), and multiple scaling exponents are required.

To analyze multifractal systems (Kantelhardt *et al.* 2002) purposed Multifractal Detrended Fluctuation Analysis (MF-DFA) which is an extension of the DFA. MF-DFA methodology has been successfully applied to many nonstationary time series datasets in

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the literature (Kantelhardt *et al.* 2003; Movahed *et al.* 2006; Telesca *et al.* 2004). The literature demonstrates that many time series from various fields exhibit multifractal properties, and a single scaling exponent is not sufficient to describe these datasets (Matia *et al.* 2003; Chen and He 2010; He and Chen 2010b,a; Zunino *et al.* 2009).

Afterwards, by developing DFA methodology Podobnik and Stanley (2008) introduced the detrended cross-correlation analysis (DCCA) methodology for studying cross correlations between two systems. Subsequently, Zhou (2008) combined MF-DFA and DCCA to propose the multifractal detrended cross-correlation analysis (MF-DCCA) methodology for investigating multifractal properties of two correlated nonstationary time series. MF-DCCA methodology has been successfully applied to numerous economic and financial datasets from foreign exchange market (Xie *et al.* 2017; Li *et al.* 2016), the stock market (Ma *et al.* 2013a; Yue *et al.* 2017), the crude oil market (Ma *et al.* 2013b, 2014; Wang *et al.* 2011b), carbon market (Zhuang *et al.* 2014, 2015) and the commodity market (Wang *et al.* 2011a; Lu *et al.* 2017).

Furthermore foreign exchange market is of great importance to global economy. This market connects economies around the world without geographic and temporal boundaries. Exchange rates are vital macroeconomic variables for policy makers, investors, researchers and economists. Instabilities of exchange rates can have devastating effects on the economies. Therefore, researchers and economists have attempted to model exchange rates using various methodologies. These studies have revealed that predicting and explaining fluctuations in exchange rates is challenging. Efficient market hypothesis suggested by (Fama 1965) indicated, share prices follow random walk and are unpredictable. However, this hypothesis challenged by different authors subsequently (Yen and Lee 2008; Lim and Brooks 2011). An alternative to EMH is fractal market hypothesis (FMH) which is suggested by (Lim and Brooks 2011; Peters 1994). This hypothesis suggests that markets exhibit the same structure on different scales (daily, weekly, monthly, etc.). The EMH has led to investigations into the fractal and multifractal properties of economic and financial time series.

To the best of our knowledge, there is only one study in the literature that investigates the multifractal properties of USD/TRY exchange rates (Gülbaş and Gazanfer 2013). This study detected multifractality in USD/TRY exchange rates but did not provide a time varying analysis to investigate how multifractality and sources of multifractality change over time. There are other studies in the literature that examine the multifractal properties of various exchange rates as well (Stošić *et al.* 2015; Schmitt *et al.* 1999; Caraiani and Haven 2015; Han *et al.* 2019). While these studies have detected multifractality in other exchange rates, they have not shed light on how multifractality and its sources change over time.

MF-DFA and MF-DCCA methods are important methods in the field of time series analysis, particularly for studying complex and non-linear behaviors in financial data and other complex systems. The importance of these methods is presented below:

a) Capturing Nonlinear Behavior: Financial and economic data often exhibit nonlinear behaviors that cannot be adequately captured by traditional linear methods. MF-DFA and MF-DCCA are designed to detect and quantify these nonlinear characteristics, providing a more accurate representation of the underlying dynamics.

b) Multiscale Analysis: MF-DFA and MF-DCCA allow for the analysis of data across multiple time scales. This is important because financial data often exhibit different patterns and behaviors at different scales. By analyzing multiple scales, these methods offer a more comprehensive view of the system's complexity.

c) Multifractality: These methods are specifically designed to identify and characterize multifractal behavior in time series data. Multifractality refers to the property where different scales of observation exhibit different levels of self-similarity and irregularity. This is a common feature in financial data and other complex systems.

d) Cross-Correlation Analysis: MF-DCCA goes beyond traditional correlation analysis by accounting for cross-correlations that exist at different time scales. This is crucial in understanding how different variables interact and influence each other over different horizons.

MF-DFA and MF-DCCA methods have some differences from other methods. These differences are summarized as below:

a) Fractal vs. Multifractal Analysis: Traditional fractal analysis focuses on self-similarity at a single fractal dimension. In contrast, multifractal analysis considers multiple fractal dimensions, which allows for a more nuanced understanding of complex systems.

b) Nonlinear vs. Linear Methods: While linear methods assume a linear relationship between variables, MF-DFA and MF-DCCA are designed to capture nonlinear and multifractal behaviors. This is particularly important in financial markets where linearity often fails to explain the full complexity.

c) Time Scale Consideration: MF-DFA and MF-DCCA analyze data across multiple time scales, which provides insights into the dynamics at different levels. Traditional methods might overlook these multiscale interactions.

d) Cross-Correlation Consideration: MF-DCCA specifically addresses cross-correlations between multiple variables at different time scales. This is a feature that many traditional methods lack.

e) Complexity: MF-DFA and MF-DCCA are more complex and sophisticated methods compared to traditional linear analysis. They require a deeper understanding of their underlying principles and assumptions.

In recent years Turkey has become integrated into international economic markets. According to the general trade system in Turkey, in the January-April period of 2022, exports increased by 21.6% compared to the previous year and reached 83.5 billion dollars, while imports increased by 40.2% and reached 116 billion 85 million dollars. Therefore USD/TRY and EUR/TRY exchange rates are of great importance to the Turkish economy and have significant effects on other macroeconomic variables such as GDP, current account deficit, inflation and unemployment. The selection of the preferred dataset, specifically the USD/TRY and EUR/TRY exchange rates, was based on careful consideration of several criteria that these currency pairs satisfy, making them ideal candidates for multifractality analysis. We chose to test the multifractality of USD/TRY and EUR/TRY exchange rates because of the several reasons. Firstly, USD/TRY and EUR/TRY are important currency pairs involving major global currencies (US Dollar and Euro) and the Turkish Lira.

These exchange rates reflect economic relationships between Turkey and the United States or the Eurozone. Studying their multifractality can provide insights into the dynamics of these economic relationships. Secondly, these currency pairs are among the most actively traded pairs in the foreign exchange market due to Turkey's significant economic activities and its geopolitical positioning. High trading activity often results in complex and multifractal price behaviors, making them interesting candidates for analysis. Thirdly, the Turkish Lira has historically exhibited notable volatility in comparison to major currencies. Such volatility often results in intricate, non-linear, and multifractal price movements. Studying these complex behaviors is vital for understanding the underlying dynamics and interactions in the market. Given the potential volatility of the Turkish Lira, individuals, businesses, and investors involved in transactions or investments with Turkey have a vested interest in understanding the multifractal nature of these exchange rates for effective risk management. Fourthly, exchange rates have policy implications for governments and central banks.

Understanding the multifractality of USD/TRY and EUR/TRY can aid in policy decisions related to trade, investment, and monetary policy. Finally, in the literature time-varying multifractality of USD/TRY and EUR/TRY exchange rates are not investigated in the literature. The selection of USD/TRY and EUR/TRY exchange rates is motivated by their substantial economic importance. The USD/TRY exchange rate is a key benchmark for Turkey's foreign exchange market, and the EUR/TRY exchange rate represents another critical currency pair in the region. Both are integral to international trade, investment, and financial stability within the Turkish economy.

In this study time-varying multifractal properties of exchange rates are analyzed using MFDFA and MF-DCCA methodologies. In this context, two different types of analysis were conducted. These are whole period analysis and rolling window analysis. In the whole period analysis MFDFA and MF-DCCA methodologies are applied to the entire dataset to investigate multifractality over the entire period. In the rolling window analysis MFDFA and MF-DCCA methodologies are applied to data windows and by sliding the window changes in multifractality are examined. Our study addresses seven research questions:

1. Whether USD/TRY and EUR/TRY exchange rates are multifractal?

2. How the multifractality levels of USD/TRY and EUR/TRY exchange rates change over time?

3. Whether cross-correlations between USD/TRY and EUR/TRY exchange rates are multifractal?

4. How the multifractality level of cross correlation between USD/TRY and EUR/TRY exchange rates changes over time?

5. How the fat-tailed distribution's contribution to the level of multifractality of USD/TRY and EUR/TRY exchange rates changes over time?

6. How the long-range correlation's contribution to the level of multifractality of USD/TRY and EUR/TRY exchange rates changes over time?

7. Which cause of multifractality of USD/TRY and EUR/TRY exchange rates is more prevalent over time: long-range autocorrelation or fat-tailed distribution?

Studying multifractality in exchange rates serves several purposes:

a) Better Understanding of Market Behavior: Multifractal analysis helps researchers and analysts delve deeper into the underlying structure of exchange rate movements. It allows them to identify complex patterns and irregularities that are not apparent through traditional methods.

b) Risk Management: Exchange rate movements can have significant implications for international trade, investment, and risk management. Understanding multifractality can aid in developing more accurate risk assessment models, which is crucial for businesses and financial institutions exposed to currency fluctuations.

c) Model Improvement: Traditional financial models often assume certain levels of linearity and Gaussian (normal) distribution of returns. However, exchange rates frequently exhibit fat tails, extreme events, and time-varying volatility. Studying multifractality can lead to the development of more accurate models that capture these characteristics.

d) Algorithmic Trading: Many financial institutions use algorithmic trading strategies to make investment decisions. Understanding multifractality can lead to the development of more sophisticated trading algorithms that adapt to the nonlinear and irregular behavior of exchange rates.

e) Policy Formulation: Central banks and governments make policy decisions based on economic conditions, including exchange rates. Multifractal analysis can provide insights into the underlying dynamics of exchange rates, which can inform more effective policy decisions.

f) Academic Research: Academics study multifractality in exchange rates to contribute to the theoretical understanding of financial markets and to advance the field of financial economics.

In conclusion our study makes several contributions to the literature. Firstly, as far as we know fractal properties of hourly exchange rates are not investigated in the literature. We used hourly data in our multifractal analysis because hourly data provides a higher frequency of observations compared to daily or weekly data. This increased frequency allows for a more detailed analysis of price movements and captures finer nuances in market behavior. Also, financial markets exhibit distinct intraday patterns and volatility changes and hourly data captures these patterns. Additionally, multifractal analysis involves studying patterns at various scales or time horizons. Hourly data allows for a broader range of scales to be analyzed, from short-term fluctuations to longer-term trends.

Usage of hourly data distinguish our study from other studies since hourly data offers a finer level of granularity, captures intraday price movements, reveals higher-frequency fluctuations and volatility changes, and enables researchers to study the immediate market reactions. Secondly, in the literature fractal analysis is usually applied to one or few time periods. However, we presented a time-varying analysis in a rolling window framework. Thirdly, we also presented how the contributions of multifractality sources have changed over time in a rolling window framework. The following is how our study is set up. Section 2 presents the MF-DFA and MF-DCCA techniques. Data is provided in Section 3. In Section 4, empirical findings are given. And Section 5 provides conclusions.

METHODOLOGY

Multifractal Detrended Fluctuation Analysis (MF-DFA)

Suppose x_t denotes a time series where $t = 1, 2, \ldots, N$ The MF-DFA method consist of five steps.

Step1: In the first step the profile is calculated as follows:

$$X_i = \sum_{t=1}^{i} (x_t - \bar{x}) \tag{1}$$

In the expression above $\bar{x}$ is calculated as below:

$$\bar{x} = \frac{1}{N} \sum_{t=1}^{N} x_t \tag{2}$$

Step 2: In the next step the profile X_i is divided into $N_s = int(N/s)$ equal-length parts that don't overlap. There might be a little residue at the end of the profile since the length of the series x_t might not be multiple of the time scale s. The identical process used at the end of the series was repeated in order to account for this residue. As a result of this procedure $2N_s$ total segments are obtained.

Step 3: The variance is calculated by following two formulas for segments $v = 1, 2, \ldots, N_s$ and for segments $v = N_s + 1, N_s + 2, \ldots, 2N_s$ respectively:

$$F_X^2(s, v) = \frac{1}{s} \sum_{j=1}^{s} \left(X_{(v-1)s+j} - \widehat{X}_j^v \right)^2 \tag{3}$$

$$F_X^2(s, v) = \frac{1}{s} \sum_{j=1}^{s} \left(X_{N-(v-N_s)s+j} - \widehat{X}_j^v \right)^2 \tag{4}$$

In the above formulas $\hat{X}_j^v$ denotes the fitting polynomial in segment v with order m. In this study fitting polynomial order m is selected as one.

Step 4: In the next step qth order fluctuation function $F_X^q(s)$ is computed by averaging all segments using following two formulas for $q \neq 0$ and $q = 0$ respectively:

$$F_X^q(s) = \left(\frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[F_X^2(s, v) \right]^{\frac{q}{2}} \right)^{\frac{1}{q}} \tag{5}$$

$$F_X^q(s) = \exp \left(\frac{1}{4N_s} \sum_{v=1}^{2N_s} \left[F_X^2(s, v) \right] \right) \tag{6}$$

Step 5: By analyzing logarithm plots of $F_X^q(s)$ versus logarithms of s for each q value the scaling behavior of the fluctuation function is determined. If long-range power-law correlation exists between the series, there is a power-law relationship expressed as below:

$$F_X^q(s) \sim s^{h(q)} \tag{7}$$

The generalized Hurst exponent, or $h(q)$, in the expression above reflects the correlation with power-law. The expression $h(q)$ represents the scaling behavior of segments with large fluctuations for positive values of q, whereas for negative values of q, it represents the scaling behavior of segments with smaller variations. To describe a multifractal series the singularity spectrum $f(\alpha)$ can be used which is calculated as below:

$$\alpha(q) = h(q) + qh'(q) \tag{8}$$

$$f(\alpha) = q[\alpha(q) - h(q)] + 1 \tag{9}$$

The derivative of $h(q)$ with respect to q is denoted by $h\prime(q)$ in the expression above. The Hölder exponent, denoted by the symbol $\alpha(q)$, measures the singularity's power, while the singularity spectrum, denoted by the symbol $f(\alpha)$, measures the Hausdorff dimension of the subset of the series that is characterized by $\alpha(q)$. Multifractal mass function can be calculated as below:

$$\tau(q) = qh(q) - 1 \tag{10}$$

The width of the multifractal spectrum $(\Delta\alpha)$, which is calculated as follows, can be used to gauge the level of multifractality:

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min} \tag{11}$$

Higher $\Delta\alpha$ values indicate higher levels of multifractality and lower $\Delta\alpha$ values indicate lower levels of multifractality. The singularity spectrum possesses an α_0 value which corresponds to maximum $f(\alpha)$, i.e. $f(\alpha_0) = 1$. Skewness of the spectrum indicates information on the dominant fluctuations. Right-skewed spectrum suggests that minor variations will predominate, while left-skewed spectrum suggests that huge fluctuations will.

Multifractal Detrended Cross-Correlation Analysis (MF-DCCA)

The MF-DCCA methodology combines two methods namely DCCA and MF-DFA. The MF-DCCA methodology can be utilized to demonstrate multifractal properties of two power-law correlated time series. Suppose x_t and y_t represent two time series with $t = 1, 2, \ldots, N$. The MF-DCCA method consist of following five steps:

Step1: In the first step the profiles are calculated as follows:

$$X_i = \sum_{t=1}^{i} (x_t - \bar{x}) \tag{12}$$

$$Y_i = \sum_{t=1}^{i} (y_t - \bar{y}) \tag{13}$$

In the expressions above $\bar{x}$ and $\bar{y}$ are the average values of the series.

Step 2: In the second step each profile is divided into $2N_s$ segments as in MF-DFA.

Step 3: Next covariance is calculated by following two formulas for segments $v = 1, 2, \ldots, N$ and for segments $v = N_s + 1, N_s + 2, \ldots, 2N_s$ respectively:

$$F_{XY}^2(s, v) = \frac{1}{s} \sum_{j=1}^{s} \left| X_{(v-1)s+j} - \widehat{X_j^v} \right| \cdot \left| Y_{(v-1)s+j} - \widehat{Y_j^v} \right| \tag{14}$$

$$F_{XY}^2(s, v) = \frac{1}{s} \sum_{j=1}^{s} \left| X_{N-(v-N_s)s+j} - \widehat{X_j^v} \right| \cdot \left| Y_{N-(v-N_s)s+j} - \widehat{Y_j^v} \right| \tag{15}$$

In the above formulas $\hat{X}_j^v$ and $\hat{Y}_j^v$ denote the fitting polynomials in segment v with order m. In this study fitting polynomial order m is selected as one.

Step 4: In the next step fluctuation function with order q, $F_{XY}^q(s)$, is computed by averaging all segments using following two formulas for $q \neq 0$ and $q = 0$ respectively:

$$F_{XY}^q(s) = \left(\frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[F_{XY}^2(s, v) \right]^{q/2} \right)^{1/q} \tag{16}$$

$$F_{XY}^q(s) = \exp \left(\frac{1}{4N_s} \sum_{v=1}^{2N_s} \left[F_{XY}^2(s, v) \right] \right) \tag{17}$$

Step 5: By analyzing logarithm plots of $F_{XY}^q(s)$ versus logarithm s the scaling behavior of the fluctuation function is determined for each value of q. If the considered series are power-law cross-correlated, there is a power-law relationship expressed as below:

$$F_X Y^q \ (s) \ s^(h_X Y \ (q)) \tag{18}$$

In the expression above $h_{XY}(q)$ represents generalized correlation exponent which reflects the power-law relationship. If $h_{XY}(q)$ depends on q then correlation between the two time series is multifractal. However, if $h_{XY}(q)$ is independent of q then correlation is monofractal.

Similar to MF-DFA multifractal spectrum $f_{XY}(\alpha)$ can be obtained from following formulas:

$$\alpha_{XY}(q) = h_{XY}(q) + qh'_{XY}(q) \tag{19}$$

$$f_{XY}(\alpha) = q[\alpha_{XY}(q) - h_{XY}(q)] + 1 \tag{20}$$

The term $h'_{XY}(q)$ in the expression above refers to the derivative of $h_{XY}(q)$ with regard to q. The $\alpha_{XY}(q)$ is called Hölder exponent and reflects the power of the singularity. Also, width of the multifractal spectrum $(\Delta\alpha)$ indicates strength of multifractality.

DATA AND PRELIMINARY ANALYSIS

In this study hourly data for EUR/TRY and USD/TRY exchange rates are utilized. The dataset comprises 23600 observations and spans period between 2018-05-31 13:01 and 2022.03.21 08:01:00. The data is sourced from GCM Forex company. Two types of data analyses were conducted in this study: whole-period analysis and rolling window analysis. In the rolling window analysis, a window size of 4,000 observations was selected, with a sliding step of 400 observations. The changes in exchange rates in the whole period are depicted in Figure 1 and Figure 2. Summary statistics for exchange rates are also presented in Table 1. As shown in Table 1 both exchange rates exhibit right-skewed distributions. Additionally, both exchange rates are leptokurtic and possess fat tailed distributions. Since one source of multifractality is fat tailed distribution, we can anticipate multifractality in both exchange rates. In Figure 3 and Figure 4 autocorrelations for exchange rates are plotted.

As evident in these figures, significant autocorrelations are observed in EUR/TRY and USD/TRY exchange rates up to lags 8,653 and 8,804, respectively. Therefore, there are long-range autocorrelations in both exchange rates. Since another source of multifractality is long-range autocorrelation, we can expect multifractality in these exchange rates. Long-range autocorrelation can lead to multifractality because it can create a heterogeneous distribution of the values of the time series. This heterogeneous distribution can lead to different scaling behaviors over different time intervals. For example, if the values of a time series are clustered together above the mean, then the time series will be more volatile over short time intervals. This is because the values of the time series are more likely to change rapidly when they are clustered together.

On the other hand, if the values of a time series are clustered together below the mean, then the time series will be more volatile over long time intervals. This is because the values of the time series are more likely to change slowly when they are clustered together. Therefore, long-range autocorrelation can lead to multifractality by creating a heterogeneous distribution of the values of the time series. This heterogeneous distribution can lead to different scaling behaviors over different time intervals (Jafari *et al.* 2007; Dashtian *et al.* 2011; Tanna and Pathak 2014). In our preliminary analysis we calculated fractal dimensions for USD/TRY and EUR/TRY exchange rates using Box-count estimator, Hall-Wood estimator, Wavelet estimator and DCT-II estimator (Gneiting *et al.* 2012) and presented the results in Table 2. To illustrate how these fractal dimensions change over time, we applied a rolling window analysis and displayed the findings in Figure 5 and Figure 6. As evident from Table 2 and Figures 5-6, both USD/TRY and EUR/TRY exchange rates exhibit fractal (non-integer) dimensions.

EMPIRICAL RESULTS

In this study firstly MFDFA is applied to exchange rates individually. In individual analyzes firstly, multifractality is investigated for the whole dataset. Secondly, a rolling window methodology is used to investigate how multifractal properties change over time and to assess the contributions of long-range autocorrelation and fat-tailed distribution to multifractality. Afterwards, MF-DCCA is applied to both EUR/TRY and USD/TRY exchange rates. In this Section firstly MF-DCCA is applied to whole dataset to examine the multifractal properties of the complete dataset. Secondly, using a rolling window methodology, changes in the cross-correlation multifractality between the exchange rates over time are examined. Additionally, the study explores how contributions of long-range autocorrelation and fat-tailed distribution to cross-correlation multifractality change over time.

In order to apply MFDFA and MF-DCCA methods three parameter values must be determined: vector of scales, q-order of the moment (q) and polynomial order for the detrending (m). In both whole period analysis and rolling window analysis q-order of the moment values are selected from -10 to +10 in steps of 1 including zero and polynomial order for the detrending is set to 1. However, in whole period analysis scales values are selected from 100 to 5900 in steps of 10 and in rolling window analysis scales

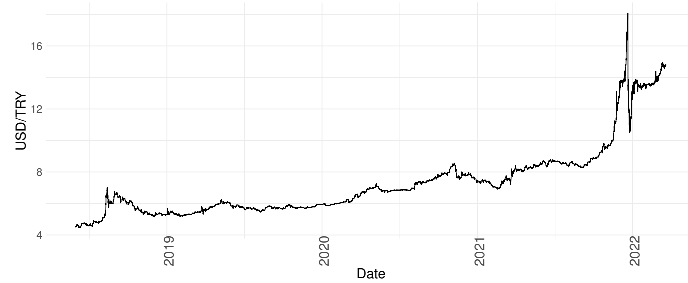

Figure 1 USD/TRY Exchange rate

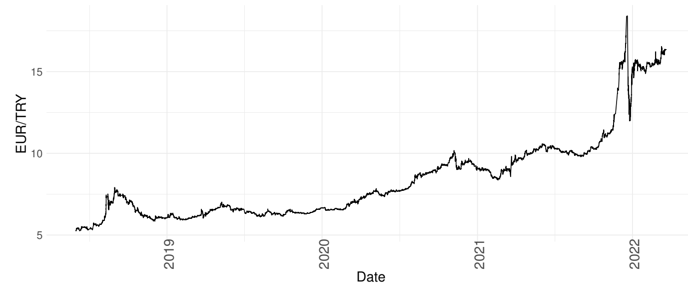

Figure 2 EUR/TRY Exchange rate

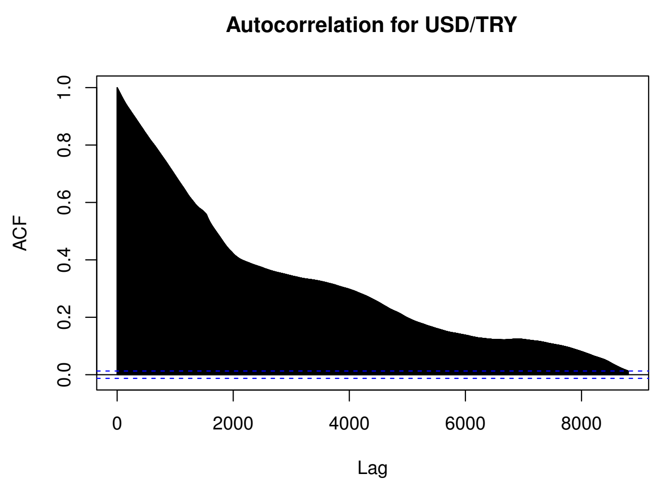

Figure 3 Autocorrelation for USD/TRY exchange rate

Table 1 Descriptive Statistics

Exchange Rate	Min	1st Q.	Median	Mean	3st Q.	Max	Std. Dev.	Skewness	Kurtosis
USD/TRY	4.451	5.738	6.792	7.302	8.203	18.080	2.3026	1.72126	5.7037
EUR/TRY	5.253	6.387	7.532	8.392	9.747	18.413	2.6457	1.44015	4.6592

Table 2 Fractal Dimensions

Method	USD/TRY	EUR/TRY
Box-count estimator	1.328052	1.316235
Hall-Wood estimator	1.510337	1.480936
Wavelet estimator	1.518846	1.405108
DCT-II estimator	1.526528	1.435383

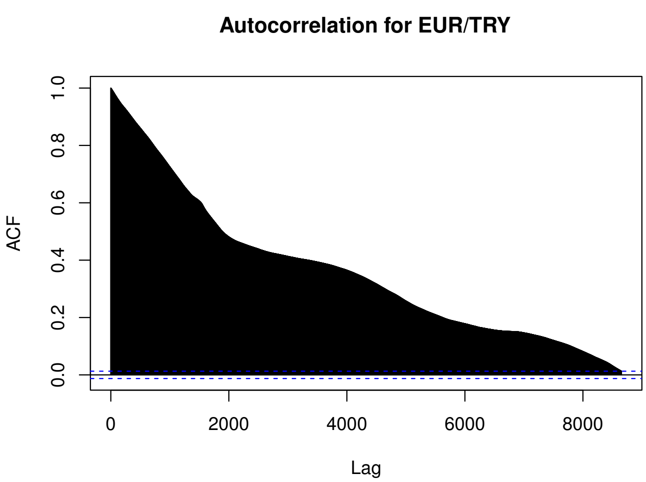

Figure 4 Autocorrelation for EUR/TRY exchange rate

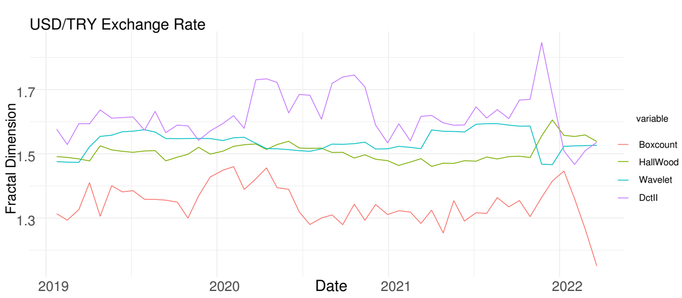

Figure 5 Change in fractal dimensions for USD/TRY exchange rate

values are selected from 10 to 400 in steps of 10. In our analysis to measure the level of multifractality $(\Delta\alpha)$ values are utilized. To illustrate how individual and cross correlated multifractality levels of the exchange rates change over time we presented the changes in $(\Delta\alpha)$ values within a rolling window framework.

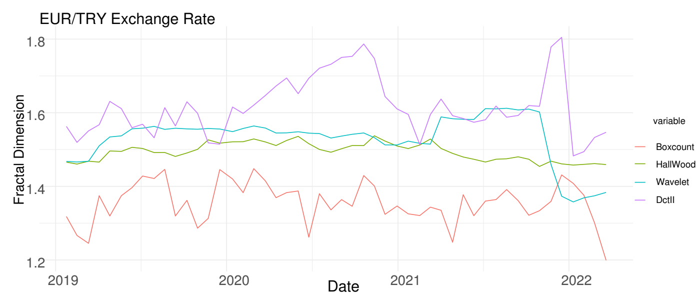

Figure 6 Change in fractal dimensions for EUR/TRY exchange rate

In the literature, not only the level of multifractality but also the factors contributing to multifractality has been investigated. Multifractality is primarily influenced by two factors. These are fat-tailed distribution and long-range autocorrelation. To measure the contribution of these two causes to the multifractality, surrogate and shuffled data are generated and utilized. In the generation of shuffled data autocorrelations are destroyed but the distribution is preserved. After generation of shuffled data, $(\Delta\alpha_Shuffled)$ Shuffled value is calculated from this shuffled data. Eventually, when $(\Delta\alpha_Shuffled)$ Shuffled is subtracted from original $(\Delta\alpha)$ value, long-range autocorrelations' contribution to the multifractality are obtained.

Another factor that contributes to multifractality is the presence of a fat-tailed distribution. To assess the multifractality's contribution from the fat-tailed distribution, surrogate data is employed. Surrogate data is generated by using a phase randomization procedure. In this procedure fat-tails in the distribution is eliminated but linear properties of the distribution are preserved. To evaluate contribution of fat tails to the multifractality, $(\Delta\alpha_Surrogate)$ surrogate value is calculated from surrogate data. Subsequently, $(\Delta\alpha_Surrogate)$ Surrogate value is subtracted from original $(\Delta\alpha)$ value to calculate contribution of fat tails to the multifractality.

In next sections to illustrate how the contributions of long-range autocorrelation factor and fat-tailed distribution factor to multifractality change over time fifty shuffled time series and fifty surrogate time series are generated for each time window and $(\Delta\alpha)$ values are calculated for each of the fifty series. Subsequently, mean and standard deviation values of fifty $(\Delta\alpha)$ parameters for shuffled and surrogate series are calculated in each time window. Since MF-DCCA method requires two time series we generated fifty pairs of surrogate and shuffled time series to explore the contributions of fat-tailed distribution and long-range autocorrelation to multifractality of the cross-correlations. By utilizing the means and standard deviations of $(\Delta\alpha)$ values calculated from surrogate and shuffled time series, contributions of two factors to the multifractality are examined. We generated multiple shuffled and surrogate series because in each realization different series are obtained. Therefore, multiple surrogate and shuffled series are required for robust results.

MF-DFA of USD/TRY Exchange Rate

Firstly, we analyzed multifractality of USD/TRY exchange rate by using whole period data. The results are presented in Figure 7. Upper left panel of Figure 7 indicates logarithm–logarithm plots of fluctuation function $F_q(s)$ versus time scale s for q values equal to 10, 0 and -10. The linearity of points in this graph reveals presence of power-law cross-correlations between time scale and fluctuation function. The upper right panel of Figure 7 illustrates how the Hurst exponent changes for various values of q. The Hurst exponents do not remain constant across a range of q values, leading us to the conclusion that the USD/TRY exchange rate exhibits multifractality.

Additionally, for q = 2 Hurst exponent is computed as 0.5268 which is slightly higher than 0.5, indicating a very weakly persistent time series. Lower left panel of Figure 7 shows how mass exponent change for different values of q. Since mass exponent nonlinearly depends on q, this provides further evidence of multifractality of USD/TRY exchange rate. Lower right panel of Figure 7 presents multifractal spectrum of USD/TRY exchange rate. Here width of the multifractal spectrum $(\Delta\alpha)$ reveals the level of multifractality and a positive $(\Delta\alpha)$ value indicates the existence of multifractality. Also, since α_0 value is higher than 0.5 there is persistent long-range correlations in the USD/TRY exchange rate series. Left-skewed spectrum implies that large fluctuations are dominant in the time series.

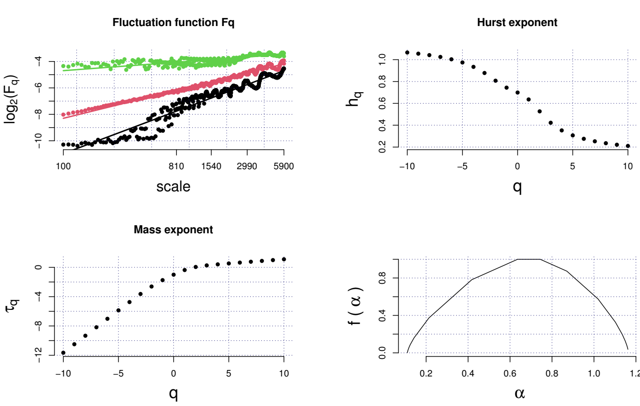

Figure 7 Change in fractal dimensions for USD/TRY exchange rate

To explore how the level of multifractality for USD/TRY exchange rate change over time we illustrated how multifractal spectrum $(\Delta\alpha)$ change over time in a rolling window framework. Results are depicted in Figure 8 and Figure 9 with black curves. In these figures with dots on black curve fifty original $(\Delta\alpha)$ values are presented and each of these corresponds to single time window. When the original $(\Delta\alpha)$ values are examined three different regimes in terms of multifractality are distinguished. In period between 2018-05-31 13:01 and 2020-06-24 13:01 and in period between 2020-07-17 06:01 and 2022-03-21 08:01 multifractality levels of USD/TRY exchange are higher than the period between 2019-11-22 19:01 and 2021-02-16 12:01. Also, there is a noticeable peak in the multifractality in the period between 2018-08-09 13:01 and 2019-04-04 00:01. Moreover, there is a collapse in the multifractality in the period between 2021-04-27 15:01 and 2021-12-16 10:01.

In our analysis we generated 50 shuffled and 50 surrogate series for each time window to illustrate how the contribution of long-range autocorrelation and fat-tailed distribution to multifractality change over time. Mean $(\Delta\alpha_S urrogate)$ values of surrogate series computed in each time window are shown with a blue curve in Figure 8 and mean $(\Delta\alpha_S huffled)$ values of shuffled series computed in each time window are shown with a blue curve in Figure 9. Red error bars represent ±1 standard deviations of generated surrogate and shuffled series in each time window.

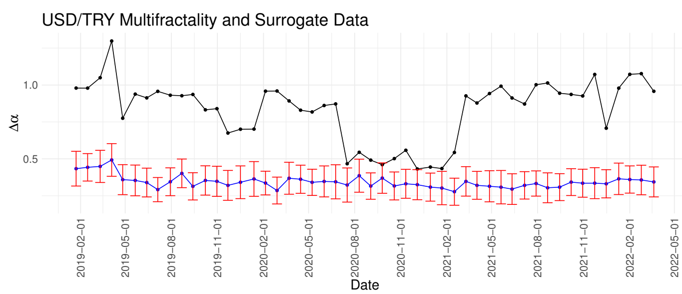

Figure 8 Change in $(\Delta\alpha)$ calculated from original data and change in $(\Delta\alpha)$ calculated from surrogate data

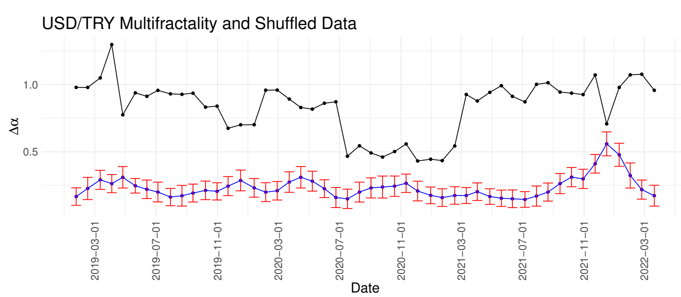

Figure 9 Change in $(\Delta\alpha)$ calculated from original data and change in $(\Delta\alpha)$ calculated from shuffled data

To assess the change in the contribution of fat-tailed distribution to multifractality we subtracted mean values of $(\Delta\alpha_S urrogate)$ obtained from surrogate data from original $(\Delta\alpha)$ values and presented this in Figure 10. In this figure, high values indicate a strong fat-tailed distribution's contribution to the multifractality, while low values indicate a low fat-tailed distribution's contribution to the multifractality. As seen from Figure 10 fat-tailed distribution's contribution to the multifractality is weakened in the period between 2019-11-22 19:01 and 2021-02-16 12:01.

Furthermore, to assess the change in long-range correlation's contribution to multifractality mean $(\Delta\alpha_S huffled)$ values obtained from shuffled data are subtracted from original $(\Delta\alpha)$ values. The results are illustrated in Figure 11. In this figure each value represents contribution level of long-range autocorrelation to multi-

Table 3 Multifractality regimes in USD/TRY exchange rate

$(\Delta\alpha)$	1. Regime	2. Regime	3. Regime
Mean	0.8966261	0.4876900	0.9534882
Variance	0.016327789	0.002052737	0.007530036

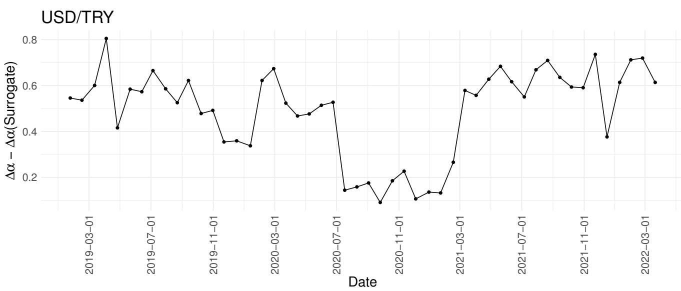

Figure 10 Change in the fat-tailed distribution's multifractality contribution

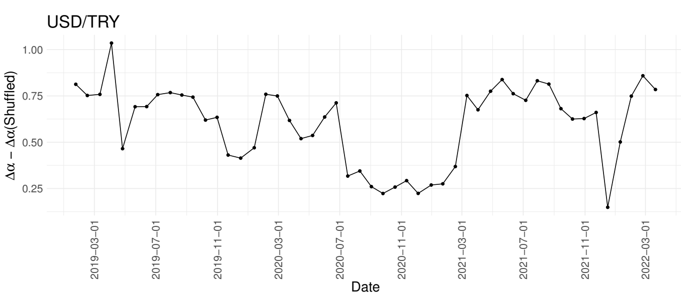

Figure 11 Change in the multifractality's long-range autocorrelation contribution

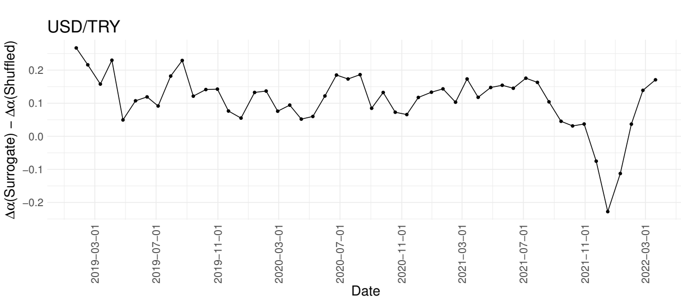

Figure 12 Examining the impacts of fat-tailed distribution and long-range autocorrelation on multifractality

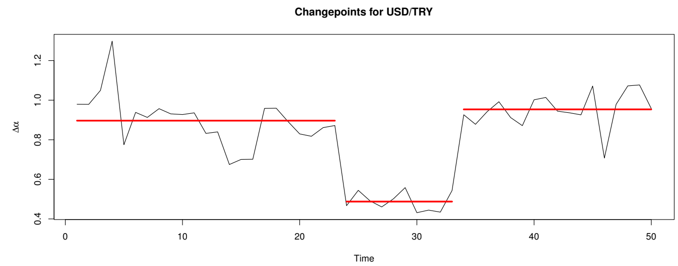

Figure 13 Change points in multifractality of USD/TRY exchange rate

fractality. This figure reveals that the long-range autocorrelation's contribution to multifractality is once again weakened between 2019-11-22 19:01 and 2021-02-16 12:01. Additionally, the contribution of long-range autocorrelation to multifractality shows a striking decline between 2021-04-27 15:01 and 2021-12-16 10:01.

Figure 12 is presented to compare the contributions of the fat-tailed distribution and long-range autocorrelation to the multifractality. This figure illustrates the difference between mean $(\Delta\alpha_S urrogate)$ value obtained from surrogate data and mean $(\Delta\alpha_S huffled)$ value obtained from shuffled data. Positive values in Figure 12 indicate that the long-range autocorrelation has a greater contribution to the multifractality than the fat-tailed distribution. Figure 12 reveals that, except for the time period from 2021-04-02 23:01 to 2022-01-10 06:01, long-range autocorrelation contributes more to multifractality than the fat-tailed distribution.

To detect change points and regimes in the level of multifractality in USD/TRY exchange rate binary segmentation algorithm is applied (Scott and Knott 1974; Sen and Srivastava 1975). We identified two change points in the 23rd and 33rd windows, resulting in three regimes. Results are presented in Table 3 and Figure 13.

MF-DFA of EUR/TRY Exchange Rate

Multifractal analysis results for EUR/TRY exchange rate covering whole period data are presented in Figure 14. Upper left panel of Figure 14 displays power-law cross-correlations between time scale s and fluctuation function $F_q(s)$ for q values equal to 10, 0 and -10. Upper right panel of Figure 14 reveals a varying Hurst exponent according to value of q, providing evidence for multifractality. Additionally, Hurst exponent for q = 2 is computed as 0.5637, slightly higher than 0.5, indicating a weakly persistent time series. Notably, this Hurst exponent value of 0.5637 is greater than the Hurst exponent value of 0.5268 for the USD/TRY exchange rate, indicating that the EUR/TRY exchange rate is more persistent than the USD/TRY exchange rate. As observed in the lower left panel of Figure 14 mass exponents are nonlinear, providing further evidence of multifractality. The lower right panel of Figure 14 displays the multifractal spectrum of the EUR/TRY exchange rate. Here positive value for $(\Delta\alpha)$ indicates evidence for multifractality.

Additionally, since α_0 value is higher than 0.5 there is persistent long-range correlations in the EUR/TRY exchange rate series. The left-skewed spectrum suggests that large fluctuations dominate the time series.

Similar to the USD/TRY exchange rate, to illustrate how multifractality level for the EUR/TRY exchange rate change over time Figure 15 and Figure 16 presented. In these figures black curves represents $(\Delta\alpha)$ values calculated from original data. As observed in these figures level of multifractality is maximum in the period between 2018-05-31 13:01 and 2019-01-23 22:01. After 2018-05-31 13:01 there is steady decline in multifractality until 2019-07-30 08:01. In the period between 2018-12-29 00:01 and 2019-10-31 01:01 slightly higher values and a horizontal trend are observed for multifractality. In the period between 2019-04-04 01:01 and 2020-09-02 13:01 multifractality remains relatively flat and low. After 2020-02-05 11:01 an upward trend is observed until 2021-10-30 01:00. However, in the period between 2021-04-27 15:01 and 2021-12-16 10:01 a collapse in the multifractality is observed.

To reveal contributions of long-range autocorrelation and fat-tailed distribution to multifractality 50 shuffled time series and 50 surrogate time series are generated in each time window. Mean values of $(\Delta\alpha_S urrogate)$ calculated from surrogate series in each time window is presented in Figure 15 with blue curve and mean values of $(\Delta\alpha_S huffled)$ calculated from shuffled series in each time window is also presented in Figure 16 with blue curve. In these figures error bars represent ± 1 standard deviations of $(\Delta\alpha_S urrogate)$ and $(\Delta\alpha_S huffled)$ values obtained from surrogate and shuffled series.

To demonstrate how contribution of fat-tailed distribution to multifractality is change over time Figure 17 is plotted. To obtain this figure mean $(\Delta\alpha_S urrogate)$ values obtained from surrogate series are subtracted from original $(\Delta\alpha)$ values. As observed in Figure 17 contribution of fat-tailed distribution to multifractality is highest in period between 2018-05-31 13:01 and 2019-01-23 22:01. Following this period, the fat-tailed distribution's contribution to multifractality decreased. After 2020-02-05 11:01 a steady increase in the fat-tailed distribution's contribution to multifractality is observed. However, between 2021-04-27 15:01 and 2021-12-16 10:01, there appears to have been a decline in the fat-tailed distribution's contribution to multifractality.

The change in the contribution of long-range autocorrelation to multifractality over time is presented in Figure 18. In this figure, it can be observed that the long-range correlation's contribution to multifractality is highest in the early period and gradually decreases untill 2019-05-21 8:01. After this date two relatively horizontal trend periods are distinguished. First horizontal trend period is between 2019-04-04 01:01 and 2021-03-11 04:01. Second horizontal trend period is between 2020-08-10 22:01 and 2021-10-30 01:00. Additionally, between 2021-04-27 15:01 and 2021-12-16 10:01, there is a collapse in the long-range correlation's contribution to multifractality. This period also corresponds to a decline in the contribution of long-range autocorrelation to multifractality.

Comparison between contributions of long-range autocorrelation and fat-tailed distribution to multifractality is presented in Figure 19. Positive values in this figure indicate that the long-range autocorrelation has a greater contribution to the multifractality than the fat-tailed distribution. Figure 19 remains relatively flat and have positive values until the date 2021-11-23 18:01. This indicates that long-range autocorrelation has been the primary source of multifractality up to this point. However negative values are observed in this figure during the period between 2021-04-27 15:01 and 2022-01-10 06:01. These negative values indicate that the fat-tailed distribution now contributes more to multifractality than long-range autocorrelation does.

To detect change points and regimes in the level of multifractality in EUR/TRY exchange rate, a binary segmentation algorithm is applied (Scott and Knott 1974; Sen and Srivastava 1975). We detected three change points in 4th, 13th and 34th windows, resulting in four regimes. Results are presented in Table 4 and Figure 20.

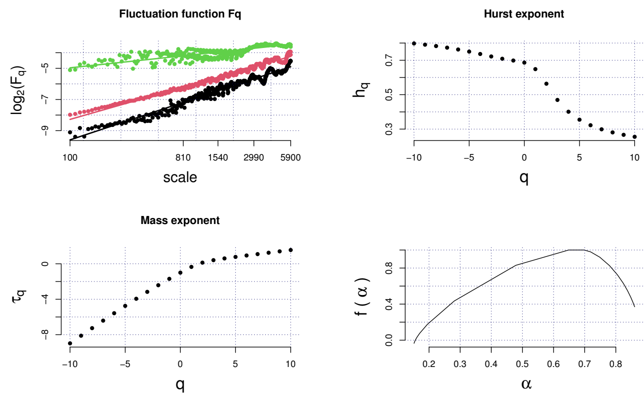

Figure 14 Whole period multifractality of EUR/TRY exchange rate

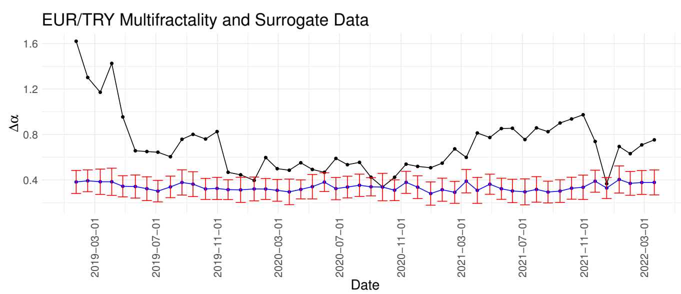

Figure 15 Change in $(\Delta\alpha)$ calculated from original data and change in $(\Delta\alpha)$ calculated from surrogate data

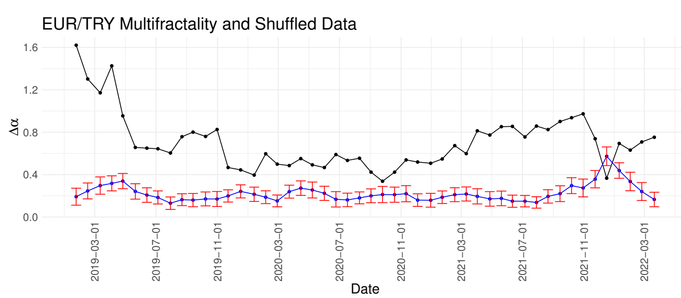

Figure 16 Change in $(\Delta\alpha)$ calculated from original data and change in $(\Delta\alpha)$ calculated from shuffled data

MF-DCCA of USD/TRY and EUR/TRY Exchange Rates

In this stage, the EUR/TRY and USD/TRY exchange rates are studied using multifractal detrended cross-correlation analysis. Firstly, results from whole dataset are presented.

Table 4 Multifractality regimes in EUR/TRY exchange rate

$(\Delta\alpha)$	1. Regime	2. Regime	3. Regime	4. Regime
Mean	1.3802250	0.7392889	0.5069429	0.7771875
Variance	0.027409312	0.011176401	0.005778175	0.019095466

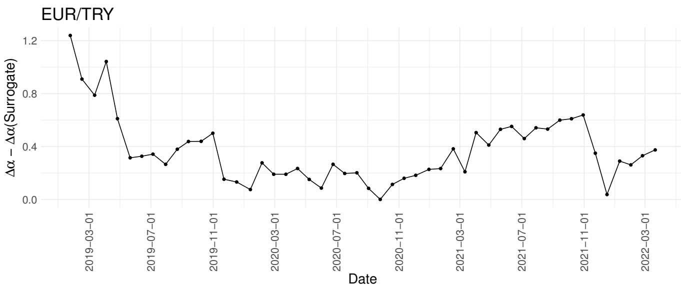

Figure 17 Change in the fat-tailed distribution's multifractality contribution

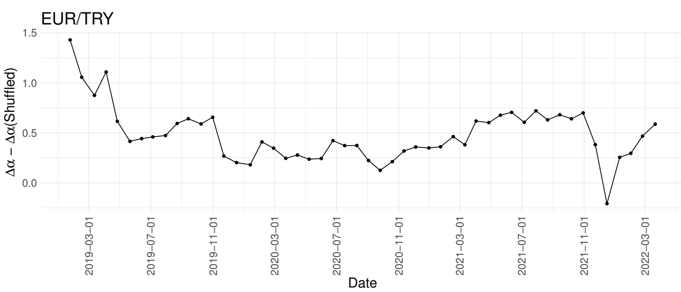

Figure 18 Change in the multifractality's long-range autocorrelation contribution

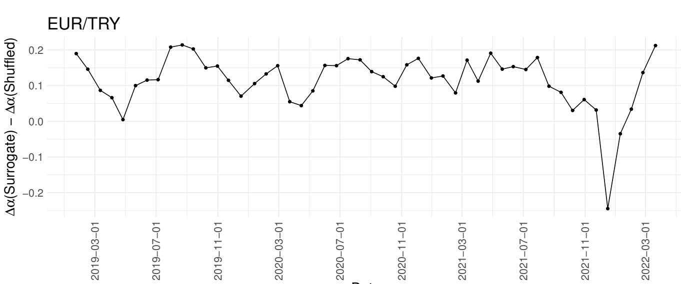

Figure 19 Examining the impacts of fat-tailed distribution and long-range autocorrelation on multifractality

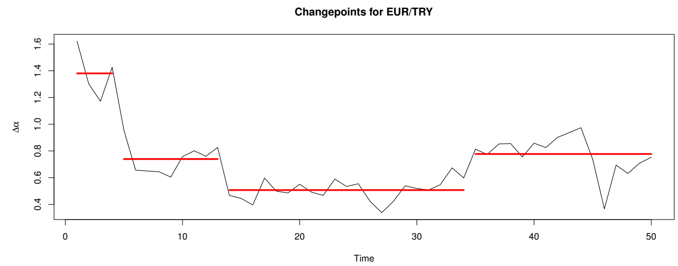

Figure 20 Change points in multifractality of EUR/TRY exchange rate

In Figure 21 relationships between time scale s and fluctuation function for q values equal to 10, 0 and -10 are plotted. The linearity of these points indicates that there is a power-law relationship between these two values. In Figure 22 generalized cross-correlation exponent between the two exchange rates are presented. In this figure, since generalized cross-correlation exponents are dependent on q values, it suggests that the cross-correlation between the exchange rates is multifractal. Additionally, for logarithm difference data, generalized cross-correlation exponent for q=2 is computed as 0.5393, slightly higher than 0.5, indicating that the cross-correlated series has a weak persistent structure. The multifractal spectrum for cross-correlation between USD/TRY and EUR/TRY exchange rates is shown in Figure 23. In this figure it can be observed that width of the multifractal spectrum $(\Delta\alpha)$ is positive, providing further evidence for multifractality in the cross-correlation. Moreover, since α_0 value is greater than 0.5, it indicates the presence of persistent long-range correlations.

In MF-DCCA level of correlation multifractality between two exchange rate series can be measured with the multifractal spectrum's width $(\Delta\alpha)$. In this part we demonstrated how multifractality level of cross correlation between the two exchange rates and source of multifractality change over time in a rolling window framework.

Long-range autocorrelation and fat-tailed distribution are the two sources of multifractality for cross correlation. To measure the contribution of these two sources shuffled time series and surrogate time series are utilized. However, in MF-DCCA, since there must be two series, 50 pairs of surrogate series and 50 pairs of shuffled series are generated for each time window. The $(\Delta\alpha_{S}urrogate)$ values obtained from pairs of surrogate series are presented in Figure 24 and $(\Delta\alpha_{S}huffled)$ values obtained from pairs of shuffled series are presented in Figure 25 with blue curves. In these figures red error bars represent ±1 standard deviation.

When Figure 24 and Figure 25 are examined, a downward trend in multifractality for original series is observed in the period between 2018-05-31 13:01 and 2021-01-22 20:01. In the period between 2020-06-01 22:01 and 2021-04-02 22:01 there is a rapid rise in multifractality. Also, in the period between 2020-08-10 22:01 and 2021-10-30 01:01 a gradual increase in multifractality is observed. However, there is a collapse in multifractality during the period between 2021-04-27 15:01 and 2021-12-16 10:01.

Figure 26 is presented to examine how the fat-tailed distribution's contribution to the multifractality changes over time. Additionally, Figure 27 is presented to reveal how the contribution of long-range correlation to multifractality changes over time. These two figures display similar pattern. In both Figure 26 and Figure 27 there are significant collapse in contributions to multifractality during the period between 2021-04-27 15:01 and 2021-12-16 10:01.

To compare fat-tailed distribution's and long-range autocorrelation's contributions to multifractality Figure 28 is presented. Positive values in this figure indicate that the long-range autocorrelation has a greater contribution to the multifractality than the fat-tailed distribution. When examining this figure, a negative value is observed for the period between 2021-04-27 15:01 and 2021-12-16 10:01. This negative value suggests that the fat-tailed distribution's contribution to multifractality has surpassed the long-range correlation's contribution. Apart from this period, dominant source of multifractality is long-range correlation.

To identify change points and regimes in the level of cross correlation multifractality between exchange rates a binary segmentation algorithm is applied (Scott and Knott 1974; Sen and Srivastava 1975). We detected seven change points in 5th, 13th, 16th, 23th, 34th, 42th, and 47th windows, resulting in eight regimes. Results are presented in Table 5 and Figure 29.

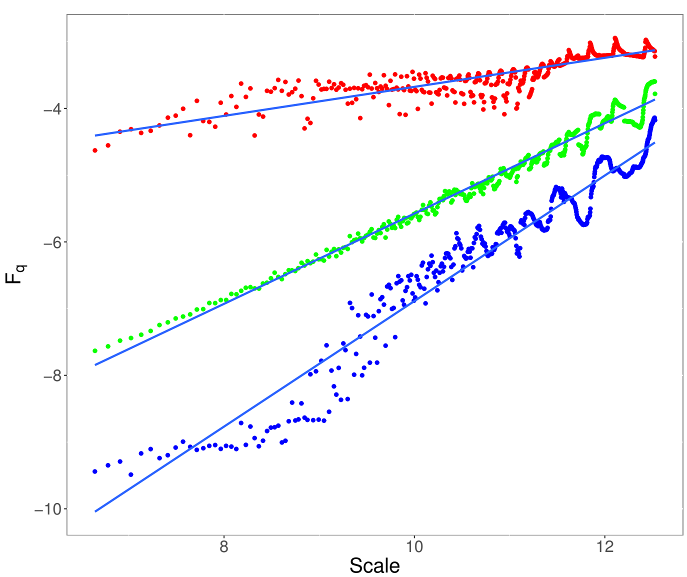

Figure 21 Fluctuation function for cross correlation

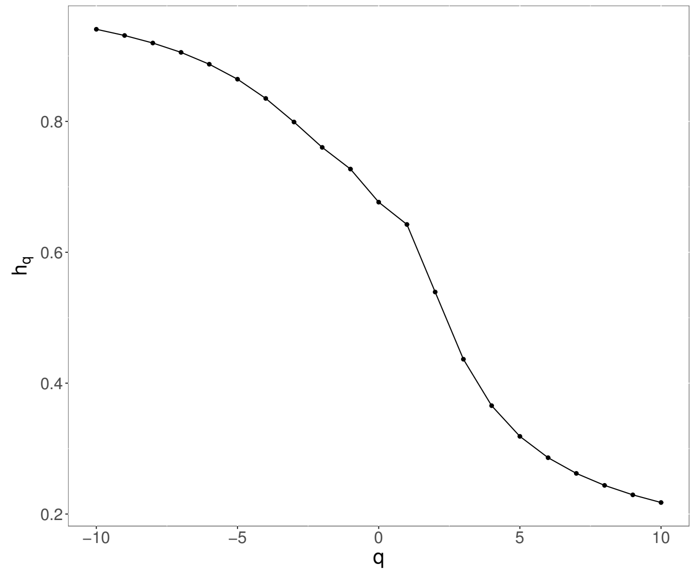

Figure 22 Generalized cross-correlation exponent between EUR/TRY and USD/TRY exchange rates

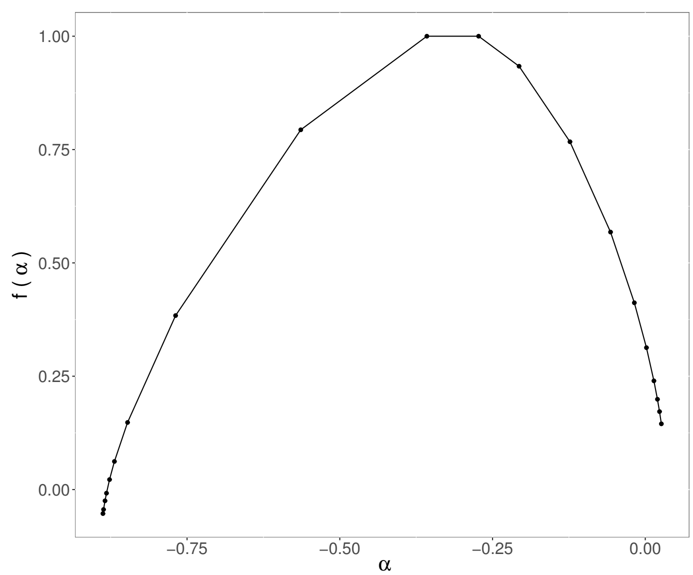

Figure 23 Multifractal spectrum for cross-correlation between EUR/TRY and USD/TRY exchange rates

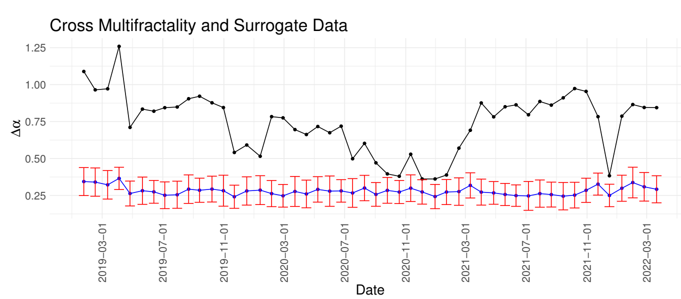

Figure 24 Change in $(\Delta\alpha)$ calculated from original data and change in $(\Delta\alpha)$ calculated from surrogate data

Table 5 Multifractality regimes in EUR/TRY exchange rate

$(\Delta\alpha)$	1. Regime	2. Regime	3. Regime	4. Regime	5. Regime	6. Regime	7. Regime	8. Regime
Mean	0.9991	0.8620	0.5493	0.7182	0.4773	0.8532	0.7763	0.8518
Variance	3.22e-02	1.11e-03	1.00e-03	1.86e-03	1.13e-02	1.68e-03	4.50e-02	9.31e-05

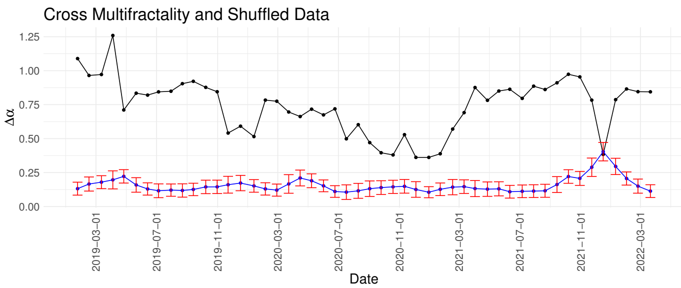

Figure 25 Change in $(\Delta\alpha)$ calculated from original data and change in $(\Delta\alpha)$ calculated from shuffled data

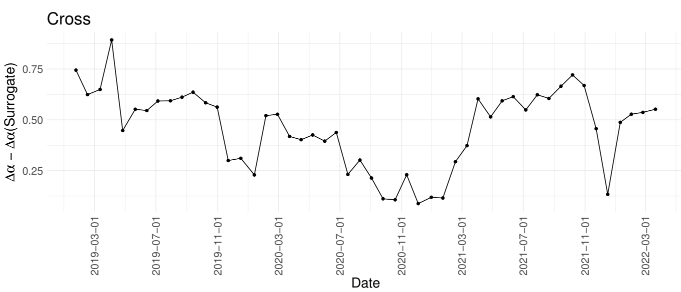

Figure 26 Change in the fat-tailed distribution's multifractality contribution

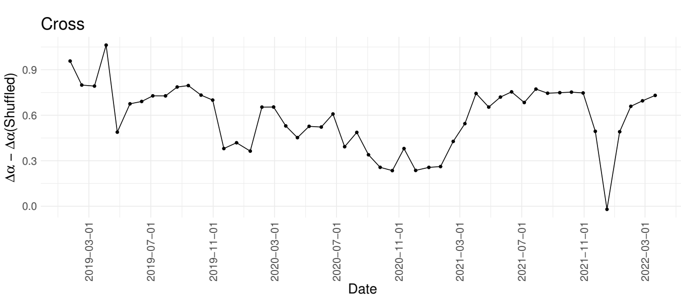

Figure 27 Change in the multifractality's long-range autocorrelation contribution

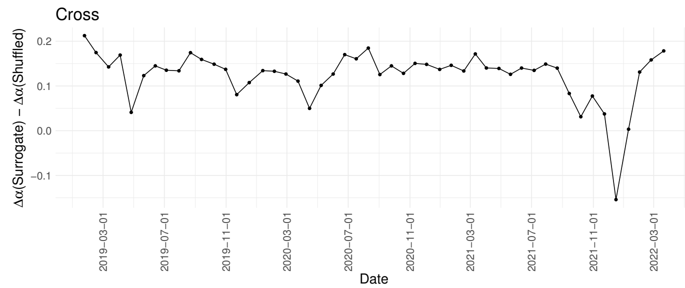

Figure 28 Examining the impacts of fat-tailed distribution and long-range autocorrelation on multifractality

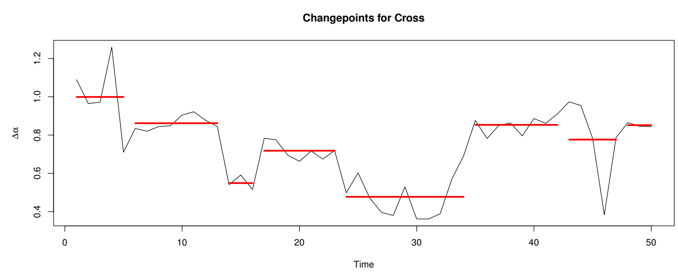

Figure 29 Change points in cross-correlation multifractality

CONCLUSION

A multifractal system is a general type of fractal system in which the system cannot be adequately described by a single exponent. In the literature, it has been demonstrated that many systems from different fields exhibit multifractality. In this study individual and cross correlation multifractality of EUR/TRY and USD/TRY exchange rates are explored with MF-DFA and MF-DCCA methodologies. In the analysis both whole period data and rolling window data are utilized. Whole period analyses reveal that the two exchange rates as well as correlation between the exchange rates are multifractal.

Multifractality in these exchange rates implies presence of inefficiencies which can be exploited by investors. These inefficiencies can be exploited by investors who are able to identify them and trade accordingly. For example, investors who believe that the volatility of a particular exchange rate is about to increase may choose to sell that currency, while investors who believe that the volatility is about to decrease may choose to buy that currency. Advanced trading algorithms can be designed to detect and act upon multifractal patterns in exchange rates. Multifractality can create arbitrage opportunities where an asset's price differs on different time scales or in different markets.

Arbitrageurs can profit from these price differentials by buying low and selling high. By using rolling window method, we illustrated how multifractal properties of the exchange rates change over time. As indicated by $(\Delta\alpha)$ values multifractality levels of the exchange rates change over time and higher multifractal levels implies higher complexity, higher risks and more violent fluctuations. Additionally, we examined how contributions of long-range autocorrelation and fat-tailed distribution to multifractality change over time. Shape of the singularity spectra for exchange rates suggests that large fluctuations are more dominant in EUR/TRY exchange rate than USD/TRY exchange rate.

Our results suggest that long-range autocorrelation's contribution to multifractality is higher than the fat-tailed distribution's contribution except during the period between 2021-04-27 15:01 and 2021-12-16 10:01. Therefore, dominant source of multifractality is the long-range autocorrelation. However, when the multifractality of the two exchange rates are examined a collapse in the multifractality is observed during in the period between 2021-04-27 15:01 and 2021-12-16 10:01. Moreover, in this period, contribution of fat-tailed distribution to multifractality become dominant. As evident from Figure 1 and Figure 2, during this period, both USD/TRY and EUR/TRY exchange rates exhibit significant instability, and there is substantial government intervention in the foreign exchange market. Since USD/TRY and EUR/TRY exchange rates are multifractal and characterized by autocorrelation, non-linearity, and long memory (persistence), traditional efficient markets hypothesis which assumes normal distribution and linearity is not appropriate for these exchange rates.

The implications of multifractality of USD/TRY and EUR/TRY exchange rates are significant and can impact various areas within finance, economics, and decision-making. Multifractal behavior suggests that exchange rate movements are not only random but also characterized by irregular patterns and fluctuations across different time scales. This complexity can lead to unexpected and extreme price movements, which are important considerations for risk assessment and management. Multifractality for these exchange rates implies that the volatility of these exchange rates can vary depending on the time scale being considered. This makes it difficult to predict the future volatility of these exchange rates, and it can also make it difficult to trade these exchange rates profitably. Also, the multifractality of these exchange rates suggests that they are not efficient markets. This means that there are opportunities to make profits by exploiting the inefficiencies in these markets.

However, these opportunities are often difficult to find and exploit, and they can also be risky. Multifractal analysis can provide insights for traders and algorithmic trading systems. By understanding the non-linear dynamics of exchange rates, traders can develop strategies that adapt to the multifractal nature of the market, potentially improving trading outcomes. Traditional linear models may not fully capture the complexities of multifractal behavior. The findings from multifractal analysis can lead to the development of more sophisticated models that better reflect the true nature of exchange rate movements. Multifractal behavior can affect portfolio diversification strategies. Investors need to consider how different assets, including USD/TRY and EUR/TRY exchange rates, interact and exhibit multifractal patterns to effectively manage risk and optimize returns. Multifractality in exchange rates can have policy implications for central banks and governments. Understanding the intricate and non-linear behaviors of currencies can inform decisions related to monetary policy, trade agreements, and economic interventions. The recognition of multifractal behavior can influence how financial markets are regulated. Regulators might need to consider the implications of non-linear and complex behaviors for market stability and investor protection.

In the future studies how multifractality and its sources evolve over longer time periods can be investigated. Comparative analysis with other currency pairs or financial assets can be conducted to identify commonalities and differences in multifractal behavior. The impact of external factors, such as geopolitical events, economic policies, or global financial crises, on the multifractality of exchange rates can be explored. Machine learning techniques to enhance the prediction and forecasting capabilities based on multifractal properties can be incorporated.

Availability of data and material

Not applicable.

Conflicts of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

LITERATURE CITED

Ashkenazy, Y., P. C. Ivanov, S. Havlin, C.-K. Peng, A. L. Goldberger, *et al.*, 2001 Magnitude and sign correlations in heartbeat fluctuations. Physical Review Letters **86**: 1900.

Blesić, S., S. Milošević, D. Stratimirović, and M. Ljubisavljević, 1999 Detrended fluctuation analysis of time series of a firing fusimotor neuron. Physica A: Statistical Mechanics and its Applications **268**: 275–282.

Buldyrev, S., N. Dokholyan, A. Goldberger, S. Havlin, C.-K. Peng, *et al.*, 1998 Analysis of dna sequences using methods of statistical physics. Physica A: Statistical Mechanics and its Applications **249**: 430–438.

Bunde, A., S. Havlin, J. W. Kantelhardt, T. Penzel, J.-H. Peter, *et al.*, 2000 Correlated and uncorrelated regions in heart-rate fluctuations during sleep. Physical review letters **85**: 3736.

Caraiani, P. and E. Haven, 2015 Evidence of multifractality from cee exchange rates against euro. Physica A: Statistical Mechanics and its Applications **419**: 395–407.

Chen, S.-P. and L.-Y. He, 2010 Multifractal spectrum analysis of nonlinear dynamical mechanisms in china's agricultural futures markets. Physica A: Statistical Mechanics and its Applications **389**: 1434–1444.

Dashtian, H., G. R. Jafari, M. Sahimi, and M. Masihi, 2011 Scaling, multifractality, and long-range correlations in well log data of large-scale porous media. Physica A: Statistical Mechanics and its Applications **390**: 2096–2111.

Fama, E. F., 1965 The behavior of stock-market prices. The journal of Business **38**: 34–105.

Gneiting, T., H. Ševčíková, and D. B. Percival, 2012 Estimators of fractal dimension: Assessing the roughness of time series and spatial data. Statistical Science pp. 247–277.

Gülbaş, E. and Ü. Gazanfer, 2013 Multifractal analysis of the dynamics of turkish exchange rate. International Journal of Economics and Finance Studies **5**: 96–107.

Han, C., Y. Wang, and Y. Ning, 2019 Comparative analysis of the multifractality and efficiency of exchange markets: Evidence from exchange rates dynamics of major world currencies. Physica A: Statistical Mechanics and its Applications **535**: 122365.

He, L.-Y. and S.-P. Chen, 2010a Are crude oil markets multifractal? evidence from mf-dfa and mf-ssa perspectives. Physica A: Statistical Mechanics and its Applications **389**: 3218–3229.

He, L.-Y. and S.-P. Chen, 2010b Are developed and emerging agricultural futures markets multifractal? a comparative perspective.

Physica A: Statistical Mechanics and its Applications **389**: 3828–3836.
Hu, K., P. C. Ivanov, Z. Chen, P. Carpena, and H. E. Stanley, 2001 Effect of trends on detrended fluctuation analysis. Physical Review E **64**: 011114.
Hurst, H. E., 1951 Long-term storage capacity of reservoirs. Transactions of the American society of civil engineers **116**: 770–799.
Hurst, H. E., 1957 A suggested statistical model of some time series which occur in nature. Nature **180**: 494–494.
Jafari, G. R., P. Pedram, and L. Hedayatifar, 2007 Long-range correlation and multifractality in bach's inventions pitches. Journal of Statistical Mechanics: Theory and Experiment **2007**: P04012.
Kantelhardt, J. W., E. Koscielny-Bunde, H. H. Rego, S. Havlin, and A. Bunde, 2001 Detecting long-range correlations with detrended fluctuation analysis. Physica A: Statistical Mechanics and its Applications **295**: 441–454.
Kantelhardt, J. W., D. Rybski, S. A. Zschiegner, P. Braun, E. Koscielny-Bunde, *et al.*, 2003 Multifractality of river runoff and precipitation: comparison of fluctuation analysis and wavelet methods. Physica A: Statistical Mechanics and its Applications **330**: 240–245.
Kantelhardt, J. W., S. A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, *et al.*, 2002 Multifractal detrended fluctuation analysis of nonstationary time series. Physica A: Statistical Mechanics and its Applications **316**: 87–114.
Li, J., X. Lu, and Y. Zhou, 2016 Cross-correlations between crude oil and exchange markets for selected oil rich economies. Physica A: Statistical Mechanics and its Applications **453**: 131–143.
Lim, K.-P. and R. Brooks, 2011 The evolution of stock market efficiency over time: A survey of the empirical literature. Journal of economic surveys **25**: 69–108.
Liu, Y., P. Gopikrishnan, H. E. Stanley, *et al.*, 1999 Statistical properties of the volatility of price fluctuations. Physical review e **60**: 1390.
Lu, X., J. Li, Y. Zhou, and Y. Qian, 2017 Cross-correlations between rmb exchange rate and international commodity markets. Physica A: Statistical Mechanics and its Applications **486**: 168–182.
Ma, F., Y. Wei, and D. Huang, 2013a Multifractal detrended cross-correlation analysis between the chinese stock market and surrounding stock markets. Physica A: Statistical Mechanics and its Applications **392**: 1659–1670.
Ma, F., Y. Wei, D. Huang, and L. Zhao, 2013b Cross-correlations between west texas intermediate crude oil and the stock markets of the bric. Physica A: Statistical Mechanics and its Applications **392**: 5356–5368.
Ma, F., Q. Zhang, C. Peng, and Y. Wei, 2014 Multifractal detrended cross-correlation analysis of the oil-dependent economies: Evidence from the west texas intermediate crude oil and the gcc stock markets. Physica A: Statistical Mechanics and its Applications **410**: 154–166.
Mandelbrot, B. B., 1982 *The fractal geometry of nature*, volume 1. WH freeman New York.
Matia, K., Y. Ashkenazy, and H. E. Stanley, 2003 Multifractal properties of price fluctuations of stocks and commodities. Europhysics letters **61**: 422.
Movahed, M. S., G. Jafari, F. Ghasemi, S. Rahvar, and M. R. R. Tabar, 2006 Multifractal detrended fluctuation analysis of sunspot time series. Journal of Statistical Mechanics: Theory and Experiment **2006**: P02003.
Peng, C.-K., S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, *et al.*, 1994 Mosaic organization of dna nucleotides. Physical review e **49**: 1685.
Peters, E. E., 1994 *Fractal market analysis: applying chaos theory to investment and economics*, volume 24. John Wiley & Sons.
Schmitt, F., D. Schertzer, and S. Lovejoy, 1999 Multifractal analysis of foreign exchange data. Applied stochastic models and data analysis **15**: 29–53.
Scott, A. J. and M. Knott, 1974 A cluster analysis method for grouping means in the analysis of variance. Biometrics pp. 507–512.
Sen, A. and M. S. Srivastava, 1975 On tests for detecting change in mean. The Annals of statistics pp. 98–108.
Stošić, D., D. Stošić, T. Stošić, and H. E. Stanley, 2015 Multifractal analysis of managed and independent float exchange rates. Physica A: Statistical Mechanics and its Applications **428**: 13–18.
Talkner, P. and R. O. Weber, 2000 Power spectrum and detrended fluctuation analysis: Application to daily temperatures. Physical Review E **62**: 150–160.
Tanna, H. and K. Pathak, 2014 Multifractality due to long-range correlation in the l-band ionospheric scintillation s 4 index time series. Astrophysics and Space Science **350**: 47–56.
Telesca, L., V. Lapenna, and M. Macchiato, 2004 Mono-and multifractal investigation of scaling properties in temporal patterns of seismic sequences. Chaos, Solitons & Fractals **19**: 1–15.
Wang, Y., Y. Wei, and C. Wu, 2011a Analysis of the efficiency and multifractality of gold markets based on multifractal detrended fluctuation analysis. Physica A: Statistical Mechanics and its Applications **390**: 817–827.
Wang, Y., Y. Wei, and C. Wu, 2011b Detrended fluctuation analysis on spot and futures markets of west texas intermediate crude oil. Physica A: Statistical Mechanics and its Applications **390**: 864–875.
Xie, C., Y. Zhou, G. Wang, and X. Yan, 2017 Analyzing the cross-correlation between onshore and offshore rmb exchange rates based on multifractal detrended cross-correlation analysis (mf-dcca). Fluctuation and Noise Letters **16**: 1750004.
Yen, G. and C.-f. Lee, 2008 Efficient market hypothesis (emh): past, present and future. Review of Pacific Basin Financial Markets and Policies **11**: 305–329.
Yue, P., H.-C. Xu, W. Chen, X. Xiong, and W.-X. Zhou, 2017 Linear and nonlinear correlations in the order aggressiveness of chinese stocks. Fractals **25**: 1750041.
Zhuang, X., Y. Wei, and F. Ma, 2015 Multifractality, efficiency analysis of chinese stock market and its cross-correlation with wti crude oil price. Physica A: Statistical Mechanics and its Applications **430**: 101–113.
Zhuang, X., Y. Wei, and B. Zhang, 2014 Multifractal detrended cross-correlation analysis of carbon and crude oil markets. Physica A: Statistical Mechanics and its Applications **399**: 113–125.
Zunino, L., A. Figliola, B. M. Tabak, D. G. Pérez, M. Garavaglia, *et al.*, 2009 Multifractal structure in latin-american market indices. Chaos, Solitons & Fractals **41**: 2331–2340.

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