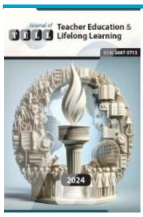

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Pre-service Teachers' Understanding of Graphs in Statistics in the Context of Lesson Study

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ABSTRACT

This study aims to examine pre-service teachers'(PTs') understandings of graphs and how their understandings about graphs have transformed into instructional practices as they participated in a lesson study. It was employed case study and three PTs participated. They initially viewed graphs as instruments for organizing and representing data. As the lesson study progressed, they began to view graphs as instruments to answer statistical questions. This fundamental transformation in their understanding resulted in a change in how they designed and implemented their lesson plans. They designed tasks around a statistical question and made connections with this question while deciding the type of graph and interpreting the graph. In addition, they anticipated students' possible conceptions. Hence, for these PTs, understanding the function graphs as instruments to answer statistical questions can be considered as a key development understanding (KDU), which was built on multiple experiences of designing, implementing and evaluating lessons.

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INTRODUCTION

Graphs are important instruments in analysing statistical data. Representing data with graphs helps to make sense of the data and answer the statistical question. Different graphical displays allow answering different questions (Bright & Friel, 2011; Chick, 2004; Monk, 2003; Pfannkuch & Wild, 2004; Shah & Hoeffner, 2002; Wild & Pfannkuch, 1999).

Teaching graphs has been one of the major objectives in Turkish middle school mathematics curriculum since 1949. While the early curriculums paid greater attention to drawing and interpreting graphs mostly independent of their function in the statistical problemsolving process (SPSP) (Ministry of National Education [MoNE], 1968; 1983; 1990; 1998), recent curriculum documents put much emphasis on SPSP (MoNE, 2005; 2013; 2018). This process emphasizes that all components of doing statistics (i.e., formulating research question, collecting and analysing data and interpreting the results) should be addressed holistically and the function of graphs in answering statistical questions should be stressed (Franklin, et al., 2015; National Council of Mathematics [NCTM], 2000). Hence, teachers are expected to design activities so that students can decide which type of graph to draw as well as interpret the graph by taking into account the statistical question or the purpose being set at the beginning of the statistical inquiry. However, research shows that teachers and PTs generally ignore the SPSP and habitually focus on procedural aspects of teaching graphs (e.g., making calculations, drawing graphs) (Chick & Pierce, 2008; Heaton & Mickelson, 2002; İjeh, 2012; Lee, et al., 2014; Reston, et al., 2006). They mostly bring the function of graphs as instruments to display data set to the fore and ignore their function as instruments to answer statistical questions (Burgess, 2007; Mercimek, 2013; Sorto, 2004). Even though the current curriculum documents in Turkey emphasize the SPSP as a general goal, and emphasize that each component of SPSP is included in terms of specific objectives, how the links between these components could be established in classroom practices are implicit or left to the teachers (Ader, 2018; Ari, 2010). In order to teach graphs by taking into account SPSP, teachers need to develop an understanding that graphs are not only used to represent data but also used to answer statistical research questions. When instructional activities are based on this understanding, using graphs in statistics could be more meaningful for students (Bargagliotti, et al, 2020). In this study, we focused on three PTs' understandings of graphs in the context of a lesson study. We explored their initial understandings of graphs and the changes in their understandings as they engaged in planning, implementing, reflecting and revising lessons related to teaching graphs in the context of lesson study. We observed that PTs' limited understandings of graphs resulted in some difficulties during the planning and implementation of the lessons. Reflections on these difficulties as a part of the lesson study process, helped them to view the function of graphs in SPSP. Hence we came to realize that "viewing graphs as instruments to answer formulated statistical questions" could be considered as a KDU for learning and teaching graphs. Simon (2006) defined KDU as "conceptual advance that is important to the development of a concept" (p. 363). Two characteristics of a KDU are (1) it involves "a change in students' ability to think about and/or perceive particular mathematical relationships" and (2) "the transition requires a building up of the understanding through students' activity and reflection and usually comes about over multiple experiences." (p. 362). Hence developing KDUs require more than explanations or demonstrations (Simon, 2006). It can be said that the teacher, who has become aware of KDUs related to the field of statistics learning, can organize the teaching process in this direction (Groth, 2013). As students' KDUs develop, they can perceive the "big ideas" at the core of the focused mathematical concepts and progressive conceptual development takes place (Silverman & Thompson, 2008; Simon, 2006). When PTs learn cognitive landmarks that play a critical role in students' thinking, they can become aware of the importance of the elements involved in students' learning process (Llinares et al, 2016). Moreover, PTs can develop perspective on how students make sense of concepts (Simon, 2006). This may impact PTs' instructional decisions (Buforn & Fernández, 2014). All these show the importance of teachers having KDUs. When the literature is reviewed, it is seen that although key developmental understandings (KDUs) related to different concepts (e.g., Llinares et al., 2016, classification of quadrilaterals; Sánchez-Matamoros et al., 2015, derivative concept) have been researched, limited KDU research on statistical concepts shows why the current study is needed. In the study, we purposed to examine how PTs understand the function of graphs in SPSP and the transformation of their understandings into instructional practices as they have participated in the lesson study. In this regard, an answer was sought for the following research question.

How PTs understand the function of graphs in SPSP and the transformation of their understandings into instructional practices as they have participated in the lesson study?

METHOD

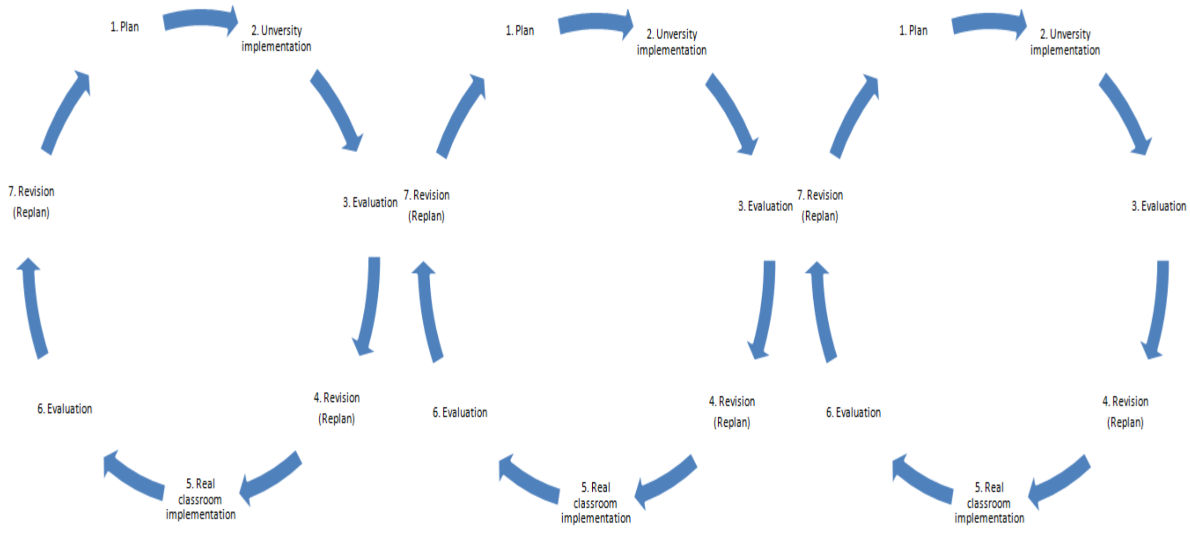
A case study design was employed because in this study, it is aimed to reveal how PTs participating in the lesson study understand the function of graphs in the SPSP process and how they transform these understandings into instructional practices. This requires a long and detailed examination of PTs. Case study allows examining the focused situation in detail and making sense of it (Merriam, 2009; Yin, 2003) led us to prefer this method. In this study, the understandings of PTs were examined in detail in their natural process for approximately four months and an attempt was made to reveal what they meant.

Participants

The context of the study consisted of 12 senior PTs enrolled in a micro teaching course offered in the last semester of an undergraduate program aimed to certificate mathematics teachers for middle schools (5th-8th grade). The PTs were mainly required to take courses related to mathematics (e.g., calculus, analytic geometry, statistics and probability) and general pedagogy (e.g., educational psychology, measurement and evaluation) during the first two years of the four-year program. They were mainly required to take courses related to teaching mathematics (e.g., Teaching methods, School experience, Teaching practice) for the last two years of the program. The participants of the study were determined by using the criterion sampling method. First, attention was paid to the PTs' willingness to participate in the study. Secondly, care was taken to ensure that PTs successfully completed the courses that were considered necessary for teaching statistics (e.g., Statistics and Probability, Methods of Teaching Mathematics). Three PTs created the data for this study. There are several reasons why these PTs were chosen. The first is that a pilot study was conducted at the school where these PTs attended for teaching practice. The classroom teacher gained experience with lesson study during this pilot study. In addition, when the observations and discussion records were examined, it was revealed that these three selected PTs took a more active role in the process, and therefore it was thought that they would provide richer data. The grade point averages of these PTs ranged between 3.21 and 3.29. It was determined that two of the PTs received their high school education at an Anatolian Teacher Traning High School and and one of them at an Anatolian High School. It was observed that these PTs primarily aimed to become academicians and were interested in the lesson study process.

Context of the Study

The PTs carried out the lesson study process in the course of Micro Teaching in Mathematics Education, which was opened in the 2016-2017 spring term. In the first three weeks, the PTs read and discussed articles and book chapters about learning and teaching for data processing (Ben-Zvi, 2011; Cobb & McClain, 2004; Van de Walle, et al., 2010). They were then informed about the lesson study model and conducted one lesson study cycle as a group of three for the part of the pilot study. The PTs implemented the lesson study for the last seven weeks of the course duration. Data from one group (Gamze, Şirin and Beyza) were analysed for the current study. Groups were asked to design, conduct and revise three lesson plans for teaching seventh grade objectives related to graphs. Three types of graphs (bar, pie and line) are included in the middle school curriculum in Turkey (MoNE, 2018). Bar graphs are presented at the 6th grade whereas pie and line graphs are presented at the 7th grade. The PTs were asked to use a format with four components when planning their lessons: 1) learning activities and key questions, 2) expected student reactions, 3) teacher's responses to student reactions, and 4) goals and method(s) of evaluation (Mathews, et al., 2009). Immediately after they had completed their lesson plans, they initially implemented these plans in the university classroom to their classmates and then in the real classroom. The lesson plans were carried out by Gamze, Şirin and Beyza respectively. After each implementation, evaluation of the lessons was made, and the lesson plan was revised. In the university classroom environment, an academician in the function of the expert (the second author) and the researcher (the first author) took part in the lesson as observers, and then evaluated the lesson. On the other hand, the classroom teacher and the researcher (the first author) participated in the lesson as observers, and afterwards evaluated the lesson in the real classroom environment. In addition, in the micro-teaching environment, the other PTs in the university class and the other group members in the real classroom environment evaluated the lesson plan and made suggestions about how to improve it. Each lesson study process lasted three weeks. This procedure is illustrated in Figure 1.



First lesson study Second lesson study Third lesson study

Figure 1. *Lesson study process (Adapted from Zhang & Cheng, 2011)*

Data Collection Tools and Data Analyses

The aforementioned implementations that took place at the university and real classroom as well as the meetings that were carried out for planning, evaluating and revising the lesson plans were video-recorded and transcribed into documents. The lesson plans organized by the PTs, video-recordings and transcribed lesson study meetings (for planning, implementation and evaluation) and classroom implementations at the university and real classroom environment, observation forms completed by the observers, field notes prepared by the first author and reflective papers written by the PTs were used to collect data. In order to data analysis, the descriptive analysis method was employed. By using the data obtained from the documents, video recordings, observation notes and reflective journals, a thorough analysis was conducted on the PTs' instructional decisions (recognized or disregard) and actions related to their teaching statistics. By analysing all the activities carried out during the lesson study process (planning the lessons, university and real classroom implementations, evaluation and revision meetings, reflections), researchers examined how the PTs made sense of the graphs, whether and to what extent they transformed their understandings about graphs into instructional practices as the lesson study implementations progressed. The findings are clustered around three themes that reflect changes in PTs' understandings of graphs as they worked for teaching graphs. These are namely establishing task requirements, deciding the appropriate type of graph, and interpreting graphs. In order to enhance the trustworthiness of the study, several strategies were adopted. The researchers spent a long time with the participants both in the university and the real classroom environments, collected data from many sources, used triangulation of data sources and data analysis (e.g., coded data in different time periods, consulted an expert review and examined intercoder reliability). In order to enhance the transferability of the study, the researcher provided detailed information about the participants and the context to be investigated and presented in-depth descriptions of the instructions and direct quotations of the participants. The intercoder reliability of the study was found to be 85%. The research study that underpins this publication was approved by the ….. University Institutional Review Board for Human Subjects Research (Protocol Number 433-1358, 02.05.2016).

FINDINGS / RESULTS

The findings are presented in three sections (a) the PTs' initial understandings of graphs, (b) the PTs' struggles with teaching graphs as a result of the lack of understanding that graphs are instruments to answer statistical research questions, and (c) the PTs' development of an understanding about the function of graphs as instruments to answer statistical questions.

PTs' Initial Understandings of Graphs

The PTs initially viewed graphs as instruments for organizing and representing data. They overlooked the fact that graphs can also be used to answer a statistical question in a SPSP. This was observed while they were designing the tasks for their lesson plans, deciding the appropriate type of graph with their students and guiding their students for reading and interpreting graphs.

During the first phases of the study, the requirement of the tasks designed was mainly for the purpose of expressing a data set with a particular type of graph or transforming graphs into each other. For instance, they planned to collect data from the class about students' favourite football teams and then ask students to express the collected data with a graph: "Which graph would you use if we organized the answers you gave and showed them on a graph?" (1st University class plan). They planned to introduce pie graph after students draw a bar graph that they learned at the 6th grade as a new way to display data: "Let's convert the bar graph into a pie graph" (1st University class planning). Next, they planned to present a larger data set (n=360) with the same context (favourite football teams) and focused on procedural aspects of drawing pie graph (e.g., working with degrees and percentages). Likewise, in the 2nd lesson study, during the university classroom implementation, they presented a table showing average temperature of a city for 5 days and asked students: "I want to express this data [average temperature of a city for 5 days] in a graph. What kind of graph should I use?". These cases initially indicated that PTs mainly viewed graphs as instruments to represent data and ignored their function as instruments to answer statistical research questions.

Their view of graphs as instruments to represent was also observed while they were discussing the appropriate type of graph to be used with their students. For instance, during the 1st university implementation, when a student asked why they need to learn pie graphs as they already knew one way to represent data (i.e., line graph), Gamze explained that "each type of graph provides a different way of representing data" (1st university class implementation). Also, they usually chose prototypical examples and contexts for the tasks and made superficial connections between these examples and types of graphs (e.g., degrees of temperature by line graph, distribution of votes by pie graph, favourite sports teams by bar graph). While designing the tasks and deciding the appropriate type of graph, they also considered the type of the variable (categorical versus quantitative variables). To put it in a nutshell, the PTs focused on the connections between types of graphs and particular contexts or type of the variable rather than making connections with a statistical research question or a purpose for drawing the graph.

Furthermore, the PTs did not take the purpose that should have been set at the beginning while reading and interpreting graphs into account. For instance, when reading and interpreting pie graphs, the PTs asked students questions related to comparing groups rather than focusing on what makes a pie graph distinct as a instrument to represent data (i.e. observing the relative contribution of each category within the whole data set, comparing two different sized-data sets). A sample dialogue between Gamze and the students presents a class discussion about a pie graph showing the number of siblings in a class.

1st University Class Implementation

"Gamze: By looking at this graph, what can you tell me about the number of people? Let's see if there are any relations between them [slices of the graph] and then interpret.

Student: For example, half of 20 is 10. The number of people with one sibling is 9. The total number of people with no sibling and those with 2 siblings is 9. Thus, this gives us a total of 18 people.

…

Gamze: Ok, can you make comparisons among groups?

Student: The number of people with one sibling is the largest.

Gamze: Yes, the number of people with one sibling is the largest, isn't it? It has a largest slice. Which group has the smallest number of people?

Student: [people with] 3 and more [siblings]."

As can be seen, the PT did not include expressions regarding the purpose for drawing the graph while interpreting the pie chart. It was observed that the PT included the column chart, which is used more appropriately for the purpose of comparing data, in some of his/her questions regarding interpreting the pie chart (e.g., can you make comparisons among groups?).

It can be said that the PTs initially tended to see graphs as a tool used only to organize and represent data because the PTs did not make a connection between the purpose of drawing graphs and a statistical research question, both when preparing tasks for students and in discussions about deciding the appropriate type of graph and interpreting graphs.

PTs' Struggles with Teaching Graphs

The fact that the PTs view the graphs only as a means of representation posed difficulties in answering students' questions during their class implementations. These difficulties were observed especially in their first and second lesson study implementations. During the first lesson study implementation, Gamze transformed the bar graph into a pie graph without presenting a rationale or a clear explanation. As the task lacked a specific statistical research question, she struggled with a student's question asking whether they can also display degrees of temperatures by using a pie graph. Here Gamze focused on the variable type rather than the research question, which should be formulated at the beginning of the statistical investigation. The dialogue is presented below.

1st University class implementation

"Student: "Can we convert all bar graphs into pie graphs? (referring to the degrees of temperature)

Gamze: Since the variable (degrees of temperature) is quantitative, it is not appropriate to represent it by using a pie graph.

…

Gamze: Let's assume that it is a graph for degrees of temperatures. Tell me the values.

Student: It can be -2°C and 5°C.

Gamze: Hım, then there would be different temperatures in two different days? Do you think it would be meaningful to display these values [into a pie graph]? What does it mean here [referring to the slices of pie graph]? They represent the number of people. For example, the largest slice in the pie graph shows the highest frequency. Here, how do we show 25 degrees of temperature.

….

Student: So, this means we cannot use the pie graph in every situation, right?

Gamze: Yes. There are certain cases for which different graphs would be more suitable.

From this dialogue, it can be seen that the PT attempted to create a pie chart based on the data provided by the student, but realized that this would not be meaningful. Since she did not go through a problem situation related to the example based on the intended use of the pie chart, she could not give a satisfactory answer to the student's question and could not structure the lesson effectively. In this example, different problem situations related to the context could be created and which representation would be more appropriate for each problem situation could be discussed. For example, in the case of a problem that involves comparing the highest daytime temperatures observed for each day in a particular month in a province, a bar graph would be an appropriate representation, while a pie graph would be a more appropriate representation for a problem situation in which the distribution of the highest temperatures observed in a month in a province within the total month is examined.A similar situation was observed in the dialogue between the student and Gamze given below.

1st University class Implementation

"Student: Why do we need to draw a pie graph? We already know the bar graph.

Gamze: Yes, we know the bar graph. It is a different representation."

The phrase "different representation" used by the PT was discussed in detail during the evaluation meeting. Gamze questioned her statement "I said that, but is it really just a different representation? Then why do we teach different types of graphs?" Similar difficulties were also observed in the second lesson study implementation. The sample excerpts from the class discussion are presented below:

2nd University class Implementation

"Student: Do we use it [line graph] only for temperature?

Şirin: No, we do not use it only for temperature. Is there anything else that comes to your mind? For example, what else can we use it for? Think about it.

…

Şirin: Why do you think we might have made these connections [points of change]?

Student: To see, for example, whether it decreased like this or whether it increased.

Şirin: Yes, we can see the increase and decrease more easily in this way, can't we? For example, it decreased from Monday to Tuesday here.

Student: It [the change] can also be seen by using a bar graph. Why are we drawing this [line graph]?

Şirin: Yes, then, let's talk about it later…"

The student asking "Why are we drawing this [line graph]?" was responded by the PT as "let's talk about it later", which can be considered as an indication of a difficulty arising from not being able to establish a connection with the purposes of drawing graphs.

The fact that the PTs see graphics only as a means of representation caused them to have difficulty answering questions from students in their practices. These difficulties triggered the PTs to question the role of graphics in SPSP.

PTs' Development of an Understandings About the Function of Graphs as Instruments to Answer Statistical Questions

When the PTs struggled with answering students' questions during the implementations, these issues came up in the evaluation meetings. They discussed the purpose of drawing graphs and began to ask questions such as "Why do we draw graphs?", "Is our purpose just to represent data?". These discussions led them to realize that the tasks they initially formulated lacked a clear purpose or a statistical research question. For the 2nd lesson study related to line graphs, they revised their task so that it included a problem situation given in a context solved by collecting and analysing data (Figure 2).

Initial Task

"I want you to Express the data presented in the table with a graph. What kind of graph could we use?"

Average temperature of Ankara for 5 hours	
Time	Temp. (^{0}C)
07.00	1
10.00	5
13.00	9
16.00	9
19.00	7

Revised Task

"Uncle Hasan will plant a vegetable in his garden. In order to get the highest yield from this vegetable, the temperature change should be the least for 2 days after the day it is planted. Below is the weather forecast for the next 14 days. Let's find the best days for Uncle Hasan to plant."

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
20°C	*23°C*	*25°C*	*22°C*	*19°C*	*23°C*	*26°C*	*24°C*	*25°C*	*26°C*	*25°C*	*23°C*	*20°C*	*24°C*

Figure 2. *Initial task and revised task about line graph-2nd lesson study*

It was noted that the revised task in Figure 3 was structured around a problem situation and a purpose. A similar situation was observed in the tasks prepared for the 3rd lesson study process (Figure 3).

Initial Task

Favorite courses of Class 7/A		
Courses	Number of students	The table presents the favorite courses of students in Class 7/A. Draw the appropriate type of graph and select three courses and compare each course's state within the whole, find the percentages.
Mathematics	4	
Turkish	3	
English	2	
Physical Education	5	
Visual Arts	1	
Science	3	
Social Studies	2	

Revised Task

Favorite courses of Class 7/A		
Courses	Number of students	The table presents the favorite courses of students in Class 7/A. The teacher wants to learn how much mathematics is liked among all courses. Please draw the appropriate type of graph and interpret the state of mathematics in comparison to two other courses you selected.
Mathematics	4	
Turkish	3	
English	2	
Physical Education	5	
Visual Arts	1	
Science	3	
Social Studies	2	

Figure 3. *Initial task and revised task about pie graph-3rd lesson study*

As stated in Figure 2, the revised task includes a clear purpose: the teacher wanted to learn how much mathematics was liked among all courses. In this way, the task required the students to focus on the place of a category (mathematics course) within the whole (all courses). In addition, it has been observed that the contexts in the tasks have diversified and the PTs were more likely to move away from prototypical examples when designing tasks. They also developed a task to show that students needed to consider the purpose or the statistical question when deciding which type of graph would be the most appropriate even if the context of the problem seemed to be related to a specific type of graph (e.g., vote counts are usually shown with pie graphs). (Figure 3).

Number of votes of the candidates for student council		
Candidates	Number of votes	The table presents the number of votes of the candidates for school council in our school. Draw a graph that compares the data and decide who the president and the vice president of the council are.
Pelin	78	
Veli	57	
Alparslan	99	
Aysel	127	

Figure 4. *Task about bar graph- 3rd lesson study*

While this task still required a revision as it presents a small sample, it showed that the PTs began to consider students' thinking process. With regard to this task, the PTs stated that "When it comes to vote counts, it is always thought that the pie graph should be used. We deliberately chose this context and wanted them [students] to learn that it [data set of vote counts] can also be expressed by using a bar graph depending on the purpose." (3rd University class evaluation). During the evaluation meetings, the PTs began to understand the function of graphs as instruments to answer the statistical research questions and considered their purpose when deciding the appropriate type of graphs. They designed tasks around a purpose or a statistical question and encouraged students to evaluate which type of graph is the most appropriate to answer the formulated question. They asked questions such as "The bar graph is used for this purpose: to compare different situations [categories]. So, what about the pie graph? What do we use it [pie graphs] for?" The example presented below shows Gamze's explanation regarding the purpose of using pie graphs in the context of favourite football teams (Number of supporters for each team was presented as Galatasaray: 90 Fenerbahçe: 180 Beşiktaş: 72 and Other teams: 18).

1st Real Class Implementation

"Gamze: Can I say that pie graph allows me to see the ratio of a piece to the whole? Does everybody agree? In this case, I can say that the number of supporters of Galatasaray occupies 90 pieces among the whole,360 pieces. Here, we have 360 data. Some of you could easily convert this into a bar graph. However, sometimes there are so much data that we need to use a pie graph to see the ratio [between parts to the whole]. So, what are pie graphs used for? What do they help us see?"

As mentioned earlier, the PTs initially evaluated the suitability of using the line graph in a given context by focusing on the variable type and carried out their lesson plans and implementations in this direction. They were unable to answer students' questions like "Do we use line graphs only for temperature?", "We can display temperatures in a bar graph, so why do we need to display with a line graph?". While they had previously taken the type of variable or generalizations about contexts as the main consideration in the selection of the most appropriate type of graph, they realized that they should also take the research question or purpose into consideration. In fact, they were able to guide the discussion to focus students' attention to the function of graphs as instruments to answer research questions. The dialogue below showed that Şirin made comments on students' suggestions and emphasized the main purpose of using line graphs as observing change across time.

2nd Real Class Implementation

"Şirin: Now where else can we use line graphs? For example, do you have any problem context that comes to your mind? Where can we use it?

Student: For example, for the number of cars sold in 2016.

Şirin: Yes, the number of cars sold across the years because we want to see increase or decrease. Increase or decrease; that is, when we want to see the change [over time], we can use a line graph.

….

Student: The number of bagels sold on different days.

Şirin: How many bagels were sold on each day? Thus, we can see the change across days."

The development of PTs' understanding of graphs as instruments to answer research questions was observed especially in the third lesson study process. During the university implementation, Beyza emphasized the importance of the purpose in determining the appropriate type of graph and stated that the decision should not be made only based on the type of variable.

3rd University class Implementation

"Beyza: Ok, can you give me an example? You can give me an example as to why a particular graph should be used to represent a data set.

Student: For temperature change or population change, I can use a line graph.

Beyza: Hımm. For only temperature or population change? What else, what is important here, the temperature or the change?

Student: The change.

Beyza: Yes, the change in something.

…

Student: If we are given a frequency table showing the frequency of number of siblings, then we can show it with a bar graph.

Beyza: Ok. You can also show it by using a pie graph.

Student: We can.

Beyza: Which one would you choose and why?

Student: If we compare everyone with each other, we can use a bar graph because we want to show their relative status. If we are interested in the ratio of parts to the whole like showing each category for the number of siblings in the whole class, then we need to use a pie graph."

The PTs also consulted the difficulties students faced and made additional arrangements in their lesson plans. For example, they started designing tasks deliberating that a problem situation suitable to be represented by using a pie graph could also be represented with a bar graph. Moreover, they used the same approach when comparing the use of line graphs with bar graphs as well as the line graphs with pie graphs at this stage.

In addition, they asked questions and made explanations that would draw more attention to the purpose of the graphs during the real classroom implementation. For example, they gave a specific data set and asked the students to ponder upon which graph would be the most appropriate to represent the given data set. During the class discussion, the PTs emphasized that students should consider its purpose when deciding the appropriate type of graph.

3rd Real Class Implementation

"Beyza: We sometimes say we should draw a bar graph here and sometimes we say that we should draw a pie graph there. Can we clearly decide which one to use? For example, can I say that this set of data should be represented with a bar graph or that set of data should be represented with a pie graph. For example, I have a set of data here [referring to the votes of student council example]…Can't we use a pie graph to draw what we have shown here instead of using a bar graph?

Student: We can draw that.

Beyza: So, why haven't we drawn it?

Student: Here it asks us to select the president and vice president. So, we just need to know the number of votes, we don't need their percentages. That is, we need to show how many votes have been given to each candidate and who received the highest number of votes in order to find this [we can use bar graphs].

Beyza: Yes, we compared them [number of votes for each candidate] with each other. Here there is no certain difference between these types of graphs…For example, I can draw either a bar graph or a pie graph for these data sets [referring to the two tasks: student council and favourite courses]. However, in one of them, I am asked to find the ratio of the piece to the whole [referring to the favourite courses task] while the other [referring to the student council task] asks me to make comparisons. That is, what is important here is what I am asked to do. What should I see in the graph? That is, depending on what I will see in the graph, I will select the graph.

The PTs also ignored the purpose for drawing the graphs while they were interpreting the graphs at the beginning. For instance, when they interpreted a pie graph, they merely focused on comparing groups rather than emphasizing on what makes a pie graph distinct as a instrument to represent data (e.g., see the relative contribution of each category within the whole data set). During the evaluation meetings, such questions as “Let's consider what our purpose for drawing the graph was” were asked by the expert. “Based on this purpose, what can we say about the graph?" led the PTs to reflect on this issue. During the revision meetings, the statements like “Here we should attend that they [students]make comparisons” can be seen as indicators that the PTs started to consider the purpose of drawing graphs during the interpretation of findings. After these discussions, they became more attentive to the function of graphs as instruments to answer research questions. For example, in the first real classroom implementation related to the favourite football teams, Gamze emphasized comparing the categories by asking questions like “Can you see the number of supporters which is two or three times higher than the number of other supporters?” (1st real class implementation). Likewise, they included sample statements related to interpreting pie graphs in their lesson plan about pie graphs: “English is the most favourite course for 10% of the students. Mathematics is the most favourite course for 20% of the class.” (3rd university class plan).

DISCUSSION, CONCLUSION, RECOMMENDATIONS

The current documents in teaching statistics emphasize that students should be given opportunities to engage in the SPSP (Bargagliotti, et al, 2020; Franklin, et al, 2015). This perspective expects teachers to address all components of the SPSP (i.e., formulating research question, collecting, and analysing data and interpreting the results) in their instructional practices. However, neither teachers (örn, Litke & Hill, 2020; Şeker-Akın, 2023) nor PTs are familiar with teaching statistics from the statistical problem-solving process. In this study, we focused on 7th grade objectives related to teaching graphs in middle school curriculum and examined PTs’ understanding of graphs and the transformation of their understanding about graphs into instructional practices as they participated in the lesson study. The participants’ initial understandings of graphs are based on viewing graphs as instruments to represent a data set. This naturally caused them to emphasize the representation function of graphs and focused on the procedural aspects of drawing graphs. The lack of understanding that graphs are instruments to answer statistical research questions formulated at the beginning of the SPSP resulted in certain inadequacies while responding to students’ questions. Given that making sense of graphs is considered as higher-order thinking and that today’s world requires individuals who can comprehend and interpret different graphical representations, the importance of drawing attention to these difficulties experienced by pre-service teachers becomes evident (Boote, 2014; Patahuddin & Lowrie, 2019).

When the participants began to acknowledge the function of graphs as instruments to answer statistical questions, they were more likely to find different, yet conceptually related ideas and problems understandable, solvable and sometimes even trivial. This fundamental transformation in their understanding resulted in a change in how they designed and implemented their lesson plans. They designed tasks around a problem situation or a statistical research question and made connections with this research question while deciding on the type of graph or interpreting the graphs. In addition, they anticipated students’ possible conceptions and misconceptions related to graphs. Hence, they also realized that they need to use this understanding of graph as a criterion when evaluating student learning.

To sum up, understanding that graphs are not only instruments to represent data, but also instruments to answer statistical research questions is an important conceptual development or key development understanding (KDU) for these PTs, which was built on multiple experiences of designing, implementing and evaluating lesson plans. KDUs are effective objectives of mathematics instruction (Simon, 2006) and can play as “powerful springboards for learning” (Silverman & Thompson, 2008, p. 502). When the literature is reviewed, it is seen that researchers are trying to define KDUs that guide statistical concepts (e.g., unconventional modifications to conventional statistical representations, hat plots (placed above dot plots) as transitional representations between uncondensed and condensed data displays) (Groth, 2013). It is argued that

KDUs constructed in relation to mathematical concepts are directly associated with mathematical knowledge for teaching and that KDUs will guide teachers in the process of structuring the instruction (Huang, 2014; Silverman & Thompson, 2008). In this context, the KDU structure proposed by Simon (2006) was used to investigate how PTs established connections to their content knowledge in the process of planning, implementing, and evaluating lesson plans.

The participation of the PTs in the lesson study program contributed initially to the development of their understanding of the function of graphs in SPSP and in the later stages of the process, they set the purpose of drawing graphs as a learning goal. They realized that this is necessary for students to understand the SPSP. In addition, the PTs' discussions on how they could make more satisfying explanations in response to student answers are thought to support this development. Research has shown that teacher training programs should provide teachers with opportunities to establish links between the conceptual and procedural understandings of mathematical contexts, test these understandings and foster the dissemination of these understandings. When the literature is examined, it is seen that teaching strategies (e.g., writing prompts) for the emergence of KDUs have been integrated into teacher training programs in various studies (e.g., Groth, 2013). It is argued that PTs need continuous support to realize profound changes such as the development of KDUs. The promising results obtained from the current study and the activities conducted can be considered to be important steps for this development to emerge (Rhee, 2012). Provision of opportunities for PTs to participate in training programs focused on KDUs and opportunities to work cooperatively to understand how these KDUs will be integrated into the teaching process will help support their development.

Although there is a need for more research to define KDUs needed by PTs / and for teachers in the teaching process, it is believed that the current study will certainly contribute to the recognition of KDUs needed in SPSPand to teach the function of graphs in this process. Furthermore, it can be said that these and similar KDUs revealed can have direct benefits in the determination of instructional goals. While constructing the curriculums, consideration of the KDUs determined by research can have some positive effects on the development of students' mathematics (Silverman, 2005). Recognition of KDUs by PTs can help them understand how students make sense of mathematical concepts. When PTs focus on the KDU of the related mathematical concept, they can better predict and interpret the development of this concept (Fernández, et al., 2018). Hence, they can turn them into pedagogically powerful ideas in the teaching process.

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