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Some Novel Fractional Integral Inequalities for Different Kinds of Convex Functions

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Abstract

In this paper, some novel integral inequalities for different kinds of convex functions have been proved by using Caputo-Fabrizio fractional integral operators. The findings includes several new integral inequalities $h-$convex functions, $s-$convex functions in the second sense. We have used the properties of Caputo-Fabrizio fractional operator, definitions of different kinds of convex functions and elemantery analysis methods.

Keywords: Caputo-Fabrizio fractional integral operator, $h-$convex functions, $s-$convex functions.

1. Introduction

Inequality theory is a field in which many researchers work, with new findings that can be given applications in many disciplines such as mathematical analysis, statistics, approximation theory and numerical analysis together with convex functions. Although the concept of convex function is a concept intertwined with inequalities by definition, it has also formed the main motivation of many researches with its aesthetic structure, features and different types. Let's start with the definition of this important class of functions.

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2. Materials and Methods

Definition 2.1. Let I be on interval in R. Then $f: I \to R$ is said to be convex, if

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y)$$

holds for all $x, y \in I$ and $t \in [0,1]$ (Pečarić et al. 1992).

Definition 2.2. A function $f: R^+ \to R$, where $R^+ = [0, \infty)$, is said to be $s-$convex in the second sense if

$$f(\alpha x + \beta y) \le \alpha^s f(x) + \beta^s f(y)$$

for all $x, y \in [0, \infty)$, $\alpha, \beta \ge 0$ with $\alpha + \beta = 1$ and for some fixed $s \in (0,1]$. We denote by K_s^2 the class of all $s-$convex functions (Breckner 1978).

Definition 2.3. (Varosanec 2007) Let $h: J \subseteq R \to R$ be a non-negative function. We say that $f: I \subseteq R \to R$ is an $h-$convex function or that f belongs to the class $SX\ (h, I)$, if f is non-negative and for all $x, y \in I$ and $\alpha \in [0,1]$, we have

$$f(\alpha x + (1-\alpha)y) \le h(\alpha)f(x) + h(1-\alpha)f(y).$$

Definition 2.4. Let $f \in H^1(0,b), b > a, \alpha \in [0,1]$ then, the definition of the left and right side of Caputo-Fabrizio fractional integral is:

$$\left({}_{a}^{CF}I^{\alpha}\right)(t) = \frac{1-\alpha}{B(\alpha)} f(t) + \frac{\alpha}{B(\alpha)} \int_a^t f(y)dy,$$

and

$$\left({}^{CF}I_b^{\alpha}\right)(t) = \frac{1-\alpha}{B(\alpha)} f(t) + \frac{\alpha}{B(\alpha)} \int_t^b f(y)dy$$

where $B(\alpha) > 0$ is normalization function (Abdeljawad and Baleanu 2017).

In the sequel of the paper, we will denote normalization function as $B(\alpha)$ with $B(0) = B(1) = 1$.

In (Tariq et al. 2022), the authors provided an integral inequality of Hermite-Hadamard type for preinvex functions via Caputo-Fabrizio fractional integral inequality as follows.

Theorem 2.1. Let $f: I = [k_1, k_1 + \mu(k_2, k_1)] \to (0, \infty)$ be a preinvex function on I° and $f \in$

$L[k_1, k_1 + \mu(k_2, k_1)]$. If $\alpha \in [0,1]$, then the following inequality holds

$$f\left(\frac{2k_1 + \mu(k_2, k_1)}{2}\right) \le \frac{B(\alpha)}{\alpha\mu(k_2, k_1)} \times \left[{}^{CF}_{k_1}I^{\alpha}\{f(k)\} + {}^{CF}I^{\alpha}_{k_1+\mu(k_2,k_1)}\{f(k)\} - \frac{2(1-\alpha)}{B(\alpha)}\, f(k)\right] \le \frac{f(k_1) + f(k_2)}{2}$$

where $k \in [k_1, k_1 + \mu(k_2, k_1)]$.

Atangana and Baleanu produced a new derivative operators using Mittag-Leffler function in Caputo-Fabrizio derivative operator as following.

Definition 2.5. (Atangana and Baleanu 2016). Let $f \in H^1(0,b), b > a, \alpha \in [0,1]$ then, the definition of the new fractional derivative is given:

(1.1) $$\left({}^{ABC}_{\ a}D^{\alpha}_t\right)[f(t)] = \frac{B(\alpha)}{1-\alpha}\int_a^t f'(x) E_\alpha\left[-\alpha\frac{(t-x)^\alpha}{(1-\alpha)}\right]dx.$$

Definition 2.6. (Atangana and Baleanu 2016). Let $f \in H^1(0,b), b > a, \alpha \in [0,1]$ then, the definition of the new fractional derivative is given:

(1.2) $$\left({}^{ABR}_{\ a}D^{\alpha}_t\right)[f(t)] = \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\int_a^t f(x) E_\alpha\left[-\alpha\frac{(t-x)^\alpha}{(1-\alpha)}\right]dx.$$

Equations (1.1) and (1.2) have a non-local kernel. Also in equation (1.1) when the function is constant we get zero.

The related fractional integral operator has been defined by Atangana-Baleanu as follows.

Definition 2.7. The fractional integral associate to the new fractional derivative with non-local kernel of a function $f \in H^1(a,b)$ as defined:

$${}^{AB}_{\ a}I^{\alpha}\{f(t)\} = \frac{1-\alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_a^t f(y)(t-y)^{\alpha-1}dy$$

where, $b > a$, $\alpha \in [0,1]$ (Atangana and Baleanu 2016).

Abdeljawad and Baleanu introduced right hand side of integral operator as following; The right fractional new integral with ML kernel of order $\alpha \in [0,1]$ is defined by

$${}^{AB}I^{\alpha}_b\{f(t)\} = \frac{1-\alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_t^b f(y)(y-t)^{\alpha-1}dy.$$

where $, b > a,\ \alpha \in [0,1]$ (Abdeljawad and Baleanu 2017).

For more information related to different kinds of fractional operators, we recommend to the readers the following papers (Abdeljawad 2015, Abdeljawad and Baleanu 2016, Akdemir et al. 2021- Akdemir et al. 2017, Butt et al. 2020, Caputo and Fabrizio 2015- Gürbüz et al. 2020, Rashid et al 2020-Samko et al. 1993, Set 2012, Set et al. 2017).

3. *Results*

Theorem 3.1. Let $I \subseteq \mathbb{R}$. Suppose that $f: [a,b] \subseteq I \longrightarrow \mathbb{R}$ is a $h-$convex function on $[a,b]$ such that $f \in L_1[a,b]$. Then, we have following inequality for Caputo-Fabrizio fractional integrals:

$$\left({}^{CF}_{\ a}I^{\alpha} f\right)(k) + \left({}^{CF}I^{\alpha}_b f\right)(k) \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)f(a)}{B(\alpha)}\int_0^1 h(t)\,dt + \frac{\alpha(b-a)f(b)}{B(\alpha)}\int_0^1 h(1-t)\,dt$$

where $B(\alpha) > 0$ is normalization function and $\alpha \in [0,1]$.

Proof. By usining the definition of $h-$convex function, we can write

$$f(ta + (1-t)b) \le h(t)f(a) + h(1-t)f(b).$$

By integrating both sides of the inequality over $[0,1]$ with respect to t, we get

$$\int_0^1 f(ta + (1-t)b)\,dt \le f(a)\int_0^1 h(t)\,dt + f(b)\int_0^1 h(1-t)\,dt.$$

By changing of the variable as $x = ta + (1-t)b$, we obtain

$$\frac{1}{b-a}\int_a^b f(x)\,dx \le f(a)\int_0^1 h(t)\,dt + f(b)\int_0^1 h(1-t)\,dt.$$

By multiplying both sides of the above inequality with $\frac{\alpha(b-a)}{B(\alpha)}$ and adding $\frac{2(1-\alpha)}{B(\alpha)}f(k)$, we have

$$\frac{2(1-\alpha)}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_a^b f(x)\,dx \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)f(a)}{B(\alpha)}\int_0^1 h(t)\,dt + \frac{\alpha(b-a)f(b)}{B(\alpha)}\int_0^1 h(1-t)\,dt.$$

By simplfying the inequality, we get the result

$$\left(\frac{1-\alpha}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_a^k f(x)\,dx\right)+\left(\frac{1-\alpha}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_k^b f(x)\,dx\right) \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)f(a)}{B(\alpha)}\int_0^1 h(t)\,dt + \frac{\alpha(b-a)f(b)}{B(\alpha)}\int_0^1 h(1-t)\,dt$$

Namely,

$$\left({}_{a}^{CF}I^{\alpha} f\right)(k)+\left({}^{CF}I_{b}^{\alpha} f\right)(k) \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)f(a)}{B(\alpha)}\int_0^1 h(t)\,dt + \frac{\alpha(b-a)f(b)}{B(\alpha)}\int_0^1 h(1-t)\,dt.$$

This completes the proof.

Theorem 3.2. Let $I \subseteq \mathbb{R}$. Suppose that $f:[a,b] \subseteq I \longrightarrow \mathbb{R}$ is a $s-$convex function in the second sense on $[a,b]$ such taht $f \in L_1[a,b]$. Then, we have the following inequality for Caputo-Fabrizio fractional integrals:

$$\left({}_{a}^{CF}I^{\alpha} f\right)(k)+\left({}^{CF}I_{b}^{\alpha} f\right)(k) \le \frac{2(1-\alpha)f(k)(s+1)+\alpha(b-a)\big(f(a)+f(b)\big)}{B(\alpha)(s+1)}.$$

where $B(\alpha) > 0$ is normalization function $s \in (0,1]$ and $\alpha \in [0,1]$.

Proof. By usining the definition of $s-$convex function in the second sense, we can write

$$f(ta+(1-t)b) \le t^s f(a) + (1-t)^s f(b).$$

By integrating both sides of the inequality over $[0,1]$ with respect to t, we get

$$\int_0^1 f(ta+(1-t)b)\,dt \le \int_0^1 t^s f(a)\,dt + \int_0^1 (1-t)^s f(b)\,dt.$$

By changing of the variable as $x = ta + (1-t)b$, and by calculating the right hand side, we obtain

$$\frac{1}{b-a}\int_a^b f(x)\,dx \le \frac{f(a)+f(b)}{s+1}.$$

By multiplying both sides of the above inequality with $\frac{\alpha(b-a)}{B(\alpha)}$ and adding $\frac{2(1-\alpha)}{B(\alpha)}f(k)$, we have

$$\frac{2(1-\alpha)}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_a^b f(x)\,dx \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)}{B(\alpha)}\frac{f(a)+f(b)}{s+1}.$$

By simplfying the inequality, we get the result.

$$\left(\frac{1-\alpha}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_a^k f(x)\,dx\right)+\left(\frac{1-\alpha}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_k^b f(x)\,dx\right) \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)}{B(\alpha)}\frac{f(a)+f(b)}{s+1}$$

Then, it is easy to see

$$\left({}_{a}^{CF}I^{\alpha} f\right)(k)+\left({}^{CF}I_{b}^{\alpha} f\right)(k) \le \frac{2(1-\alpha)f(k)(s+1)+\alpha(b-a)\big(f(a)+f(b)\big)}{B(\alpha)(s+1)}.$$

This completes the proof

Theorem 3.3. Let $I \subseteq \mathbb{R}$. Suppose that $f:[a,b] \subseteq I \longrightarrow \mathbb{R}$ is a $s-$convex function in the second sense on $[a,b]$ such that $f \in L_1[a,b]$. Then, we have the following inequality for Caputo-Fabrizio fractional integrals:

$$\left({}_{a}^{CF}I^{\alpha} f\right)(k)+\left({}^{CF}I_{b}^{\alpha} f\right)(k) \le \frac{2(1-\alpha)f(k)(ps+1)^{\frac{1}{p}}+\alpha(b-a)\big(f(a)+f(b)\big)}{B(\alpha)(ps+1)^{\frac{1}{p}}}$$

where $B(\alpha) > 0$ is normalization function $s \in (0,1]$, $q > 1, \frac{1}{p}+\frac{1}{q} = 1$ and $\alpha \in [0,1]$.

Proof : By usining the definition of $s-$convex function in the second sense, we can write

$$f(ta+(1-t)b) \le t^s f(a) + (1-t)^s f(b).$$

By integrating both sides of the inequality over $[0,1]$ with respect to t, we get

$$\int_0^1 f(ta+(1-t)b)\,dt \le f(a)\int_0^1 t^s\,dt + f(b)\int_0^1 (1-t)^s\,dt.$$

If we apply the Hölder's inequality to the right-hand side of the inequality, we get

$$\int_0^1 |f(ta+(1-t)b)|\,dt \le f(a)\left(\left(\int_0^1 t^{ps}\,dt\right)^{\frac{1}{p}}\left(\int_0^1 1^q dt\right)^{\frac{1}{q}}\right) + f(b)\left(\left(\int_0^1 (1-t)^{ps}\,dt\right)^{\frac{1}{p}}\left(\int_0^1 1^q dt\right)^{\frac{1}{q}}\right)$$
$$= f(a)\left(\left(\frac{1}{ps+1}\right)^{\frac{1}{p}}(1^q)^{\frac{1}{q}}\right) + f(b)\left(\left(\frac{1}{ps+1}\right)^{\frac{1}{p}}(1^q)^{\frac{1}{q}}\right).$$

By changing of the variable as $x=ta+(1-t)b$ and by calculating the right hand side, we obtain

$$\frac{1}{b-a}\int_a^b f(x)\,dx \le \frac{f(a)+f(b)}{(ps+1)^{\frac{1}{p}}}.$$

By multiplying both sides of the above inequality with $\frac{\alpha(b-a)}{B(\alpha)}$ and adding $\frac{2(1-\alpha)}{B(\alpha)}f(k)$, we have

$$\frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha}{B(\alpha)}\int_a^b f(x)\,dx \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)}{B(\alpha)}\frac{f(a)+f(b)}{(ps+1)^{\frac{1}{p}}}.$$

By simplfying the inequality, we get the result

$$\left(\frac{1-\alpha}{B(\alpha)}f(k) + \frac{\alpha}{B(\alpha)}\int_a^k f(x)\,dx\right) + \left(\frac{1-\alpha}{B(\alpha)}f(k) + \frac{\alpha}{B(\alpha)}\int_k^b f(x)\,dx\right) \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)}{B(\alpha)}\frac{f(a)+f(b)}{(ps+1)^{\frac{1}{p}}}.$$

Then, we can concludethat

$$\left({}_a^{CF}I^{\alpha} f\right)(k) + \left({}^{CF}I_b^{\alpha} f\right)(k) \le \frac{2(1-\alpha)f(k)(ps+1)^{\frac{1}{p}} + \alpha(b-a)\big(f(a)+f(b)\big)}{B(\alpha)(ps+1)^{\frac{1}{p}}}.$$

Theorem 3.4. Let $I \subseteq \mathbb{R}$. Suppose that $f:[a,b] \subseteq I \longrightarrow \mathbb{R}$ is a $s-$convex function in the second sense on $[a,b]$ such taht $f \in L_1[a,b]$. Then ,we have following inequality for Caputo-Fabrizio fractional integrals:

$$\left({}_a^{CF}I^{\alpha} f\right)(k) + \left({}^{CF}I_b^{\alpha} f\right)(k) \le \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)(f(a)+f(b))\big(q+p(ps+1)\big)}{B(\alpha)pq(ps+1)}$$

where $B(\alpha) > 0$ is normalization function $s \in (0,1]$, $q > 1, \frac{1}{p}+\frac{1}{q} = 1$ and $\alpha \in [0,1]$.

Proof : By usining the definition of $s-$convex function in the second sense, we can write

$$f(ta+(1-t)b) \le t^s f(a) + (1-t)^s f(b).$$

By integrating both sides of the inequality over $[0,1]$ with respect to t, we get

$$\int_0^1 f(ta+(1-t)b)\,dt \le \int_0^1 t^s f(a)\,dt + \int_0^1 (1-t)^s f(b)\,dt.$$

If we apply the Young's inequality to the right-hand side of the inequality, we get

$$\int_0^1 |f(ta+(1-t)b)|\,dt \le f(a)\left(\frac{1}{p}\left(\int_0^1 t^{ps}\,dt\right) + \frac{1}{q}\left(\int_0^1 1^q\,dt\right)\right) + f(b)\left(\frac{1}{p}\left(\int_0^1 (1-t)^{ps}\,dt\right) + \frac{1}{q}\left(\int_0^1 1^q\,dt\right)\right)$$
$$\le f(a)\left(\frac{1}{p(ps+1)} + \frac{1}{q}\right) + f(b)\left(\frac{1}{p(ps+1)} + \frac{1}{q}\right).$$

By changing of the variable as $x=ta+(1-t)b$ and by calculating the right hand side, we obtain

$$\frac{1}{b-a}\int_a^b f(x)\,dx \le \frac{(f(a)+f(b))\big(q+p(ps+1)\big)}{pq(ps+1)}.$$

By multiplying both sides of the above inequality with $\frac{\alpha(b-a)}{B(\alpha)}$ and adding $\frac{2(1-\alpha)}{B(\alpha)}f(k)$, we have

$$\frac{2(1-\alpha)}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_a^b f(x)\,dx \leq \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)}{B(\alpha)}\frac{\big(f(a)+f(b)\big)\big(q+p(ps+1)\big)}{pq(ps+1)}.$$

By simplfying the inequality, we get the result

$$\left(\frac{1-\alpha}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_a^k f(x)\,dx\right) + \left(\frac{1-\alpha}{B(\alpha)}f(k)+\frac{\alpha}{B(\alpha)}\int_k^b f(x)\,dx\right) \leq \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)\big(f(a)+f(b)\big)\big(q+p(ps+1)\big)}{B(\alpha)pq(ps+1)}$$

Namely,

$$\left({}_{a}^{CF}I^{\alpha}f\right)(k)+\left({}^{CF}I_b^{\alpha}f\right)(k) \leq \frac{2(1-\alpha)}{B(\alpha)}f(k) + \frac{\alpha(b-a)\big(f(a)+f(b)\big)\big(q+p(ps+1)\big)}{B(\alpha)pq(ps+1)}.$$

Remark 1. If we set $s=1$ in the main findings, we obtain new estimations for convex functions via Caputo-Fabrizio fractional integrals.

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