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STRUCTURED ROBUST STABILITY ANALYSIS OF AN INVERTED PENDULUM SYSTEM WITH A FIXEDFEEDBACK

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ABSTRACT: *Robust stability analysis of inverted pendulum system with a fixed linear quadratic feedback is carried out using μ analysis tools. A mathematical model of inverted pendulum system is presented and linearized about the desired equilibrium point. A linear quadratic control feedback matrix is used in the configuration of the nominal inverted pendulum system for its stabilization. After this, uncertainties in the inverted pendulum with a fixed feedback, is modeled in linear fractional transformation form , which is suitable for structured singular value computation. Both parametric and modeling uncertainties are considered in the inverted pendulum system. After deriving out uncertain system model, mixed μ analysis method which is a structured uncertainty analysis method, is used to compute the uncertainty bound that does not cause instability of the inverted pendulum under feedback. A simulation test for validity of the results is provided.*

KEYWORDS: Structured singular value, Uncertainty, Robustness, Inverted pendulum

SABİT GERİBESLEMELİ TERS SARKAÇ SİSTEMİNİN YAPISAL GÜRBÜZ KARARLILIK ANALİZİ

ÖZET: *Sabit doğrusal-kuadratik kontrol altındaki ters sarkaç sistemindeki belirsizliklere karşı gürbüzlük analizi, μ analiz araçları kullanılarak yapılmıştır. Ters sarkaç sisteminin matematiksel modeli verilip, çalışma noktası etrafında doğrusallaştırılmaktadır. Nominal ters sarkaç sisteminin kararlaştırılması için doğrusal-kuadratik kontrol geri besleme matrisi kullanılmaktadır. Yapısal tekil değer hesaplanmasında kullanılmak üzere, sabit doğrusal-kuadratik kontrol altında olan ters sarkaç sistemindeki belirsizlikler doğrusal kesirli dönüşümler kullanılarak modellenmektedir. Ters sarkaç sistemindeki parametrik ve modellemeden kaynaklanan belirsizlikler dikkate alınmıştır. Belirsizlik modeli oluşturulduktan sonra, karışık μ analizi (bir yapısal belirsizlik analiz yöntemi) yöntemi kullanılarak, geri belsem altındaki ters sarkaç sisteminde kararsızlığa neden olmayan belirsizlik sınırı bulunmaktadır. Bulunan sonuçları doğrulayan simülasyonlar da verilmektedir.*

ANAHTAR KELİMELER: Yapısal tekil değer, Belirsizlik, Gürbüzlük, Ters sarkaç

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I. INTRODUCTION

In this paper, structured robust stability of inverted pendulum system with a fixed linear quadratic feedback is analyzed. Uncertainties in the pendulum system consist of both parametric and modeling uncertainties. We model these uncertainties suitably for structured robustness analysis, and compute uncertainty bounds for the stability of the inverted pendulum system.

The inverted pendulum system, which has a rich dynamic structure, is used in numerous studies to test validity of new control strategies. In the context of μ analysis, [1] utilizes an inverted pendulum model to test a controller in the presence of uncertainties in both pendulum mass and length. In another study, [2] describes an efficient LMI-based methodology to ensure various specifications of stability and this methodology is applied to the control of inverted pendulum with an uncertain model. In [3], loop shaping design procedure is used to design a robust controller for stabilizing an inverted pendulum system. In another significant article, [4] shows a design of robust digital controller using a blend of state space and frequency response methods for balancing an inverted pendulum on a moving cart.

Our control objective is to keep the inverted pendulum at its vertically upright equilibrium position on a moving cart while controlling the cart position in the presence of uncertainties. In the analysis, we firstly present a mathematical model of inverted pendulum system. Secondly, we linearize this model about the desired equilibrium point. Following this, we use a fixed linear quadratic feedback to control the nominal system. After this, we model the inverted pendulum as an uncertain system with a known structure. We consider masses of the cart and the pendulum, length of the pendulum as parametric uncertainties, modeling errors as dynamic uncertainty in our model. In the concluding section, we use the mixed μ analysis method, a structured uncertainty analysis method, to compute the uncertainty bound that does not cause instability of the inverted pendulum under feedback.

II. NOMINAL MODEL OF INVERTED PENDULUM SYSTEM

The inverted pendulum system is a pendulum attached to a moving cart (Figure 1) and is intrinsically unstable, that is, it may fall over any time in any direction unless a suitable control force u is applied. Here we consider only a two-dimensional problem so that the pendulum moves only in the x-y plane. For this inverted pendulum system; M_p denotes mass of the pendulum which is assumed to be concentrated at the end of the pendulum. The symbols *l*, M_c, and *b* denote the length of the pendulum, mass of the cart, and the friction constant respectively. The angle of the pendulum from the vertical line is denoted by θ and the distance in horizontal plane from the reference point is denoted by *x*.

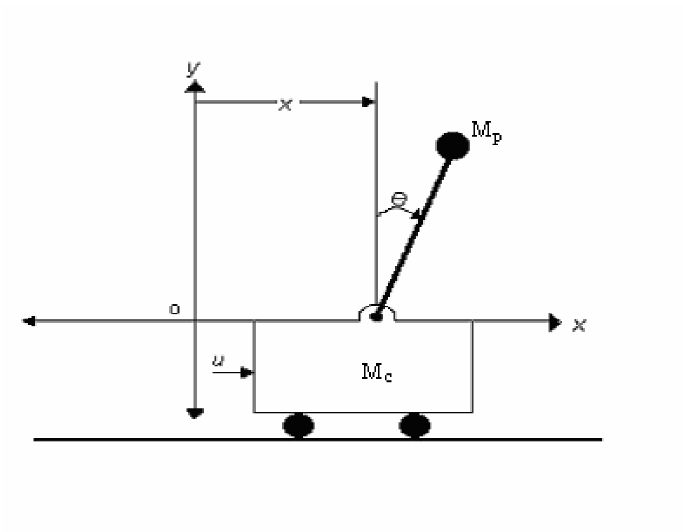

Figure 1. Inverted Pendulum system

Nonlinear model of the inverted pendulum system shown in Figure 1 is as follows [5]:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-bx_2 + M_p l \sin(x_3)x_4^2 - M_p g \sin(x_3)\cos(x_3) + u}{M_c + M_p - M_p \cos^2(x_3)} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{(bx_2 - u - M_p l \sin(x_3)x_4^2)\cos(x_3) + (M_c + M_p)g \sin(x_3)}{l(M_c + M_p - M_p \cos^2(x_3))}
\end{aligned}
\qquad (1)
$$

where $x_1 := x$, $x_2 := \dot{x}$, $x_3 := \theta$, $x_4 := \dot{\theta}$ and *u* is the control input. The control objective is to keep the pendulum vertically upright about the equilibrium point $(x_3, x_4) = (0,0)$ on a moving cart. This may be termed as stabilization of the pendulum. As an initial step of satisfying the control objective we linearize (1) about the equilibrium point as

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{M_p g}{M_c} x_3 - \frac{b}{M_c} x_2 + \frac{1}{M_c} u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{(M_c + M_p) g}{M_c l} x_3 + \frac{b}{M_c l} x_2 - \frac{1}{M_c l} u
\end{aligned} \quad . \tag{2}
$$

In the sequel, we use a typical set of parameter values given in Table 1 for the nominal system.

Table 1. Typical parameter values for an inverted pendulum system.

Parameters	Symbol	Value	Unit
Mass of the cart	M_c	3	kg
Mass of the pend.	M_p	0.5	kg
Length of the pend.	l	0.5	m
Friction constant	b	2	kg/s
Gravitational force	g	9.8	m/s^2

III. LINEAR QUADRATIC CONTROL OF THE INVERTED PENDULUM

In this section linear quadratic controller (LQC), [5] is applied to nominal linear model of the inverted pendulum system. Linear model of the inverted pendulum system given by expression (2) can be represented compactly as

$$
\begin{aligned}
\dot{X} &= AX + Bu \\
y &= CX
\end{aligned} \tag{3}
$$

where $X = [x_1 \;\; x_2 \;\; x_3 \;\; x_4]^T$. Let r be the reference signal that is required to be followed by the cart position x. Then the system states must follow $x_d=[r\ 0\ 0\ 0]^T$ from the initial state $x_0=[0\ 0\ 0\ 0]^T$. This may be done by constructing the error dynamics of the system. Therefore, we define the error in terms of system states and desired trajectory x_d as

$$e := X - x_d . \quad (4)$$

Using this in expression (3), error dynamics can be written as

$$\begin{aligned} \dot{e} &= Ae + Bu \\ y &= Ce + r \end{aligned} . \quad (5)$$

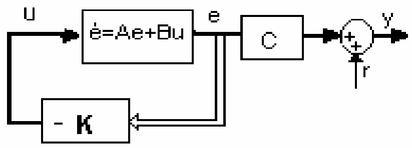

Figure 2: Block diagram of the feedback system with error states

Block diagram of feedback system with error states is given in Figure 2, and we use the control input u, which is common to systems (3) and (5), of the form

$$u = -Ke \quad (6)$$

where $K := [k_1 \quad k_2 \quad k_3 \quad k_4]$. Clearly, driving e to the origin of the state space is equivalent to system states' tracking of the desired trajectory. Using MATLAB, an LQC feedback matrix that drives e to the origin can be found [6] as

$$\mathrm{K} = [-15.82 \quad -22.46 \quad -150.45 \quad -36.09] \quad (7)$$

Robustness of the inverted pendulum system with respect to a single parameter perturbation at a time, under a fixed linear quadratic controller, is considered in [6]. In the sequel, we use μ analysis to analyze robustness of the inverted pendulum system with respect to many parameter perturbations occurring simultaneously. This requires a specific modeling technique, namely linear fractional transformation (LFT) technique, which decomposes the system into known and uncertain parts.

IV. A BRIEF BACKGROUND OF THE STRUCTURED ROBUST STABILITY ANALYSIS

A linear uncertain system can be represented as an interconnection of completely known linear block *M* and the uncertainty block Δ (Figure 3).

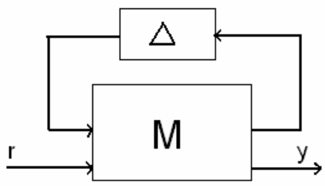

Figure 3. Macro model of inverted pendulum system with uncertainties

The uncertainty block has the form $\Delta = \text{diag}(\Delta_1, \ldots, \Delta_m)$, where each subblock Δ_i is either a real scalar blocks $\Delta_i = \delta_i I_k, \delta_i \in R$, complex scalar block $\Delta_i = \delta_i I_k, \delta_i \in C$ or a full complex block $\Delta_i \in C^{k_i \times k_i}$. Transfer function matrix M of the completely known block can be partitioned as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{8}$$

such that the uncertainty block Δ interacts only with submatrix M_{11}.

The structured singular value, $\mu_\Delta(M_{11})$, of a matrix $M_{11} \in C^{n \times n}$ with respect to a block structure Δ is defined as

$$\mu_\Delta(M_{11}) \overset{\Delta}{=} \frac{1}{min\{\bar{\sigma}(\Delta) : det(I - M_{11}\Delta) = 0\}}. \tag{9}$$

If no Δ makes $I - M_{11}\Delta$ singular, the structured singular value is defined as $\mu_\Delta(M_{11}) \overset{\Delta}{=} 0$. In (9), $\bar{\sigma}(\Delta)$ denotes the maximum singular value of Δ. We may interpret this definition as, given a certain class of uncertainties, μ is the inverse of the size of the smallest uncertainty that causes the system to become unstable. Unfortunately, (9) is not suitable for computing μ since the implied optimization problem may have multiple local minima. The optimization problem given in (9) is known as NP hard and μ computation has some complexity [7], [8]. So, in recent years, new formulas are developed to calculate structured singular value for robustness analysis. The mixed problems, which has both real and complex uncertainties in its uncertain block, can have fundamentally different properties from the purely complex μ problem, and these properties have important implications for computation. A large body of work focused on the development of efficient bound analysis of considering both parametric and dynamic uncertainty [9], [10], [11], [12].

For the mixed uncertainty case, let $\rho_R(M_{11}) = \max\{|\lambda| : \lambda$ is a real eigenvalue of $M_{11}\}$, with $\rho_R(M_{11}) = 0$ if M_{11} has no real eigenvalue. Also let for an uncertainty block Δ , m_r denotes number of real scalar blocks, m_c denotes number of complex scalar blocks and m_C denotes number of full complex blocks with $m = m_r + m_c + m_C \le n$. Given a block structure

$$\kappa = (k_1, \cdots, k_{m_r}; k_{m_r+1}, \cdots, k_{m_r+m_c}; k_{m_r+m_c+1}, \cdots, k_m) \tag{10}$$

such that $\sum_{q=1}^{m} k_q = n$, uncertainty matrix Δ is a subset of diagonal matrices

$$L_\kappa = \begin{cases} \Delta = \text{block diag}(\delta_1^r I_{k1}, \cdots, \delta_{m_r}^r I_{k_{m_r}}, \delta_1^c I_{k_{m_r+1}}, \cdots, \delta_{m_c}^c I_{k_{m_r+m_c}}, \Delta_1^C, \cdots, \Delta_{m_C}^C) : \\ \delta_i^r \in R, \quad \delta_i^c \in C, \quad \Delta_i^C \in C^{k_{m_r+m_c+i} x k_{m_r+m_c+i}} \end{cases} \tag{11}$$

where for any integer k, I_k denotes the $k \times k$ identity matrix. In order to develop lower bounds for μ we need to define some sets of block diagonal scaling matrices:

$$Q_L = \left\{ \Delta \in L_\kappa : \delta_i^r \in [-1,1], \quad \delta_i^{c*} \delta_i^c = 1, \quad \Delta_i^{c*} \Delta_i^c = I_{k_{m_r+m_c+i}} \right\} \tag{12.a}$$

$$D_L = \begin{cases} \Delta = \text{block diag}(D_1, \cdots, D_{m_r+m_c}, \quad d_1 I_{k_{m_r+m_r+1}}, \cdots, d_{m_C} I_{k_m}) : \\ 0 < D_i = D_i^* \in C^{k_i \times k_i}, \quad 0 < d_i \in R \end{cases} \tag{12.b}$$

$$G_L = \left\{ \text{block diag}(G_1, \ldots, G_{m_r}, 0_{k_{m_r+1}}, \ldots, 0_{k_m}) : G_i = G_i^* \in C^{k_i \times k_i} \right\} \tag{12.c}$$

For any matrix $M_{11} \in C^{n \times n}$, and any compatible block structure κ the lower and upper bounds can be given as [10]:

$$\max_{Q \in Q_L} \rho_R(QM_{11}) \le \mu_\Delta(M_{11}) \tag{13}$$

$$\mu_\Delta(M_{11}) \le \inf_{\substack{D \in D_L \\ G \in G_L}} \quad \inf_{0 \le \beta \in R} \left\{ \beta : \overline{\sigma}\left(\left(\frac{DM_{11}D^{-1}}{\beta} - jG \right) (I + G)^{-1/2} \right) \le 1 \right\}. \tag{14}$$

Note that, the lower bound provides only a sufficient condition for instability and also returns a worst case Δ, that is a worst case combination of uncertain parameters for the problem given in Figure 3. On the other hand, the upper bound provides a sufficient condition for stability in the presence of a specified level of structured uncertainty.

V. UNCERTAINTY MODELING OF INVERTED PENDULUM

The term uncertainty refers to the differences or errors between models and reality and various types of uncertainties can arise in physical systems [13]. In the unstructured robust control design methods, the unmodeled dynamics usually covers the parametric uncertainties, which results in highly conservative results. Representing uncertainties in a structured form gives rise to the most unconservative results in terms of system stability. Uncertainty structure in this study consists of both unmodeled dynamics and parametric uncertainties. In this paper, we analyze effect of the structured uncertainties on the stability of the inverted pendulum in the mixed μ analysis context. The mixed μ analysis has mature tools to deal with both unmodeled dynamics and parametric uncertainties [9].

In order to exhibit the steps of obtaining the linear fractional transformation (LFT) [14] model of the inverted pendulum system, we firstly express its nominal model under LQC feedback in block diagram. Under the LQC input given by (6), and using $e=x-x_d$, one can write system dynamics (3) as

$$\begin{aligned} \dot{X} &= (A - BK)X + BKx_d \\ y &= Cx \end{aligned} \tag{15}$$

Noting that $x_d=[r\ 0\ 0\ 0]^T$, the system state dynamics above can be written in terms of system component quantities as

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_1}{M_c} & -\dfrac{b+k_2}{M_c} & -\dfrac{M_p g + k_3}{M_c} & -\dfrac{k_4}{M_c} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_1}{M_c l} & \dfrac{b+k_2}{M_c l} & \dfrac{(M_c + M_p)g + k_3}{M_c l} & \dfrac{k_4}{M_c l} \end{bmatrix} X + \begin{bmatrix} 0 \\ \dfrac{k_1}{M_c} \\ 0 \\ -\dfrac{k_1}{M_c l} \end{bmatrix} r \tag{16}$$

$$y = x_1.$$

The dynamics in (16) contains suitable details for block diagram modeling. By using SIMULINK, the block diagram corresponding to (16) is presented in Figure 4.

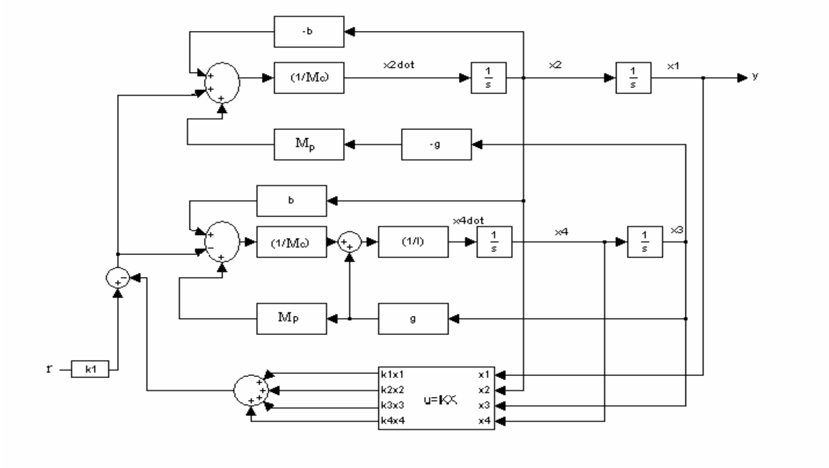

Figure 4. Block diagram of the inverted pendulum system under LQC feedback

In our uncertainty modeling we consider the parametric uncertainties in M_c, M_p, l, and unmodeled dynamics arising from several simplifications in the original model and the linearization. The uncertain parameters' possible variations about their nominal values can be represented as

$$\frac{1}{M_c}=\frac{1}{\overline{M}_c(1+M_{cv}\delta_{M_c})},\ M_p=\overline{M}_p(1+M_{pv}\delta_{M_p}),\ \frac{1}{l}=\frac{1}{\bar{l}(1+l_v\delta_l)} \qquad (17)$$

with $-1\le\delta_{M_c},\delta_{M_p},\delta_l\le 1$, the normalization quantities $M_{c_v}=1,\ M_{p_v}=1,\ l_v=1$ and nominal parameters $\overline{M}_c=3,\ \overline{M}_p=0.5,\ \bar{l}=0.5$ (Table 1). Parametric uncertainties in equation (17) can be written using LFTs as

$$\frac{1}{M_c}=F_U(P_{M_c},\delta_{M_c}),\ M_p=F_L(P_{M_p},\delta_{M_p}),\ \ \frac{1}{l}=F_U(P_{li},\delta_l) \qquad (18)$$

where $P_{M_{ci}}=\begin{bmatrix}-M_{cv} & \frac{1}{\overline{M}_c}\\ -M_{cv} & \frac{1}{\overline{M}_c}\end{bmatrix}$, $P_{M_p}=\begin{bmatrix}\overline{M}_p & m_{pv}\overline{M}_p\\ 1 & 0\end{bmatrix}$, and $P_{li}=\begin{bmatrix}-l_{var} & \frac{1}{\bar{l}}\\ -l_{var} & \frac{1}{\bar{l}}\end{bmatrix}$.

One can use the block diagrams given in Figures 5.a and 4.b to represent $1/P$ and P in LFT forms respectively.

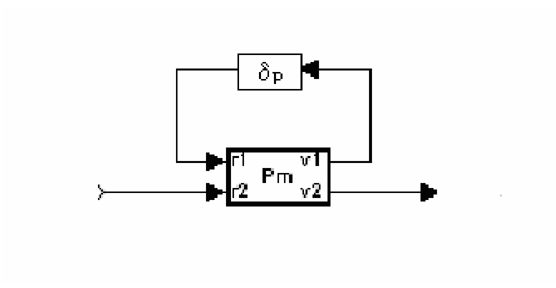

Figure 5.a. LFT model of uncertainty $1/P$

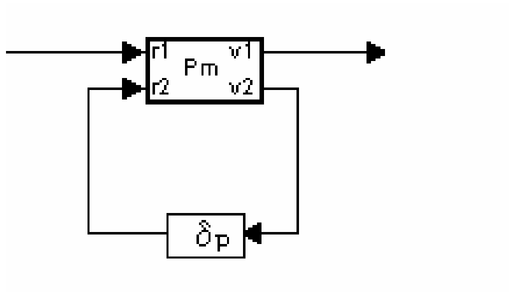

Figure 5.b. LFT model of uncertainty P

Unmodeled dynamics in our model, a multiplicative uncertainty added serially at the input of the system, has the form

$$u = \bar{u}(1 + W_m \delta_{Wm}) \tag{19}$$

with nominal system input $\bar{u}$ and uncertainty weighting function $W_m = 0.1\frac{(s/10+1)}{(s/100+1)}$.

Multiplicative uncertainty weighting function $W_m(s)$ adds 10% and 100% multiplicative uncertainty at low and high frequencies respectively. Block diagram for this uncertainty is given in Figure 5.c.

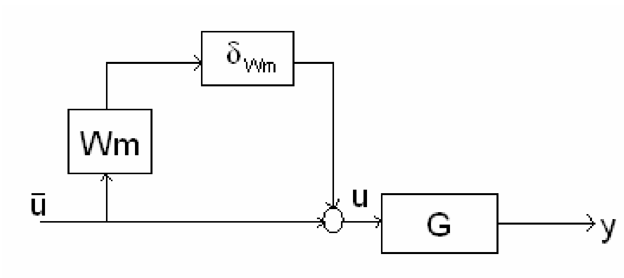

Figure 5.c Unmodeled uncertainty at the system input

Using Figures 5.a-b for parametric uncertainties and Figure 5.c for unmodeled uncertainties, known part M and the uncertain block Δ of the overall system can be formed as in Figure 6. Input-output relationship of this system is characterized by an upper linear fractional transformation

$$y = F_U(M, \Delta)r, \tag{20}$$

which we utilize to perform simulation that verifies validity of our μ computation in the next section.

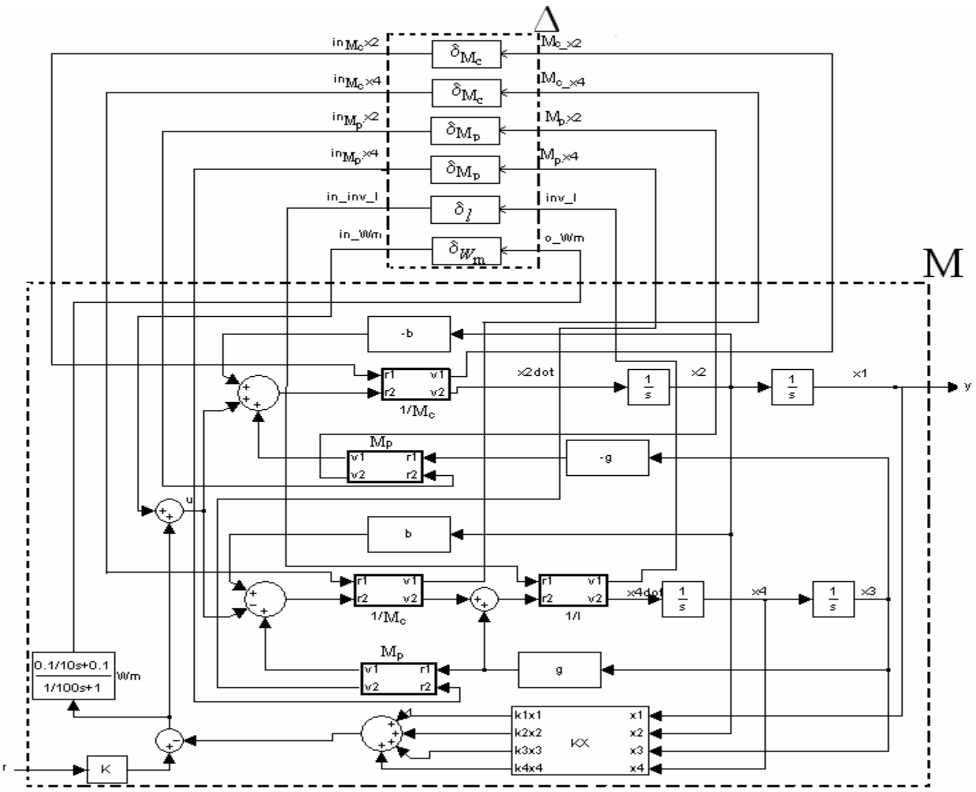

Figure 6. A detailed block diagram of the interconnection M and Δ

VI. STRUCTURED ROBUST STABILITY ANALYSIS OF THE SYSTEM

For the inverted pendulum system modeled in the previous section, we compute the μ bounds versus frequency using the μ toolbox of MATLAB. In the computation, we provide algebraic representation of the uncertain block to MATLAB as

$$\Delta = \begin{bmatrix} \delta_{M_c} I_2 & & & \\ & \delta_{M_p} I_2 & & \\ & & \delta_l & \\ & & & \delta_{Wm}(s) \end{bmatrix}; \quad \delta_{M_c} \in R, \delta_{M_p} \in R, \ \delta_l \in R, \delta_{Wm}(s) \in C. \tag{21}$$

We also provide the algebraic representation of the known block M_{11} as

$$M_{11}(s) = \left(\frac{1}{s^5 + 117s^4 + 1796s^3 + 7347s^2 + 13480s + 10350} \right) *$$

$$\left[\begin{matrix} -s^5 - 124s^4 - 2483s^3 - 7741s^2 & -24s^4 - 2503s^3 - 9703s^2 & 0.17s^5 + 20.7s^4 + 414s^3 + 1290s^2 \\ 6.8s^4 + 687s^3 + 393s^2 - 13480s - 10350 & -s^5 - 93s^4 + 706s^3 + 2356s^2 - 13480s - 10350 & -1.14s^4 - 114s^3 - 65.6s^2 + 2247s + 1724 \\ -133.s^2 - 13480s - 10350 & 19.7s^3 + 1828s^2 - 13480s - 10350 & 22.3s^2 + 2247s + 1724 \\ 133s^2 + 13480s + 10350 & -19.7s^3 - 1828s^2 + 13480s + 10350 & -22.3s^2 - 2247s - 1724 \\ 13.7s^4 + 1375s^3 + 1055s^2 & -2s^5 - 186s^4 + 1375s^3 + 1055s^2 & 3.7s^4 + 48s^3 + 54.3s^2 - 582s - 603.5 \\ -22.5s^4 - 288s^3 - 325s^2 + 3497s + 3621 & -72.s^4 - 1071s^3 - 3617s^2 - 1220s + 517 & 4s^4 + 417s^3 + 1617s^2 \end{matrix} \right.$$

$$\left. \begin{matrix} 4s^4 + 417s^3 + 1617s^2 & -12s^4 - 1252s^3 - 4852s^2 & 0.33s^5 + 49s^4 + 1662s^3 + 5815s^2 \\ 0.17s^5 + 15.6s^4 - 117s^3 - 392s^2 + 2247s + 1724 & 12s^4 + 1252s^3 + 4852s^2 & 0.33s^5 + 28s^4 - 464s^3 - 916s^2 + 8990s + 6898 \\ -3.3s^3 - 304s^2 + 2247s + 1724 & 9.8s^3 + 914s^2 - 6742s - 5173 & -6.5s^3 - 564s^2 + 8990s + 6898 \\ 3.3s^3 + 304s^2 - 2247s - 1724 & -9.8s^3 - 914s^2 + 6742s + 5173 & 6.54s^3 + 564s^2 - 8990s - 6898 \\ 0.33s^5 + 31s^4 - 229s^3 - 175s^2 & -s^5 - 93s^4 + 687s^3 + 527s^2 & 0.67s^5 + 57.6s^4 - 916s^3 - 703s^2 \\ 12s^4 + 178s^3 + 602s^2 + 203s - 86 & -36s^4 - 535s^3 - 1809s^2 - 609s + 258.7 & 31.6s^4 + 453s^3 + 1314s^2 - 759s - 1380 \end{matrix} \right] \quad (22)$$

Using (21) and (22), MATLAB commands [14] can be used for generation of μ graphics as a function of frequency. In Figure 7, lower and upper lines represent lower and upper μ bounds respectively. The graphics shows that lower and upper μ values over the considered frequency range are $\gamma_1 = 1.5619$ and $\gamma_2 = 1.6074$ respectively.

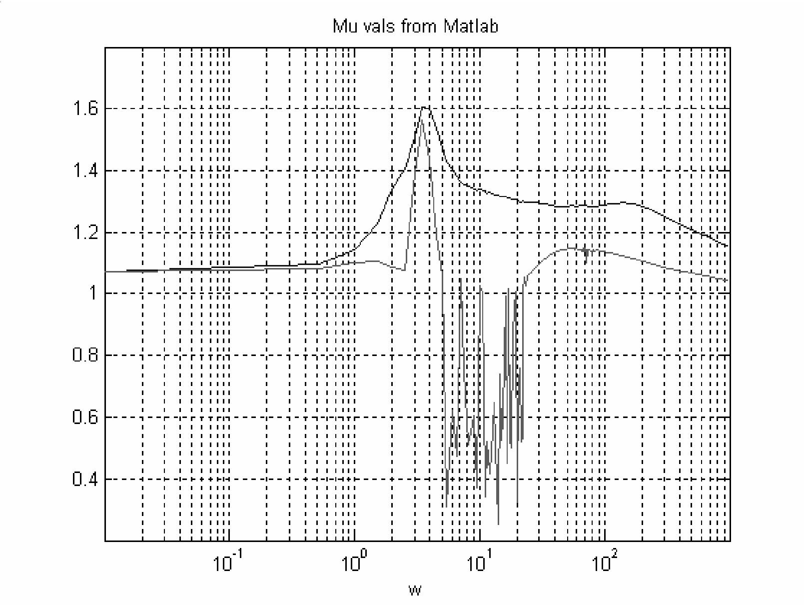

Figure 7: μ bounds versus frequency graphics

For a particular perturbation matrix Δ that corresponds $\mu = 1.5619$ the perturbed system (20) is unstable. For all perturbation matrices Δ that satisfies $\max_w \bar{\sigma}(\Delta(jw)) < \frac{1}{\gamma_2}$ the perturbed system (20) is stable. For instance, the following perturbation matrix

$$\Delta = \begin{bmatrix} 0.6221 I_2 & & & \\ & 0.6221 I_2 & & \\ & & 0.6221 & \\ & & & \frac{-0.6221s + 6.549}{s + 10.53} \end{bmatrix} \tag{23}$$

satisfies $\max_w \bar{\sigma}(\Delta(jw)) < \frac{1}{\gamma_2}$, hence this does not cause instability. Step response of the inverted pendulum for this uncertainty matrix is given in Figure 8.

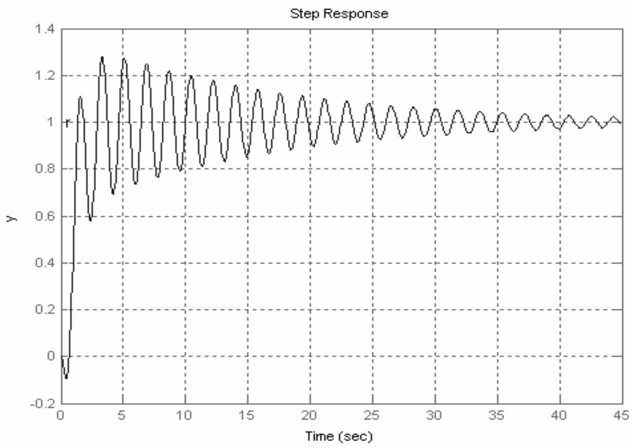

Figure 8. Step response of the perturbed system (20)

VII. CONCLUSION

In this paper, a structured robust stability analysis of inverted pendulum system with a fixed LQC feedback is presented. We provide a LFT model of the system which allows analysis of simultaneous parametric perturbations in masses of the pendulum and the cart, length of the pendulum, and the unmodeled dynamics. For the system under consideration, simulations, as one of the them shown in Figure 8, have shown that a 62 percent simultaneous perturbation from the nominal model does not harm the stability of the system. Under the given control law, this level of robustness can be viewed as satisfactory for a real time application.

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