Mathematical Modelling and Numerical Simulation with Applications, 2023, 3(2), 101–110

https://dergipark.org.tr/en/pub/mmnsa
ISSN Online: 2791-8564 / Open Access
https://doi.org/10.53391/mmnsa.1317989

RESEARCH PAPER

A study on the solutions of (1+1)-dimensional Mikhailov-Novikov-Wang equation

Seyma Tuluce Demiray [1,‡] and **Ugur Bayrakci** [1,*,‡]

[1]Department of Mathematics, Faculty of Science and Letters, Osmaniye Korkut Ata University, Osmaniye, Türkiye

*Corresponding Author
‡seymatuluce@gmail.com (Seyma Tuluce Demiray); ubayrakci42@gmail.com (Ugur Bayrakci)

Abstract

The basic principle of this study is to obtain various solutions to the (1+1) dimensional Mikhailov-Novikov-Wang integrable equation (MNWIE). For this purpose, the generalized exponential rational function method (GERFM) is applied to this equation. Thus, several trigonometric functions, hyperbolic functions, and dark soliton solutions to the studied equation are acquired. In this way, some new solutions to the equation that have not been presented before have been obtained. In addition, 2D and 3D graphics of the acquired solutions are drawn for specific values. The obtained results and the graphic drawings of the results have been provided by using Wolfram Mathematica 12.

Keywords: Generalized exponential rational function method; (1+1)-dimensional Mikhailov-Novikov-Wang integrable equation; trigonometric function solution; hyperbolic function solution; dark soliton solution

AMS 2020 Classification: 35C07; 35A25; 35C08

1 Introduction

In this study, GERFM has been used, the solution methods of nonlinear evolution equations (NLEEs), and this method has been applied to the (1+1)-dimensional MNWIE, which is a variant of NLEEs. NLEEs have very important applications in areas such as mathematical physics, optical fibers, mathematical chemistry, hydrodynamics, fluid dynamics, geochemistry, control theory, meteorology, optics, mechanics, chemical kinematics, biophysics, biogenetics, and so on. A number of methods have been developed by various researchers in order to obtain solutions for NLEEs, which have such important areas of use in the scientific world:
Modified direct algebraic, modified Kudryashov and trigonometric-quantic B-spline methods [1], improved Bernoulli sub-equation function method [2], modified extended tanh-function

➤ **Received:** 23.06.2022 ➤ **Revised:** 25.08.2022 ➤ **Accepted:** 27.08.2022 ➤ **Published:** 30.06.2023

method [3], new extended generalized Kudryashov and generalized new Kudryashov methods [4], new function method [5], exp $(-\phi(\xi))$ method [6], double $(G'/G, 1/G)$-expansion method [7], modified extended tanh-expansion based method [8], modified simple equation method [9], Jacobi elliptic function expansion method [10], modified $(1/G')$-expansion method [11].

(1+1)-dimensional MNWIE is given as [12]:

$$u_{tt} - u_{xxxt} - 8u_x u_{xt} - 4u_{xx}u_t + 2u_x u_{xxxx} + 4u_{xx}u_{xxx} + 24u_x^2 u_{xx} = 0. \tag{1}$$

Eq. (1) was derived by using the perturbative symmetry approach to the classification of integrable equations. Also, this equation belongs to a hierarchy where the Boussinesq equation is a member of this class. Hence, studying this equation can provide a good understanding of various nonlinear scientific phenomena in physics [12, 13].

(1+1)-dimensional MNWIE has been studied by some researchers recently. Ray and Singh got the kink-type multisoliton solutions of the equation with the help of simplified Hirota's method [14]. Raza et al. obtained new soliton solutions of the equation using the exp $(-\phi(\xi))$-expansion method, singular manifold method, and generalized projective Riccati equations method [15]. Akbulut et al. found exact solutions of the equation generalized Kudryashov method, exponential rational function method, and modified extended tanh-function method [16]. Bekir et al. obtained new exact soliton solutions of the equation using (G'/G)-expansion method [17]. This study, which was prepared to specify the solutions of the (1+1)-dimensional MNWIE using GERFM [18–23] was designed as follows: In Section 2, GERFM's basic principles are presented. In Section 3, some solutions of (1+1)-dimensional MNWIE have been obtained by applying GERFM. In Section 4, results and discussion are provided and finally, in Section 5, the concluding remarks are given.

2 Brief description of the GERFM

This section introduces an algorithm of the simplest version of the GERFM method, which is used for finding integrable solutions, whose essential steps are given as follows:

Step1: We consider NLPDE given below:

$$P(u, u_x, u_t, u_{xx}, ...) = 0. \tag{2}$$

We first apply the wave transform given below to Eq. (2);

$$u(x,t) = u(\eta), \eta = x - ct, \tag{3}$$

where c values that are not taken into account will be calculated later. Eq. (2) is transformed into an ordinary differential equation by using Eq. (3):

$$R(u, u', u'', \cdots) = 0. \tag{4}$$

Step2: Assume that we think that the solutions of Eq. (4) as:

$$u(\eta) = a_0 + \sum_{i=1}^{M} a_i \Phi(\eta)^i + \sum_{i=1}^{M} \frac{b_i}{\Phi(\eta)^i}, \tag{5}$$

where

$$\Phi(\eta) = \frac{p_1 e^{q_1 \eta} + p_2 e^{q_2 \eta}}{p_3 e^{q_3 \eta} + p_4 e^{q_4 \eta}}. \tag{6}$$

Here the value of M is determined through the homogeneous balance principle. $p_1, p_2, p_3, p_4, q_1, q_2, q_3, q_4$ are real or complex constants, $a_0, a_1, a_2, \cdots, a_M, b_1, b_2, \cdots, b_M$ are scalars and will be determined.
Step3: If Eq. (5) is taken into account in Eq. (4), $P(e^{q_1\eta}, e^{q_2\eta}, e^{q_3\eta}, e^{q_4\eta}) = 0$ equation system is obtained. A system of equations is obtained by equating all coefficients of P to zero.
Step4: If we solve the obtained system of equations and the found values consider in Eq. (5), the solutions of the discussed NLPDE are obtained.

3 Application of GERFM

To get the exact solutions of Eq. (1) we consider the following transformation:

$$u(x,t) = u(\eta), \eta = x - ct. \tag{7}$$

Replacing Eq. (7) into Eq. (1) and the resulting equation is integrated assuming the integration constant is zero. So the following equation is obtained,

$$c^2 u' + cu''' + 6c(u')^2 + 2u'''u' + (u'')^2 + 8(u')^3 = 0. \tag{8}$$

$u' = v$ transform is written in Eq. (8). Then take the integral and by considering the integration constant as zero, we obtain

$$c^2 v + cv'' + 6cv^2 + 2vv'' + (v')^2 + 8v^3 = 0. \tag{9}$$

By using the balance principle in Eq. (9), we obtain

$$M = 2. \tag{10}$$

If $M = 2$ is taken into account in Eq. (5), the following equality is achieved:

$$u(\eta) = a_0 + a_1\Phi(\eta) + a_2\Phi^2(\eta) + \frac{b_1}{\Phi(\eta)} + \frac{b_2}{\Phi^2(\eta)}. \tag{11}$$

So the obtained different states of the considered equation via GERFM are as follows:

Family one: For $p = [-2-i, 2-i, -1, 1]$ and $q = [i, -i, i, -i]$ values, Eq. (6) is converted into,

$$\Phi(\eta) = \frac{cos(\eta) + 2sin(\eta)}{sin(\eta)}. \tag{12}$$

$$a_0 = -10, a_1 = 0, a_2 = 0, b_1 = 40, b_2 = -50, c = 4. \tag{13}$$

Embedding Eq. (13) in Eqs. (11) and (12), The trigonometric function solution of Eq. (1) is acquired as,

$$u_1(x,t) = \frac{10sin\left[4t - x\right]}{cos\left[4t - x\right] - 2sin\left[4t - x\right]}. \tag{14}$$

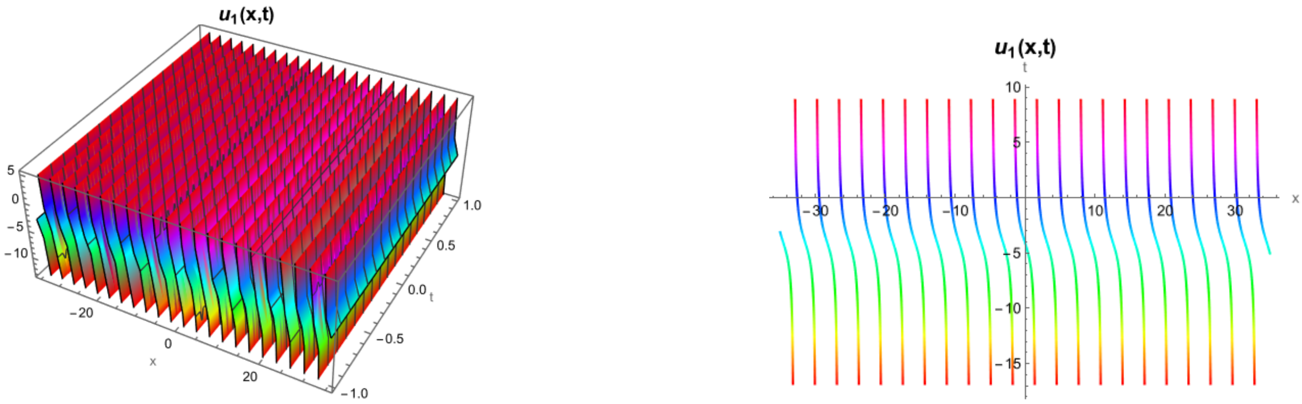

Figure 1. 3D plot of solution (14) for $-35 \leq x \leq 35, -1 \leq t \leq 1$ ranges and 2D plot of solution for $t = 0.5$ with this range.

Family two: For $p = [i, -i, 1, 1]$ and $q = [i, -i, i, -i]$ values, Eq. (6) is converted into,

$$\Phi(\eta) = \frac{-sin(\eta)}{cos(\eta)}. \tag{15}$$

$$a_0 = -4, a_1 = 0, a_2 = -2, b_1 = 0, b_2 = -2, c = 16. \tag{16}$$

Embedding Eq. (16) in Eqs. (11) and (15), The trigonometric function solution of Eq. (1) is acquired as,

$$u_2(x,t) = 4cot\left[2x - 32t\right]. \tag{17}$$

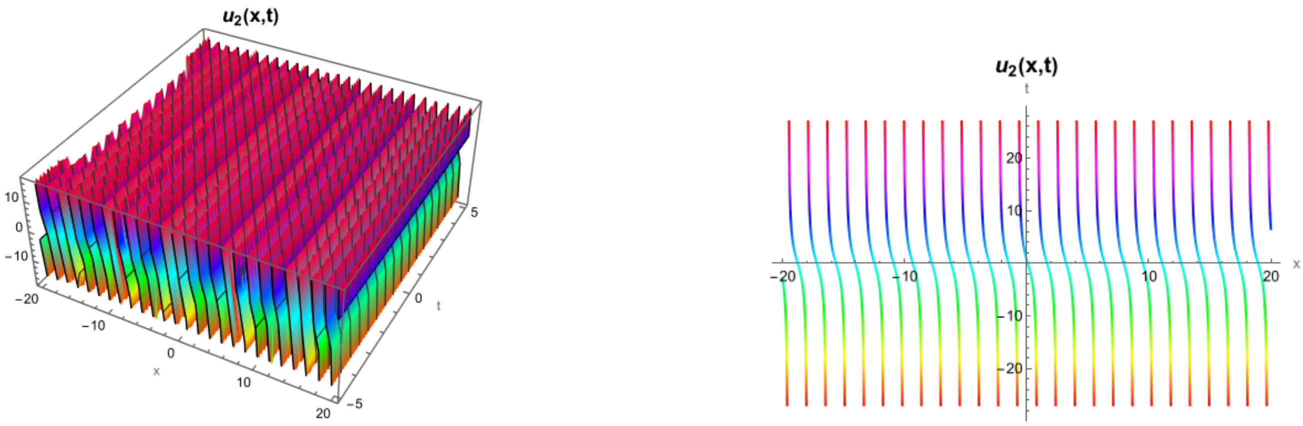

Figure 2. 3D plot of solution (17) for $-20 \leq x \leq 20, -5 \leq t \leq 5$ ranges and 2D plot of solution for $t = 3$ with this range.

Family three: For $p = [1, 1, -1, 1]$ and $q = [1, -1, 1, -1]$ values, Eq. (6) converted into,

$$\Phi(\eta) = \frac{-cosh(\eta)}{sinh(\eta)}. \tag{18}$$

Case-1:

$$a_0 = 4, a_1 = 0, a_2 = -2, b_1 = 0, b_2 = -2, c = -16. \tag{19}$$

Embedding Eq. (19) in Eqs. (11) and (18), The dark soliton solution of Eq. (1) is acquired as,

$$u_3(x,t) = 4coth\left[2x + 32t\right]. \tag{20}$$

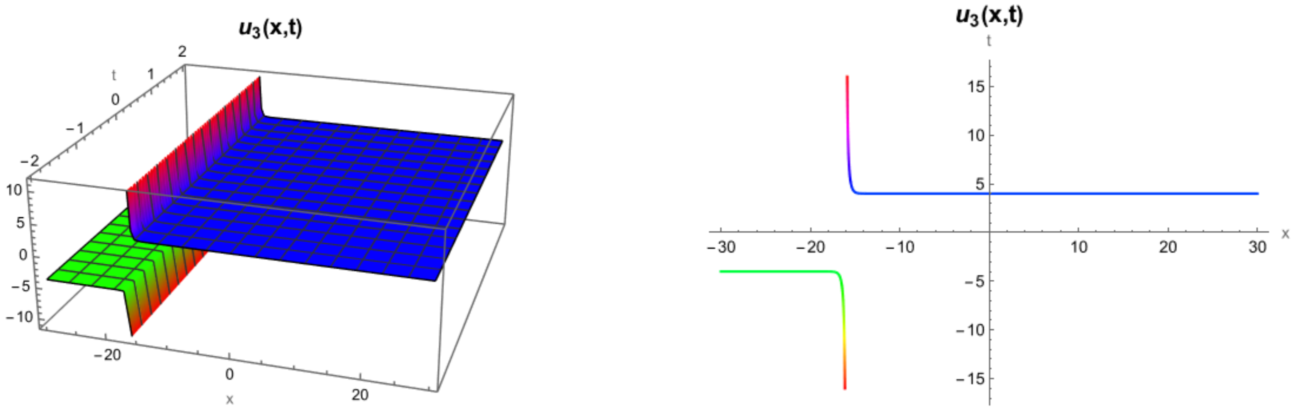

Figure 3. 3D plot of solution (20) for $-30 \le x \le 30, -2 \le t \le 2$ ranges and 2D plot of solution for $t = 1$ with this range.

Case-2:

$$a_0 = 2, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -2, c = -4. \tag{21}$$

Embedding Eq. (21) in Eqs. (11) and (18), The dark soliton solution of Eq. (1) is acquired as,

$$u_4(x,t) = 2tanh\left[x + 4t\right]. \tag{22}$$

Family four: For $p = [-2, -3, 1, 1]$ and $q = [1, 0, 1, 0]$ values, Eq. (6) converted into,

$$\Phi(\eta) = \frac{-3 - 2e^{\eta}}{1 + e^{\eta}}, \tag{23}$$

$$a_0 = -12, a_1 = 0, a_2 = 0, b_1 = -60, b_2 = -72, c = -1. \tag{24}$$

Embedding Eq. (24) in Eqs. (11) and (23), The soliton solution of Eq. (1) is acquired as,

$$u_5(x,t) = -\frac{6}{3 + 2e^{(x+t)}}. \tag{25}$$

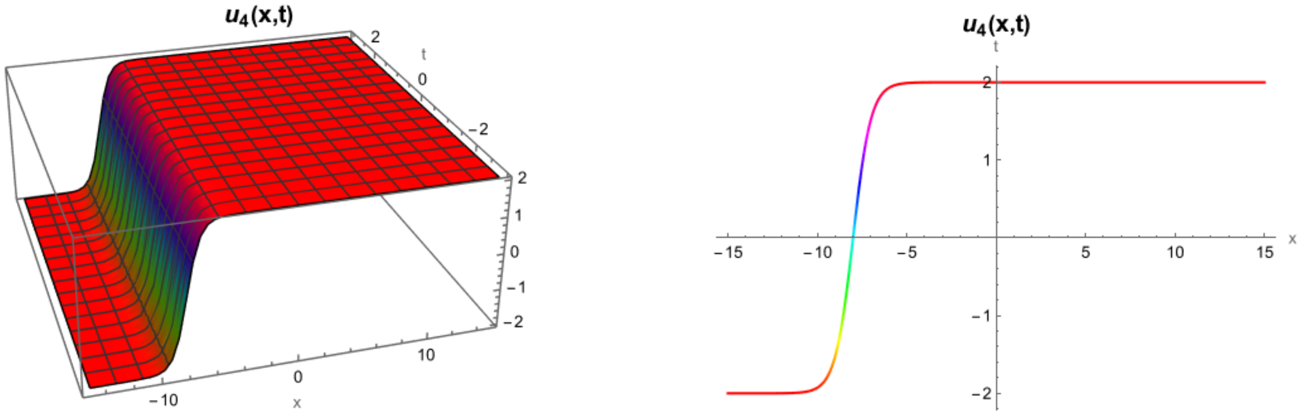

Figure 4. 3D plot of solution (22) for $-15 \leq x \leq 15, -3 \leq t \leq 3$ ranges and 2D plot of solution for $t = 2$ with this range.

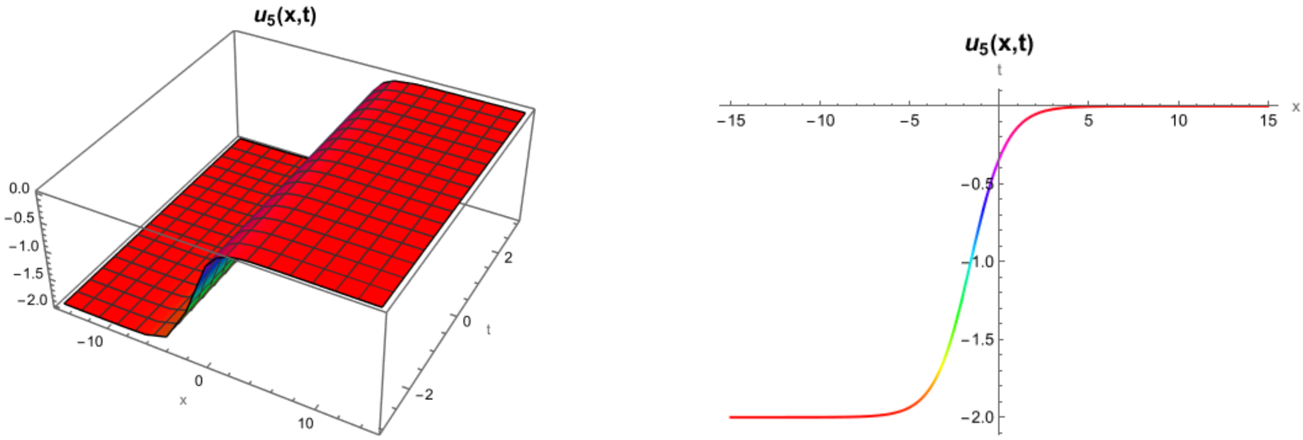

Figure 5. 3D plot of solution (25) for $-15 \leq x \leq 15, -3 \leq t \leq 3$ ranges and 2D plot of solution for $t = 2$ with this range.

Family five: For $p = [-2-i, -2+i, 1, 1]$ and $q = [i, -i, i, -i]$ values, Eq. (6) converted into,

$$\Phi(\eta) = \frac{-2cos(\eta) + sin(\eta)}{cos(\eta)}. \tag{26}$$

$$a_0 = -10, a_1 = -8, a_2 = -2, b_1 = 0, b_2 = 0, c = 4. \tag{27}$$

Embedding Eq. (27) in Eqs. (11) and (26), The trigonometric function solution of Eq. (1) is acquired as,

$$u_6(x,t) = 2tan\left[4t - x\right]. \tag{28}$$

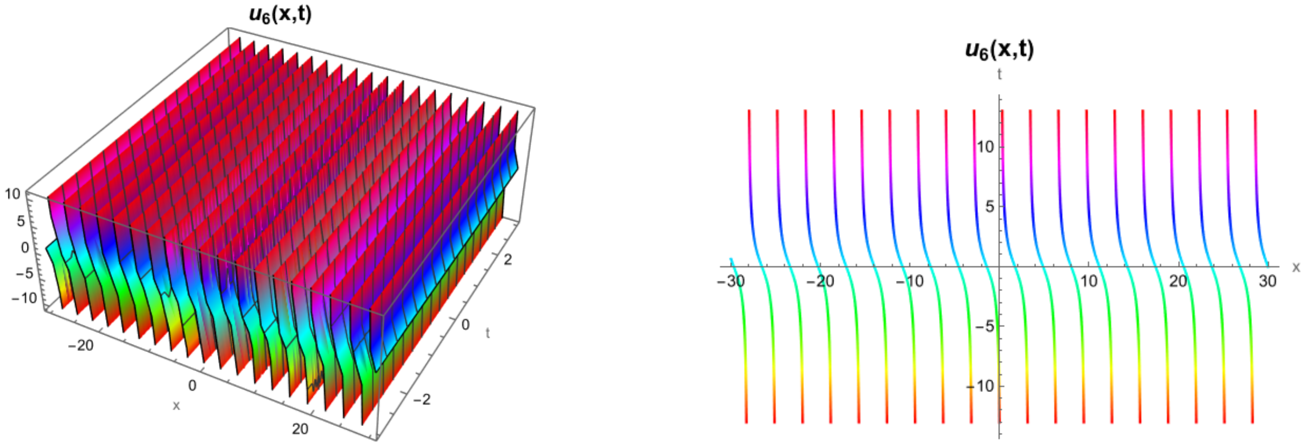

Figure 6. 3D plot of solution (28) for $-30 \leq x \leq 30, -3 \leq t \leq 3$ ranges and 2D plot of solution for $t = 2$ with this range.

Family six: For $p = [-3, -1, 1, 1]$ and $q = [1, -1, 1, -1]$ values, Eq. (6) converted into,

$$\Phi(\eta) = \frac{-2cosh(\eta) - sinh(\eta)}{cosh(\eta)}. \tag{29}$$

$$a_0 = -6, a_1 = 0, a_2 = 0, b_1 = -24, b_2 = -18, c = -4. \tag{30}$$

Embedding Eq. (30) in Eqs. (11) and (29), the hyperbolic function solution of Eq. (1) is acquired as,

$$u_7(x,t) = \frac{3sinh\,[x+4t]}{2cosh\,[x+4t] + sinh\,[x+4t]}. \tag{31}$$

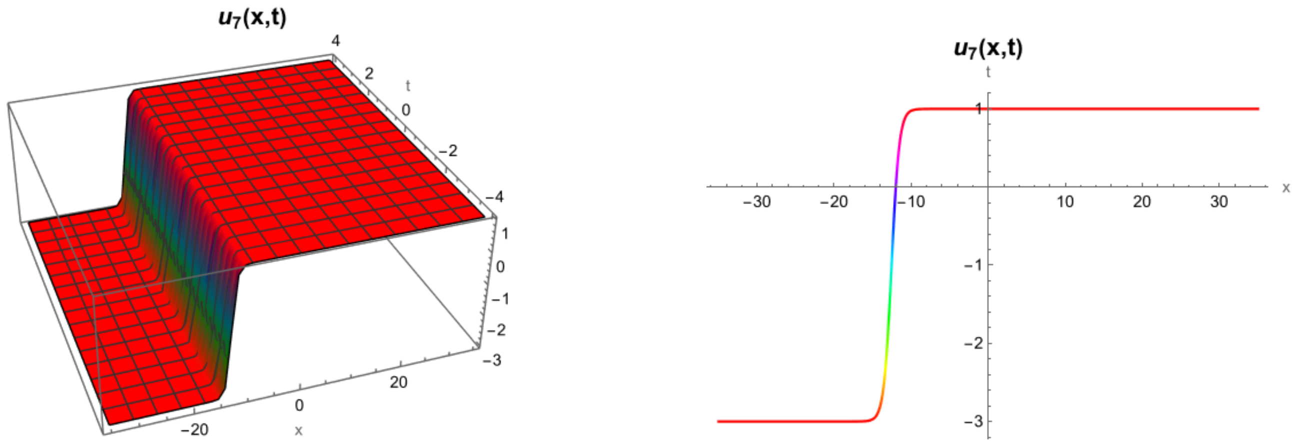

Figure 7. 3D plot of solution (31) for $-35 \leq x \leq 35, -4 \leq t \leq 4$ ranges and 2D plot of solution for $t = 3$ with this range.

4 Results and discussions

We have obtained seveeral trigonometric function, hyperbolic function and dark soliton solutions of the (1+1)-dimensional MNWIE by applying the GERFM. Several methods were previously applied by some authors to obtain the solutions of the (1+1)-dimensional MNWIE. When we compare the solutions we found with those of previously published papers, our $u_1(x,t)$ solution is similar to solution (31) given by Raza et al. [15]. In addition to our $u_3(x,t)$ solution is similar to (33) solution given by Raza et al. [15], and solutions (23)-(24) given by Akbulut et al. [16]. Our $u_4(x,t)$ solution is similar to (27) and (28) solutions given by Akbulut et al. [16]. Our $u_7(x,t)$ solution is similar to solution (24) given by Raza et al. [15]. According to our research, our other solutions have not been provided before. Thus, the GERFM appears to be an effective method for finding solutions to NLEEs.

5 Conclusion

In this study, (1+1)-dimensional MNWIE was investigated. GERFM, which is the solution method of NLEEs, was applied to this equation. Thus, several trigonometric function, hyperbolic function and dark soliton solutions of the equation were obtained. In order to understand the physical appearance of the found solutions, 2D and 3D graphics were drawn. These obtained results can be further extended and investigated to solve other equations of the Boussinesq type due to their significance in making sense of various nonlinear phenomena. In addition, this considered method can be applied to obtain solutions of equations used for various models. The most important advantage of the method used in this study is that a wide variety of solution families can be created. It is a more general method compared to other methods, as it offers a wide variety of solution families. Despite these advantages, since a different algebraic equation system is created for each solution family, the processing density increases.

Declarations

Consent for publication

Not applicable.

Conflicts of interest

The authors declare that they have no conflict of interests.

Funding

Not applicable.

Author's contributions

S.T.D.: Investigation, Resources, Data Curation, Writing - Review & Editing. U.B.: Conceptualization, Methodology, Writing - Original Draft. All authors discussed the results and contributed to the final manuscript.

Acknowledgements

Not applicable.

References

[1] Khater, M.M.A. and Alabdali, A.M. Multiple novels and accurate traveling wave and numerical solutions of the (2+ 1) dimensional Fisher-Kolmogorov-Petrovskii-Piskunov equation. *Mathematics*, 9(12), 1-13, (2021). [CrossRef]

[2] Dusunceli, F., Celik, E., Askin, M. and Bulut, H. New exact solutions for the doubly dispersive equation using the improved Bernoulli sub-equation function method. *Indian Journal of Physics*, 95(2), 309-314, (2021). [CrossRef]

[3] Bakıcıerler, G., Alfaqeih, S. and Mısırlı, E. Analytic solutions of a (2+1)-dimensional nonlinear Heisenberg ferromagnetic spin chain equation. *Physica A: Statistical Mechanics and its Applications*, 582, 126255, (2021). [CrossRef]

[4] Malik, S., Kumar, S., Nisar, K.S. and Saleel, C.A. Different analytical approaches for finding novel optical solitons withgeneralized third-order nonlinear Schrödinger equation. *Results in Physics*, 29, 104755, (2021). [CrossRef]

[5] Aktürk, T., Gurefe, Y. and Pandir, Y. An application of the new function method to the Zhiber–Shabat equation. *An International Journal of Optimization and Control: Theories & Applications*, 7(3), 271–274, (2017). [CrossRef]

[6] Akbulut, A., Kaplan, M. and Tascan, F. The investigation of exact solutions of nonlinear partial differential equations by using exp $(-\phi(\xi))$ method. *Optik*, 132, 382–387, (2017). [CrossRef]

[7] Ünal, M. and Ekici, M. The double $(G'/G, 1/G)$-expansion method and its applications for some nonlinear partial differential equations. *Journal of the Institute of Science and Technology*, 11(1), 599-608, (2021). [CrossRef]

[8] Nuruddeen, R.I., Aboodh, K.S. and Ali, K.K. Analytical investigation of soliton solutions to three quantum Zakharov-Kuznetsov equations. *Communications in Theoretical Physics*, 70(4), 405-412, (2018). [CrossRef]

[9] Tahir, M. and Awan, A.U. Optical singular and dark solitons with Biswas–Arshed model by modified simple equation method. *Optik*, 202, 163523, (2020). [CrossRef]

[10] Tasbozan, O., Çenesiz, Y. and Kurt, A. New solutions for conformable fractional Boussinesq and combined KdV-mKdV equations using Jacobi elliptic function expansion method. *The European Physical Journal Plus*, 131, 244, (2016). [CrossRef]

[11] Yokuş, A., Durur, H. and Duran, S. Simulation and refraction event of complex hyperbolic type solitary wave in plasma and optical fiber for the perturbed Chen-Lee-Liu equation. *Optical and Quantum Electronics*, 53, 402, (2021). [CrossRef]

[12] Mikhailov, A.V., Novikov, V.S. and Wang, J.P. On classification of integrable nonevolutionary equations. *Studies in Applied Mathematics*, 118(4), 419–457, (2007). [CrossRef]

[13] Mikhailov, A.V. and Novikov, V.S. Perturbative symmetry approach. *Journal of Physics A: Mathematical and General*, 35(22), 4775, (2002). [CrossRef]

[14] Ray, S.S. and Singh, S. New various multisoliton kink-type solutions of the (1+1)-dimensional Mikhailov–Novikov–Wang equation. *Mathematical Methods in the Applied Sciences*, 44(18), 14690-14702, (2021). [CrossRef]

[15] Raza, N., Seadawy, A.R., Arshed S. and Rafiq, M.H. A variety of soliton solutions for the Mikhailov-Novikov-Wang dynamical equation via three analytical methods. *Journal of Geometry and Physics*, 176, 104515, (2022). [CrossRef]

[16] Akbulut, A., Kaplan, M. and Kaabar, M.K.A. New exact solutions of the Mikhailov-Novikov-Wang equation via three novel techniques. *Journal of Ocean Engineering and Science*, 8(1), 103-110, (2021). [CrossRef]

[17] Bekir, A., Shehata, M.S.M. and Zahran, E.H.M. Comparison between the new exact and numerical solutions of the Mikhailov–Novikov–Wang equation. *Numerical Methods for Partial Differential Equations*, (2021). [CrossRef]

[18] Srivastava, H.M., Günerhan, H. and Ghanbari, B. Exact traveling wave solutions for resonance nonlinear Schrödinger equation with intermodal dispersions and the Kerr law nonlinearity. *Mathematical Methods in the Applied Sciences*, 42(18), 7210-7221, (2019). [CrossRef]

[19] Ghanbari, B., Osman, M.S. and Baleanu, D. Generalized exponential rational function method for extended Zakharov Kuzetsov equation with conformable derivative. *Modern Physics Letters A*, 34(20), 1950155, (2019). [CrossRef]

[20] Ismael, H.F., Bulut, H. and Baskonus, H.M. W-shaped surfaces to the nematic liquid crystals with three nonlinearity laws. *Soft Computing*, 25, 4513-4524, (2021). [CrossRef]

[21] Sağlam Özkan, Y. The generalized exponential rational function and Elzaki–Adomian decomposition method for the Heisenberg ferromagnetic spin chain equation. *Modern Physics Letters B*, 35(12), 2150200, (2021). [CrossRef]

[22] Duran, S. Breaking theory of solitary waves for the Riemann wave equation in fluid dynamics. *International Journal of Modern Physics B*, 35(09), 2150130, (2021). [CrossRef]

[23] Duran, S. An investigation of the physical dynamics of a traveling wave solution called a bright soliton. *Physica Scripta*, 96(12), 125251, (2021). [CrossRef]

Mathematical Modelling and Numerical Simulation with Applications (MMNSA)
(https://dergipark.org.tr/en/pub/mmnsa)

How to cite this article: Tuluce Demiray, S. & Bayrakci, U. (2023). A study on the solutions of (1+1)-dimensional Mikhailov-Novikov-Wang equation. *Mathematical Modelling and Numerical Simulation with Applications*, 3(2), 101-110. https://doi.org/10.53391/mmnsa.1317989