Journal of Amasya University the Institute of Sciences and Technology 4(1) (2023) 1-19

JOURNAL OF AMASYA UNIVERSITY THE INSTITUTE OF SCIENCES AND TECHNOLOGY (JAUIST)

ISSN: 2717-8900 https://doi.org/10.54559/jauist.1312656

A contribution to complementary soft binary piecewise plus and gamma operations

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Abstract — Molodtsov, in 1999, introduced soft set theory as mathematical a tool to deal with uncertainty. Since then, different kinds of soft set operations have been defined and used in various types. In this paper, it is aimed to contribute to the soft set literature by obtaining the distributions of soft binary piecewise operations over complementary soft binary piecewise plus and gamma operations.

Keywords: *Soft sets, conditional complements, soft set operations*

Subject Classification (2020): 03E20, 03E72

1. Introduction

The existence of some types of uncertainty in the problems of many fields such as economics, environmental and health sciences, engineering prevents us from using classical methods to solve the problems successfully. There are three well-known basic theories that we can consider as a mathematical tool to deal with uncertainties, which are probability theory, fuzzy set theory, and interval mathematics. But since all these theories have their own shortcomings, Molodtsov [1] introduced Soft Set Theory as mathematical tools to overcome these uncertainties. Since then, this theory has been applied to many fields including information systems, decision making, optimization theory, game theory, operations research, measurement theory and so on. In [2,3], first contributions as regards soft set operations were made. After then, Ali et al. [4] introduced and investigated several soft set operations such as restricted and extended soft set operations. in Sezgin and Atagün [5] discussed the basic properties of soft set operations and illustrated the interconnections of soft set operations with each other. They also defined the notion of restricted symmetric difference of soft sets and investigated its properties. Sezgin et al. [6] defined a new soft set operation called extended difference of soft sets and Stojanovic [7] proposed the extended symmetric difference of soft sets and investigated its properties. The two main categories into which the operations of soft set theory fall, according to the research, are restricted soft set operations and extended soft set operations.

Çağman [8] proposed two conditional complements of sets as a new concept of set theory, i.e., inclusive complement and exclusive complement and explored the relationships between them. By the inspiration of this study, Sezgin et al. [9] defined some new binary operations on sets and investigated their basic properties together with their interconnections. Aybek [10] transferred these complements to soft set theory and defined

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Article History: Received: 10 Jun 2023 — Accepted: 26 Jun 2023 — Published: 02 Jul 2023

some new restricted soft set operations and extended soft set operations. Demirci [11], Sarıalioğlu [12], and Akbulut [13] introduced a new type of extended operation by changing the form of extended soft set operations using the complement at the first and second row of the piecewise function of extended soft set operations and studied the basic properties of them in detail. Moreover, Eren [14] defined a new type of soft difference operations and with the inspiration of this study, Yavuz [15] defined some new soft set operations called soft binary piecewise operations and studied their basic properties. Also, by introducing a new type of soft binary piecewise operation, studies on soft set operations were studied [16-21] by changing the form of soft binary piecewise operation by using the complement at the first row of the soft binary piecewise operations.

Sezgin and Atagün [16] and Sezgin and Aybek [17] defined complementary soft binary piecewise plus and gamma operation, respectively. The algebraic properties of these new operations were investigated. Especially the distributions of these operations over extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations, and restricted soft set operations were handled. In this study, we aim to contribute to the literature of soft set theory by obtaining the distributions of soft binary piecewise operations over complementary soft binary piecewise plus and gamma operations.

2. Preliminaries

Definition 2.1. [1] Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U and $N \subseteq E$. A pair (K, N) is called a soft set over U where K is a set-valued function such that $K: N \to P(U)$.

The set of all the soft sets over U is designated by $S_E(U)$, and throughout this paper, all the soft sets are the elements of $S_E(U)$. Çağman [8] defined two conditional complements of sets, for the ease of illustration, we show these complements as $+$ and θ, respectively. These complements are defined as following: Let P and C be two subsets of U. C-inclusive complement of P is defined by, $P + C = P' \cup C$ and C-exlusive complement of P is defined by $P\theta C = P' \cap C$. Here, U refers to a universe, P' is the complement of P over U. Sezgin et al. [9] introduced such new three complements as binary operations of sets as following: Let P and C be two subsets of U. Then, $P * C = P' \cup C'$, $P\gamma C = P' \cap C$, $P\lambda C = P \cup C'$ [9]. Aybek [10] conveyed these classical sets to soft sets, and they defined restricted and extended soft set operations and investigated their properties.

As a summary for soft set operations, we can categorize all types of soft set operations as following: Let "∇" be used to represent the set operations (i.e., here ∇ can be $\cap$, $\cup$, $-$, Δ, $+$, θ, $*$, λ, γ), then restricted operations, extended operations, complementary extended operations, soft binary piecewise operations, complementary soft binary piecewise operations are defined in soft set theory as following:

Definition 2.2. [4,6,10] Let (K, P) and (G, C) be soft sets over U. The restricted operation ∇ (restricted intersection, union, difference, symmetric difference, plus, theta, star, gamma, and lambda) of (K, P) and (G, C) is the soft set (Y, S), denoted by $(K, P)\nabla_R(G, C) = (Y, S)$ where $S = P \cap C \neq \emptyset$ and for all $v \in S$, $Y(v) = K(v)\nabla G(v)$.

Definition 2.3. [2,4,6,7,10] Let (K, P) and (G, C) be soft sets over U. The extended operation ∇ (extended union, intersection, difference, symmetric difference, plus, theta, gamma, lambda, and star) of (K, P) and (G, C) is the soft set (Y, S), denoted by $(K, P)\nabla_\varepsilon(G, C) = (Y, S)$ where $S = P \cup C$ and for all $v \in S$,

$$Y(v) = \begin{cases} K(v), & v \in P - C \\ G(v), & v \in C - P \\ K(v)\nabla G(v), & v \in P \cap C \end{cases}$$

Definition 2.4. [11-13] Let (K, P) and (G, C) be soft sets over U. The complementary extended operation ∇ (complementary extended gamma, intersection, star, plus, union, theta, difference, and lambda) of (K, P) and (G, C) is the soft set (Y, S), denoted by $(K, P)\overset{*}{\nabla}_\varepsilon(G, C) = (Y, S)$ where $S = P \cup C$ and for all $v \in S$,

$$Y(v)=\begin{cases} K'(v), & v\in P-C\\ G'(v), & v\in C-P\\ K(v)\nabla G(v), & v\in P\cap C\end{cases}$$

Definition 2.5. [14,15] Let (K,P) and (G,C) be soft sets over U. The soft binary piecewise operation ∇ (soft binary piecewise difference, intersection, union, plus, gamma, theta, lambda, and star) of (K,P) and (G,C) is the soft set (Y,P), denoted by $(K,P)\widetilde{\nabla}(G,C)=(Y,P)$ where for all $v\in P$,

$$Y(v)=\begin{cases} K(v), & v\in P-C\\ K(v)\nabla G(v), & v\in P\cap C\end{cases}$$

Definition 2.6. [16-21] Let (K,P) and (G,C) be soft sets over U. The complementary soft binary piecewise operation ∇ (complementary soft binary piecewise star, theta, plus, intersection, union, gamma, lambda, and difference) of (K,P) and (G,C) is the soft set (Y,P), denoted by $(K,P)\overset{*}{\widetilde{\nabla}}(G,C)=(Y,P)$ where for all $v\in P$,

$$Y(v)=\begin{cases} K'(v), & v\in P-C\\ K(v)\nabla G(v), & v\in P\cap C\end{cases}$$

Definition 2.7. [16] Let (K,P) and (G,C) be soft sets over U. The complementary soft binary piecewise plus $(+)$ operation of (K,P) and (G,C) is the soft set (Y,P), denoted by $(K,P)\overset{*}{\tilde{+}}(G,C)=(Y,P)$ where for all $v\in P$,

$$Y(v)=\begin{cases} K'(v), & v\in P-C\\ K'(v)\cup G(v), & v\in P\cap C\end{cases}$$

Definition 2.8. [17] Let (K,P) and (G,C) be soft sets over U. The complementary soft binary piecewise gamma (γ) operation of (K,P) and (G,C) is the soft set (Y,P), denoted by $(K,P)\overset{*}{\tilde{\gamma}}(G,C)=(Y,P)$ where for all $v\in P$,

$$Y(v)=\begin{cases} K'(v), & v\in P-C\\ K'(v)\cap G(v), & v\in P\cap C\end{cases}$$

3. Distribution Rules

In this section, distributions of soft binary piecewise operations over complementary soft binary piecewise plus and gamma operation are investigated in detail, and many interesting results are obtained.

Theorem 3.1. Let (K,P), (G,C), and (L,R) be soft sets over U. Then, we have the following distributions of soft binary piecewise operations over complementary soft binary piecewise plus $(+)$ operation:

i. $(K,P)\,\widetilde{\cap}\left[(G,C)\overset{*}{\tilde{+}}(L,R)\right]=\left[(K,P)\,\tilde{\setminus}\,(G,C)\right]\widetilde{\cap}\,[(L,R)\,\widetilde{\cap}\,(K,P)]$ where $P\cap C\cap R=\emptyset$

Proof.

Handle the left-hand side of the equality and let $(G,C)\overset{*}{\tilde{+}}(L,R)=(M,C)$ where for all $I\in C$

$$M(I)=\begin{cases} G'(I), & I\in C-R\\ G'(I)\cup L(I), & I\in C\cap R\end{cases}$$

Let $(K,P)\,\widetilde{\cap}\,(M,C)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} K(I), & I\in P-C\\ K(I)\cup M(I), & I\in P\cap C\end{cases}$$

and thus

$$N(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cup G'(I), & I\in P\cap(C-R)=P\cap C\cap R' \\ K(I)\cap[G'(I)\cup L(I)], & I\in P\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.1}$$

Handle the left-hand side of the equality: $\left[(K,P)\,\tilde{\setminus}\,(G,C)\right]\tilde{\cap}\,[(L,R)\,\tilde{\cap}\,(K,P)]$. Let $(K,P)\,\tilde{\setminus}\,(G,C)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cap G'(I), & I\in P\cap C \end{cases}$$

Suppose that $(L,R)\,\tilde{\cap}\,(K,P)=(W,R)$ where for all $I\in R$

$$W(I)=\begin{cases} L(I), & I\in R-P \\ L(I)\cap K(I), & I\in R\cap P \end{cases}$$

Let $(V,P)\,\tilde{\cap}\,(W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cap W(I), & I\in P\cap R \end{cases}$$

and thus

$$T(I)=\begin{cases} K(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K(I)\cap G'(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K(I)\cap L(I), & I\in(P-C)\cap(R-P)=\emptyset \\ K(I)\cap[L(I)\cap K(I)], & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cap G'(I)]\cap L(I), & I\in(P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cap G'(I)]\cap[L(I)\cap K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases}$$

Therefore,

$$T(I)=\begin{cases} K(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K(I)\cap G'(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K(I)\cup L(I), & I\in(P-C)\cap(R-P)=\emptyset \\ K(I), & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cap G'(I)]\cup L(I), & I\in(P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cap G'(I)]\cup[L(I)\cap K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases} \tag{3.2}$$

Handle $I\in P-C$ in the first equation. Since $P-C=P\cap C'$, if $I\in C'$, then $I\in L-C$ or $I\in(C\cup R)'$. Hence, if $I\in P-C$, then $I\in P\cap C'\cap R'$ or $I\in P\cap C'\cap R$. Thus, it can be observed that (3.1)=(3.2). □

ii. $\left[(K,P)\overset{*}{\tilde{\mp}}(G,C)\right]\tilde{\cap}\,(L,R)=\left[(K,P)\overset{*}{\tilde{\gamma}}(L,R)\right]\tilde{\cup}\,[(G,C)\,\tilde{\cap}\,(L,R)]$

Proof.

Handle the left-hand side of the equality and let $(K,P)\overset{*}{\tilde{\mp}}(G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases} K'(I), & I\in P-C \\ K'(I)\cup G(I), & I\in P\cap C \end{cases}$$

Let $(M,P)\,\tilde{\cap}\,(L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} M(I), & I\in P-R \\ M(I)\cap L(I), & I\in P\cap R \end{cases}$$

Thus,

$$N(I)=\begin{cases} K'(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K'(I)\cup G(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K'(I)\cap L(I), & I\in(P-C)\cap R=P\cap C'\cap R \\ [K'(I)\cup G(I)]\cap L(I), & I\in(P\cap C)\cap R=P\cap C\cap R \end{cases} \tag{3.3}$$

Handle the left-hand side of the equality: $\left[(K,P)\overset{*}{\tilde{\gamma}}(L,R)\right]\,\tilde{\cup}\,[(G,C)\,\tilde{\cap}\,(L,R)]$. Let $(K,P)\overset{*}{\tilde{\gamma}}(L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K'(I), & I\in P-R \\ K'(I)\cap L(I), & I\in P\cap R \end{cases}$$

Suppose that $(G,C)\,\tilde{\cap}\,(L,R)=(W,C)$ where for all $I\in C$

$$W(I)=\begin{cases} G(I), & I\in C-R \\ G(I)\cap L(I), & I\in C\cap R \end{cases}$$

Let $(V,P)\,\tilde{\cup}\,(W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cup W(I), & I\in P\cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K'(I), & I\in(P-R)-C=P\cap C'\cap R' \\ K'(I)\cap L(I), & I\in(P\cap R)-C=P\cap C'\cap R \\ K'(I)\cup G(I), & I\in(P-R)\cap(C-R)=P\cap C\cap R' \\ K'(I)\cup[G(I)\cap L(I)], & I\in(P-R)\cap(C\cap R)=\emptyset \\ [K'(I)\cap L(I)]\cup G(I), & I\in(P\cap R)\cap(C-R)=\emptyset \\ [K'(I)\cap L(I)]\cup[G(I)\cap L(I)], & I\in(P\cap R)\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.4}$$

It can be observed that (3.3)=(3.4). □

iii. $(K,P)\,\tilde{\cup}\left[(G,C)\overset{*}{\tilde{\mp}}(L,R)\right]=\left[(K,P)\tilde{\lambda}(G,C)\right]\tilde{\cup}\,[(L,R)\,\tilde{\cup}\,(K,P)]$ where $P\cap C'\cap R=\emptyset$.

Proof.

Handle the left-hand side of the equality and let $(G,C)\overset{*}{\tilde{\mp}}(L,R)=(M,C)$ where for all $I\in C$

$$M(I)=\begin{cases} G'(I), & I\in C-R \\ G'(I)\cup L(I), & I\in C\cap R \end{cases}$$

Let $(K,P)\,\tilde{\cup}\,(M,C)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cup M(I), & I\in P\cap C \end{cases}$$

Thus,

$$N(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cup G'(I), & I\in P\cap(C-R)=P\cap C\cap R' \\ K(I)\cup[G'(I)\cup L(I)], & I\in P\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.5}$$

Handle the left-hand side of the equality: $\left[(K,P)\tilde{\lambda}(G,C)\right]\tilde{\cup}\,[(L,R)\,\tilde{\cup}\,(K,P)]$. Let $(K,P)\tilde{\lambda}(G,C)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cup G'(I), & I\in P\cap C \end{cases}$$

Suppose that $(L,R)\ \tilde{\cup}\ (K,P)=(W,R)$ where for all $I\in R$

$$W(I)=\begin{cases} L(I), & I\in R-P \\ L(I)\cup K(I), & I\in R\cap P \end{cases}$$

Let $(V,P)\ \tilde{\cup}\ (W,R)=(T,P)$. Then, for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cup W(I), & I\in P\cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K(I), & I\in (P-C)-R=P\cap C'\cap R' \\ K(I)\cup G'(I), & I\in (P\cap C)-R=P\cap C\cap R' \\ K(I)\cup L(I), & I\in (P-C)\cap(R-P)=\emptyset \\ K(I)\cup[L(I)\cup K(I)], & I\in (P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cup G'(I)]\cup L(I), & I\in (P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cup G'(I)]\cup[L(I)\cup K(I)], & I\in (P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases} \tag{3.6}$$

It can be observed that (3.5)=(3.6). □

iv. $\left[(K,P)\ \overset{*}{\tilde{\mp}}\ (G,C)\right]\ \tilde{\cup}\ (L,R)=\left[(K,P)\ \overset{*}{\tilde{\mp}}\ (L,R)\right]\ \tilde{\cup}\ [(G,C)\ \tilde{\cup}\ (L,R)]$

Proof.

Handle the left-hand side of the equality and let $(K,P)\ \overset{*}{\tilde{\mp}}\ (G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases} K'(I), & I\in P-C \\ K'(I)\cup G(I), & I\in P\cap C \end{cases}$$

Let $(M,P)\ \tilde{\cup}\ (L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} M(I), & I\in P-R \\ M(I)\cup L(I), & I\in P\cap R \end{cases}$$

Thus,

$$N(I)=\begin{cases} K'(I), & I\in (P-C)-R=P\cap C'\cap R' \\ K'(I)\cup G(I), & I\in (P\cap C)-R=P\cap C\cap R' \\ K'(I)\cup L(I), & I\in (P-C)\cap R=P\cap C'\cap R \\ [K'(I)\cup G(I)]\cup L(I), & I\in (P\cap C)\cap R=P\cap C\cap R \end{cases} \tag{3.7}$$

Handle the left-hand side of the equality: $\left[(K,P)\ \overset{*}{\tilde{\mp}}\ (L,R)\right]\ \tilde{\cup}\ [(G,C)\ \tilde{\cup}\ (L,R)]$. Let $(K,P)\ \overset{*}{\tilde{\mp}}\ (L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K'(I), & I\in P-R \\ K'(I)\cup L(I), & I\in P\cap R \end{cases}$$

Suppose that $(G,C)\ \tilde{\cup}\ (L,R)=(W,C)$ where for all $I\in C$

$$W(I)=\begin{cases} G(I), & I\in C-R \\ G(I)\cup L(I), & I\in C\cap R \end{cases}$$

Let $(V,P)\ \tilde{\cup}\ (W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cup W(I), & I\in P\cap R \end{cases}$$

and thus

$$T(I)=\begin{cases} K'(I), & I\in(P-R)-C=P\cap C'\cap R' \\ K'(I)\cup L(I), & I\in(P\cap R)-C=P\cap C'\cap R \\ K'(I)\cup G(I), & I\in(P-R)\cap(C-R)=P\cap C\cap R' \\ K'(I)\cup[G(I)\cup L(I)], & I\in(P-R)\cap(C\cap R)=\emptyset \\ [K'(I)\cup L(I)]\cup G(I), & I\in(P\cap R)\cap(C-R)=\emptyset \\ [K'(I)\cup L(I)]\cup[G(I)\cup L(I)], & I\in(P\cap R)\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.8}$$

It can be observed that (3.7)=(3.8). □

v. $(K,P)\tilde{\setminus}\left[(G,C)\overset{*}{\tilde{\mp}}(L,R)\right]=[(K,P)\,\tilde{\cap}\,(G,C)]\,\tilde{\cap}\,[(L,R)\tilde{\gamma}(K,P)]$ where $P\cap C'\cap R=\emptyset$

Proof.

Handle the left-hand side of the equality and let $(G,C)\overset{*}{\tilde{\mp}}(L,R)=(M,C)$ where for all $I\in C$

$$M(I)=\begin{cases} G'(I), & I\in C-R \\ G'(I)\cup L(I), & I\in C\cap R \end{cases}$$

Let $(K,P)\tilde{\setminus}(M,C)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cap M'(I), & I\in P\cap C \end{cases}$$

Thus,

$$N(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cap G(I), & I\in P\cap(C-R)=P\cap C\cap R' \\ K(I)\cap[G(I)\cup L'(I)], & I\in P\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.9}$$

Handle the left-hand side of the equality: $[(K,P)\,\tilde{\cap}\,(G,C)]\,\tilde{\cap}\,[(L,R)\,\tilde{\gamma}(K,P)]$. Let $(K,P)\,\tilde{\cap}\,(G,C)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cap G(I), & I\in P\cap C \end{cases}$$

Suppose that $(L,R)\tilde{\gamma}(K,P)=(W,R)$ where for all $I\in R$

$$W(I)=\begin{cases} L(I), & I\in R-P \\ L'(I)\cap K(I), & I\in R\cap P \end{cases}$$

Let $(V,P)\,\tilde{\cap}\,(W,R)=(T,P)$. Then, for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cap W(I), & I\in P\cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K(I)\cap G(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K(I)\cap L(I), & I\in(P-C)\cap(R-P)=\emptyset \\ K(I)\cap[L'(I)\cap K(I)], & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cap G(I)]\cap L(I), & I\in(P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cap G(I)]\cap[L'(I)\cap K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases} \tag{3.10}$$

It can be observed that (3.9)=(3.10). □

vi. $\left[(K,P)\overset{*}{\tilde{\mp}}(G,C)\right]\tilde{\setminus}(L,R)=\left[(K,P)\overset{*}{\tilde{\theta}}(L,R)\right]\tilde{\cup}\left[(G,C)\tilde{\setminus}(L,R)\right]$

Proof.

Handle the left-hand side of the equality and let $(K,P)\overset{*}{\tilde{\mp}}(G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases}K'(I), & I\in P-C\\ K'(I)\cup G(I), & I\in P\cap C\end{cases}$$

Let $(M,P)\tilde{\setminus}(L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases}M(I), & I\in P-R\\ M(I)\cap L'(I), & I\in P\cap R\end{cases}$$

Thus,

$$N(I)=\begin{cases}K'(I), & I\in(P-C)-R=P\cap C'\cap R'\\ K'(I)\cup G(I), & I\in(P\cap C)-R=P\cap C\cap R'\\ K'(I)\cap L'(I), & I\in(P-C)\cap R=P\cap C'\cap R\\ [K'(I)\cup G(I)]\cap L'(I), & I\in P\cap(C\cap R)=P\cap C\cap R\end{cases} \quad (3.11)$$

Handle the left-hand side of the equality: $\left[(K,P)\overset{*}{\tilde{\theta}}(L,R)\right]\tilde{\cup}\left[(G,C)\tilde{\setminus}(L,R)\right]$. Let $(K,P)\overset{*}{\tilde{\theta}}(L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases}K'(I), & I\in P-R\\ K'(I)\cap L'(I), & I\in P\cap R\end{cases}$$

Suppose that $(G,C)\tilde{\setminus}(L,R)=(W,C)$ where for all $I\in C$

$$W(I)=\begin{cases}G(I), & I\in C-R\\ G(I)\cap L'(I), & I\in C\cap R\end{cases}$$

Let $(V,P)\tilde{\cup}(W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases}V(I), & I\in P-R\\ V(I)\cup W(I), & I\in P\cap R\end{cases}$$

Thus,

$$T(I)=\begin{cases}K'(I), & I\in(P-R)-C=P\cap C'\cap R'\\ K'(I)\cap L'(I), & I\in(P\cap R)-C=P\cap C'\cap R\\ K'(I)\cup G(I), & I\in(P-R)\cap(C-R)=P\cap C\cap R'\\ K'(I)\cup[G(I)\cap L'(I)], & I\in(P-R)\cap(C\cap R)=\emptyset\\ [K'(I)\cap L'(I)]\cup G(I), & I\in(P\cap R)\cap(C-R)=\emptyset\\ [K'(I)\cap L'(I)]\cup[G(I)\cap L'(I)], & I\in(P\cap R)\cap(C\cap R)=P\cap C\cap R\end{cases} \quad (3.12)$$

It can be observed that (3.11)=(3.12). □

vii. $(K,P)\tilde{\lambda}\left[(G,C)\overset{*}{\tilde{\mp}}(L,R)\right]=[(K,P)\tilde{\cup}(G,C)]\tilde{\cup}[(L,R)\tilde{\mp}(K,P)]$ where $P\cap C'\cap R=\emptyset$

Proof.

Handle the left-hand side of the equality and let $(G,C)\overset{*}{\tilde{\mp}}(L,R)=(M,C)$ where for all $I\in C$

$$M(I)=\begin{cases}G'(I), & I\in C-R\\ G'(I)\cup L(I), & I\in C\cap R\end{cases}$$

Let $(K,P)\tilde{\lambda}(M,C)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases}K(I), & I\in P-C\\ K(I)\cup M'(I), & I\in P\cap C\end{cases}$$

Thus,

$$N(I)=\begin{cases}K(I), & I\in P-C\\ K(I)\cup G(I), & I\in P\cap(C-R)=P\cap C\cap R'\\ K(I)\cup[G(I)\cup L'(I)], & I\in P\cap(C\cap R)=P\cap C\cap R\end{cases} \quad (3.13)$$

Handle the left-hand side of the equality: $[(K,P)\,\tilde{\cup}\,(G,C)]\,\tilde{\cup}\,[(L,R)\tilde{\mp}(K,P)]$. Let $(K,P)\,\tilde{\cup}\,(G,C)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases}K(I), & I\in P-C\\ K(I)\cup G(I), & I\in P\cap C\end{cases}$$

Suppose that $(L,R)\tilde{\mp}(K,P)=(W,R)$ where for all $I\in R$

$$W(I)=\begin{cases}L(I), & I\in R-P\\ L'(I)\cup K(I), & I\in R\cap P\end{cases}$$

Let $(V,P)\,\tilde{\cup}\,(W,R)=(T,P)$. Then, for all $I\in P$

$$T(I)=\begin{cases}V(I), & I\in P-R\\ V(I)\cup W(I), & I\in P\cap R\end{cases}$$

Thus,

$$T(I)=\begin{cases}K(I), & I\in(P-C)-R=P\cap C'\cap R'\\ K(I)\cup G(I), & I\in(P\cap C)-R=P\cap C\cap R'\\ K(I)\cup L(I), & I\in(P-C)\cap(R-P)=\emptyset\\ K(I)\cup[L'(I)\cup K(I)], & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R\\ [K(I)\cup G(I)]\cup L(I), & I\in(P\cap C)\cap(R-P)=\emptyset\\ [K(I)\cup G(I)]\cup[L'(I)\cup K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R\end{cases} \quad (3.14)$$

It can be observed that (3.13)=(3.14). □

viii. $\left[(K,P)\overset{*}{\tilde{\mp}}(G,C)\right]\tilde{\lambda}(L,R)=\left[(K,P)\overset{*}{\tilde{*}}(L,R)\right]\tilde{\cup}\left[(G,C)\tilde{\lambda}(L,R)\right]$

Proof.

Handle the left-hand side of the equality and let $(K,P)\overset{*}{\tilde{\mp}}(G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases}K'(I), & I\in P-C\\ K'(I)\cup G(I), & I\in P\cap C\end{cases}$$

Let $(M,P)\tilde{\lambda}(L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases}M(I), & I\in P-R\\ M(I)\cup L'(I), & I\in P\cap R\end{cases}$$

Thus,

$$N(I)=\begin{cases}K'(I), & I\in(P-C)-R=P\cap C'\cap R'\\ K'(I)\cup G(I), & I\in(P\cap C)-R=P\cap C\cap R'\\ K'(I)\cup L'(I), & I\in(P-C)\cap R=P\cap C'\cap R\\ [K'(I)\cup G(I)]\cup L'(I), & I\in(P\cap C)\cap R=P\cap C\cap R\end{cases} \quad (3.15)$$

Handle the left-hand side of the equality: $\left[(K,P)\overset{*}{\tilde{*}}(L,R)\right]\tilde{\cup}\left[(G,C)\tilde{\lambda}(L,R)\right]$. Let $(K,P)\overset{*}{\tilde{*}}(L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases}K'(I), & I\in P-R\\ K'(I)\cup L'(I), & I\in P\cap R\end{cases}$$

Suppose that $(G,C)\tilde{\lambda}(L,R)=(W,C)$ where for all $I\in C$

$$W(I)=\begin{cases}G(I), & I\in C-R\\ G(I)\cup L'(I), & I\in C\cap R\end{cases}$$

Let $(V,P)\tilde{\cup}(W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases}V(I), & I\in P-R\\ V(I)\cup W(I), & I\in P\cap R\end{cases}$$

Thus,

$$T(I)=\begin{cases}K'(I), & I\in(P-R)-C=P\cap C'\cap R'\\ K'(I)\cup L'(I), & I\in(P\cap R)-C=P\cap C'\cap R\\ K'(I)\cup G(I), & I\in(P-R)\cap(C-R)=P\cap C\cap R'\\ K'(I)\cup[G(I)\cup L'(I)], & I\in(P-R)\cap(C\cap R)=\emptyset\\ [K'(I)\cup L'(I)]\cup G(I), & I\in(P\cap R)\cap(C-R)=\emptyset\\ [K'(I)\cup L'(I)]\cup[G(I)\cup L'(I)], & I\in(P\cap R)\cap(C\cap R)=P\cap C\cap R\end{cases}\tag{3.16}$$

It can be observed that (3.15)=(3.16). □

Theorem 3.2. Let (K,P), (G,C), and (L,R) be soft sets over U. Then, we have the following distributions of soft binary piecewise operations over complementary soft binary piecewise gamma (γ) operation:

i. $(K,P)\tilde{\cap}\left[(G,C)\overset{*}{\tilde{\gamma}}(L,R)\right]=\left[(K,P)\tilde{\setminus}(G,C)\right]\tilde{\cup}\left[(L,R)\tilde{\cap}(K,P)\right]$ where $P\cap C\cap R=\emptyset$

Proof.

Let first handle the left-hand side of the equality and let $(G,C)\overset{*}{\tilde{\gamma}}(L,R)=(M,C)$ where for all $I\in C$

$$M(I)=\begin{cases}G'(I), & I\in C-R\\ G'(I)\cap L(I), & I\in C\cap R\end{cases}$$

Let $(K,P)\tilde{\cap}(M,C)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases}K(I), & I\in P-C\\ K(I)\cap M(I), & I\in P\cap C\end{cases}$$

Thus,

$$N(I)=\begin{cases}K'(I), & I\in P-C\\ K(I)\cap G'(I), & I\in P\cap(C-R)=P\cap C\cap R'\\ K(I)\cap[G'(I)\cap L(I)], & I\in P\cap(C\cap R)=P\cap C\cap R\end{cases}\tag{3.17}$$

Handle the left-hand side of the equality: $\left[(K,P)\tilde{\setminus}(G,C)\right]\tilde{\cup}\left[(L,R)\tilde{\cap}(K,P)\right]$. Let $(K,P)\tilde{\setminus}(G,C)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases}K(I), & I\in P-C\\ K(I)\cap G'(I), & I\in P\cap C\end{cases}$$

Suppose that $(L,R)\tilde{\cap}(K,P)=(W,R)$ where for all $I\in R$

$$W(I)=\begin{cases} L(I), & I\in R-P \\ L(I)\cap K(I), & I\in R\cap P \end{cases}$$

Let $(V,P)\,\tilde{\cup}\,(W,R)=(T,P)$. Then, for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cup W(I), & I\in P\cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K(I), & I\in (P-C)-R=P\cap C'\cap R' \\ K(I)\cap G'(I), & I\in (P\cap C)-R=P\cap C\cap R' \\ K(I)\cup L(I), & I\in (P-C)\cap(R-P)=\emptyset \\ K(I)\cup[L(I)\cap K(I)], & I\in (P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cap G'(I)]\cup L(I), & I\in (P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cap G'(I)]\cup[L(I)\cap K(I)], & I\in (P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases}$$

Therefore,

$$T(I)=\begin{cases} K(I), & I\in (P-C)-R=P\cap C'\cap R' \\ K(I)\cap G'(I), & I\in (P\cap C)-R=P\cap C\cap R' \\ K(I)\cup L(I), & I\in (P-C)\cap(R-P)=\emptyset \\ K(I), & I\in (P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cap G'(I)]\cup L(I), & I\in (P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cap G'(I)]\cup[L(I)\cap K(I)], & I\in (P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases} \quad (3.18)$$

Handle $I\in P-C$ in the first equation. Since $P-C=P\cap C'$, if $I\in C'$, then $I\in R-C$ or $I\in (C\cup R)'$. Hence, if $I\in P-C$, then $I\in P\cap C'\cap R'$ or $I\in P\cap C'\cap R$. Thus, it can be observed that (3.17)=(3.18). □

ii. $\left[(K,P)\overset{*}{\tilde{\gamma}}(G,C)\right]\tilde{\cap}\,(L,R)=\left[(K,P)\overset{*}{\tilde{\gamma}}(L,R)\right]\tilde{\cap}\,[(G,C)\,\tilde{\cap}\,(L,R)]$

Proof.

Handle the left-hand side of the equality and let $(K,P)\overset{*}{\tilde{\gamma}}(G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases} K'(I), & I\in P-C \\ K'(I)\cap G(I), & I\in P\cap C \end{cases}$$

Let $(M,P)\,\tilde{\cap}\,(L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} M(I), & I\in P-R \\ M(I)\cap L(I), & I\in P\cap R \end{cases}$$

Thus,

$$N(I)=\begin{cases} K'(I), & I\in (P-C)-R=P\cap C'\cap R' \\ K'(I)\cap G(I), & I\in (P\cap C)-R=P\cap C\cap R' \\ K'(I)\cap L(I), & I\in (P-C)\cap R=P\cap C'\cap R \\ [K'(I)\cap G(I)]\cap L(I), & I\in P\cap(C\cap R)=P\cap C\cap R \end{cases} \quad (3.19)$$

Handle the left-hand side of the equality: $\left[(K,P)\overset{*}{\tilde{\gamma}}\,(L,R)\right]\tilde{\cap}\,[(G,C)\,\tilde{\cap}\,(L,R)]$. Let $(K,P)\overset{*}{\tilde{\gamma}}(L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K'(I), & I\in P-R \\ K'(I)\cap L(I), & I\in P\cap R \end{cases}$$

Suppose that $(G,C)\,\tilde{\cap}\,(L,R)=(W,C)$ where for all $I\in C$

$$W(I) = \begin{cases} G(I), & I \in C - R \\ G(I) \cap L(I), & I \in C \cap R \end{cases}$$

Let $(V, P) \tilde{\cap} (W, R) = (T, P)$ where for all $I \in P$

$$T(I) = \begin{cases} V(I), & I \in P - R \\ V(I) \cap W(I), & I \in P \cap R \end{cases}$$

Thus,

$$T(I) = \begin{cases} K'(I), & I \in (P - R) - C = P \cap C' \cap R' \\ K'(I) \cap L(I), & I \in (P \cap R) - C = P \cap C' \cap R \\ K'(I) \cap G(I), & I \in (P - R) \cap (C - R) = P \cap C \cap R' \\ K'(I) \cap [G(I) \cap L(I)], & I \in (P - R) \cap (C \cap R) = \emptyset \\ [K'(I) \cap L(I)] \cap G(I), & I \in (P \cap R) \cap (C - R) = \emptyset \\ [K'(I) \cap L(I)] \cap [G(I) \cap L(I)], & I \in (P \cap R) \cap (C \cap R) = P \cap C \cap R \end{cases} \quad (3.20)$$

It can be observed that (3.19)=(3.20). □

iii. $(K, P) \tilde{\cup} \left[(G, C) \overset{*}{\tilde{\gamma}} (L, R)\right] = \left[(K, P) \tilde{\lambda} (G, C)\right] \tilde{\cap} [(L, R) \tilde{\cup} (K, P)]$

Proof.

Handle the left-hand side of the equality and let $(G, C) \overset{*}{\tilde{\gamma}} (L, R) = (M, C)$ where for all $I \in C$

$$M(I) = \begin{cases} G'(I), & I \in C - R \\ G'(I) \cap L(I), & I \in C \cap R \end{cases}$$

Let $(K, P) \tilde{\cup} (M, C) = (N, P)$ where for all $I \in P$

$$N(I) = \begin{cases} K(I), & I \in P - C \\ K(I) \cup M(I), & I \in P \cap C \end{cases}$$

Thus,

$$N(I) = \begin{cases} K(I), & I \in P - C \\ K(I) \cup G'(I), & I \in P \cap (C - R) = P \cap C \cap R' \\ K(I) \cup [G'(I) \cap L(I)], & I \in P \cap (C \cap R) = P \cap C \cap R \end{cases} \quad (3.21)$$

Handle the left-hand side of the equality: $\left[(K, P) \tilde{\lambda} (G, C)\right] \tilde{\cap} [(L, R) \tilde{\cup} (K, P)]$. Let $(K, P) \tilde{\lambda} (G, C) = (V, P)$ where for all $I \in P$

$$V(I) = \begin{cases} K(I), & I \in P - C \\ K(I) \cup G'(I), & I \in P \cap C \end{cases}$$

Suppose that $(L, R) \tilde{\cup} (K, P) = (W, R)$ where for all $I \in R$

$$W(I) = \begin{cases} L(I), & I \in R - P \\ L(I) \cup K(I), & I \in R \cap P \end{cases}$$

Let $(V, P) \tilde{\cap} (W, R) = (T, P)$. Then, for all $I \in P$

$$T(I) = \begin{cases} V(I), & I \in P - R \\ V(I) \cap W(I), & I \in P \cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K(I)\cup G'(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K(I)\cap L(I), & I\in(P-C)\cap(R-P)=\emptyset \\ K(I)\cap[L(I)\cup K(I)], & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cup G'(I)]\cap L(I), & I\in(P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cup G'(I)]\cap[L(I)\cup K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases}$$

Therefore,

$$T(I)=\begin{cases} K(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K(I)\cup G'(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K(I)\cap L(I), & I\in(P-C)\cap(R-P)=\emptyset \\ K(I), & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cup G'(I)]\cap L(I), & I\in(P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cup G'(I)]\cap[L(I)\cup K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases} \tag{3.22}$$

It can be observed that (3.21)=(3.22). □

iv. $\left[(K,P)\overset{*}{\tilde{\gamma}}(G,C)\right]\tilde{\cup}\,(L,R)=\left[(K,P)\overset{*}{\tilde{\mp}}(L,R)\right]\tilde{\cap}\,[(G,C)\,\tilde{\cup}\,(L,R)]$

Proof.

Handle the left-hand side of the equality and let $(K,P)\overset{*}{\tilde{\gamma}}(G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases} K'(I), & I\in P-C \\ K'(I)\cap G(I), & I\in P\cap C \end{cases}$$

Let $(M,P)\,\tilde{\cup}\,(L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} M(I), & I\in P-R \\ M(I)\cup L(I), & I\in P\cap R \end{cases}$$

Thus,

$$N(I)=\begin{cases} K'(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K'(I)\cap G(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K'(I)\cup L(I), & I\in(P-C)\cap R=P\cap C'\cap R \\ [K'(I)\cap G(I)]\cup L(I), & I\in P\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.23}$$

Handle the left-hand side of the equality: $\left[(K,P)\overset{*}{\tilde{\mp}}\,(L,R)\right]\tilde{\cap}\,[(G,C)\,\tilde{\cup}\,(L,R)]$. Let $(K,P)\overset{*}{\tilde{\mp}}(L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K'(I), & I\in P-R \\ K'(I)\cup L(I), & I\in P\cap R \end{cases}$$

Suppose that $(G,C)\,\tilde{\cup}\,(L,R)=(W,C)$ where for all $I\in C$

$$W(I)=\begin{cases} G(I), & I\in C-R \\ G(I)\cup L(I), & I\in C\cap R \end{cases}$$

Let $(V,P)\,\tilde{\cap}\,(W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cap W(I), & I\in P\cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K'(I), & I\in(P-R)-C=P\cap C'\cap R' \\ K'(I)\cup L(I), & I\in(P\cap R)-C=P\cap C'\cap R \\ K'(I)\cap G(I), & I\in(P-R)\cap(C-R)=P\cap C\cap R' \\ K'(I)\cap[G(I)\cup L(I)], & I\in(P-R)\cap(C\cap R)=\emptyset \\ [K'(I)\cup L(I)]\cap G(I), & I\in(P\cap R)\cap(C-R)=\emptyset \\ [K'(I)\cup L(I)]\cap[G(I)\cup L(I)], & I\in(P\cap R)\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.24}$$

It can be observed that (3.23)=(3.24). □

v. $(K,P)\tilde{\setminus}\left[(G,C)\overset{*}{\tilde{\gamma}}(L,R)\right]=[(K,P)\,\tilde{\cap}\,(G,C)]\,\tilde{\cup}\,[(L,R)\tilde{\gamma}(K,P)]$ where $P\cap C\cap R=\emptyset$

Proof.

Handle the left-hand side of the equality and let $(G,C)\overset{*}{\tilde{\gamma}}(L,R)=(M,C)$ where for all $I\in C$

$$M(I)=\begin{cases} G'(I), & I\in C-R \\ G'(I)\cap L(I), & I\in C\cap R \end{cases}$$

Let $(K,P)\tilde{\setminus}(M,C)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cap M'(I), & I\in P\cap C \end{cases}$$

Thus,

$$N(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cap G(I), & I\in P\cap(C-R)=P\cap C\cap R' \\ K(I)\cap[G(I)\cup L'(I)], & I\in P\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.25}$$

Handle the left-hand side of the equality: $[(K,P)\,\tilde{\cap}\,(G,C)]\,\tilde{\cup}\,[(L,R)\tilde{\gamma}(K,P)]$. Let $(K,P)\,\tilde{\cap}\,(G,C)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K(I), & I\in P-C \\ K(I)\cap G(I), & I\in P\cap C \end{cases}$$

Suppose that $(L,R)\tilde{\gamma}(K,P)=(W,R)$ where for all $I\in R$

$$W(I)=\begin{cases} L(I), & I\in R-P \\ L'(I)\cap K(I), & I\in R\cap P \end{cases}$$

Let $(V,P)\,\tilde{\cup}\,(W,R)=(T,P)$. Then, for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cup W(I), & I\in P\cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K(I)\cap G(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K(I)\cup L(I), & I\in(P-C)\cap(R-P)=\emptyset \\ K(I)\cup[L'(I)\cap K(I)], & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cap G(I)]\cup L(I), & I\in(P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cap G(I)]\cup[L'(I)\cap K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases}$$

Therefore,

$$T(I)=\begin{cases} K(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K(I)\cap G(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K(I)\cup L(I), & I\in(P-C)\cap(R-P)=\emptyset \\ K(I), & I\in(P-C)\cap(R\cap P)=P\cap C'\cap R \\ [K(I)\cap G(I)]\cup L(I), & I\in(P\cap C)\cap(R-P)=\emptyset \\ [K(I)\cap G(I)]\cup[L'(I)\cap K(I)], & I\in(P\cap C)\cap(R\cap P)=P\cap C\cap R \end{cases} \tag{3.26}$$

It can be observed that (3.25)=(3.26). □

vi. $\left[(K,P)\overset{*}{\tilde{\gamma}}(G,C)\right]\tilde{\setminus}(L,R)=\left[(K,P)\overset{*}{\tilde{\theta}}(L,R)\right]\tilde{\cap}\left[(G,C)\tilde{\setminus}(L,R)\right]$

Proof.

Let first handle the left-hand side of the equality and let $(K,P)\overset{*}{\tilde{\gamma}}(G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases} K'(I), & I\in P-C \\ K'(I)\cap G(I), & I\in P\cap C \end{cases}$$

Let $(M,P)\tilde{\setminus}(L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases} M(I), & I\in P-R \\ M(I)\cap L'(I), & I\in P\cap R \end{cases}$$

Thus,

$$N(I)=\begin{cases} K'(I), & I\in(P-C)-R=P\cap C'\cap R' \\ K'(I)\cap G(I), & I\in(P\cap C)-R=P\cap C\cap R' \\ K'(I)\cap L'(I), & I\in(P-C)\cap R=P\cap C'\cap R \\ [K'(I)\cap G(I)]\cap L'(I), & I\in(P\cap C)\cap R=P\cap C\cap R \end{cases} \tag{3.27}$$

Handle the left-hand side of the equality: $\left[(K,P)\overset{*}{\tilde{\theta}}(L,R)\right]\tilde{\cap}\left[(G,C)\tilde{\setminus}(L,R)\right]$. Let $(K,P)\overset{*}{\tilde{\theta}}(L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases} K'(I), & I\in P-R \\ K'(I)\cap L'(I), & I\in P\cap R \end{cases}$$

Suppose that $(G,C)\tilde{\setminus}(L,R)=(W,C)$ where for all $I\in C$

$$W(I)=\begin{cases} G(I), & I\in C-R \\ G(I)\cap L'(I), & I\in C\cap R \end{cases}$$

Let $(V,P)\tilde{\cap}(W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases} V(I), & I\in P-R \\ V(I)\cap W(I), & I\in P\cap R \end{cases}$$

Thus,

$$T(I)=\begin{cases} K'(I), & I\in(P-R)-C=P\cap C'\cap R' \\ K'(I)\cap L'(I), & I\in(P\cap R)-C=P\cap C'\cap R \\ K'(I)\cap G(I), & I\in(P-R)\cap(C-R)=P\cap C\cap R' \\ K'(I)\cap[G(I)\cap L'(I)], & I\in(P-R)\cap(C\cap R)=\emptyset \\ [K'(I)\cap L'(I)]\cap G(I), & I\in(P\cap R)\cap(C-R)=\emptyset \\ [K'(I)\cap L'(I)]\cap[G(I)\cap L'(I)], & I\in(P\cap R)\cap(C\cap R)=P\cap C\cap R \end{cases} \tag{3.28}$$

It can be observed that (3.27)=(3.28). □

vii. $(K,P)\tilde{\lambda}\left[(G,C)\overset{*}{\tilde{\gamma}}(L,R)\right] = [(K,P)\,\tilde{\cup}\,(G,C)]\,\tilde{\cap}\,[(L,R)\tilde{\mp}(K,P)]$ where $P \cap C \cap R = \emptyset$

Proof.

Handle the left-hand side of the equality and let $(G,C)\overset{*}{\tilde{\gamma}}(L,R) = (M,C)$ where for all $I \in C$

$$M(I) = \begin{cases} G'(I), & I \in C - R \\ G'(I) \cap L(I), & I \in C \cap R \end{cases}$$

Let $(K,P)\tilde{\lambda}(M,C) = (N,P)$ where for all $I \in P$

$$N(I) = \begin{cases} K(I), & I \in P - C \\ K(I) \cup M'(I), & I \in P \cap C \end{cases}$$

Thus,

$$N(I) = \begin{cases} K(I), & I \in P - C \\ K(I) \cup G(I), & I \in P \cap (C - R) = P \cap C \cap R' \\ K(I) \cup [G(I) \cup L'(I)], & I \in P \cap (C \cap R) = P \cap C \cap R \end{cases} \tag{3.29}$$

Handle the left-hand side of the equality: $[(K,P)\,\tilde{\cup}\,(G,C)]\,\tilde{\cap}\,[(L,R)\tilde{\mp}(K,P)]$. Let $(K,P)\,\tilde{\cup}\,(G,C) = (V,P)$ where for all $I \in P$

$$V(I) = \begin{cases} K(I), & I \in P - C \\ K(I) \cup G(I), & I \in P \cap C \end{cases}$$

Suppose that $(L,R)\tilde{\mp}(K,P) = (W,R)$ where for all $I \in R$

$$W(I) = \begin{cases} L(I), & I \in R - P \\ L'(I) \cup K(I), & I \in R \cap P \end{cases}$$

Let $(V,P)\,\tilde{\cap}\,(W,R) = (T,P)$. Then, for all $I \in P$

$$T(I) = \begin{cases} V(I), & I \in P - R \\ V(I) \cap W(I), & I \in P \cap R \end{cases}$$

Thus,

$$T(I) = \begin{cases} K(I), & I \in (P - C) - R = P \cap C' \cap R' \\ K(I) \cup G(I), & I \in (P \cap C) - R = P \cap C \cap R' \\ K(I) \cap L(I), & I \in (P - C) \cap (R - P) = \emptyset \\ K(I) \cap [L'(I) \cup K(I)], & I \in (P - C) \cap (R \cap P) = P \cap C' \cap R \\ [K(I) \cup G(I)] \cap L(I), & I \in (P \cap C) \cap (R - P) = \emptyset \\ [K(I) \cup G(I)] \cap [L'(I) \cup K(I)], & I \in (P \cap C) \cap (R \cap P) = P \cap C \cap R \end{cases}$$

Therefore,

$$T(I) = \begin{cases} K(I), & I \in (P - C) - R = P \cap C' \cap R' \\ K(I) \cup G(I), & I \in (P \cap C) - R = P \cap C \cap R' \\ K(I) \cap L(I), & I \in (P - C) \cap (R - P) = \emptyset \\ K(I), & I \in (P - C) \cap (R \cap P) = P \cap C' \cap R \\ [K(I) \cup G(I)] \cap L(I), & I \in (P \cap C) \cap (R - P) = \emptyset \\ [K(I) \cup G(I)] \cap [L'(I) \cup K(I)], & I \in (P \cap C) \cap (R \cap P) = P \cap C \cap R \end{cases} \tag{3.30}$$

It can be observed that (3.29)=(3.30). □

viii. $\left[(K,P)\overset{*}{\tilde{\gamma}}(G,C)\right]\tilde{\lambda}(L,R) = \left[(K,P)\overset{*}{\tilde{*}}(L,R)\right]\,\tilde{\cap}\,\left[(G,C)\tilde{\lambda}(L,R)\right]$

Proof.

Handle the left-hand side of the equality and let $(K,P)\overset{*}{\tilde{\gamma}}(G,C)=(M,P)$ where for all $I\in P$

$$M(I)=\begin{cases}K'(I), & I\in P-C\\ K'(I)\cap G(I), & I\in P\cap C\end{cases}$$

Let $(M,P)\tilde{\lambda}(L,R)=(N,P)$ where for all $I\in P$

$$N(I)=\begin{cases}M(I), & I\in P-R\\ M(I)\cup L'(I), & I\in P\cap R\end{cases}$$

Thus,

$$N(I)=\begin{cases}K'(I), & I\in(P-C)-R=P\cap C'\cap R'\\ K'(I)\cap G(I), & I\in(P\cap C)-R=P\cap C\cap R'\\ K'(I)\cup L'(I), & I\in(P-C)\cap R=P\cap C'\cap R\\ [K'(I)\cap G(I)]\cup L'(I), & I\in(P\cap C)\cap R=P\cap C\cap R\end{cases}\tag{3.31}$$

Handle the left-hand side of the equality: $\left[(K,P)\overset{*}{\tilde{*}}(L,R)\right]\tilde{\cap}\left[(G,C)\tilde{\lambda}(L,R)\right]$. Let $(K,P)\overset{*}{\tilde{*}}(L,R)=(V,P)$ where for all $I\in P$

$$V(I)=\begin{cases}K'(I), & I\in P-R\\ K'(I)\cup L'(I), & I\in P\cap R\end{cases}$$

Suppose that $(G,C)\tilde{\lambda}(L,R)=(W,C)$ where for all $I\in C$

$$W(I)=\begin{cases}G(I), & I\in C-R\\ G(I)\cup L'(I), & I\in C\cap R\end{cases}$$

Let $(V,P)\tilde{\cap}(W,R)=(T,P)$ where for all $I\in P$

$$T(I)=\begin{cases}V(I), & I\in P-R\\ V(I)\cap W(I), & I\in P\cap R\end{cases}$$

Thus,

$$T(I)=\begin{cases}K'(I), & I\in(P-R)-C=P\cap C'\cap R'\\ K'(I)\cup L'(I), & I\in(P\cap R)-C=P\cap C'\cap R\\ K'(I)\cap G(I), & I\in(P-R)\cap(C-R)=P\cap C\cap R'\\ K'(I)\cap[G(I)\cup L'(I)], & I\in(P-R)\cap(C\cap R)=\emptyset\\ [K'(I)\cup L'(I)]\cap G(I), & I\in(P\cap R)\cap(C-R)=\emptyset\\ [K'(I)\cup L'(I)]\cap[G(I)\cup L'(I)], & I\in(P\cap R)\cap(C\cap R)=P\cap C\cap R\end{cases}\tag{3.32}$$

It can be observed that (3.31)=(3.32). □

4. Conclusion

In this paper, we explore more about complementary soft binary piecewise plus and gamma operation by investigating the relationships between these soft set operations and soft binary piecewise operations. In this paper, it is aimed to contribute to the soft set literature by obtaining the distributions of soft binary piecewise operations over complementary soft binary piecewise plus and gamma operations. This is a theoretical study for soft sets and some future studies may continue by investigating the distributions of soft binary piecewise operations over other complementary soft piecewise operations.

Author Contributions

All the authors equally contributed to this work. They all read and approved the final version of the paper.

Conflict of Interest

All the authors declare no conflict of interest.

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