The Types of Examples Teachers Use in Teaching Function Concept*

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Abstract

Although the concept of function is one of the important subject of mathematics courses, many studies show that learners have difficulty in this subject. The basis of these difficulties is that most students encode the prototypes of examples, representations and algebraic rules used for explaining function concept, which match with their own thinking, instead of the definition of function at learning the functions. Therefore, the examples which teachers present in the functions have an important roles in students' learning on this subject. In this study, it is aimed to determine the types of examples which two teachers use in the functions. In the scope of the study, it is made use of unstructured observations and informal interviews. The theoretical framework which Bills et al (2006) use in classification of examples, is utilized in analyzing the data. In the findings, it is found that the teachers use generic examples and non-examples in their lessons, despite that they don't use counter examples.

Keywords: *Example, Types of Examples, Functions*

Öğretmenlerin Fonksiyonlar Konusunda Kullandıkları Örnek Türleri

Öz

Fonksiyon kavramı matematik dersinin önemli konularından biri olmasına rağmen yapılan birçok araştırma, bu konunun öğrenilmesinde öğrencilerin zorluk yaşadıklarını göstermektedir. Bu zorlukların temelini öğrencilerin önemli bir çoğunluğunun fonksiyonlar konusunu öğrenirken fonksiyonun tanımını değil, fonksiyonu açıklamak için kullanılan örnekler, temsiller ve cebirsel kurallar arasından kendi düşünce yapılarıyla uyuşan prototipleri zihinlerine kodlamaları oluşturmaktadır. Bu nedenle öğretmenlerin fonksiyon konusunda sundukları örnekler öğrencilerin fonksiyonlar konusunu anlamalarında oldukça önemli bir yere sahiptir. Bu çalışmada, iki matematik öğretmenin fonksiyonlar konusunda kullandığı örnek türlerinin belirlenmesi amaçlanmıştır. Çalışma kapsamında yapılandırılmamış gözlemlerden ve informal mülakatlardan yararlanılmıştır. Verilerin analizinde Bills vd. (2006)' nin, örnekleri sınıflandırmak için kullanmış oldukları teorik çatıdan faydalanılmıştır. Elde edilen bulgularda öğretmenlerin derslerinde jenerik örneklerden ve örnek olmayan örneklerden sıklıkla yararlandıkları, buna karşın derslerinde karşıt örneklere hiç vermedikleri tespit edilmiştir.

Anahtar Kelimeler: *Örnek, Örnek Türleri, Fonksiyon*

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Introduction

Function concept is one of the most basic concepts in mathematics. So, besides being the basis of the mathematics curriculum, function concept has a duty as ensuring integrity between the mathematics subjects (Yerushalmy and Schwarz, 1993). Many subjects in primary education curriculum have the features of preparation to function concept (patterns, rate-ratio, etc.) and later the concept is studied in details in high school years (Özdemir-Erdoğan, Erdoğan and Yanık, 2012). Due to this importance, functions gained the interest of researchers and many studies have been done about this subject so far. In these studies, it is seen that even the students with good knowledge of mathematics seem to lack the knowledge of functions and functions is a difficult subject to understand for students (Breidenbach et al, 1992; Carlson, 1998; Özdemir-Erdoğan et al, 2012; Tall and Bakar, 1992), furthermore, students have many misconceptions of functions (Dubinsky and Harel, 1992; Vinner, 1983).

Students have some difficulties on the function concept, because they place the examples about functions in their minds and have problems at understanding the functions thought behind these examples (Bayazıt and Gray, 2004; Vinner, 1983). At the same time when the studies are examined, it seems that students focus on the visual and formal features related to function concept rather than the meaning of function concept (Bayazıt and Aksoy, 2010). While learning function subject, many students encode the first examples which match their frame of minds from the examples, representations, algebraic uses rather than the meaning of function concept. While solving the problems about functions, students start to think with the examples in their minds rather than the definition of function and thus they may be unsuccessful. It seems that the reasons of these difficulties are due to the fact that students don't learn the definition of function but they encode the first examples which match their frame of minds from the examples, representations, algebraic rules used to explain functions (Vinner, 1983). Thus, the examples teachers use to teach functions are of vital importance in students' learning function concept.

Theoretical Framework

Under this title, the concept of example and the types of example are mentioned briefly since it is thought that it will contribute to understanding the results of the research and interpreting the data obtained within the scope of research.

Concept of example and types of example

Throughout the years, as well as suggesting many different ideas about how to teach mathematics better, one of the important common points of these different ideas is the fact that the examples are strong pedagogical instruments to teach mathematics (Watson and Mason, 2002). Because people tend to configure the concepts and relations through the examples and this case requires using the examples efficiently

in learning environments (Bills et al, 2006; Zazkis and Chernoff, 2008; Zodik and Zaslavsky, 2008; Watson and Mason, 2002). Especially examples support us to understand the concepts better by turning the abstract concepts into concrete structures (Gökbulut, 2010). Due to the importance of examples in teaching and learning mathematics, researchers have reported to make various classifications by studying the definition and features of the concept of example. When literature is investigated, we see that there have been many definitions related to the concept of example. In these definitions, Watson and Mason (2005) expressed the concept of example in mathematics as everything which is used to describe the principles and concepts; Tsamir, Tirosh and Levenson (2008) defined it as description of the definitions of the concepts or features. Gökbulut and Ubuz (2013) named it as the explanations used to express the general principals, the samples or examples of concepts. For instance, taking the opinions about the definitions into consideration, the concept of example in mathematics is defined as the special cases to explain the definitions or the principals of the concepts.

Examples help students' knowledge of concepts be more meaningful by making the definitions more meaningful, classifying the mathematical expressions and associating the similar situations of these expressions (Watson and Mason, 2002). It is difficult for just one example to always express all the meanings of a concept (Lakoff, 1987). In this sense, examples differ in terms of their intended use.

Firstly, Polya (1973; cited in Mittal& Paris, 1993), Michener (1978) and later Bills and et al (2006) classified the example types in accordance with their intended use (Table 1).

Table 1.
Classification of the Example Types

Polya (1973)	*Leading example* Simple examples used for expressing the concept, or the features of the concept *Suggestive example* Examples which help in providing the qualities of the concept and at the same time in putting forward the boundaries of the concept more clearly *Counter example* Examples used for disproving any assumption
Michener (1978)	*Introduction example* Examples which help in supporting the basic definitions and results, create a simple perception about the concept for the learner *Reference Example* Standart examples mentioned several times in development of a concept, a result or a theory *Model Example* Examples summarizing the general case of the concept *Counter example* Examples used for sharpening the boundaries of the distinction between the concepts and for showing that the results can not be generalized all the time

Bills et al (2006)	*Generic example* Examples that clearly show the overall situation of the concept *Non Example* Examples used to express the equivalent of a concept (highlighting the features they don't have), describe the limits, express the conditions in a theorem *Counter Example* Examples used to demonstrate that an idea or a claim is false

Considering the importance of examples at mathematics education, as seen in Table 1, examples are classified by different researchers in different ways depending on their uses. Bills et al (2006) explained in their work that although examples were named under different sample names by different researchers, in the most general sense they could be grouped under three specific descriptive names as generic, counter and non-examples. When the samples of Bills et al (2006) are analyzed, it is seen that generic example is same as the model example of Michener and counter example is found to be present in all researchers. While all of the example types of other researchers studies present the examples belong to the concept, it is significant that Bill et al (2006) give place to the examples types which belong to the concept and also which don't. Because it is important to study both examples which belong to the concept and which don't, besides the visual presentations, impressions and experiences to create conceptual images (Özyürek, 1984; Senemoğlu, 1997; Tsamir, Tirosh and Leverson, 2008). If students see the examples that don't belong to the concept beside the one belong to the concept, they can understand better the qualities that define the concept and differentiate the taught concept from the other concepts (Gökkurt, 2014). This may decrease the number of possible misconceptions that may occur in students' minds by making students have healthier concept image (Gökbulut and Ubuz, 2013). In the framework of this research, example types that teachers use are decided to be analyzed according to Table 3 which was developed by Bills et al (2006).

Aim of the Study

There are many studies that examine the challenges and misconceptions that students have about functions subject (Breidenbach et al, 1992; Bayazıt and Gray, 2004; DeMarois and Tall, 1996; Güler et al, 2015; Ural, 2006; Vinne, 1983). When these researches are examined, it has been observed that students pay much more attention to representations and visual elements of this concept rather than the function concept. Vinner (1983) stated that students encode the first examples of the concept in their minds and this causes misconceptions, and this situation may cause students have conceptual images incomplete or wrong (Bayazıt and Aksoy, 2010). Therefore, it is important how teachers use the examples which are of vital importance in creating students' conceptual images.

With this study, it is aimed to determine the example types teachers use in teaching function concept considering that the examples teachers offer in the functions subject are quite important in students' understanding the subject.

Method

In this research, it is aimed to examine the example types used in functions subject by the 9th grade teachers. For this purpose, determining what the example types 9th grade mathematics teachers use are, includes questions oriented to 'case study'. This method is preferred because of the idea that teachers participated in the research can give detailed information about the example types they use while teaching functions. Because case study is used to reveal and report the details about individual, group or community (Libarkin and Kurdziel, 2002). Case study is also considered appropriate for the research (Libarkin and Kurdziel, 2002; Merriam, 1998) due to the fact that it allows to deal with the process of the determination of the examples teachers' use, in a short time and examining them deeply.

Sample of Research

Two mathematics teachers (1 Male and 1 Female) working at an Anatolian High School in Trabzon form the sample of this research. Teachers were selected by the purposive sampling method. The teachers were selected on the basis of their willingness to participate in the study. The professional experience of the teachers participating in the study was between 12-24 years. Especially teachers with much time of service were preferred because it was thought that the examples and example types teachers use due to their professional experience would be different. T1 and T2 were involved in different types of schools before this school. Each of the educational institutions the teachers who participated in the study graduated from, information on gender and professional experience are given in Table 2. Teachers participating in the study are coded as T1 and T2 in order to keep the names secret.

Table 2.
Demographic Characteristics of the Participants Involved in the Study

Type of School	Participants	Gender	Undergraduate Graduation	Education Level	Work Experience
Anatolian High School	T1	F	Faculty of Education	Bachelor's degree	24
	T2	M	Faculty of Arts and Sciences	Bachelor's degree	16

Surveyed teachers were conducted informal interviews before the lessons in order to know them better. In these conversations, it was aimed to obtain information about the types of examples these teachers use. T1 teacher stated that she often uses examples of the concept and also examples which don't belong to the concept in her lessons. She also stated that she always interacts with her students during the lessons, and the number of examples she uses depends on the performance of her students. T2 teacher expressed that he creates his questions in accordance with the questions in the course books and he uses examples of the concept more than the ones which don't belong to the concept in his lessons.

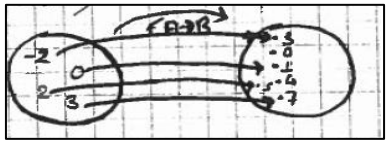
Data Collection Tools

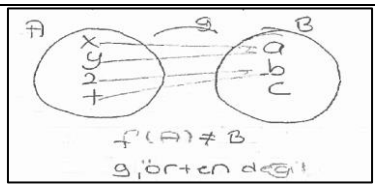
Unstructured observation was used to determine the types of examples 9th grade mathematics teachers use in functions subject and after observations, informal interviews were used to determine the example types used. The researcher didn't interfere in the lessons, she just made observation. Examples used in the classroom by the teachers and the reasons of using these examples were observed in detail with unstructured observation and informal interviews, respectively. In these interviews, the teachers were asked the objectives of use of the examples in their lessons. So this contributes to the correct analysis of the examples by taking teachers' views of why they prefer these examples.

Data Analysis

In the analysis of the collected data, example types which were developed by Bills et al (2006) as the theoretical framework for classification of the examples were used. Descriptive analysis technique was used because the types of examples in the research were predetermined. According to this, while the data obtained from the observation and informal interviews were being analyzed, the examples used by the teachers were analyzed individually and the interviews about the intended uses of these examples made after the lessons, were analyzed. The researchers classified the examples that teachers used in functions subject taking the characteristics of the example types shown in Table 3 into consideration. To ensure the reliability of this classification, another researcher with a Ph. D. in mathematics education was informed about the example types and then he was asked to classify the same examples. As a result of the classification, 87% compliance was identified between the researchers and the other mathematics educator. The resulting differences are discussed and examined again by the researchers. The classification and the example types belonging to this classification are shown in Table 3.

Table 3.
Example Types and Examples Belong to These Example Types

Example Type	Explanation	Example
Generic Example	examples showing the overall situation of the concept	 This is a generic example of function concept.

Non Example	examples used to clearly demonstrate the boundaries of the concept by highlighting the features that concept does not have	 This is an example of a function which is not onto.
Counter examples	examples used to demonstrate that a claim or a thought is false	Claim. Let f be a function from A to B. Then the inverse function of f exists. Example. Let f be from R to R, $f(x)= x^2$. The inverse function of f doesn't exist.

Findings

In this section, the findings of the observation notes regarding the example types used in the functions by T1 and T2 teaching at the 9th grades and the findings of the informal interviews made after the lessons are presented.

T1's Examples and Explanations About The Functions

T1 aimed at drawing students' attention to the subject by mentioning the place and importance of functions in daily life before starting the functions subject. After these explanations, the teacher T1 made the definition of the concept to her students as follows:

> *Let A and B are two sets which are not empty. A relation which maps every element of A to a unique element of B is called a function from A to B. Functions are shown such as f, g, h, etc.*

After this definition, he made the following presentation:

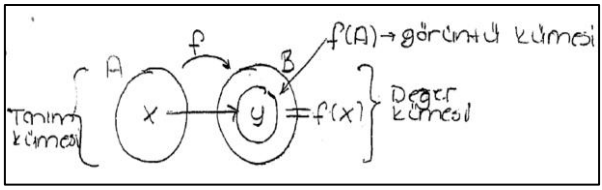


Figure 1. The teacher T1's representation on the definition of function

With Figure 1, T1 expressed that A is called the domain, the B is called the codomain and the subset of codomain, which is created by the elements in the domain is called the image set. In Figure 1, the definitions about the concept of function are explained. T1 expressed as follows that for a relation to be a function, the expressions in Figure 1 is not enough and there are two necessary conditions:

For the relation f to be a function from A to B, firstly there won't be any element in A which doesn't go to any element of B and also there will be a unique image of every element of the domain.

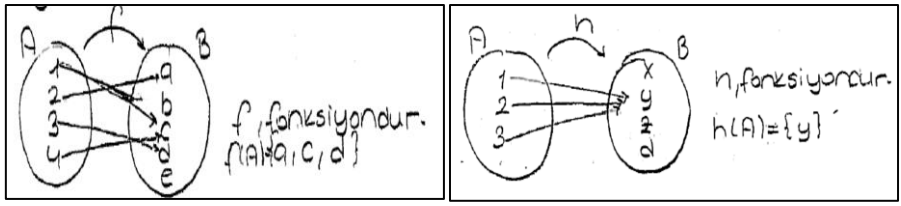


Figure 2. Generic examples

After T1 expressed to her students the definition of function and the necessary conditions to be a function, she used generic examples in Figure 2 while explaining what is meant by this expression. In Figure 2, in the left example it is emphasized that no elements should remain free in the domain and in the right example it is emphasized that each element will only have one image. In the right one, in addition to the explanations in the left one, it is explained that all elements in the domain can have the same image. It was observed that T1 explained the definition and the must have features for a relation to be function with these generic examples.

As well as using the generic examples to explain the required conditions for a relation to be a function, it was observed that T1 used non examples as in Figure 3 to express in which conditions relations are not functions.

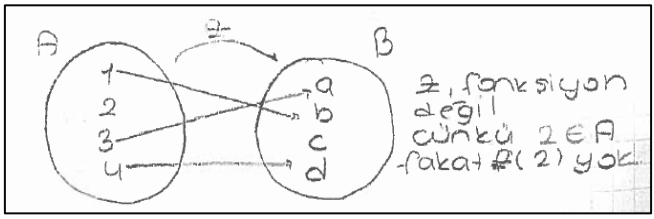


Figure 3. Non-example

T1: *One of the requirements for a relation to be a function is not having any free elements in the domain. As shown in this example, the relation is not a function because there is a free element in the domain. Free elements in the codomain are not important.*

T1 distinctly presented the conditions to be a function to the students with the example which belongs to the type of non-examples in Figure 3. T1 intended to provide her students a better understanding of function concept by showing a case with this example which doesn't belong to function.

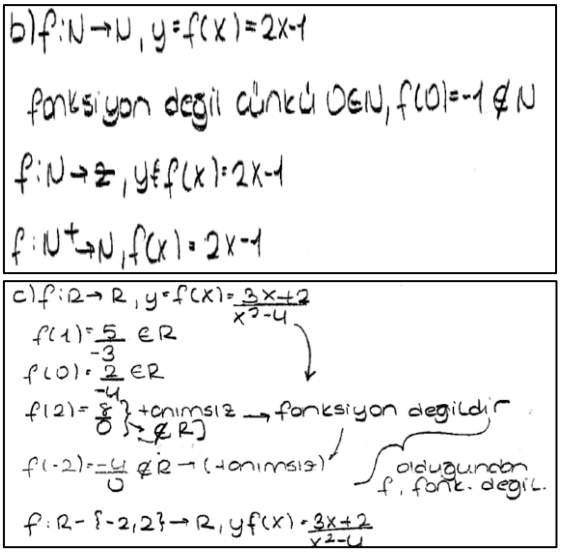


Figure 4. Non-examples

T1*: The mathematical expressions defined from the set of natural number to the set of natural numbers shown in left example is not a function. Because when each element in the domain is written in its place in this operation, its image is meant to be a natural number but in case of bringing the number zero from the domain and writing in its place in this operation, the result is -1, which is a number that does not belong to the codomain. And this means that this statement does not specify a function. If the domain consisted of positive natural numbers, or if the codomain was the set of integers, it would work as a function. Similarly, in the other example, there is a rational expression, and see that here, if the values which make the denominator zero are the elements of the domain, it doesn't make a function because you will not find image, and you need to see the image of each element. However, if you remove the set of values that make the expressions undefined from the domain, it indicates a function.*

By providing the non-examples in Figure 4, T1 explained to the students that every relation with algebraic expression is not a function and the domains should be organized for these relations to indicate functions. With these non-examples, as different from the example in Figure 3, T1 wanted to explain in which cases mathematical equations could be functions.

In summary, it was observed that T1 made use of non-examples as well as generic examples, but never used counter examples. Besides, it was seen that T1

used generic and non-examples belonging to function types as well as the definition of function.

T2's Examples and Explanations About The Functions

It was seen that T2 began his lesson writing the definition of function and he made use of generic examples to demonstrate what the definition meant. Accordingly, T2 defined a function as follows:

Let A and B two non-empty sets. A relation f which maps every element of A to a unique element of B is called a function from A to B. After giving the definition of function to his students, T2 showed what he meant with this definition by drawing Figure 5 on the board.

> *A and B are two non-empty sets, a relation f from A to B, which matches each element of A to only one element of B is called a function from A to B.*

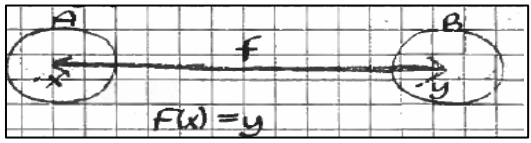


Figure 5. Representation on the definition of T2

T2 stated that A is the domain and B is the codomain. With this representation, T2 tried to explain what students should understand from the definition of a function. It was observed that T2 used the generic example in Figure 6 to draw attention to some certain features belonging to the definition of function.

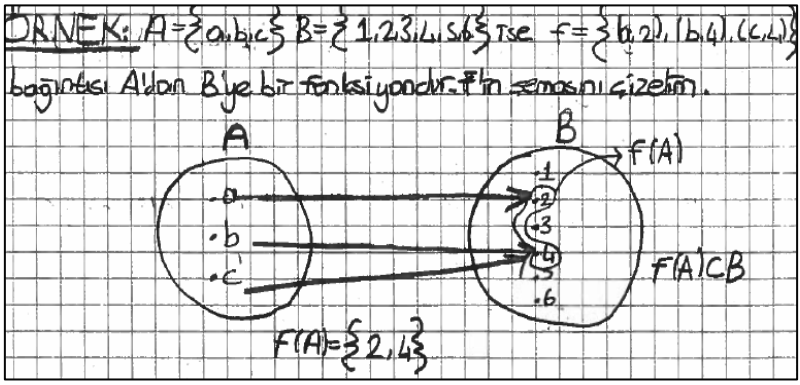


Figure 6. Generic example

> *T2: The set A is our domain and we can match each element in here with any element in B. When we look at the definition, we see that*

each element in A can only match one single element in B. For example, we can match a to 2, b and c to 4. These are the elements of our image set. As seen, image set is a subset of the domain. It is important that there is no free element in the domain and we match to only one element in the codomain.

T2 aimed to draw attention to the image set in function concept and to emphasize that the image set and the codomain can be different from each other by using the generic example in Figure 6. So, it is observed that T2 used the generic example to emphasize that the image set doesn't have to be equal to the codomain set B and it can be any subset of B.

It is seen that T2 firstly used the generic examples that emphasize the necessary features related to the definition in order to explain what the definition of a function means to his students, and then he used non examples to draw attention on the features that do not belong to function concept.

T2 used the non-examples in Figure 7 to emphasize the features that do not belong to this concept.

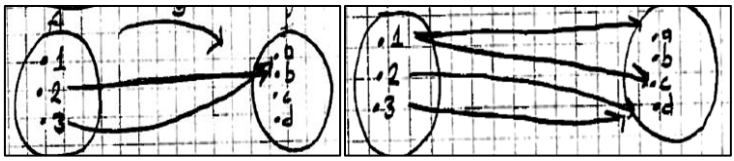


Figure 7. Non-examples

T2: If we analyze the left and right examples, for a relation to be a function there shouldn't have been a free element in the domain, that is A set. But if we look at the left example, we see that there is an unmatched element, so this relation is not a function. Similarly, in our right example, there is no free element in A set, but one element has got 2 images in the codomain. Look, every elements in the domain should have only one single image. The elements 2 and 3 match to d, no problem; b may be free in the codomain set.

It is observed that T2 used the non-examples to explain the absence of the features while drawing attention to the features of the concept. T2 explained under which circumstances a relation might be a function by using Figure 7.

In summary, while teaching functions subject, as well as using the generic examples to express the definition and each type of functions, T2 used the non-examples to demonstrate the features that do not belong to this definition and each type of functions. It is also observed that T2 did not use counter examples to teach functions.

In Table 4, the example types and the frequencies of them which the 9th grade teachers (T1 and T2) used are shown.

Table 4.

The Frequencies of the Example Types Which Teachers Used in Their Lessons

	Example Types		
Teachers	Generic Example	Non Example	Counter Example
T1	50	19	0
T2	27	4	0

As seen in Table 4, the teachers used mostly generic examples, however, they didn't use any counter examples. Besides, it has been observed that there is a big difference between the numbers of the examples the teachers used, and this gap follows from the generic and non-examples.

Discussion and Conclusion

With this research, it is aimed to determine the example types two mathematics teachers, working at an Anatolian High School in Trabzon, use to teach functions. The data obtained from the research demonstrates that the teachers use mostly generic examples in the subject of functions and they don't use counter examples. It is found out that the teachers use generic examples to demonstrate the definitions and types of functions. In the related literature, Mason and Pimm (1984) defined the generic example as examples used to express their only general situations ignoring their special features. Besides, apart from general situations belonging to the concept, Bill et al stated that sometimes it is used to prove a theorem and to show how a procedure is applied. In this research, it is found out that teachers mostly use generic examples while explaining the definition of function, its types and how the four operations with functions are made. It was stated that it can be useful to express what they think about a principal or a general situation to students by means of generic examples (Zaslavsky, 2010). Similarly, for this study, it can be said that using generic examples may be beneficial to let students understand the concept of function. However, generic examples may cause students to ignore the special features of functions since they only inform about the general situation of the concepts. Therefore, it can be said that presenting the examples that do not belong to the concept, that is non-example, in order to draw attention to the special situations that belong to the concept may be useful.

It is found out that teachers use non examples to explain under which circumstances an algebraic equation is not a function and in which conditions it doesn't belong to the function types. Teachers' using non examples while demonstrating out of concept situations (or out of definition) may help students understand the function concept better. Because non examples are the examples that are used to explain the boundaries of a concept or to distinctly express the conditions

belonging to a concept (Bills et al, 2006). Supporting this condition, Özdemir Erdoğan et al. (2012) stated that there is a significant deficiency in students' knowledge in defining the function concept and recognition of the algebraic representations of the concept. Dubinsky and Harel (1992) and Breidenbach et al (1992) stated in their research that due to the fact that the definition of functions is not given with all of its features, there are problems in understanding the function concept. In respect to this, it is concluded that teachers using non examples in their lessons draw attention to the irrelevant features as well as the relevant ones may provide students to understand the function concept better.

Another important finding obtained from the research is that T1 uses more non-examples than T2. This difference is due to the fact that while T2 uses the non-examples only to express the conditions of the functions clearly, T1, besides these examples, emphasizes that not each equation written algebraically like in Figure 4 is a function. We can anticipate that this will support students significantly while learning the basics and features of the function concept. Generally, the number of examples used by T1 is higher than the number of examples used by T2. The difference between the numbers of the examples is seen in both generic and non-examples. This may be due to the difference of the pedagogical approach of the teachers. Bayazıt and Aksoy (2010) expressed that teachers with the same quality and content of knowledge with respect to students' difficulties in learning and misconceptions about functions may show quite different pedagogical approaches towards the teaching of this subject. This situation can be shown as one of the reasons of the difference between the numbers of the examples the teachers used because as observed, T1 organized her lessons and examples in accordance with the attitudes and behaviors of students in the class while T2 used the examples he prepared in advance and he didn't pay attentions his students' needs. Furthermore, it is found out that the teachers didn't use any counter examples in their lessons.

With this research, it can be suggested that besides the generic examples, teachers should use the non-examples in order to expand the boundaries of the concept by highlighting the features of it and they should make use of these kinds of examples as well. Besides, we believe that it is necessary to make a similar research with a larger sample group and in the context of different mathematical topics. It can be also said that the results will contribute to teacher training.

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Genişletilmiş Özet

Fonksiyon kavramı matematik dersinin önemli konularından biri olmasına rağmen yapılan birçok araştırma, bu konunun öğrenilmesinde öğrencilerin zorluk yaşadıklarını göstermektedir. Bu zorlukların temelini öğrencilerin önemli bir çoğunluğunun fonksiyonlar konusunu öğrenirken fonksiyonun tanımını değil, fonksiyonu açıklamak için kullanılan örnekler, temsiller ve cebirsel kurallar arasından kendi düşünce yapılarıyla uyuşan prototipleri zihinlerine kodlamaları oluşturmaktadır. Fonksiyonlar konusunda öğrenci zorluklarını ve kavram yanılgılarını inceleyen çok sayıda çalışma bulunmaktadır (Bayazit ve Gray, 2004; Breidenbach v.d., 1992; DeMarois ve Tall, 1996; Güler vd., 2015; Ural 2006; Vinner, 1983). Bu çalışmalar incelendiğinde; öğrencilerin fonksiyon kavramından ziyade, bu kavrama ait temsil ve görsel öğelere daha çok dikkat ettikleri görülmüştür. Vinner (1983), öğrencilerin fonksiyon kavramının tanımından ziyade, bu kavrama ait ilk örnekleri zihinlerine kaydettiklerini ve kavram yanılgılarına sebep olduğunu ifade etmiştir. Özellikle kavramla ilgili doğru seçilmeyen örnekler öğrencilerin kavram imgelerinin eksik veya yanlış oluşmasına yol açabilir (Bayazıt ve Aksoy, 2010). Bu yüzden öğrencilerin kavram imgelerinin oluşmasında önemli bir yeri olan örneklerin öğretmenler tarafından nasıl kullanıldığının incelenmesi önemlidir. Benzer şekilde Evangelidou ve arkadaşları (2004) öğrencilerin aldıkları eğitim ile kalıplaşan öğrenilmiş davranışlar içerisinde yapılan örneklendirmelere ve anlatımlara göre öğrenmelerinin konunun öğrenilmesini zorlaştırabileceğini belirtmişlerdir. Örnekler, öğrenme ve öğretme sürecinde özellikle kavramsallaştırma, genelleştirme, soyutlama ve tartışma bakımından matematiksel

düşünmenin gelişimine katkı sağlar. Ayrıca örnekler genelleme yapmak, matematiksel ilişkileri ve tümevarımsal sorgulatmayı başlatmak, ilke ve kavramları gösteren daha geniş bir sınıfı belirtmek, kavramları ve sonuçları desteklemek (Michener, 1978) ve matematiksel tekniklerin nasıl uygulandığını göstermek gibi birçok durumda kullanılır (Muir, 2007; Watson ve Mason, 2002a, 2002b; Zaslavsky, 2010). Örneklerin bu özellikleri, zihnimizde soyut birer düşünce olan kavramları somut bir yapıya dönüştürmemizi sağlamakla birlikte kavramları (Gökbulut, 2010) ve tanımları daha anlamlı hale getirmemize, matematiksel ifadeleri sınıflandırmamıza ve bu ifadelerin birbirleriyle olan benzer durumlarını ilişkilendirmemize yardımcı olur (Watson ve Mason, 2002b). Ayrıca Leinhardt ve Schwarz (1997) öğretmenlerin açıklamalarının temelinde örneklerin önemli bir rolü olduğunu öğretmenlerin örnek kullanımı ile ilgili yapmış oldukları çalışmalarında vurgulamaktadırlar. Bu nedenle öğretmenlerin fonksiyon konusunda sundukları örneklerin öğrencilerin fonksiyonlar konusunu anlamalarında oldukça önemli bir yere sahip olduğu söylenebilir. Bu araştırmada; 9. sınıf matematik öğretmenlerinin, fonksiyonlar konusunda kullanmış oldukları örnek türlerinin incelenmesi amaçlanmıştır. Bu amaç doğrultusunda, 9. sınıf matematik öğretmenlerinin kullandıkları örnek türlerinin tespit edilmesi özel durum yöntemine yönelik sorular içermektedir. Bu yöntemin tercih edilmesinde; araştırmaya katılan öğretmenlerin fonksiyonlar konusunda kullanmış oldukları örnek çeşitleri hakkında detaylı bilgi verebileceği düşüncesi etkili olmuştur. Çalışmanın örneklemini Trabzon ilinde bir Anadolu lisesinde çalışan 2 matematik öğretmeni (1 Bayan ve 1 Bay) oluşturmaktadır. Çalışmaya katılan öğretmenlerin mesleki tecrübeleri 12-24 yıl arasındadır. Araştırmada özellikle mesleki tecrübesi yüksek olan öğretmenler tercih edilmiştir. Bunun sebebi ise Rowland'ın (2008) mesleki tecrübesi az olan öğretmenlerin kullandıkları örneklerin öğrencilerin konuları daha iyi kavraması için yeterli olmadığını belirtmesidir. Araştırma kapsamında özellikle hizmet süresi fazla olan öğretmenler tercih edilmiştir. 9. sınıf matematik öğretmenlerinin fonksiyonlar konusunda kullandıkları örnek türlerinin tespit edilmesi için yapılandırılmamış gözlem ve gözlemlerden sonra kullanılan örnek türlerinin tespiti için informal mülakatlar kullanılmıştır. Yapılandırılmamış gözlem ile sınıf içinde öğretmenin kullandığı örnekler ve bu örnekleri kullanma sebepleri ayrıntılı bir şekilde gözlenmiştir. İnformal mülakatlarda öğretmenlerin derslerinde kullandıkları örneklerin kullanım amaçları sorulmuştur. Böylelikle öğretmenlerin derste kullanmış oldukları örnekleri neden tercih ettikleri hakkında ayrıntılı görüş alınarak, örneklerin analizinin doğru bir şekilde yapılmasına katkı sağlanmıştır. İlgili literatürde örnek türlerinin yer aldığı çeşitli sınıflandırmalara rastlanılmıştır. Bu sınıflandırmalar içerisinde diğer sınıflandırmaları da kapsayan genel olarak örneklerin üç kategori altında toplanabileceğini ifade eden Bills ve arkadaşlarının (2006) yılındaki sınıflandırması tercih edilmiştir. Örnekleri pedagojik açıdan, kavramlara ait tanımların ya da kuralların örnekleri (üçgenin tanımı, 3 ile bölünebilmenin kuralı ve polinomun tanımı, vb…) ve bir prosedürün uygulamasındaki örnekler (bir üçgenin alanın bulunması, bir tamsayının üç ile bölünebilmesinin bulunması, bir polinomun

köklerinin bulunması, vb...) olmak üzere Bills ve diğ. (2006) yaptıkları araştırmada ikiye ayırmışlardır. Bu kategoriyi ise, örneklerin işlevlerine göre '*jenerik'*, '*karşıt* 've '*örnek dışı'* örnekler olarak üç özel tanımlayıcı isim altında oluşturmuşlardır. Araştırmada örnek türleri önceden belli olduğu için betimsel analiz tekniği kullanılmıştır. Buna göre gözlem ve informal mülakatlardan elde edilen veriler analiz edilirken öğretmenlerin kullandığı örnekler tek tek incelenmiş ve bu örnekleri kullanım amaçları ile ilgili ders sonrası yapılmış olan mülakatlar analiz edilmiştir. Elde edilen veriler araştırmacıların dışında başka bir matematik eğitimcisi tarafından da analiz edilmiştir. Sınıflandırma sonucunda örneklerin örnek türlerine göre sınıflandırılmasında araştırmacılar ile diğer matematik eğitimcisi arasında % 87 uyum tespit edilmiştir. Ortaya çıkan farklılıklar yeniden araştırmacılar tarafından ele alınmış ve incelenmiştir. Yapılan çalışmada elde edilen bulgular öğretmenlerin fonksiyonlar konusunda en fazla jenerik örnek kullandıkları ve karşıt örnek ise hiç kullanmadıklarını göstermiştir. Bu araştırma ile öğretmenlerin derslerinde jenerik örneklerin yanı sıra kavrama ait özellikleri vurgulayan kavramın sınırlarını genişletmek için kavrama ait olmayan örnekleri kullanmalarının yararlı olabileceği önerilebilir. Ayrıca burada sunulan çalışmanın bir benzerinin daha geniş bir örneklem grubuyla ve farklı matematik konuları bağlamında yapılmasının gerekli olduğuna inanmaktayız. Bu araştırmalardan da elde edilen sonuçların öğretmen eğitimine katkı sağlayabileceği söylenebilir.