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Eksenel yönde tabakalı kirişlerin temel frekansı üzerinde tabaka dizilişinin etkileri

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ÖZ

Bu çalışmada eksenel yönde tabakalara sahip kirişlerin serbest titreşim davranışı üzerinde tabaka dizilişinin etkisi Timoshenko kiriş teorisine göre sonlu elemanlar programı (ANSYS) kullanılarak incelenmiştir. Her tabaka Alüminyum/Alüminyum oksit, Alüminyum/Zirkonyum ve Alüminyum/Nikel gibi farklı sistemlere sahiptir. Tabaka dizilişi Taguchi Metodunda L9 orthogonal dizi kullanılarak yürütülmüştür. Optimum tabaka dizilişini elde edebilmek için Taguchi Metodu ve optimum tabaka kombinasyonu kullanıldı. Yanıtlar üzerinde önemli tabakaları ve katkı yüzdelerini gerçekleştirebilmek için Varyans Analizi (ANOVA) kullanıldı. Sonuçlara göre yanıtlar üzerinde en etkili parametreler sırasıyla %67.94 ile Alüminyum/Alüminyum oksit, %31.08 ile Alüminyum/Nikel ve %0.95 ile Alüminyum/Zirkonyum için elde eddilmiştir. İlk mod olarak da bilinen temel frekans değerleri tabakalardaki Alüminyum/Alüminyum oksit ve Alüminyum/Zirkonyum içeriklerinin artmasıyla artmış ve Alüminyum/Nikel içeriğinin artması ile azalmıştır.

Anahtar Kelimeler: Temel Frekans, Tabakalı Kiriş, ANSYS

The effects of layer arrangements on fundamental frequency of layered beams in axial direction

ABSTRACT

In this study, the influence of layer arrangements on free vibration behavior of beams which have layers in the axial direction is investigated according to Timoshenko Beam Theory by using finite element program (ANSYS). Each layer has different systems such as Aluminum/Alumina, Aluminum/Zirconia and Aluminum/Nickel. Layer arrangements are conducted using the L9 orthogonal array in Taguchi Method. In order to obtain the sorting order of optimum layers, Taguchi Method and optimum layer combination are utilized. Analysis of Variance (ANOVA) is used to carry out the significant layers and percentage of contribution on the responses. According to the results, the most effective parameters on the responses are obtained for Aluminum/Alumina with 67.94%, Aluminum/Nickel with 31.08% and Aluminum/Zirconia with 0.95%, respectively. Fundamental frequency values, also known as the first mode frequency values, increase with the increasing Aluminum/Alumina and Aluminum/Zirconia contents and decrease with the increasing Aluminum/Nickel content in layers.

Keywords: Fundamental Frequency, Layered Beams, ANSYS

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1. INTRODUCTION

In engineering areas, free vibration behaviors of the beams are of great importance. There are many beams made of materials such as metal, ceramic, composite, polymer etc. for different usage environments. But beams fabricated using a single material may not be suitable for the ambient conditions. So beams produced depending on different usage rates of different materials may be needed. In the literature, there are many studies consist of free vibration analysis of different beam types. Chandrashekhara and Bangera [1] evaluated the free vibration analysis of laminated composite beams using a finite element model depended on a higher-order shear deformation theory. Li [2] presented a study about the free vibration analysis of beams, which have generally boundary conditions. Abbas [3] submitted a study consist of the free vibration analysis of Timoshenko beams according to elastically restrained ends. Lee and Ke [4] investigated of the problem consisting of the free vibration analysis of a non-uniform beam for general elastically restrained boundary conditions. Mao and Pietrzko [5] studied the free vibration analysis of a stepped Euler–Bernoulli beams which have two uniform sections according to Adomian decomposition method. Jang and Bert [6] presented a study dealing with the exact and numerical solutions of free vibration of stepped beams. Jaworski and Dowell [7] investigated the theoretical and experimental flexural-free vibration analysis of a cantilevered beam consisting of multiple cross-section steps. Ju *et al.* [8] pulished a study involving the free vibration analysis of stepped beams. Zheng and Kessissoglou [9] observed the study about the free vibrations of a cracked beam according to the finite element method. Kisa and Arif Gurel [10] presented the study consist of the free vibrations of stepped and uniform cracked beams which have circular cross sections. Chandrashekhara et al. [11] studied of the free vibration analysis of composite beams according to rotary inertia and shear deformation. Kapuria et al. [12] carried out the static and free vibration response of layered FGM beams consist of systems such as Al/SiC and Ni/Al_2O_3. They used powder metallurgy for producing the layered FGM beams consist of Al/SiC and powder thermal spray processes for fabricating the layered FGM beams consist of Ni/Al_2O_3. Aydogdu and Taskin [13] determined the free vibration of FG beams varied Young's modulus in the thickness direction for simply supported boundary conditions. Pradhan and Chakraverty [14] presented a study about the free vibration of Euler and Timoshenko functionally graded beams varied material properties in the thickness direction continuously for different sets of boundary conditions. Şimşek [15] evaluated the fundamental frequency analysis of functionally graded beams of which vary the material properties in the thickness direction continuously according to the power-law form under different boundary conditions depending on the different higher-order beam theories. Sina et al. [16] presented a study consist of the free vibration of functionally graded beams assumed to vary in the thickness direction according to a simple power law distribution in terms of volume fraction of material constituents under different boundary conditions. Alshorbagy et al. [17] evaluated the free vibration characteristics of and dynamic behavior of a functionally graded beam which have material graduation in axially or transversally in the thickness direction depended on the power law under various boundary conditions using finite element method numerically. Huang and Li [18] published a study consist of free vibration of axially functionally graded beams which have non-uniform cross-section with different boundary conditions. Şimşek [19] performed the vibration analysis of a functionally graded beam varied material properties in the thickness direction continuously based on the power-law form with simply-supported using Euler–Bernoulli, Timoshenko and the third order shear deformation beam theories under a moving mass. Yilmaz and Evran [20] produced axially layered functionally graded short beams having different Al/SiC content using powder metallurgy technique and investigated the free vibration behavior of the beams by using experimental and finite element methods. As can be seen from literature mentioned, there is no study consist of the effects of layer arrangements in free vibration analysis of axially layered beams using L9 orthogonal array based on Taguchi Method. The vibrations studied are the bending vibrations

2. MATERIALS AND METHODS

2.1. Materials

In the study, the axially layered beams consist of three layers, totally. Each layer has different systems such as Al/Al_2O_3, Al/ZrO_2 and Al/Ni and its content is modelled as 95%Al, 90%Al and %85Al which are in arithmetic fashion. The effective material properties P_{ef} for each layer of the beams, such as Elasticity modulus E_{ef} and mass density ρ_{ef} are carried out using a simple rule of mixture of composite materials as given in Equation 1 [21]. Poisson's ratio is taken to be constant.

$$P_{ef} = \sum_{k=1} P_k V_{f_k} \tag{1}$$

P_k and V_{fk} in the equations are the material properties and volume fraction of the constituent material k respectively. The sum of the volume fractions of all the constituent materials also equal to one, i.e.,

(2)

$$\sum_{k=1} V_{f_k} = 1 \tag{2}$$

Elasticity modulus, densities and poisson's ratios of Aluminium (Al), Alumina (Al_2O_3), Zirconia (ZrO_2) and Nickel (Ni) are given in Table 1. The effective material properties such as Elasticity modulus E_{ef}, mass density ρ_{ef} and poisson's ratio value υ_{ef} of the systems such as Al/Al_2O_3, Al/ZrO_2 and Al/Ni are presented as Table 2 and these values were used to obtain optimum layer agreements for free vibration behavior of axially layered beams.

Table 1. Elasticity modulus, densities and poisson's ratios of Al, Al_2O_3, ZrO_2 and Ni

Materials Properties	Materials			
	Al [22]	Al_2O_3 [22]	ZrO_2 [23]	Ni [24]
Elasticity modulus E (GPa)	70	380	151	199.5
Density ρ (kg/m^3)	2707	3800	3000	8900
Poisson's ratios υ	0.3	0.3	0.3	0.3

Table 2. Elasticity modulus, densities and poisson's ratios of Al/Al_2O_3, Al/ZrO_2 and Al/Ni

Material Properties	Material Combinations								
	Al/Al_2O_3			Al/ZrO_2			Al/Ni		
	95% Al	90% Al	85% Al	95% Al	90% Al	85% Al	95% Al	90% Al	85% Al
Elasticity modulus E (GPa)	85.5	101	116.5	74.05	78.1	82.15	76.475	82.95	89.425
Density ρ (kg/m^3)	2761.65	2816.3	2870.95	2721.65	2736.3	2750.95	3016.65	3326.3	3635.95
Poisson's Ratios υ (-)	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3

2.2. Method

The purpose of this study is to carry out the effects of layer arrangements in free vibration analysis of axially layered beams and to determine the optimum layer arrangements. The analysis was performed by using L9 orthogonal array based on Taguchi Methodology. The orthogonal array consist of 3 levels and 3 factors. Layers which have systems such as Al/Al_2O_3, Al/ZrO_2 and Al/Ni were determined as control factors in the analysis. In order to obtain fundamental frequency values of the beams, materials properties of the systems in Table 2 was also used as control factors. Each factor has 3 different levels as shown in Table 3 and thus 9 analysis were performed as shown in Table 4.

Table 3. Control factors and their levels.

Code	Control Factors	Level 1	Level 2	Level 3
A	Layers consist of Al/Al_2O_3	%95Al/%5Al_2O_3	%90Al/%10Al_2O_3	%85Al/%15Al_2O_3

B	Layers consist of Al/ZrO_2	%95Al/%5ZrO_2	%90Al/%10ZrO_2	%85Al/%15ZrO_2
C	Layers consist of Al/Ni	%95Al/%5Ni	%90Al/%10Ni	%85Al/%15Ni

In the Taguchi method, quality loss function is used for three characteristics such as the nominal-the-best, smaller-the-better, larger-the better [25]. Larger is the better characteristic as $(S/N)_{LB}$ in the study is used as given by Equation 3 [26].

$$(S/N)_{LB} = -10.\log\left(n^{-1}\sum_{i=1}^{n}(y_i^2)^{-1}\right) \qquad (3)$$

in which n shows number of analysis in a trial and y_i is observed ith data.

3. NUMERICAL ANALYSIS

Free vibration analysis are carried out using ANSYS finite element program according to Timoshenko Beam Theory. BEAM3 element type which has three degrees of freedom at each node: translations in the nodal x and y directions and rotation about the nodal z-axis was used in the modelling to obtained fundamental frequencies. BEAM3 element in the ANSYS program is also a uniaxial element consist of tension, compression, and bending capabilities. It can be seen detailed information for the BEAM3 element using the ANSYS help menu. Shear deflection influence are often important in the lateral deflection of short beams and thus the value of shear deflection constant (SHEARZ) is defined for rectangle cross-section as 6/5. For the eigenvalue extraction, the Block Lanczos method was carried out. The beams have 10 mm cross section height, 12 mm cross section base and 60 mm length and layer sizes are equal each other. In addition, 20 elements where ANSYS for each layer are calculated. The analysis were performed under clamped-free boundary condition as shown in Figure 1.

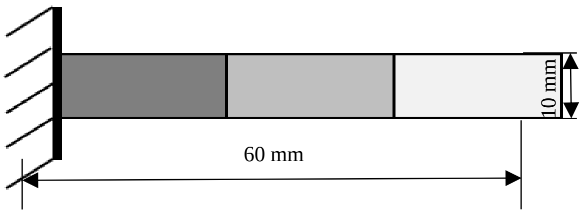


Figure 1. Axially layered beam with clamped-free boundary condition

4. RESULTS AND DISCUSSIONS

In order to obtain optimum result among fundamental frequency results of axially layered beams, MINITAB 15 packet program was used. S/N ratio values corresponding to the fundamental frequency results are calculated by using Equation 3. Results obtained and S/N ratio values are given in Table 4 according to L9 orthogonal array. In addition, Table 4 shows that overall mean of the fundamental frequency results of all beam and the arithmetic mean of the results is in the interval of 2172.7 and 2628.0 Hz.

Table 4. Results and S/N ratio values of fundamental frequencies.

Test No	Designation	Control Factors			Results ω (Hz)	S/N Ratio η (dB)
		A	B	C		
1	$A_1B_1C_1$	%95Al/%5Al_2O_3	%95Al/%5ZrO_2	%95Al/%5Ni	2325.2	67.3292
2	$A_1B_2C_2$	%95Al/%5Al_2O_3	%90Al/%10ZrO_2	%90Al/%10Ni	2244.6	67.0228
3	$A_1B_3C_3$	%95Al/%5Al_2O_3	%85Al/%15ZrO_2	%85Al/%15Ni	2172.7	66.7400
4	$A_2B_1C_2$	%90Al/%10Al_2O_3	%95Al/%5ZrO_2	%90Al/%10Ni	2379.2	67.5286
5	$A_2B_2C_3$	%90Al/%10Al_2O_3	%90Al/%10ZrO_2	%85Al/%15Ni	2305.3	67.2545
6	$A_2B_3C_1$	%90Al/%10Al_2O_3	%85Al/%15ZrO_2	%95Al/%5Ni	2505.3	67.9772
7	$A_3B_1C_3$	%85Al/%15Al_2O_3	%95Al/%5ZrO_2	%85Al/%15Ni	2413.7	67.6537
8	$A_3B_2C_1$	%85Al/%15Al_2O_3	%90Al/%10ZrO_2	%95Al/%5Ni	2628.0	68.3925

9	$A_3B_3C_2$	%85Al/%15Al_2O_3	%85Al/%15ZrO_2	%90Al/%10Ni	2538.5	68.0915
		Overall Mean			2390.3	

4.1. Effect on Free Vibration Analysis

In order to see the influence of control factors on free vibration analysis, tests were conducted using L9 orthogonal array. According to results obtained using ANSYS program in Table 4, the average values of fundamental frequencies for all of levels such as level 1, level 2 and level 3 of control factors for raw data are plotted in Figure 2. Figure 2 shows that fundamental frequencies increase with increase of Al_2O_3 and ZrO_2 contents and decrease with the increasing Ni content in layers. Mechanical properties of each layer consist of Al/Al_2O_3, Al/ZrO_2 and Al/Ni systems increase depend on increase of contents such as Al_2O_3, ZrO_2 and Ni. This situation can be seen in Table 2 clearly. But layers with Al/Ni systems decrease fundamental frequencies of axially layered beams and has created the effect in the opposite direction according to layers consist of Al/Al_2O_3 and Al/ZrO_2. Therefore, it can be said that increase of fundamental frequencies related to both mechanical properties such as Elasticity modulus and density and layer agreements.

4.2. Determination of Optimum Layers

In the Figure 2, optimum layers of axially layered beams are third level of layers with Al/Al_2O_3 and Al/ZrO_2, first level of layer consist of Al/Ni. These results can be seen from Table 5.

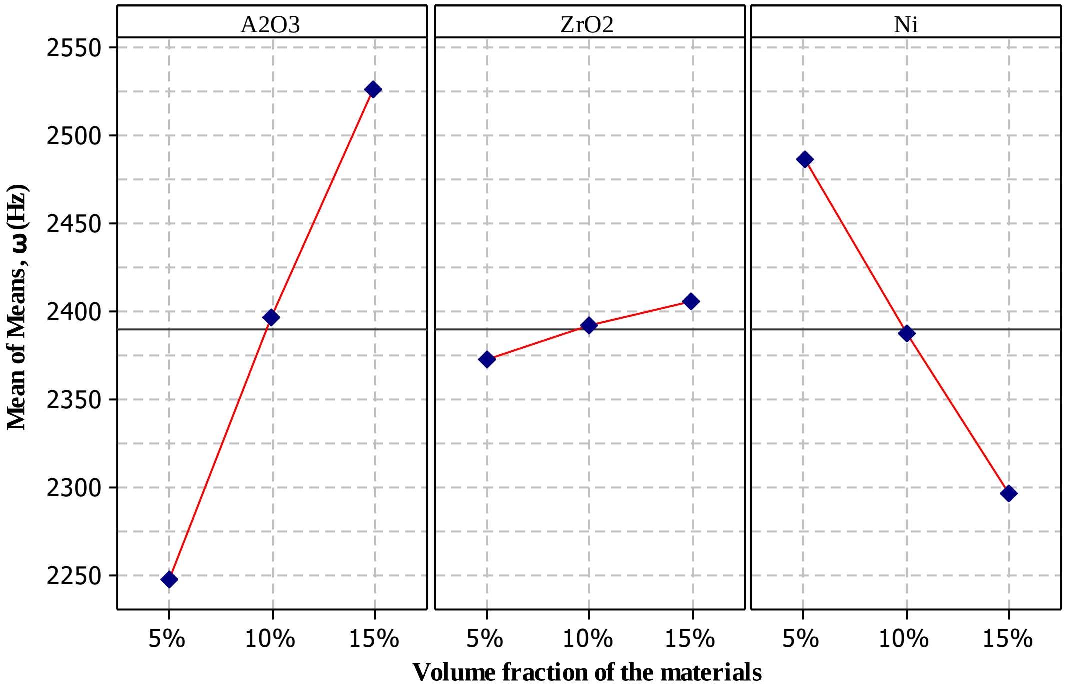


Figure 2. Main influence plots for fundamental frequencies

Table 5. Response table for fundamental frequencies

Level	S/N ratios (dB)			Means (Hz)		
	A	B	C	A	B	C
1	67.03	67.50	67.90[a]	2247	2373	2486[a]
2	67.59	67.56	67.55	2397	2393	2387
3	68.05[a]	67.60[a]	67.22	2527[a]	2406[a]	2297
Δ	1.02	0.10	0.68	279	33	189

Rank	1	3	2	1	3	2

[a] Optimum level, Δ = Difference between maximum and minimum values

Table 6. Analysis of Variance for fundamental frequencies (Raw Data)

Source	DF	Sum of squares	Variance	F	P	Cont. (%)
A	2	117137	58568	2548.54[a]	0.000	67.94
B	2	1639	819	35.65[a]	0.027	0.95
C	2	53580	26790	1165.74[a]	0.001	31.08
Error	2	46	23			0.03
Total	8	172402				100

[a]Significant for 95% confidence level R-Sq = 99.97% R-Sq(adj) = 99.89%

In order to determine the effects of control factors on fundamental frequency results, analysis of variance (ANOVA) was performed. ANOVA results are given in Table 6. According to the table, all of layers have significant effect on fundamental frequency results in the 95% reliability interval. In addition, the most effective control parameter on results is obtained using layer with Al/Al_2O_3 which have 67.94% contribution ratio and other effective control parameters are layer made of Al/Ni with 31.08% contribution ratio and layer consist of Al/ZrO_2 with 0.95% contribution ratio, respectively.

4.3. Estimation of Optimum Fundamental Frequency

The optimum value of fundamental frequency is predicted at the optimal levels of significant variables such as level 3 of layer consist of Al/Al_2O_3 and Al/ZrO_2 and level 1 of layer made of Ni. The estimated mean of the response characteristic named fundamental frequency can be defined as Equation 4 [26].

$$\overline{\omega_{Hz}} = \bar{A}_3 + \bar{B}_3 + \bar{C}_1 - 2\bar{T}_{\omega Hz} \quad (4)$$

in which $\bar{T} = 2390.3$ is overall mean of fundamental frequencies and is taken from Table 4. In addition, average values of fundamental frequencies at the third level of Al/Al_2O_3 and Al/ZrO_2 are $\bar{A}_3 = 2527$ and $\bar{B}_3 = 2406$, respectively. Also, average value of fundamental frequency at the first level of Ni is $\bar{C}_1 = 2486$. These values are taken from Table 5. 95% confidence intervals (CI) are calculated using Equation 5 [27].

$$CI_{\omega Hz} = \sqrt{F_{\alpha;1;n_2} V_{error}\left[\frac{1}{n_{eff}} + \frac{1}{R}\right]} \quad (5)$$

where, $F_{0.05;1;2}$ = 18.513 is taken from listed F values [26] according to 95% confidence intervals at DOF 1 and error DOF n_2 and R=1 is the repetition number for verification tests. V_{error} = 23 is variance of error term and is taken from Table 6. Effective number of repetitions (n_{eff}) is calculated as [27],

$$n_{eff} = N/[1+T_{DF}] \quad (6)$$

where, T_{DF} = 6 and N = 9 are sum of degrees of freedom for meaningful control parameters and total number of tests, respectively. T_{DF} value is also taken from Table 6. Hence neff = 1.2857. According to the values calculated for Equation 5, $CI_{\omega Hz} = \pm 27.51$. Estimation of optimum fundamental frequency for 95% CI is expressed as,

$$\overline{\omega_{Hz}} - CI_{\omega Hz} < \overline{\omega_{Hz}} < \overline{\omega_{Hz}} + CI_{\omega Hz} \quad (7)$$

Comparison of optimum results is shown in Table 7 for 95% confidence intervals.

Table 7. Comparison of optimum results

Optimum Level	Predicted Result	ANSYS Result	Predicted Confidence Intervals for 95% Confidence Level
	(Hz)	(Hz)	(Hz)
$A_3B_3C_1$	2638.40	2643.60	$2610.89 < \overline{\omega_{Hz}} < 2665.91$

4.4. Regression Analysis

Regression technique was employed for estimating relationships among control parameters and fundamental frequencies results obtained using ANSYS program. In the equation, all control parameters were used since the p-value is larger than 0.05. Prediction model for R-Sq = 99.8% and R-Sq (adj) = 99.7% is shows as,

$$\omega_{Hz}=2267+140Al/A_2O_3+16.4Al/ZrO_2–94.5\ Al/Ni \quad (8)$$

in which, control factors such as Al/A_2O_3 and Al/ZrO_2 have positive effect on fundamental frequency. But Al/Ni is has negative effect. In order words, increase of Al/A_2O_3 and Al/ZrO_2 increase fundamental frequency. Increase of Al/Ni also decrease. ANOVA result for regression analysis is presented in Table 8.

Table 8. ANOVA result for regression analysis

Source	DF	Sum of squares	Variance	F	P
Regression	3	172114	57371	998.78	0.000
Residual Error	5	287	57		
Total	8	172402			

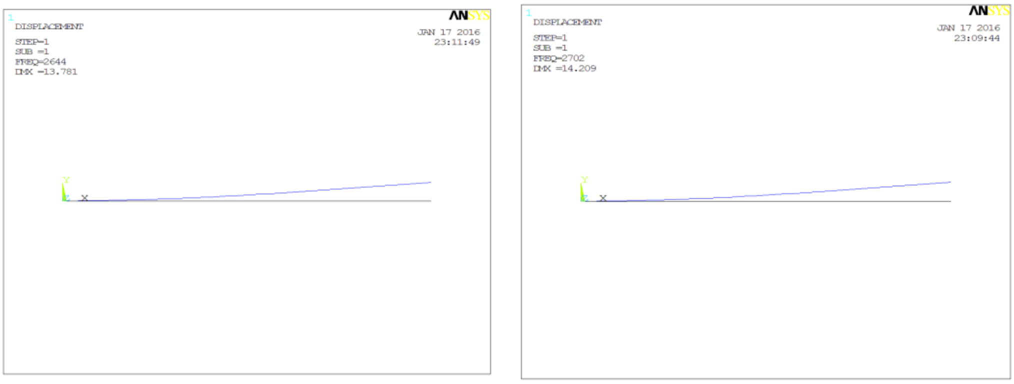
4.5. Optimum Layer Arrangements

In the study, free vibration behavior of axially layered beams consist of 3 layers is investigated. Each layer is made of different contents such as Al_2O_3, ZrO_2 and Ni. Optimum layers for fundamental frequency results are determined by using %85Al-%15Al_2O_3, %85Al-%15ZrO_2, and % 95Al-% 5Ni, respectively. In order words, maximum fundamental frequency is obtained using these layers. In order to determine whether effect of the sorting order of optimum layers for fundamental frequency, different combination of these layers are analyzed by using ANSYS program. Different combinations of the layers are tabled in Table 9.

Table 9. Different combination of optimum layers

Beam Type	Beam Configurations			ω (Hz)
Type 1	%85Al-%15Al_2O_3	%85Al-%15ZrO_2	% 95Al-% 5Ni	2643.6
Type 2	%85Al-%15Al_2O_3	% 95Al-% 5Ni	%85Al-%15ZrO_2	2702.1
Type 3	%85Al-%15ZrO_2	%85Al-%15Al_2O_3	% 95Al-% 5Ni	2366.6
Type 4	%85Al-%15ZrO_2	% 95Al-% 5Ni	%85Al-%15Al_2O_3	2326.8
Type 5	% 95Al-% 5Ni	%85Al-%15ZrO_2	%85Al-%15Al_2O_3	2293.4
Type 6	% 95Al-% 5Ni	%85Al-%15Al_2O_3	%85Al-%15ZrO_2	2380.9

According to results in Table 9, maximum fundamental frequency is obtained for Type 2 beam configuration which have %85Al-%15Al_2O_3, %95Al-%5Ni and %85Al-%15ZrO_2. This situation can be explained that layers have high contribution rate on fundamental frequency are located near the clamped end of the beam. In order words, increase of contribution rate of layers from clamped-end to free-end decrease fundamental frequency values. This condition can be monitored for Type 1 and Type 2 beam configurations or Type 3 and Type 4 beam configurations or Type 5 and Type 6 beam configurations. The second highest fundamental frequency is obtained for Type 1 beam configuration with %85Al-%15Al_2O_3, %85Al-%15ZrO_2 and % 95Al-% 5Ni. Type 1 beam configuration is determined according to Taguchi Methodology. Maximum fundamental frequencies obtained using Taguchi Methodology and optimum layer combinations are demonstrated as in Figure 3.



(a) (b)

Figure 3. Maximum fundamental frequency as (a) Taguchi Methodology (Type1) and (b) Optimum layer arrangements (Type 2)

5. CONCLUSIONS

In the study, the influences of layer arrangements on free vibration behavior of layered beams in the axial direction are performed by using finite element program (ANSYS) for the theoretical prediction according to Timoshenko Beam Theory and the design of trials is conducted L9 orthogonal array, which is having 9 rows and 3 columns, based on Taguchi methodology. Based on the results obtained following generalized conclusions can be draw from the study:

1) Fundamental frequencies increase with increase of Al/Al_2O_3 and Al/ZrO_2 contents and decrease with the increasing Al/Ni content in layers.

2) Contributions ratios of layers on fundamental frequency are Al/Al_2O_3 with 67.94%, Al/Ni with 31.08% and Al/ZrO_2 with 0.95%, respectively.

3) Decrease of contribution ratios of layers from clamped end to free end increase the fundamental frequencies for clamp-free boundary condition.

4) Percent contribution of layers on fundamental frequency plays important role on layer arrangements.

5) Although maximum fundamental frequency is obtained using beam with %85Al-%15Al_2O_3, % 95Al-% 5Ni and %85Al-%15ZrO_2 for optimum layer arrangement, maximum fundamental frequency according to Taguchi Methodology is determined for beam with %85Al-%15Al_2O_3, %85Al-%15ZrO_2 and % 95Al-% 5Ni.

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