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A New Soft Set Operation: Complementary Soft Binary Piecewise Difference (\) Operation

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ABSTRACT

Soft set theory is a theory of dealing with uncertainty. Since its inception, many kinds of soft set operations have been defined and used in various types. In this paper, a new kind of soft set operation called, complementary soft binary piecewise difference operation is defined and its basic properties are investigated. We obtain many striking analogous facts between difference operation in classical theory and complementary soft binary piecewise difference operation in soft set theory. Also, by obtaining the relationships between this new soft set operation and all other types of soft set operations, we aim to contribute to the soft set literature with the help of examing the distribution rules.

Yeni Bir Esnek Küme İşlemi: Tümleyenli Esnek İkili Parçalı Fark (\) İşlemi

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ÖZ

Esnek küme teorisi, belirsizlikle başa çıkan bir teoridir. Başlangıcından bu yana, birçok türde esnek küme işlemi tanımlanmış ve çeşitli şekillerde kullanılmıştır. Bu çalışmada, tümleyenli esnek ikili parçalı fark işlemi adı verilen yeni bir esnek küme işlemi tanımlanmış ve temel cebirsel özellikleri araştırılmıştır. Klasik teorideki fark işlemi ile esnek küme teorisindeki tümleyenli esnek ikili parçalı fark işlemi arasında birçok çarpıcı benzer özellikler elde edilmiştir. Ayrıca bu işlem ile diğer tüm esnek küme işlemleri arasındaki ilişkiler dağılma kurallar yardımıyla incelenerek, esnek küme literatürüne katkıda bulunma amaçlanmıştır.

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1.Introduction

Molodtsov (1999) intoduced Soft Set Theory to overcome the uncertainties. Since 1999, the theory has been applied to many fields such as decision-making as in Özlü (2022a, 2022b), and Paik and Mondal (2022), measurement theory, operations research, optimization theory, game theory, information systems and some algebraic structures as in Atagün and Aygün (2016), and Addis et al. (2022). Riaz and Hashimi (2019) and Ayub et al. (2021) studied Linear Diophantine Fuzzy Sets and Linear

Diophantine Fuzzy aggregation operators and Riaz et al. (2021, 2023) on Spherical Linear Diophantine Fuzzy Sets fuzzy modeling which are some top recent topics as novel mathematical approaches to model vagueness and uncertainty. First contributions as regards soft set operations were made by Maji et al. (2003) and Pei and Miao (2005). Then, Ali et al. (2009) introduced and examined many soft set operations such as restricted and extended soft set operations. The basic properties of soft set operations were discussed and Sezgin and Atagün (2010) illustrated the interconnections of soft set operations with each other. Sezgin et al. (2019) defined a new soft set operation called the extended difference of soft sets, and Stojanovic (2021) defined and examined the extended symmetric difference of soft sets. When the studies on the operations of soft sets are examined, it is seen that the operations in soft set theory proceed under two main headings, restricted soft set operations and extended soft set operations. Çağman (2021) defined two conditional complements of sets as a new concept of set theory. With the inspiration of this study, Sezgin et al. (2023c) defined some new complements of sets. Aybek (2024) also transferred these complements to soft set theory, and some new restricted soft set operations and extended soft set operations were defined. Demirci (2024), Sarıalioğlu (2024), and Akbulut (2024) defined a new type of extended operation by changing the form of extended soft set operations using the complement at the first and second row of the piecewise function of extended soft set operations and studied the basic properties of them in detail. Moreover, a new type of soft difference operations was defined in Eren (2019), and by being inspired by this study, Yavuz (2024) and Sezgin and Yavuz (2023a) defined some new soft set operations, which they call binary piecewise soft set operations, and they studied their basic properties in detail, too. Also, in some studies (Sezgin and Demirci, 2023; Sezgin and Sarıalioğlu, in press; Sezgin and Atagün, 2023; Sezgin and Yavuz, 2023b; Sezgin and Aybek, 2023; Sezgin et al., 2023a, 2023b), studies continued on soft set operations by defining a new type of binary piecewise soft set operation. They changed the form of soft binary piecewise operation by using the complement at the first row of the soft binary piecewise operations.

The purpose of this study is to contribute to the literature by defining a new soft set operation which we call "complementary soft binary piecewise difference operation". For this aim, the definition of the operation, and its example are given. The algebraic properties like closure, unit and inverse element, and abelian property of this new operation are examined in detail. We obtain many stunning analogous facts between the difference operation in classical theory and complementary soft binary piecewise difference operation in soft set theory. By examing the distribution rules, it is aimed to contribute to the literature by obtaining the relationship between this operation and other types of soft set operations.

2. Preliminaries

Definition 2.1. Let U be the universal set, E be the parameter set, $\mathrm{P(U)}$ be the power set of U and $\mathrm{Q} \subseteq \mathrm{E}$. A pair (C, Q) is called a soft set over U where C is a set-valued function such that $\mathrm{C}: \mathrm{Q} \rightarrow \mathrm{P(U)}$. (Molodtsov, 1999)

Throughout this paper, the set of all the soft sets over U (no matter what the parameter set is) is designated by $S_E(U)$. Let A be a fixed subset of E and $S_A(U)$ be the collection of all soft sets over U with the fixed parameters set A. Clearly, $S_A(U)$ is a subset of $S_E(U)$ and, in fact, all the soft sets are the elements of $S_E(U)$.

Definition 2.2. (C, Q) is called a relative null soft set (with respect to the parameter set Q), denoted by $\emptyset_Q$, if $C(t) = \emptyset$ for all t∈Q and (C, Q) is called a relative whole soft set (with respect to the parameter set Q), denoted by U_Q if $C(t) = U$ for all t∈Q. The relative whole soft set (with respect to the universe set of parameters E, denoted by U_E, is called the absolute soft set over U (Ali et al., 2009)

Definition 2.3. For two soft sets (C, Q) and (Y, I), we say that (C, Q) is a soft subset of (Y, I) and it is denoted by $(C, Q) \tilde{\subseteq} (Y, I)$, if Q⊆ I and $C(t) \subseteq Y(t)$, ∀t∈ Q. Two soft sets (C, Q) and (Y, I) are said to be soft equal if (C, Q) is a soft subset of (Y, I) and (Y, I) is a soft subset of (C, Q) (Pei and Miao, 2005).

Definition 2.4. The relative complement of a soft set (C, Q), denoted by $(C, Q)^r$, is defined by $(C, Q)^r = (C^r, Q)$, where $C^r: Q \to P(U)$ is a mapping given by $(C, Q)^r = U\backslash C(t)$ for all $t \in Q$ (Ali et al., 2009). From now on, $U\backslash C(t)=[C(t)]'$ will be designated by C'(t) for the sake of designation.

Two conditional complements of sets as a new concept of set theory, that is, inclusive complement and exclusive complement were defined in Çağman (2021). For ease of illustration, we show these complements as + and θ, respectively. These complements are binary operations, and are defined as follows: Let Q and I be two subsets of U. I-inclusive complement of Q is defined by, Q+I=Q'∪I, and the I-Exlusive complement of Q is defined by QθI=Q'∩I'. Here, U refers to a universe, and Q' is the complement of P over U. For more information, we refer to Çağman (2021).
The relations between these two complements were examined in detail by Sezgin et al. (2023c), and they also introduced such new three complements as binary operations of sets as follows: Let Q and I be two subsets of U. Then, Q*I=Q'∪I', QγI=Q'∩I, Q$\boldsymbol{\lambda}$I=Q∪I' (Sezgin et al., 2023c). These set operations were also conveyed to soft sets and Aybek (2024) defined restricted and extended soft set operations and examined their properties.
Now, we can categorize all types of soft set operations as follows: Let "∇" be used to represent the set operations (i.e.,∇ can be ∩,∪,\, Δ,+,θ,*, λ,γ), then restricted operations, extended operations, complementary extended operations, soft binary piecewise operations, complementary soft binary piecewise operations are defined in soft set theory as follows:
Definition 2.5. Let (C, Q) and (Y, I) be soft sets over U. The restricted ∇ operation of (C, Q) and (Y, I) is the soft set (H,S), denoted by $(C, Q)\nabla_R(Y, I) = (H, S)$, where $S = Q \cap I \neq \emptyset$ and $\forall t \in S$, $H(t) = Q(t)\nabla\, Y(t)$. (Ali et al., 2009; Sezgin and Atagün, 2011; Aybek, 2024).

Definition 2.6. Let (C, Q) and (Y, I) be soft sets over U. The extended ∇ operation of (C, Q) and (Y, I) is the soft set (H,S), denoted by $(C, Q)\nabla_\varepsilon(Y, I) = (H, S)$, where $S = Q \cup I$ and $\forall t \in S$,

$$H(t)=\begin{cases} C(t), & t\in Q\backslash I,\\ Y(t), & t\in I\backslash Q,\\ C(t)\nabla Y(t), & t\in I\cap Q.\end{cases}$$

(Maji et al. 2003; Ali et al. 2009; Sezgin et al. 2019; Stojanovic, 2021; Aybek, 2024).

Definition 2.7. Let (C,Q) and (Y,I) be soft sets over U. The complementary extended ∇ operation (C,Q) and (Y,I) is the soft set (H,S), denoted by $(C,Q)\overset{*}{\nabla_{\varepsilon}}(Y,I)=(H,S)$, where $S=Q\cup I$, $\forall t\in S$,

$$H(t)=\begin{cases} C'(t), & t\in Q\backslash I\\ Y'(t), & t\in I\backslash Q,\\ C(t)\,\nabla Y(t), & t\in Q\cap I.\end{cases}$$

(Sarıalioğlu, 2024; Demirci, 2024; Akbulut, 2024).

Definition 2.8. Let (C,Q) and (Y,I) be soft sets over U. The soft binary piecewise ∇ operation of (C,Q) and (Y,I) is the soft set (H,Q), denoted by $(C,Q)\underset{\nabla}{\sim}(Y,I)=(H,Q)$, where $\forall t\in Q$,

$$H(t)=\begin{cases} C(t), & t\in Q\backslash I\\ C(t)\,\nabla Y(t), & t\in Q\cap I\end{cases}$$

(Eren, 2019; Yavuz, 2024, Sezgin ve Yavuz, 2023a)

Definition 2.9. Let (C,Q) and (Y,I) be soft sets over U. The complementary soft binary piecewise ∇ operation of (C,Q) and (Y,I) is the soft set (H,Q), denoted by $(C,Q)\overset{*}{\underset{\nabla}{\sim}}(Y,I)=(H,Q)$, where $\forall t\in Q$,

$$H(t)=\begin{cases} C'(t), & t\in Q\backslash I\\ C(t)\,\nabla Y(t), & t\in Q\cap I\end{cases}$$

(Sezgin and Sarıalioğlu, in press; Sezgin and Demirci, 2023; Sezgin and Atagün, 2023; Sezgin and Aybek, 2023; Sezgin et al., 2023a, 2023b; Sezgin and Yavuz, 2023b; Sezgin and Dagtoros, 2023).

3. Complementary Soft Binary Piecewise Difference (\) Operation and Its PropertiesDefinition 3.1. Let (C,Q) and (Y,I) be soft sets over U. The complementary soft binary piecewise difference (\) operation of (C,Q) and (Y,I) is the soft set (A,Q), denoted by, $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)=(A,Q)$, where $\forall j\in Q$,

$$A(j)=\begin{cases} C'(j), & j\in Q\backslash I\\ C(j)\backslash Y(j), & j\in Q\cap I\end{cases}$$

Note that if the parameter sets of the soft sets are the same, say Q, then the complementary soft binary piecewise difference operation of (C, Q) and (Y, Q) is the soft set (K,Q) denoted by, $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,Q) = (K,Q)$, where ∀j∈Q,

$$K(j)=\begin{cases} C'(j), & j\in Q\backslash Q \\ C(j)\backslash Y(j), & j\in Q\cap I \end{cases}$$

Here since Q\Q=∅; we can ignore the first line of the piecewise function under these cases, and thus the complementary soft binary piecewise difference (\) operation turns out to be the restricted difference of soft sets. The same argument is valid when the parameter set of the second soft set is E.

Example 3.2. Let E={e_1,e_2,e_3,e_4}be the parameter set Q={e_1, e_3} and I={e_2, e_3,e_4}be the subsets of E and U={h_1,h_2,h_3,h_4,h_5} be the initial universe set. Assume that (C,Q) and (Y,I) are the soft sets over U defined as following:

(C,Q)={(e_1, {h_2,h_5}), (e_3,{h_1,h_2,h_5})} and (Y,I)={(e_2,{h_1, h_4,h_5}), (e_3,{h_2,h_3,h_4}),(e_4,{ h_3, h_5})}.

Let $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,I)=(A,Q)$. Then,

$$A(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ C(j)\backslash Y(j), & j\in Q\cap I \end{cases}$$

Since Q={e_1, e_3} and Q\I={e_1}, so A(e_1) = Q'(e_1)={ h_1,h_3,h_4}, and since Q∩I={e_3} so A(e_3)= C(e_3)\Y(e_3)=C(e_3)∩Y'(e_3)={h_1, h_2,h_5}∩{h_1,h_5}={h_1, h_5}.Thus, $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,I)$={(e_1,{h_1,h_3,h_4), (e_3, {h_1, h_5})}.

Theorem 3.3. (Algebraic properties of the operation)

1) The set $S_E(U)$ is closed under the operation $\underset{\backslash}{\overset{*}{\sim}}$.

Proof: It is clear that $\underset{\backslash}{\overset{*}{\sim}}$ is a binary operation in $S_E(U)$. That is,

$$\underset{\backslash}{\overset{*}{\sim}} : S_E(U) \times S_E(U) \to S_E(U)$$

$((C,Q), (Y,I)) \rightarrow (A,Q)$

Hence, when (C,Q) and (Y,I) are two soft sets over U, then so is $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,I)$.

2) $[(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,Q)] \underset{\backslash}{\overset{*}{\sim}} (A,Q) \neq (C,Q) \underset{\backslash}{\overset{*}{\sim}} [(Y,Q) \underset{\backslash}{\overset{*}{\sim}} (A,Q)]$.

Proof: Let $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,Q)=(T,Q)$, where $T(j)=C(j)\cap Y'(j)$ for all $j\in Q$. Let $(T,Q) \underset{\backslash}{\overset{*}{\sim}} (A,Q)=(M,Q)$, where $M(j)=T(j)\cap A'(j)$ for all $j\in Q$. Thus, $M(j)=[C(j) \cap Y'(j)]\cap A'(j)$ for all $j\in Q$. Let $(Y,Q) \underset{\backslash}{\overset{*}{\sim}} (A,Q)=(L,Q)$, where $L(j)=Y(j)\cap A'(j)$ for all $j\in Q$. Let $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (L,Q)=(N,Q)$, where $N(j)=C(j)\cap L'(j)$ for all $j\in Q$. Thus, $N(j)=C(j)\cap [Y'(j)\cup A(j)]$ (2) for all $j\in Q$. It is seen that (1)≠(2). That is, for the soft sets whose parameter sets are the same, the operation $\underset{\backslash}{\overset{*}{\sim}}$ does not have associativity property. Moreover, we have the following:

3) $[(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,I)] \underset{\backslash}{\overset{*}{\sim}} (A,Z) \neq (C,Q) \underset{\backslash}{\overset{*}{\sim}} [(Y,I) \underset{\backslash}{\overset{*}{\sim}} (A,Z)]$.

Proof: Let $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,I)=(T,Q)$, where $\forall j\in Q$;

$$T(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ C(j)\cap Y'(j), & j\in Q\cap I \end{cases}$$

Let $(T,Q) \underset{\backslash}{\overset{*}{\sim}} (A,Z) =(M,Q)$, where $\forall j\in Q$;

$$M(j)=\begin{cases} T'(j), & j\in Q\backslash Z \\ T(j) \cap A'(j), & j\in Q\cap Z \end{cases}$$

Thus,

$$M(j)=\begin{cases} C(j), & j\in (Q\backslash I)\backslash Z=Q\cap I'\cap Z' \\ C'(j)\cup Y(j), & j\in (Q\cap I)\backslash Z=Q\cap I\cap Z' \\ C'(j)\cap A'(j), & j\in (Q\backslash I)\cap Z=Q\cap I'\cap Z \end{cases}$$

$[C(j)\cap Y'(j)]\cap A'(j), \quad j\in(Q\cap I)\cap Z=Q\cap I\cap Z$

Let $(Y,I) \underset{\backslash}{\overset{*}{\sim}} (A,Z)=(K,I)$, where $\forall j\in I$;

$$K(j)=\begin{cases} Y'(j), & j\in I\setminus Z \\ Y(j)\cap A'(j), & j\in I\cap Z \end{cases}$$

Let $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (K,I)=(S,Q)$, where $\forall j\in Q$;

$$S(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap K'(j), & j\in Q\cap I \end{cases}$$

Thus,

$$S(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap Y(j), & j\in Q\cap(I-Z)=Q\cap I\cap Z' \\ C(j)\cap[Y'(j)\cup A(j)], & j\in Q\cap(I\cap Z)=Q\cap I\cap Z \end{cases}$$

Here let's handle $j\in Q-I$ in the second equation of the first line. Since $Q\setminus I=Q\cap I'$, if $j\in I'$, then $j\in Q\setminus I$ or $j\in(I\cup Z)'$. Hence, if $j\in Q\setminus I$, then $j\in Q\cap I'\cap Z'$ or $j\in Q\cap I'\cap Z$. Thus, it is seen that $M\neq S$. That is, for the soft sets whose parameter sets are not the same, the operation $\underset{\backslash}{\overset{*}{\sim}}$ does not have associativity property on the set $S_E(U)$.

4) $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,I)\neq(Y,I) \underset{\backslash}{\overset{*}{\sim}} (C,Q)$.

Proof: Let $(C,Q) \underset{\backslash}{\overset{*}{\sim}} (Y,I)=(A,Q)$. Then, $\forall j\in Q$;

$$A(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap Y'(j), & j\in Q\cap I \end{cases}$$

Let $(Y,I) \underset{\backslash}{\overset{*}{\sim}} (C,Q)=(T,I)$. Then $\forall j\in I$;

$$T(j)=\begin{cases} Y'(j), & j\in I\setminus Q \\ Y(j)\cap Q'(j), & j\in I\cap Q \end{cases}$$

Here, while the parameter set of the soft set of the left-hand side is Q; the parameter set of the soft set of the right-hand side is I. Thus, by the definition of soft equality

$$(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,I)\neq(Y,I)\underset{\backslash}{\overset{*}{\sim}}(C,Q).$$

Hence, the operation $\underset{\backslash}{\overset{*}{\sim}}$ does not have commutative property in the set $S_E(U)$, where the parameter sets of the soft sets are different. Moreover, the operation $\underset{\backslash}{\overset{*}{\sim}}$ does not have commutative property where the parameter sets of the soft sets are the same; since $C(j)\cap Y'(j)\neq Y(j)\cap Q'(j)$.

5) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)=\emptyset_Q$.

Proof: Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)=(A,Q)$, where $A(j)=C(j)\cap C'(j)=\emptyset$ for all $j\in Q$. Thus $(A,Q)=\emptyset_Q$. That is, the operation $\underset{\backslash}{\overset{*}{\sim}}$ does not have idempotency property on the set $S_E(U)$.

6) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}\emptyset_Q=(C,Q)$.

Proof: Let $\emptyset_Q=(S,Q)$. Then, $\forall j\in Q$; $S(j)=\emptyset$. Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(S,Q)=(A,Q)$, where $A(j)=C(j)\cap S'(j)$ for all $j\in Q$. Hence, $\forall j\in Q$; $A(j)=C(j)\cap S'(j)=C(j)\cap U=C(j)$. Thus, $(A,Q)=(C,Q)$. Note that, for the soft sets whose parameter set is Q, $\emptyset_Q$ is the right-identity element for the operation $\underset{\backslash}{\overset{*}{\sim}}$ in the set $S_E(U)$.

7) $\emptyset_Q\underset{\backslash}{\overset{*}{\sim}}(C,Q)=\emptyset_Q$.

Let $\emptyset_Q=(S,Q)$. Then, $\forall j\in Q$; $S(j)=\emptyset$. Let $(S,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)=(A,Q)$, where $A(j)=S(j)\cap C'(j)$ for all $j\in Q$. Thus, $\forall j\in Q$; $A(j)=S(j)\cap C'((j)=\emptyset\cap C(j)=\emptyset$, hence $(A,Q)=\emptyset_Q$. Note that, for the soft sets whose parameter set is Q, $\emptyset_Q$ is the left-absorbing element for the operation $\underset{\backslash}{\overset{*}{\sim}}$ in the set $S_E(U)$.

8) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}\emptyset_E=(C,Q)$.

Proof: Let $\emptyset_E=(S,E)$. Hence $\forall j\in E$; $S(j)=\emptyset$. Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(S,E)=(A,Q)$. Thus, $A(j)=C(j)\cap S'(d)$ for all $j\in Q\cap E=Q$. Hence, $\forall j\in Q$, $A(j)=C(j)\cap S'(j)=C(j)\cap U=C(j)$, so $(A,Q)=(C,Q)$.

Note that, for the soft sets (no matter what the parameter set is), $\emptyset_E$ is the right identity element for the operation $\overset{*}{\underset{\backslash}{\sim}}$ in the set $S_E(U)$.

9) $\emptyset_E\overset{*}{\underset{\backslash}{\sim}}(C,Q)=U_{Q'}$.

Proof: Let $\emptyset_E=(S,E)$. Hence $\forall j\in E$; $S(j)=\emptyset$. Let $(S,E)\overset{*}{\underset{\backslash}{\sim}}(C,Q)=(A,E)$. Thus, $\forall j\in E$,

$$A(j)=\begin{cases} S'(j), & j\in E\backslash Q=Q' \\ S(j)\cap C'(j), & j\in Q\cap E=Q \end{cases}$$

Hence, $\forall j\in Q'$, $S'(j)=U$ and for all $j\in Q$, $S(j)\cap C'(j)=\emptyset\cap Q'(j)=\emptyset$, so $(A,Q)=U_{Q'}$.

10) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}U_Q=\emptyset_Q$.

Proof: Let $U_Q=(T,Q)$. Then, $\forall j\in Q$; $T(j)=U$. Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(T,Q)=(A,Q)$, where $A(j)=C(j)\cap T'(j)$, $\forall j\in Q$. Thus, $\forall j\in Q$; $A(j)=C(j)\cap T'(j)=C(j)\cap\emptyset=\emptyset$, hence $(A,Q)=\emptyset_Q$.

11) $U_Q\overset{*}{\underset{\backslash}{\sim}}(C,Q)=(C,Q)^r$.

Proof: Let $U_Q=(T,Q)$. Then, $\forall j\in Q$; $T(j)=U$. Assume that $(T,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)=(A,Q)$, where $A(j)=T(j)\cap C'(j)$, $\forall j\in Q$. Hence, $\forall j\in Q$; $A(j)=T(j)\cap C'(j)=U\cap C'(j)=C'(j)$. Thus, $(T,Q)=(C,Q)^r$.

12) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}U_E=\emptyset_Q$.

Proof: Let $U_E=(T,E)$. Hence, $\forall j\in E$, $T(j)=U$. Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(T,E)=(A,Q)$, where $A(j)=C(j)\cap T'(j)$ for all $j\in Q\cap E=Q$. Hence, $\forall j\in Q$, $A(j)=C(j)\cap T'(j)=C(j)\cap\emptyset=C(j)$, so $(A,Q)=\emptyset_Q$.

13) $U_E\underset{\backslash}{\overset{*}{\sim}}(C,Q)=(C,Q)^r$.

Proof: Let $U_E=(T,E)$. Then, $\forall j\in E$; $T(j)=U$. Let $(T,E)\underset{\backslash}{\overset{*}{\sim}}(C,Q)=(A,E)$, where $\forall j\in E$;

$$A(j)=\begin{cases} T'(j), & j\in E\setminus Q=Q' \\ T(j)\cap C'(j), & j\in E\cap Q=Q \end{cases}$$

Hence $\forall j\in Q$; $A(j)=T(j)\cap C'(j)=U\cap C'(j)=C'(j)$, thus $(A,Q)=(C,Q)^r$.

14) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)^r=(C,Q)$.

Proof: Let $(C,Q)^r=(A,Q)$. Hence, $\forall j\in Q$; $A(j)=C'(j)$. Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Q)=(T,Q)$, where $T(j)=C(j)\cap A'(j)$ for all $\forall j\in Q$. Hence, $\forall j\in Q$; $T(j)=C(j)\cap A'(j)=C(j)\cap C(j)=C(j)$, thus $(T,Q)=(C,Q)$. Note that, the relative complement of every soft set is its right identity element for the operation $\underset{\backslash}{\overset{*}{\sim}}$ in the set $S_E(U)$.

15) $(C,Q)^r\underset{\backslash}{\overset{*}{\sim}}(C,Q)=(C,Q)^r$.

Proof: Let $(C,Q)^r=(A,Q)$. Hence, $\forall j\in Q$; $A(j)=C'(j)$. Let $(A,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)=(T,Q)$, where $T(j)=A(j)\cap C'(j)$ for all $j\in Q$. Hence, $\forall j\in Q$; $T(j)=A(j)\cap C'(j)=C'(j)\cap C'(j)=C'(j)$, thus $(T,Q)=(C,Q)^r$. Note that, the relative complement of a soft set is the left absorbing element of its own soft set for the operation $\underset{\backslash}{\overset{*}{\sim}}$ in the set $S_E(U)$.

16) $[(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,I)]^r=(C,Q)\tilde{\mp}(Y,I)$.

Proof: Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)=(AQ)$. Then, $\forall j\in Q$,

$$A(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ C(j)\cap Y'(j), & j\in Q\cap I \end{cases}$$

Let $(A,Q)^{r}=(T,Q)$, so $\forall j\in Q$,

$$T(j)=\begin{cases} C(j), & j\in Q\backslash I \\ C'(j)\cup Y(j), & j\in Q\cap I \end{cases}$$

Thus, $(T,Q)=(C,Q)\tilde{\mp}(Y,I)$.

In classical theory, $C\cap Y=U \Leftrightarrow C=U$ and $Y=U$. Now, we have the following:

17) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)=U_Q \Leftrightarrow (C,Q)=U_Q$ and $(Y,Q)=\emptyset_Q$.

Proof: Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)=(T,Q)$. Hence, $T(j)=C(j)\cap Y'(j)$ for all $j\in Q$. Since $(T,Q)=U_Q$, $\forall j\in Q$, $T(j)=U$. Hence, $\forall j\in Q$, $T(j)=C(j)\cap Y'(j)=U \Leftrightarrow \forall j\in Q$, $C(j)=U$ and $Y'(j)=U \Leftrightarrow \forall j\in Q$, $C(j)=U$ and $Y(j)=\emptyset$ $\Leftrightarrow (C,Q)=U_Q$ and $(Y,Q)=\emptyset_Q$.

In classical theory $\emptyset\subseteq C$ for all C. Now, we have the following:

18) $\emptyset_Q\,\tilde{\subseteq}(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)$ and $\emptyset_I\,\tilde{\subseteq}(Y,I)\overset{*}{\underset{\backslash}{\sim}}(C,Q)$.

Proof: Let $\emptyset_Q=(S,Q)$. Hence, $\forall j\in Q$, $S(j)=\emptyset$.

Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)=(A,Q)$. When considering the soft subset axioms, i) $Q\subseteq Q$ and ii) $\forall j\in Q$; $S(j)=\emptyset$, $\emptyset\subseteq C'(j)$, and $\emptyset\subseteq C(j)\cap Y'(j)$. Thus, $\emptyset_Q\,\tilde{\subseteq}((C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)$. The proof of the theorem $\emptyset_I\,\tilde{\subseteq}(Y,I)\overset{*}{\underset{\backslash}{\sim}}(C,Q)$ is similar to the above proof.

In classical theory, $C\subseteq U$ for all C. Now, we have the following:

19) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)\tilde{\subseteq}\,U_Q$ and $(Y,I)\overset{*}{\underset{\backslash}{\sim}}(C,Q)\tilde{\subseteq}\,U_I$

Proof: Let U_Q=(T,Q). Hence, ∀j∈Q, T(j)=U. Let (C,Q)$\overset{*}{\underset{\setminus}{\sim}}$(Y,I)=(A,Q). When considering the soft subset axioms, i)Q⊆Q and ii) ∀j∈Q; T(j)=U, so C'(j)⊆U and C(j) ∩ Y'(j) ⊆U. Thus, (C,Q)$\overset{*}{\underset{\setminus}{\sim}}$(Y,I) $\tilde{\subseteq}$ U_Q. The proof of the theorem (Y,I)$\overset{*}{\underset{\setminus}{\sim}}(C,Q)\tilde{\subseteq}$ U_I is similar to the above proof.

In classical theory, C\Y $\subseteq$ C and Y\C $\subseteq$ Y. Moreover, C\Y $\subseteq Y'$ and Y\C $\subseteq C'$ Now, we have the following analogy:

20) (C,Q)$\overset{*}{\underset{\setminus}{\sim}}(Y,Q)\tilde{\subseteq}$(C,Q) and (Y,Q)$\overset{*}{\underset{\setminus}{\sim}}(C,Q)\tilde{\subseteq}$(Y,Q). Moreover, (C,Q)$\overset{*}{\underset{\setminus}{\sim}}(Y,Q)\tilde{\subseteq}$(Y,Q)^r and (Y,Q)$\overset{*}{\underset{\setminus}{\sim}}(C,Q)\tilde{\subseteq}$(C,Q)^r.

Proof: Let (C,Q)$\overset{*}{\underset{\setminus}{\sim}}$(Y,Q)=(A,Q). First of all, Q⊆ Q. Moreover, A(j)=C(j)∩Y'(j) for all ∀j∈Q. Since ∀j∈Q, A(j)=C(j)∩Y'(j)⊆C(j), thus (A,Q)=(C,Q)$\overset{*}{\underset{\setminus}{\sim}}(Y,Q)\tilde{\subseteq}$(C,Q). (Y,Q)$\overset{*}{\underset{\setminus}{\sim}}(C,Q)\tilde{\subseteq}$(Y,Q) can be shown similarly. Since ∀j∈Q, A(j)=C(j)∩Y'(j)⊆Y'(j), (A,Q)=(C,Q)$\overset{*}{\underset{\setminus}{\sim}}(Y,Q)\tilde{\subseteq}$ $(G,A)^r$. (Y,Q)$\overset{*}{\underset{\setminus}{\sim}}(C,Q)\tilde{\subseteq}$(C,Q)^r can be shown similary.

In classical theory, C\Y=C∩Y'. Now we have the following analogy.

21) (C,Q)$\overset{*}{\underset{\setminus}{\sim}}$(Y,Q) =(C,Q)$\overset{*}{\underset{\cap}{\sim}}$ $(Y,Q)^r$.

Proof: Let (C,Q)$\overset{*}{\underset{\cap}{\sim}}$ $(Y,Q)^r = (A,Q)$, where A(j)=C(j)∩Y'(j) for all j∈Q. Hence, A(j)=C(j)\Y(j) for all j∈Q. Thus, (A,Q)=(C,Q)$\overset{*}{\underset{\setminus}{\sim}}$(Y,Q).

In classical theory, C∪Y=(C\Y)∪(Y\C)∪(C∩Y). Now, we have the following analogy:

22) $(C,Q)\underset{\cup}{\overset{*}{\sim}}(Y,Q)=[(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,Q)]\underset{\cup}{\overset{*}{\sim}}[(Y,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)]\underset{\cup}{\overset{*}{\sim}}[(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q)]$.

Proof: Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,Q)=(A,Q)$, $(Y,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)=(K,Q)$ and $(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q)=(T,Q)$. Then, $A(j)=C(j)\cap Y'(j)$ for all $j\in Q$ and $K(j)=Y(j)\cap C'(j)$ for all $j\in Q\cap Q=Q$. And $T(j)=C(j)\cap Y(j)$ for all $j\in Q$. Now, let $(A,Q)\underset{\cup}{\overset{*}{\sim}}(K,Q)=(M,Q)$, where $M(j)=A(j)\cup K(j)$ for all $j\in Q$. Thus, $M(j)=[C(j)\cap Y'(j)]\cup[Y(j)\cap C'(j)]$. Now let $(M,Q)\underset{\cup}{\overset{*}{\sim}}(T,Q)=(W,Q)$ where $W(j)=M(j)\cup T(j)$ for all $j\in Q$. Thus, $W(j)=[C(j)\cap Y'(j)]\cup[Y(j)\cap C'(j)]\cup[C(j)\cap Y(j)]=C(j)\cup Y(j)$. Now, assume that $(C,Q)\underset{\cup}{\overset{*}{\sim}}(Y,Q)=(D,Q)$, where $\forall j\in Q$, $D(j)=C(j)\cup Y(j)$ for all $j\in Q$. It is seen that $(D,Q)=(T,Q)$.

In classicl theory; $C=(C\backslash Y)\cup(C\cap Y)$ and $Y=(C\backslash Y)\cup(C\cap Y)$. Now, we have the following analogy:

23) $(C,Q)=[(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,Q)]\underset{\cup}{\overset{*}{\sim}}[(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q)]$ and $(Y,Q)=[(Y,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)]\underset{\cup}{\overset{*}{\sim}}[(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q)]$

Proof: Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,Q)=(A,Q)$, where $A(j)=C(j)\cap Y'(j)$ for all $j\in Q$ and $(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q)=(K,Q)$, where $K(j)=C(j)\cap Y(j)$ for all $j\in Q$. Let $(A,Q)\underset{\cup}{\overset{*}{\sim}}(K,Q)=(T,Q)$, where $T(j)=A(j)\cup K(j)$ for all $j\in Q$. Therefore, $\forall j\in Q, T(j)=[C(j)\cap Y'(j)]\cup[C(j)\cap Y(j)]=C(j)$. Hence, $(T,Q)=(C,Q)$.

In classical theory, $C\cup Y=(C\backslash Y)\cup Y$ and $C\cup Y=(Y\backslash C)\cup C$. Now, we have the following analogy.

24) $(C,Q)\underset{\cup}{\overset{*}{\sim}}(Y,Q)=[(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,Q)]\underset{\cup}{\overset{*}{\sim}}(Y,Q)$ and $(C,Q)\underset{\cup}{\overset{*}{\sim}}(Y,Q)=[(Y,Q)\underset{\backslash}{\overset{*}{\sim}}(C,Q)]\underset{\cup}{\overset{*}{\sim}}(C,Q)$.

Proof: Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,Q)=(A,Q)$, where $A(j)=C(j)\cap Y'(j)$ for all $j\in Q$ and $(A,F)\underset{\cup}{\overset{*}{\sim}}(Y,Q)=(K,Q)$, where $K(j)=A(j)\cup Y(j)$ for all $j\in Q$. Thus, $(A,Q)\underset{\cup}{\overset{*}{\sim}}(Y,Q)=(K,Q)$, where $K(j)=[C(j)\cap Y'(j)]\cup Y(j)$ for all $j\in Q$. So, $K(j)=[C(j)\cup Y(j)]$ for all $j\in Q$. Assume that $(C,Q)\underset{\cup}{\overset{*}{\sim}}(Y,Q)=(S,Q)$, hence $(K,Q)=(S,Q)=(C,Q)\underset{\cup}{\overset{*}{\sim}}(Y,Q)$.

In classical theory, $C\subseteq Y \Leftrightarrow C\backslash Y=\emptyset$. Moreover, we have the following analogy.

25) $(C,Q)\tilde{\subseteq}(Y,Q) \Leftrightarrow (C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=\emptyset_Q$.

Proof: Let $(C,Q)\tilde{\subseteq}(Y,Q)$. Then $C(j)\subseteq Y(j)$, $\forall j\in Q$. And let $(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=(A,Q)$. Then, $A(j)=C(j)\setminus Y(j)$ for all $j\in Q$. Since $\forall j\in Q$, $C(j)\subseteq Y(j)$, then $C(j)\setminus Y(j)=\emptyset$, and hence $(A,Q)=(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=\emptyset_Q$, For the converse, we need to show that when $(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=\emptyset_Q$, then $(C,Q)\tilde{\subseteq}(Y,Q)$. To show this, let $(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=(T,Q)$. Then, $T(j)=C(j)\setminus Y(j)$ for all $j\in Q$. Since, $(T,Q)=Q$, $\forall j\in Q$, $C(j)\setminus Y(j)=\emptyset$. Then, $C(j)\subseteq Y(j), \forall j\in Q$. Thus, $(C,Q)\tilde{\subseteq}(Y,Q)$.

In classical theory, if $C\cap Y=\emptyset$, then $C\setminus Y=C$. Now, we have the following analogy:

26) If $(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q) = \emptyset_Q$, then $(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=(C,Q)$.

Proof: Let $(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q)=(A,Q)$. Then, $A(j)=C(j)\cap Y(j)$ for all $j\in Q$. Since, $(A,Q)=\emptyset_Q$, $A(j)=\emptyset$ for all $j\in Q$. Thus, $A(j)=C(j)\cap Y(j)=\emptyset$, and so, $C(j)\setminus Y(j)=C(j)$, $\forall j\in Q$.

Let $(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=(S,Q)$. Then, $S(j)=C(j)\setminus Y(j)$ for all $j\in Q$. Thus, $(S,Q)=(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=(C,Q)$.

In classical theory, $(C\setminus Y)\cap Y=\emptyset$ and $(Y\setminus C)\cap C=\emptyset$. Now, we have a similar analogy:

27) $[(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)]\underset{\cap}{\overset{*}{\sim}}(Y,Q)=\emptyset_Q$ and $[(Y,Q)\underset{\setminus}{\overset{*}{\sim}}(C,Q)]\underset{\cap}{\overset{*}{\sim}}(C,Q)=\emptyset_Q$

Proof: Let $(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,Q)=(A,Q)$. Then, $A(j)=C(j)\cap Y'(j)$ for all $j\in Q$. And let $(A,Q)\underset{\cap}{\overset{*}{\sim}}(Y,Q)=(T,Q)$, where $T(j)=A(j)\cap Y(j)$ for all $j\in Q$. Thus $\forall j\in Q$, $T(j)=[C(j)\cap Y'(j)]\cap Y(j)$ for all $j\in Q$. So, $T(j)=\emptyset$ for all $j\in Q$. Since $\forall j\in Q$, $T(j)=\emptyset$, $(T,Q)=\emptyset_Q$. Moreover, $[(Y,Q)\underset{\setminus}{\overset{*}{\sim}}(C,Q)]\underset{\cap}{\overset{*}{\sim}}(C,Q)=\emptyset_Q$ can be shown similarly.

REMARK 1: $[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]\overset{*}{\underset{\backslash}{\sim}}(Y,Q)=(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)$ and $[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]\overset{*}{\underset{\backslash}{\sim}}(C,Q)=(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)$ by (26) and (27). This is an analogy of $(C\backslash Y)\backslash Y=C\backslash Y$ and $(Y\backslash C)\backslash C=Y\backslash C$.

In classical theory, $(C\backslash Y)\cap(Y\backslash C)=\emptyset$. Now, we have the following analogy.

28) $[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]\overset{*}{\underset{\cap}{\sim}}[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]=\emptyset_Q$ and $[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]\overset{*}{\underset{\cap}{\sim}}[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]=\emptyset_Q$

Proof: Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)=(A,Q)$. Then, $A(j)=C(j)\cap Y'(j)$ for all $j\in Q$. Let $(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)=(K,Q)$. Then, $K(j)=Y(j)\cap C'(j)$ for all $j\in Q$. And let $(A,Q)\overset{*}{\underset{\cap}{\sim}}(K,Q)=(T,Q)$, where $T(j)=A(j)\cap K(j)$ for all $j\in Q$. Thus, $T(j)=[C(j)\cap Y'(j)]\cap[Y(j)\cap C'(j)]$ for all $j\in Q$. Hence, $T(j)=\emptyset$ for all $j\in Q$. Since $\forall j\in Q$, $T(j)=\emptyset$, $(T,Q)=\emptyset_Q$.

Moreover $[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]\overset{*}{\underset{\cap}{\sim}}[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]=\emptyset_Q$ can be shown similarly.

REMARK 2: From the theorem (26) and (28)**,** $[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]\overset{*}{\underset{\backslash}{\sim}}[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]=[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]$ and $[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]\overset{*}{\underset{\backslash}{\sim}}[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]=[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]$. This is an anology of $(C\backslash Y)\backslash(Y\backslash C)=C\backslash Y$ and $(Y\backslash C)\backslash(C\backslash Y)=Y\backslash C$.

In classical theory, $(C\backslash Y)\cap(C\cap Y)=\emptyset$ and $(Y\backslash C)\cap(C\cap Y)=\emptyset$. Now, we have the following analogy.

29) $[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)]\overset{*}{\underset{\cap}{\sim}}[(C,Q)\overset{*}{\underset{\cap}{\sim}}G,Q)]=\emptyset_Q$ and $[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]\overset{*}{\underset{\cap}{\sim}}[(C,Q)\overset{*}{\underset{\cap}{\sim}}G,Q)]=\emptyset_Q$.

Proof: Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,Q)=(A,Q)$. Then, $A(j)=C(j)\cap Y'(j)$ for all $j\in Q$. Let $(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,Q)=(K,Q)$. Then, $K(j)=C(j)\cap Y(j)$ for all $j\in Q$.And let $(A,Q)\overset{*}{\underset{\cap}{\sim}}(K,Q)=(T,Q)$, where $T(j)=A(j)\cap K(j)$ for all $j\in Q$. So, $T(j)=[C(j)\cap Y'(j)]\cap[C(j)\cap Y(j)]$ for all $j\in Q$. Hence, $T(j)=\emptyset$ for all $j\in Q$. Since $\forall j\in Q$, $T(j)=\emptyset$, $(T,Q)=\emptyset_Q$.

Moreover, $[(Y,Q)\overset{*}{\underset{\backslash}{\sim}}(C,Q)]\overset{*}{\underset{\cap}{\sim}}[(C,Q)\overset{*}{\underset{\cap}{\sim}}G,Q)]=\emptyset_Q$ can be shown similarly.

REMARK 3: By theroem (26) and (29), $[(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)]\overset{*}{\underset{\setminus}{\sim}}([(C,Q)\overset{*}{\underset{\cap}{\sim}}G,Q)]=[(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)]$ and $[(Y,Q)\overset{*}{\underset{\setminus}{\sim}}(C,Q)]\overset{*}{\underset{\setminus}{\sim}}[(C,Q)\overset{*}{\underset{\cap}{\sim}}G,Q)]=[(Y,Q)\overset{*}{\underset{\setminus}{\sim}}(C,Q)]$. This is an analogy of $(C\setminus Y)\setminus(C\setminus Y)=C\setminus Y$ and $(Y\setminus C)\setminus(Y\setminus C)=Y\setminus C$.

In classical theory, $C\cap(Y\setminus C)=\emptyset$ and $Y\cap(C\setminus Y)=\emptyset$. Now, we have the following analogy.

30) $(C,Q)\overset{*}{\underset{\cap}{\sim}}[(Y,Q)\overset{*}{\underset{\setminus}{\sim}}(C,Q)]=\emptyset_Q$ and $(Y,Q)\overset{*}{\underset{\cap}{\sim}}[(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)]=\emptyset_Q$

Proof: Let $(Y,Q)\overset{*}{\underset{\setminus}{\sim}}(C,Q)=(A,Q)$. Then, $A(j)=Y(j)\cap C'(j)$ for all $j\in Q$. Let $(C,Q)\overset{*}{\underset{\cap}{\sim}}(A,F)=(K,Q)$. Then, $K(j)=C(j)\cap A(j)$ for all $j\in Q$. Thus, $K(j)=C(j)\cap[Y(j)\cap C'(j)]$ for all $j\in Q$. Hence $\forall j\in Q$, $K(j)=\emptyset$, for all $j\in Q$. Since $\forall j\in Q$, $K(j)=\emptyset$, $(K,Q)=\emptyset_Q$. Moreover $(Y,Q)\overset{*}{\underset{\cap}{\sim}}[(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)]=\emptyset_Q$ can be shown similarly.

REMARK 4: By (26) and (30), $(C,Q)\overset{*}{\underset{\setminus}{\sim}}[(Y,Q)\overset{*}{\underset{\setminus}{\sim}}(C,Q)]=(C,Q)$ and $(Y,Q)\overset{*}{\underset{\setminus}{\sim}}[(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)]=(Y,Q)$. This is an analogy of $C\setminus(Y\setminus C)=C$ and $Y\setminus(C\setminus Y)=Y$.

In classical theory, $C\setminus(C\setminus Y)=C\cap Y$ and $Y\setminus(Y\setminus C)=C\cap Y$. Now, we have the following:

31) $(C,Q)\overset{*}{\underset{\setminus}{\sim}}[(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)]=(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,Q)$ and $(Y,Q)\overset{*}{\underset{\setminus}{\sim}}[(Y,Q)\overset{*}{\underset{\setminus}{\sim}}(C,Q)]=(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,Q)$.

Proof: Let $(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)=(A,Q)$. Then, $A(j)=C(j)\setminus Y(j)$ for all $j\in Q$. Let $(C,Q)\overset{*}{\underset{\setminus}{\sim}}(A,Q)=(K,Q)$. Then, $K(j)=C(j)\setminus A(j)$ for all $j\in Q$. Thus, $K(j)=C(j)\setminus(C(j)\setminus Y(j))$ for all $j\in Q$. Hence $\forall j\in Q$, $K(j)=C(j)\cap Y(j)$ for all $j\in Q$. Thus $\forall j\in Q$, $(K,Q)=(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,Q)$. Moreover $(Y,Q)\overset{*}{\underset{\setminus}{\sim}}[(Y,Q)\overset{*}{\underset{\setminus}{\sim}}(C,Q)]=(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,Q)$ can be shown similarly.

In classical theory, $C\setminus(C\cap Y)=C\setminus Y$ and $Y\setminus(C\cap Y)=Y\setminus C$. Now we have the following:

32) $(C,Q)\overset{*}{\underset{\setminus}{\sim}}[(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,Q)]=(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)$ and $(Y,Q)\overset{*}{\underset{\setminus}{\sim}}[(Y,Q)\overset{*}{\underset{\cap}{\sim}}(C,Q)]=(C,Q)\overset{*}{\underset{\setminus}{\sim}}(Y,Q)$.

Proof: Let (C,Q) $\overset{*}{\underset{\cap}{\sim}}$ (Y,Q)=(A,Q). Then, A(j)=C(j)∩Y(j) for all j∈Q. Let (C,Q) $\overset{*}{\underset{\backslash}{\sim}}$ (A,Q)=(K,Q). Then, K(j)=C(j)\A(j) for all j∈Q. Thus, K(j)=C(j)\[(C(j)∩Y(j)] for all j∈Q. Hence ∀j∈Q, K(j)=C(d)\ Q(d) for all j∈Q. Thus, ∀j∈Q, (K,Q)=(C,Q) $\overset{*}{\underset{\backslash}{\sim}}$ (Y,Q). Moreover (Y,Q) $\overset{*}{\underset{\backslash}{\sim}}$ [(Y,Q) $\overset{*}{\underset{\cap}{\sim}}$ (C,Q)]=(C,Q) $\overset{*}{\underset{\backslash}{\sim}}$ (Y,Q) can be shown similarly.

4. Distribution Rules

In this section, the distribution of complementary soft binary piecewise difference (\) operation over other soft set operations are examined in detail and many interesting results are obtained.

4.1. Distribution of complementary soft binary piecewise difference (\) operation over extended soft set operations:

i) Left-distribution of complementary soft binary piecewise difference (\) operation over extended soft set operations:

The followings are satisfied, when $Q \cap I' \cap Z = \emptyset$.

1) (C,Q) $\overset{*}{\underset{\backslash}{\sim}}$ [(Y,I)$\cap_{\varepsilon}$(A,Z)=[(C,Q) $\overset{*}{\underset{\backslash}{\sim}}$ (Y,I)] $\tilde{\cup}$ [(A,Z) $\overset{*}{\underset{\gamma}{\sim}}$ (C,Q)].

Proof: Let first handle the left-hand side of the equality, and let (Y,I) $\cap_{\varepsilon}$(A,Z)=(M,I∪Z) where ∀j∈I∪Z;

$$M(j) = \begin{cases} Y(j), & j \in I \backslash Z \\ A(j), & j \in Z \backslash I \\ Y(j) \cap A(j), & j \in I \cap Z \end{cases}$$

Assume that (C,Q) $\overset{*}{\underset{\backslash}{\sim}}$ (M,I∪Z)=(N,Q), where ∀j∈Q;

$$N(j) = \begin{cases} C'(j), & j \in Q \backslash (I \cup Z) \\ C(j) \cap M'(j), & j \in Q \cap (I \cup Z) \end{cases}$$

Hence,

$$N(j)=\begin{cases} C'(j), & j\in Q\setminus (I\cup Z) = Q\cap I'\cap Z' \\ C(j)\cap Y'(j), & j\in Q\cap(I\setminus Z)= Q\cap I\cap Z' \\ C(j)\cap A'(j), & j\in Q\cap(Z\setminus I)= Q\cap I'\cap Z \\ C(j)\cap[(Y'(j)\cup A'(j)], & j\in Q\cap I\cap Z= Q\cap I\cap Z \end{cases}$$

Now let's handle the right-hand side of the equality $[(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cup}[(A,Z)\underset{\gamma}{\overset{*}{\sim}}(C,Q)]$. Assume that

$(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,I)=(V,Q)$, where $\forall j\in Q$;

$$V(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap Y'(j), & j\in Q\cap I \end{cases}$$

Let $(A,Z)\underset{\gamma}{\overset{*}{\sim}}(C,Q)=(W,Q)$, where $\forall j\in Z$;

$$W(j)=\begin{cases} A'(j), & j\in Z\setminus Q \\ A'(j)\cap C(j), & j\in Z\cap Q \end{cases}$$

Let $(V,Q)\ \tilde{\cup}\ (W,Z)=(T,Q)$, where $\forall j\in Q$;

$$T(j)=\begin{cases} V(j), & j\in Q\setminus Z \\ V(j)\cup W(j), & j\in Q\cap Z \end{cases}$$

Hence,

$$T(j)=\begin{cases} C'(j), & j\in(Q-I)-Z=Q\cap I'\cap Z' \\ C(j)\cap Y'(j), & j\in(Q\cap I)-Z=Q\cap I\cap Z' \\ C'(j)\cup A'(j), & j\in (Q-I)\cap(Z-Q)=\emptyset \\ C'(j)\cup[A'(j)\cap C(j)], & j\in(Q-I)\cap(Z\cap Q)=Q\cap I'\cap Z \\ [C(j)\cap Y'(j)\cup A'(j), & j\in(Q\cap I)\cap(Z-Q)=\emptyset \\ [C(j)\cap Y'(j)]\cup[A'(j)\cap C(j)], & j\in(Q\cap I)\cap(Z\cap Q)=Q\cap I\cap Z \end{cases}$$

It is seen that N=T.

2) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)\cup_{\varepsilon}(A,Z)=[(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\underset{\gamma}{\overset{*}{\sim}}(C,Q)]$

3) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}\ [(Y,I)\lambda_{\varepsilon}(A,Z)]=[(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\underset{\cap}{\overset{*}{\sim}}(C,Q)],$

4) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}\ [(Y,I)\backslash_{\varepsilon}(A,Z)]\ =\ [(C,Q)\underset{\backslash}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cup}\ [(A,Z)\underset{\cap}{\overset{*}{\sim}}(C,Q)]$

ii) Right-distribution of complementary soft binary piecewise difference (\) operation over extended soft set operations:

1) $[(C,Q)\cup_{\varepsilon}(Y,I)]\underset{\backslash}{\overset{*}{\sim}}(A,Z)=\ [(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)]\ \cup_{\varepsilon}\ [(Y,I)\underset{\backslash}{\overset{*}{\sim}}(A,Z)]$, where $Q\cap I\cap Z'=\emptyset$

Moreover, $[(C,Q)\cup_{\varepsilon}(Y,I)]\underset{\backslash}{\overset{*}{\sim}}(A,Z)=\ [(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)]\ \cap_{\varepsilon}\ [(Y,I)\underset{\backslash}{\overset{*}{\sim}}(A,Z)]$,where $Q\cap I\cap Z=\emptyset$.

Proof: Let's first handle the left-hand side of the equality. Let $(C,Q)\cup_{\varepsilon}(Y,I)=(M,Q\cup I)$ where $\forall j\in Q\cup I$

$$M(j)=\begin{cases} C(j), & j\in Q\backslash I \\ Y(j), & j\in I\backslash Q \\ C(j)\cup Y(j), & j\in Q\cap I \end{cases}$$

Suppose that $(M,Q\cup I)\underset{\backslash}{\overset{*}{\sim}}(A,Z)=(N,Q\cup I)$, where $\forall j\in Q\cup I$;

$$N(j)=\begin{cases} M'(j), & j\in(Q\cup I)\backslash Z \\ M(j)\cap A'(j), & j\in(Q\cup I)\cap Z \end{cases}$$

$$N(j)=\begin{cases} C'(j), & j\in(Q\backslash I)\backslash Z=Q\cap I'\cap Z' \\ Y'(j), & j\in(I\backslash Q)\backslash Z=Q'\cap I\cap Z' \\ C'(j)\cap Y'(j), & j\in(Q\cap I)\backslash Z=Q\cap I\cap Z' \\ C(j)\cap A'(j), & j\in(Q\backslash I)\cap Z=Q\cap I'\cap Z \\ Y(j)\cap A'(j), & j\in(I\backslash Q)\cap Z=Q'\cap I\cap Z \\ [C(j)\cup Y(j)]\cap A'(j), & j\in(Q\cap I)\cap Z=Q\cap I\cap Z \end{cases}$$

Now let's handle the right-hand side of the equality: $[(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)] \cup_{\varepsilon} [(Y,I)\underset{\backslash}{\overset{*}{\sim}}(A,Z)]$. Let

$(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)=(V,Q)$, where $\forall j\in Q$;

$$V(j)=\begin{cases} C'(j), & j\in Q\setminus Z \\ C(j)\cap A'(j), & j\in Q\cap Z \end{cases}$$

Let $(Y,I)\underset{\backslash}{\overset{*}{\sim}}(A,Z)=(W,I)$, where $\forall j\in I$;

$$W(j)=\begin{cases} Y'(j), & j\in I\setminus Z \\ Y(j)\cap A'(j), & j\in I\cap Z \end{cases}$$

Assume that $(V,Q)\cup_{\varepsilon}(W,I)=(T,Q\cup I)$, where $\forall j\in Q\cup I$;

$$T(j)=\begin{cases} V(j), & j\in Q\setminus I \\ W(j), & j\in I\setminus Q \\ V(j)\cup W(j), & j\in Q\cap I \end{cases}$$

Hence,

$$T(j)=\begin{cases} C'(j), & j\in (Q\setminus Z)\setminus I=Q\cap I'\cap Z' \\ C(j)\cap A'(j), & j\in (Q\cap Z)\setminus I=Q\cap I'\cap Z \\ Y'(j), & j\in (I\setminus Z)\setminus Q=Q'\cap I\cap Z' \\ Y(j)\cap A'(j), & j\in (I\cap Z)\setminus Q=Q'\cap I\cap Z \\ C'(j)\cup Y'(j), & j\in (Q\setminus Z)\cap(I\setminus Z)=Q\cap I\cap Z' \\ C'(j)\cup[Y(j)\cap A'(j)] & j\in (Q\setminus Z)\cap(I\cap Z)=\emptyset \\ [C(j)\cap A'(j)]\cup Y'(j), & j\in (Q\cap Z)\cap(I\setminus Z)=\emptyset \\ [C(j)\cap A'(j)]\cup[Y(j)\cap A'(j)], & j\in (Q\cap Z)\cap(I\cap Z)=Q\cap I\cap Z \end{cases}$$

It is seen that that N=T. $[(C,Q)\cup_{\varepsilon}(Y,I)]\overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]\cap_{\varepsilon}[(Y,I)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]$,where $Q\cap I\cap Z=\emptyset$ can be shown as well.

2) $[(C,Q)\cap_{\varepsilon}(Y,I)]\overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]\cap_{\varepsilon}[(Y,I)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]$, where$Q\cap I\cap Z'=\emptyset$.

Moreover, $[(C,Q)\cap_{\varepsilon}(Y,I)]\overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]\cup_{\varepsilon}[(Y,I)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]$,where $Q\cap I\cap Z=\emptyset$.

3) $[(C,Q)\lambda_{\varepsilon}(Y,I)]\overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]\cup_{\varepsilon}[(Y,I)\overset{*}{\underset{\theta}{\sim}}(A,Z)]$, where $Q\cap I\cap Z'=Q'\cap I\cap Z=\emptyset$.

4) $[(C,Q)\backslash_{\varepsilon}(Y,I)]\overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]\cap_{\varepsilon}[(Y,I)\overset{*}{\underset{\theta}{\sim}}(A,Z)]$,where $Q\cap I\cap Z'=Q'\cap I\cap Z=\emptyset$.

4.2. Distribution of complementary soft binary piecewise difference (\) operation over complementary extended soft set operations:

i) Left-distribution of complementary soft binary piecewise difference (\) operations over extended complementary soft set operations:

The followings are satisfied when $Q\cap I'\cap Z=\emptyset$.

1) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}[(Y,I)\overset{*}{\theta_{\varepsilon}}(A,Z)]=[(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,I)]\widetilde{\cup}[(A,Z)\overset{*}{\underset{\cap}{\sim}}(C,Q)]$.

Proof: Let's first handle the left-hand side of the equality. Assume $(Y,I)\overset{*}{\theta_{\varepsilon}}(A,Z)=(M,I\cup Z)$, so $\forall j\in I\cup Z$,

$$M(j)=\begin{cases} Y'(j), & j\in I\backslash Z \\ A'(j), & j\in Z\backslash I \\ Y'(j)\cap A'(j), & j\in I\cap Z \end{cases}$$

Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(M,I\cup Z)=(N,Q)$, then $\forall j\in Q$,

$$N(j)=\begin{cases} C'(j), & j\in Q\backslash(I\cup Z) \\ C(j)\cap M'(j), & j\in Q\cap(I\cup Z) \end{cases}$$

$$N(j)=\begin{cases} C'(j), & j\in Q\setminus(I\cup Z)=Q\cap I'\cap Z' \\ C(j)\cap Y(j), & j\in Q\cap(I\setminus Z)=Q\cap I\cap Z' \\ C(j)\cap A(j), & j\in Q\cap(Z\setminus I)=Q\cap I'\cap Z \\ C(j)\cap[(Y(j)\cup A(j)], & j\in Q\cap I\cap Z=Q\cap I\cap Z \end{cases}$$

Now let's handle the right-hand side of the equality $[(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,I)]\ \tilde{\cup}\ [(A,Z)\overset{*}{\underset{\cap}{\sim}}(C,Q)]$. Let $(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,I)=(V,Q)$, so $\forall j\in Q$,

$$V(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap Y(j), & j\in Q\cap I \end{cases}$$

Let $(A,Z)\overset{*}{\underset{\cap}{\sim}}(C,Q)=(W,Z)$, hence $\forall j\in Z$,

$$W(j)=\begin{cases} A'(j), & j\in Q\setminus Z \\ A(j)\cap C(j), & j\in Z\cap Q \end{cases}$$

Assume that $(V,Q)\tilde{\cup}(W,Z)=(T,Q)$, hence $\forall j\in Q$,

$$T(j)=\begin{cases} V(j), & j\in Q\setminus Z \\ V(j)\cup W(j), & j\in Q\cap Z \end{cases}$$

Hence,

$$T(j)=\begin{cases} C'(j), & j\in(Q\setminus I)\setminus Z=Q\cap I'\cap Z' \\ C(j)\cap Y(j), & j\in(Q\cap I)\setminus Z=Q\cap I\cap Z' \\ C'(j)\cup A'(j), & j\in(Q\setminus I)\cap(Z\setminus Q)=\emptyset \\ C'(j)\cup[A(j)\cap C(j)], & j\in(Q\setminus I)\cap(Z\cap Q)=Q\cap I'\cap Z \\ [C(j)\cap Y(j)]\cup A'(j & j\in(Q\cap I)\cap(Z\setminus Q)=\emptyset \\ [C(j)\cap Y(j)]\cup[A(j)\cap C(j)], & j\in(Q\cap I)\cap(Z\cap Q)=Q\cap I\cap Z \end{cases}$$

It is seen that that N=T.

2) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}[(Y,I)\overset{*}{*_{\varepsilon}}(A,Z)] = [(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\overset{*}{\underset{\cap}{\sim}}(C,Q)]$

3) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}[(Y,I)\overset{*}{+_{\varepsilon}}(A,Z)] = [(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\overset{*}{\underset{\gamma}{\sim}}(C,Q)]$

4) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}[(Y,I)\overset{*}{\gamma_{\varepsilon}}(A,Z)] = [(C,Q)\overset{*}{\underset{\cap}{\sim}}(Y,I)]\ \tilde{\cup}\ [(A,Z)\overset{*}{\underset{\gamma}{\sim}}(C,Q)]$

ii) Right-distribution of complementary soft binary piecewise difference (\) operation over complementary extended soft set operations:

1) $[(C,Q)\overset{*}{*_{\varepsilon}}(Y,I)]\overset{*}{\underset{\backslash}{\sim}}(A,Z) = [(C,Q)\underset{\theta}{\sim}(A,Z)]\ \cup_{\varepsilon}\ [(Y,I)\underset{\theta}{\sim}(A,Z)]$ where $Q\cap I\cap Z'=\emptyset$.

Proof: Let's first handle the left-hand side of the equality, let $(C,Q)\overset{*}{*_{\varepsilon}}(Y,I)=(M,Q\cup I)$, where $\forall j\in Q\cup I$;

$$M(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ Y'(j), & j\in I\backslash Q \\ C'(j)\cup Y'(j), & j\in Q\cap I \end{cases}$$

Let $(M,Q\cup I)\overset{*}{\underset{\backslash}{\sim}}(A,Z)=(N,Q\cup I)$, where $\forall j\in Q\cup I$;

$$N(j)=\begin{cases} M'(j), & j\in (Q\cup I)\backslash Z \\ M(j)\cap A'(j), & j\in (Q\cup I)\cap Z \end{cases}$$

Thus,

$$N(j)=\begin{cases} C(j), & j\in (Q\backslash I)\backslash Z=Q\cap I'\cap Z' \\ Y(j), & j\in (I\backslash Q)\backslash Z=Q'\cap I\cap Z' \\ C(j)\cap Y(j), & j\in (Q\cap I)\backslash Z=Q\cap I\cap Z' \\ C'(j)\cap A'(j), & j\in (Q\backslash I)\cap Z=Q\cap I'\cap Z \\ Y'(j)\cap A'(j), & j\in (I\backslash Q)\cap Z=Q'\cap I\cap Z \\ [C'(j)\cup Y'(j)]\cap A'(j), & j\in (Q\cap I)\cap Z=Q\cap I\cap Z \end{cases}$$

Now let handle the right hand side of the equality: $[(C,Q)\underset{\theta}{\sim}(A,Z)]\cup_{\varepsilon}[(Y,I)\underset{\theta}{\sim}(A,Z)]$. Assume that $(C,Q)\underset{\theta}{\sim}(A,Z)=(V,Q)$, where $\forall j\in Q$;

$$V(j)=\begin{cases} C(j), & j\in Q\setminus Z \\ C'(j)\cap A'(j), & j\in Q\cap Z \end{cases}$$

Let $(Y,I)\underset{\theta}{\sim}(A,Z)=(W,I)$, where $\forall j\in I$;

$$W(j)=\begin{cases} Y(j), & j\in I\setminus Z \\ Y'(j)\cap A'(j), & j\in I\cap Z \end{cases}$$

Assume that $(V,Q)\cup_{\varepsilon}(W,I)=(T,Q\cup I)$, where $\forall j\in Q\cup I$;

$$T(j)=\begin{cases} V(j), & j\in Q\setminus I \\ W(j), & j\in I\setminus Q \\ V(j)\cup W(j), & j\in Q\cap I \end{cases}$$

Thus,

$$T(j)=\begin{cases} C(j), & j\in (Q\setminus Z)\setminus I=Q\cap I'\cap Z' \\ C'(j)\cap A'(j), & j\in (Q\cap Z)\setminus I=Q\cap I'\cap Z \\ Y(j), & j\in (I\setminus Z)\setminus Q=Q'\cap I\cap Z' \\ Y'(j)\cap A'(j), & j\in (I\cap Z)\setminus Q=Q'\cap I\cap Z \\ C(j)\cup Y(j), & j\in (Q\setminus Z)\cap(I\setminus Z)=Q\cap I\cap Z' \\ C(j)\cup[Y'(j)\cap A'(j)], & j\in (Q\setminus Z)\cap(I\cap Z)=\emptyset \\ [C'(j)\cap A'(j)]\cup Y(j), & j\in (Q\cap Z)\cap(I\setminus Z)=\emptyset \\ [C'(j)\cap A'(j)]\cup[Y'(j)\cap A'(j)], & j\in (Q\cap Z)\cap(I\cap Z)=Q\cap I\cap Z \end{cases}$$

It is seen that that N=T.

2) $[(C,Q)\overset{*}{\theta_{\varepsilon}}(Y,I)]\ \overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\underset{\theta}{\sim}(A,Z)]\ \cap_{\varepsilon}\ [(Y,I)\underset{\theta}{\sim}(A,Z)]$, where $Q\cap I\cap Z'=\emptyset$.

3)$[(C,Q)\overset{*}{+_{\varepsilon}}(Y,I)]\ \overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\underset{\theta}{\sim}(A,Z)]\ \cup_{\varepsilon}\ [(Y,I)\underset{\backslash}{\sim}(A,Z)]$, where $Q\cap I\cap Z'=Q'\cap I\cap Z=\emptyset$.

4) $[(C,Q)\overset{*}{\gamma_{\varepsilon}}(Y,I)]\ \overset{*}{\underset{\backslash}{\sim}}(A,Z)=[(C,Q)\underset{\theta}{\sim}(A,Z)]\ \cap_{\varepsilon}\ [(Y,I)\overset{\sim}{\backslash}(A,Z)]$ where $Q\cap I\cap Z'=Q'\cap I\cap Z=\emptyset$.

4.3. Distribution of complementary soft binary piecewise difference (\) operation over soft binary piecewise operations:

i) Left -distribution of complementary soft binary piecewise difference (\) operation over soft binary piecewise operations:

The followings are satisfied when $Q\cap I'\cap Z=\emptyset$.

1) $(C,Q)\overset{*}{\underset{\backslash}{\sim}}\ [(Y,I)\tilde{\cup}(A,Z)]\ =\ [(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\ \overset{*}{\underset{\gamma}{\sim}}(C,Q)]$

Proof: Let's first handle the left-hand side of the equality, let $(Y,I)\tilde{\cup}(A,Z)=(M,I)$, where $\forall j\in I$;

$$M(j)=\begin{cases} Y(j), & j\in I\backslash Z \\ Y(j)\cup A(j), & j\in I\cap Z \end{cases}$$

Let $(C,Q)\overset{*}{\underset{\backslash}{\sim}}(M,I)=(N,Q)$, where $\forall j\in Q$;

$$N(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ C(j)\cap M'(j), & j\in Q\cap I \end{cases}$$

Thus,

$$N(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ C(j)\cap Y'(j), & j\in Q\cap(I\backslash Z)=Q\cap I\cap Z' \\ C(j)\cap[Y'(j)\cap A'(j)] & j\in Q\cap I\cap Z \end{cases}$$

Now let's handle the right-hand side of the equality: $[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\overset{*}{\underset{\gamma}{\sim}}(C,Q)]$. Let

$(C,Q)\overset{*}{\underset{\backslash}{\sim}}(Y,I)=(V,Q)$, where $\forall j\in Q$;

$$V(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ C(j)\cap Y'(j), & j\in Q\cap I \end{cases}$$

Let $(A,Z)\underset{\gamma}{\overset{*}{\sim}}(C,Q)=(W,Z)$, where $\forall j\in Z$;

$$W(j)=\begin{cases} A'(j), & j\in Z\setminus Q \\ A'(j)\cap C(j), & j\in Z\cap Q \end{cases}$$

Suppose $(V,Q)\tilde{\cap}(W,Z)=(T,Q)$, where $\forall j\in Q$;

$$T(j)=\begin{cases} V(j), & j\in Q\setminus Z \\ V(j)\cap W(j), & j\in Q\cap Z \end{cases}$$

Therefore,

$$T(j)=\begin{cases} C'(j), & j\in(Q\setminus I)\setminus Z=Q\cap I'\cap Z' \\ C(j)\cap Y'(j), & j\in(Q\cap I)\setminus Z=Q\cap I\cap Z' \\ C'(j)\cap A'(j), & j\in(Q\setminus I)\cap(Z\setminus Q)=\emptyset \\ C'(j)\cap[A'(j)\cap C(j)], & j\in(Q\setminus I)\cap(Z\cap Q)=Q\cap I'\cap Z \\ [C(j)\cap Y'(j)]\cap A'(j), & j\in(Q\cap I)\cap(Z\setminus Q)=\emptyset \\ [C(j)\cap Y'(j)]\cap[A'(j)\cap C(j)] & j\in(Q\cap I)\cap(Z\cap Q)=Q\cap I\cap Z \end{cases}$$

Here let handle $j\in Q\setminus I$ in the first equation of the first line. Since $Q\setminus I=Q\cap I'$, if $j\in I'$, then $j\in Z\setminus I$ or $j\in(I\cup Z)'$. Hence, if $j\in Q\setminus I$, then $j\in Q\cap I'\cap Z'$ or $j\in Q\cap I'\cap Z$. Thus, it is seen that that N=T, where $Q\cap I\cap Z=\emptyset$.

2)$(C,Q)\underset{\setminus}{\overset{*}{\sim}}[(Y,I)\tilde{\cap}(A,Z)]=[(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cup}\ [(A,Z)\underset{\gamma}{\overset{*}{\sim}}(C,Q)]$

3)$(C,Q)\underset{\setminus}{\overset{*}{\sim}}[(Y,I)\tilde{\lambda}(A,Z)]=\ [(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\underset{\cap}{\overset{*}{\sim}}(C,Q)]$

4)$(C,Q)\underset{\setminus}{\overset{*}{\sim}}[(Y,I)\tilde{\setminus}(A,Z)]=\ [(C,Q)\underset{\setminus}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cup}\ [(A,Z)\underset{\cap}{\overset{*}{\sim}}(C,Q)]$

ii) Right-distribution of complementary soft binary piecewise difference (\) operation over soft binary piecewise operations:

The followings are satisfied when $Q\cap I'\cap Z=\emptyset$.

1) $[(C,A)\,\tilde{\cap}\,(Y,I)]\ \overset{*}{\underset{\backslash}{\sim}}\ (A,Z)=\ [(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]\ \tilde{\cap}\ [(Y,I)\ \overset{*}{\underset{\backslash}{\sim}}(A,Z)].$

Proof: Let's first handle the left-hand side of the equality. Suppose $(C,Q)\tilde{\cap}(Y,I)=(M,Q)$, where $\forall j\in Q$,

$$M(j)=\begin{cases} C(j), & j\in Q\backslash I \\ C(j)\cap Y(j), & j\in Q\cap I \end{cases}$$

Let $(M,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)=(N,Q)$, where $\forall j\in Q$,

$$N(j)=\begin{cases} M'(j), & j\in Q\backslash Z \\ M(j)\cap A'(j), & j\in Q\cap Z \end{cases}$$

Thus,

$$N(j)=\begin{cases} C'(j), & j\in (Q\backslash I)\backslash Z = Q\cap I'\cap Z' \\ C'(j)\cup Y'(j), & j\in (Q\cap I)\backslash Z= Q\cap I\cap Z' \\ C(j)\cap A'(j), & j\in (Q\backslash I)\cap Z= Q\cap I'\cap Z \\ [C(j)\cap Y(j)]\cap A'(j), & j\in (Q\cap I)\cap Z= Q\cap I\cap Z \end{cases}$$

Now let's handle the right-hand side of the equality: $[(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)]\ \tilde{\cap}\ [(Y,I)\ \overset{*}{\underset{\backslash}{\sim}}(A,Z)]$. Let

$(C,Q)\overset{*}{\underset{\backslash}{\sim}}(A,Z)=(V,Q)$, where $\forall j\in Q$;

$$V(j)=\begin{cases} C'(j), & j\in Q\backslash Z \\ C(j)\cap A'(j), & j\in Q\cap Z \end{cases}$$

Let $(Y,I)\overset{*}{\underset{\backslash}{\sim}}(A,Z)=(W,I)$, where $\forall j\in I$;

$$W(j)=\begin{cases} Y'(j), & j\in I\backslash Z \\ Y(j)\cap A'(j), & j\in I\cap Z \end{cases}$$

Suppose that $(V,Q)\tilde{\cap}(W,I)=(T,Q)$, where $\forall j\in Q$;

$$T(j)=\begin{cases} V(j), & j\in Q\backslash I \\ V(j)\cap W(j), & j\in Q\cap I \end{cases}$$

$$T(j)=\begin{cases} C'(j), & j\in (Q\backslash Z)\backslash I=Q\cap I'\cap Z' \\ C(j)\cap A'(j), & j\in(Q\cap Z)\backslash I=Q\cap I'\cap Z \\ C'(j)\cap Y'(j), & j\in(Q\backslash Z)\cap(I\backslash Z)=Q\cap I\cap Z' \\ C'(j)\cap[Y(j)\cap A'(j)], & j\in(Q\backslash Z)\cap(I\cap Z)=\emptyset \\ [C(j)\cap A'(j)\cap Y'(j), & j\in(Q\cap Z)\cap(I\backslash Z)=\emptyset \\ [C(j)\cap A'(j)\cap[Y(j)\cap A'(j)], & j\in(Q\cap Z)\cap(I\cap Z)=Q\cap I\cap Z \end{cases}$$

It is seen that that N=T.

2) $[(C,A)\tilde{\cup}(Y,I)] \underset{\backslash}{\overset{*}{\sim}} (A,Z)= [(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)] \ \tilde{\cup}\ [(Y,I)\underset{\backslash}{\overset{*}{\sim}}(A,Z)].$

3) $[(C,A)\ \tilde{\lambda}(Y,I)] \underset{\backslash}{\overset{*}{\sim}} (A,Z)= [(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)] \ \tilde{\cup}\ [(Y,I) \underset{\theta}{\overset{*}{\sim}}(A,Z)].$

4) $[(C,A)\ \tilde{\backslash}(Y,I)] \underset{\backslash}{\overset{*}{\sim}} (A,Z)= [(C,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)] \ \tilde{\cap}\ [(Y,I) \underset{\theta}{\overset{*}{\sim}}(A,Z)].$

4.4. Distribution of complementary soft binary piecewise difference (\) operation over complementary soft binary piecewise operations:

i) Left-distribution of complementary soft binary piecewise difference (\) operation over complementary soft binary piecewise operations:

The followings are satisfied when $Q\cap I'\cap Z=\emptyset$.

1) $(C,Q)\underset{\backslash}{\overset{*}{\sim}} [(Y,I)\underset{*}{\overset{*}{\sim}}(A,Z)] = [(C,Q)\underset{\cap}{\overset{*}{\sim}} (Y,I)]\ \tilde{\cap}\ [(A,Z) \underset{\cap}{\overset{*}{\sim}} (C,Q)]$

Proof: Let's first handle the left-hand side of the equality, let $(Y,I)\underset{*}{\overset{*}{\sim}}(A,Z)=(M,I)$, where $\forall j\in I$;

$$M(j)=\begin{cases} Y'(j), & j\in I\setminus Z \\ Y'(j)\cup A'(j), & j\in I\cap Z \end{cases}$$

Let $(C,Q)\underset{\setminus}{\overset{*}{\sim}}(M,I)=(N,Q)$, where $\forall j\in Q$;

$$N(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap M'(j), & j\in Q\cap I \end{cases}$$

Thus,

$$N(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap Y(j), & j\in Q\cap(I\setminus Z)= Q\cap I\cap Z' \\ C(j)\cap[(Y(j)\cap A(j)], & j\in Q\cap I\cap z= Q\cap I\cap Z \end{cases}$$

Now let's handle the right-hand side of the equality: $[(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\underset{\cap}{\overset{*}{\sim}}(C,Q)]$. Let

$(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,I)=(V,Q)$, where $\forall j\in Q$;

$$V(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C(j)\cap Y(j), & j\in Q\cap I \end{cases}$$

Suppose that $(A,Z)\underset{\cap}{\overset{*}{\sim}}(C,Q)=(W,Z)$, where $\forall j\in Z$;

$$W(j)=\begin{cases} A'(j), & j\in Z\setminus Q \\ A(j)\cap C(j), & j\in Z\cap Q \end{cases}$$

Let $(V,Q)\tilde{\cap}(W,Z)=(T,Q)$, where $\forall j\in Q$;

$$T(j)=\begin{cases} V(j), & j\in Q\setminus Z \\ V(j)\cap W(j), & j\in Q\cap Z \end{cases}$$

Hence,

$$T(j)=\begin{cases} C'(j), & j\in(Q\backslash I)\backslash Z =Q\cap I'\cap Z' \\ C(j)\cap Y(j), & j\in(Q\cap I)\backslash Z=Q\cap I\cap Z' \\ C'(j)\cap A'(j), & j\in(Q\backslash I)\cap(Z\backslash Q)=\emptyset \\ C'(j)\cap[A(j)\cap C(j)], & j\in(Q\backslash I)\cap(Z\cap Q)=Q\cap I'\cap Z \\ [C(j)\cap Y(j)]\cap A'(j), & j\in(Q\cap I)\cap(Z\backslash Q)=\emptyset \\ [C(j)\cap Y(j)]\cap[A(j)\cap C(j)], & j\in(Q\cap I)\cap(Z\cap Q)=Q\cap I\cap Z \end{cases}$$

Take care that since $Q\backslash I= Q\cap I'$, if $j\in I'$, then $j\in Z\backslash I$ or $j\in(I\cup Z)'$. Hence, if $j\in Q\backslash I$, $j\in Q\cap I'\cap Z'$ or $j\in Q\cap I'\cap Z$. Thus, it is seen that that $N=T$.

2) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)\underset{\theta}{\overset{*}{\sim}}(A,Z)] = [(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cup}\ [(A,Z)\underset{\cap}{\overset{*}{\sim}}(C,Q)]$

3) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)\underset{+}{\overset{*}{\sim}}(A,Z)] = [(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cap}\ [(A,Z)\underset{\gamma}{\overset{*}{\sim}}(C,Q)]$

4) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I\underset{\gamma}{\overset{*}{\sim}}(A,Z)] = [(C,Q)\underset{\cap}{\overset{*}{\sim}}(Y,I)]\ \tilde{\cup}\ [(A,Z)\underset{\gamma}{\overset{*}{\sim}}(C,Q)]$

ii) Right-distribution of complementary soft binary piecewise difference (\) operation over complementary soft binary piecewise operations:

The followings are satisfied when $Q\cap I\cap Z'=\emptyset$.

1) $[(C,A)\underset{\theta}{\overset{*}{\sim}}(Y,I)]\underset{\backslash}{\overset{*}{\sim}}(A,Z)= [(C,Q)\underset{\theta}{\sim}(A,Z)]\ \tilde{\cap}\ [(Y,I)\underset{\theta}{\sim}(A,Z)]$.**Proof:** Let's first handle the left-hand side of the equality, let $(C,Q)\underset{\theta}{\overset{*}{\sim}}(Y,I)=(M,Q)$, where $\forall j\in Q$,

$$M(j)=\begin{cases} C'(j), & j\in Q\backslash I \\ C'(j)\cap Y'(j), & j\in Q\cap I \end{cases}$$

Let $(M,Q)\underset{\backslash}{\overset{*}{\sim}}(A,Z)=(N,Q)$, where $\forall j\in Q$,

$$N(j)=\begin{cases} M'(j), & j\in Q\setminus Z \\ M(j)\cap A'(j), & j\in Q\cap Z \end{cases}$$

Hence,

$$N(j)=\begin{cases} C(j), & j\in(Q\setminus I)\setminus Z=Q\cap I'\cap Z' \\ C(j)\cup Y(j), & j\in(Q\cap I)\setminus Z=Q\cap I\cap Z' \\ C'(j)\cap A'(j), & j\in(Q\setminus I)\cap I=Q\cap I'\cap Z \\ [C'(j)\cap Y'(j)]\cap A'(j), & j\in(Q\cap I)\cap Z=Q\cap I\cap Z \end{cases}$$

Now let's handle the right-hand side of the equality $[(C,Q)\tilde{\theta}(A,Z)]\ \tilde{\cap}\ [(Y,I)\tilde{\theta}(A,Z)]$. Let $(C,Q)\tilde{\theta}(A,Z)=(V,Q)$, where $\forall j\in Q$;

$$V(j)=\begin{cases} C(j), & j\in Q\setminus Z \\ C'(j)\cap A'(j), & j\in Q\cap Z \end{cases}$$

Assume that $(Y,I)\tilde{\theta}(A,Z)=(W,I)$, where $\forall j\in I$;

$$W(j)=\begin{cases} Y(j), & j\in I\setminus Z \\ Y'(j)\cap A'(j), & j\in I\cap Z \end{cases}$$

Let $(V,Q)\tilde{\cap}(W,I)=(T,Q)$, where $\forall j\in Q$;

$$T(j)=\begin{cases} V(j) & j\in Q\setminus I \\ V(j)\cap W(j) & j\in Q\cap I \end{cases}$$

$$T(j)=\begin{cases} C(j), & j\in(Q\setminus Z)\setminus I=Q\cap I'\cap Z' \\ C'(j)\cap A'(j), & j\in(Q\cap Z)\setminus I=Q\cap I'\cap Z \\ C(j)\cap Y(j), & j\in(Q\setminus Z)\cap(I\setminus Z)=Q\cap I\cap Z' \\ C(j)\cap[Y'(j)\cap A'(j)], & j\in(Q\setminus Z)\cap(I\cap Z)=\emptyset \\ [C'(j)\cap A'(j)]\cap Y(j), & j\in(Q\cap Z)\cap(I\setminus Z)=\emptyset \\ [C'(j)\cap A'(j)]\cap[Y'(j)\cap A'(j)], & j\in(Q\cap Z)\cap(I\cap Z)=Q\cap I\cap Z \end{cases}$$

It is seen that that N=T.

2) $[(C,A)\underset{*}{\overset{*}{\sim}}(Y,I)]\underset{\backslash}{\overset{*}{\sim}}(A,Z)=[(C,Q)\,\tilde{\theta}\,(A,Z)]\ \tilde{\cup}\ [(Y,I)\,\tilde{\theta}\,(A,Z)]$

3) $[(C,A)\underset{+}{\overset{*}{\sim}}(Y,I)]\underset{\backslash}{\overset{*}{\sim}}(A,Z)=[(C,Q)\,\tilde{\theta}\,(A,Z)]\ \tilde{\cup}\ [(Y,I)\,\tilde{\backslash}\,(A,Z)]$

4) $[(C,A)\underset{\gamma}{\overset{*}{\sim}}(Y,I)]\underset{\backslash}{\overset{*}{\sim}}(A,Z)=[(C,Q)\,\tilde{\theta}\,(A,Z)]\ \tilde{\cap}\ [(Y,I)\,\tilde{\backslash}\,(A,Z)]$

4.5. Distribution of complementary soft binary piecewise difference (\) operation over restricted soft set operations:

The followings are satisfied when $I\cap Z \neq \emptyset$ and $Q \cap I \cap Z = \emptyset$

1) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)\cap_R(A,Z)]=[(C,Q)\underset{\theta}{\overset{*}{\sim}}(Y,I)]\cup_R[C,Q)\underset{\theta}{\overset{*}{\sim}}(A,Z)]$.

Proof: Let's first handle the left-hand side of the equality, suppose $(Y,I)\cap_R(A,Z)=(M,I\cap Z)$, and so $\forall j\in I\cap Z$, $M(j)=Y(j)\cap A(j)$. Let $(C,Q)\underset{\backslash}{\overset{*}{\sim}}(M,I\cap Z)=(N,Q)$, so $\forall j\in Q$,

$$N(j)=\begin{cases} C'(j), & j\in Q\backslash(I\cap Z) \\ C(j)\cap M'(j), & j\in Q\cap(I\cap Z) \end{cases}$$

Thus,

$$N(j)=\begin{cases} C'(j), & j\in Q\backslash(I\cap Z) \\ C(j)\cap[Y'(j)\cup A'(j)], & j\in Q\cap(I\cap Z) \end{cases}$$

Now let's handle the right-hand side of the equality: $[(C,Q)\underset{\theta}{\overset{*}{\sim}}(Y,I)]\ \cup_R\ [(C,Q)\underset{\theta}{\overset{*}{\sim}}(A,Z)]$. Let $(C,Q)\underset{\theta}{\overset{*}{\sim}}(Y,I)=(V,Q)$ and $\forall j\in Q$,

$$V(j)=\begin{cases} C'(j), & j\in Q\setminus I \\ C'(j)\cap Y'(j), & j\in Q\cap I \end{cases}$$

Let $(C,Q)\underset{\theta}{\overset{*}{\sim}}(A,Z)=(W,Q)$ and $\forall j\in Q$,

$$W(j)=\begin{cases} C'(j), & j\in Q\setminus Z \\ C'(j)\cap A'(j), & j\in Q\cap Z \end{cases}$$

Assume that $(V,Q)\cup_R(W,Q)=(T,Q)$, so $\forall j\in T(j)=V(j)\cup W(j)$,

$$T(j)=\begin{cases} C'(j)\cup C'(j), & j\in(Q\setminus I)\cap(Q\setminus Z) \\ C'(j)\cup[C'(j)\cap A'(j)], & j\in(Q\setminus I)\cap(Q\cap Z) \\ [C'(j)\cap Y'(j)]\cup F'(j), & j\in(Q\cap I)\cap(F\setminus Z) \\ [C'(j)\cap Y'(j)]\cup[C'(j)\cap A'(j)], & j\in(Q\cap I)\cap(Q\cap Z) \end{cases}$$

Thus,

$$T(j)=\begin{cases} C'(j), & j\in Q\cap I'\cap Z' \\ C'(j), & j\in Q\cap I'\cap Z \\ C'(j), & j\in Q\cap I\cap Z' \\ [C'(j)\cap Y'(j)]\cup[C'(j)\cap A'(j)], & j\in Q\cap I\cap Z \end{cases}$$

Considering the parameter set of the first equation of the first row, that is, $Q\setminus(I\cap Z)$; since $Q\setminus(I\cap Z)=Q\cap(I\cap Z)'$, an element in $(I\cap Z)'$ may be in $I\setminus Z$, in $Z\setminus I$ or $(I\cup Z)$. Then, $Q\setminus(I\cap Z)$ is equivalent to the following 3 states: $Q\cap(I\cap Z')$, $Q\cap(I'\cap Z)$ and $Q\cap(I'\cap Z')$. Hence, that N=T.

2) $(C,Q)\underset{\setminus}{\overset{*}{\sim}}[(Y,I)\cup_R(A,Z)=[(C,Q)\underset{*}{\overset{*}{\sim}}(Y,I)]\cap_R[(C,Q)\underset{*}{\overset{*}{\sim}}(A,Z)]$.

3) $(C,Q)\underset{\setminus}{\overset{*}{\sim}}[(Y,I)\theta_R(A,Z)]=[C,Q)\underset{\gamma}{\overset{*}{\sim}}(Y,I)]\cup_R[(C,Q)\underset{\gamma}{\overset{*}{\sim}}(A,Z)]$.

4) $(C,Q)\underset{\setminus}{\overset{*}{\sim}}[(Y,I)*_R(A,Z)]=[(C,Q)\underset{+}{\overset{*}{\sim}}(Y,I)]\cap_R[(C,Q)\underset{+}{\overset{*}{\sim}}(A,Z)]$.

5) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)\gamma_R(A,Z)]=[(C,Q)\underset{*}{\overset{*}{\sim}}(Y,I)]\cap_R[(C,Q)\underset{+}{\overset{*}{\sim}}(A,Z)]$.

6) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)\lambda_R(A,Z)]=[(C,Q)\underset{\theta}{\overset{*}{\sim}}(Y,I)]\cup_R[(C,Q)\underset{\gamma}{\overset{*}{\sim}}(A,Z)]$.

7) $[(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)\backslash_R(A,Z)]=[(C,Q)\underset{\gamma}{\overset{*}{\sim}}(Y,I)]\cap_R[(C,Q)\underset{\theta}{\overset{*}{\sim}}(A,Z)]$,

8) $(C,Q)\underset{\backslash}{\overset{*}{\sim}}[(Y,I)+_R(A,Z)] =[(C,Q)\underset{+}{\overset{*}{\sim}}(Y,I)]\cap_R[C,Q)\underset{*}{\overset{*}{\sim}}(A,Z)]$.

5. Conclusion

The concept of soft set operations is a critical idea just like essential operations on numbers and primary operations on sets. Soft set operations are the operations that are applied to two or more soft sets to develop a relationship between them. The operations in soft set ideas have continued beneathneath fundamental headings as restricted soft set operations and extended soft set operations. In this paper, a new type of soft set operation which we call complementary soft binary piecewise difference operation has been defined. The algebraic properties of the operation have been investigated. We have obtained some interesting analogous facts between the difference operation in classical theory and complementary soft binary piecewise difference operation in soft set theroy. Also, we have obtained the relationships between this new soft set operation and other types of soft set operations such as extended opearation, complementary extended operations, soft binary pecewise operations, complementary soft binary piecewise operations intersectiona and restricted operations. This research is to serve as a basis for many applications, especially decision making cryptography. Since soft set is a powerful mathematical tool for uncertain object detection, with this study, researchers may suggest some new encryption or decision-making methods based on soft sets. Moreover, studies on the soft algebraic structures may be handled again as regards the algebraic properties by the operation defined in this paper.

Author's Contribution

The contributions of the authors are equal.

Conflict of Interest

The authors have declared that there are no conflicts of interest.

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