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Examination of Pre-service Science Teachers' and Science Teachers' Mathematical Modeling Competencies

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Abstract

The purpose of this study is to examine the mathematical modeling competencies (MMC) of pre-service science teachers and science teachers towards the use of mathematical modeling (MM) in science education. For this purpose, a qualitative study was conducted with 12 pre-service science teachers and science teachers. A case study design was used in the research. Data were collected through activity forms, interviews, and observation techniques. Descriptive analysis was used to determine MMC by considering the sub-competencies revised by Çakmak (2019) in line with Borromeo Ferri's (2006) MMC. Upon examining the MMC, it was concluded that MMC developed in the process, and the most successful understanding and the least successful ones were simplification and structuring MMC. Contrary to the literature, it was determined that reaching real results from mathematical results was satisfactory. It was observed that newly graduated science teachers and experienced science teachers had an equal level of MMC, while pre-service science teachers had a lower level of MMC. In the process, the least competence was shown in biology and chemistry, and the most competence was shown in physics. It is recommended to increase MM studies for science education, to provide MM courses at least at the undergraduate level, and to experience MM practices through in-service training.

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Keywords: mathematical modeling, mathematical modeling competencies, science education.

Fen Bilgisi Öğretmen Adayları ve Fen Bilgisi Öğretmenlerinin Matematiksel Modelleme Yeterliklerinin İncelenmesi

Öz

Bu çalışmanın amacı, fen eğitiminde matematiksel modelleme (MM) kullanımına yönelik olarak fen bilgisi öğretmen adayları ve fen bilgisi öğretmenlerinin matematiksel modelleme yeterliklerini (MMY) incelemektir. Bu amaçla, 12 fen öğretmen adayı ve fen öğretmeni ile nitel bir araştırma yürütülmüştür. Araştırmada durum çalışması deseni kullanılmıştır. Veriler, etkinlik formları, görüşme ve gözlem teknikleri ile toplanmıştır. MMY'ni belirlemek için, Borromeo Ferri'nin (2006) MMY'ye göre Çakmak'ın (2019) revize ettiği alt yeterlikler esas alınarak betimsel analiz kullanılmıştır. MMY'nin süreç içerisinde geliştiği, en başarılı anlamanın ve en başarısız olanın basitleştirme ve yapılandırma MMY olduğu sonucuna varılmıştır. Literatürün aksine, matematiksel sonuçlardan gerçek sonuçlara ulaşmada problem yaşanmadığı tespit edilmiştir. Yeni mezun fen bilgisi öğretmenleri ve deneyimli fen bilgisi öğretmenlerinin MMY'nin eşit düzeyde olduğu görülürken, fen öğretmen adaylarının MMY'nin daha düşük olduğu görülmüştür. Süreçte en az biyoloji ve kimya, en çok fizik konularında yeterlik gösterilmiştir. Fen eğitimine yönelik MM çalışmalarının artırılması, en azından lisans düzeyinde MM derslerinin verilmesi ve hizmet içi eğitimlerle MM uygulamalarının deneyimlendirilmesi çalışmalarının yapılması önerilmektedir.

Anahtar kelimeler: matematiksel modelleme, matematiksel modelleme yeterlikleri, fen eğitimi.

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INTRODUCTION

The interaction between science and mathematics is a dynamic and synergistic relationship. Mathematics provides the language and tools for expressing analyzing and validating scientific ideas while science in turn inspires the development of new mathematical theories and applications. This collaborative interaction has been fundamental to numerous scientific breakthroughs and technological advancements throughout history. The interactivity between science and mathematics is dynamic and continually evolving, with advancements in one field often influencing and inspiring developments in the other. This interconnectedness underscores the importance of an integrated approach to teaching and learning science and mathematics.

Mathematical modeling (MM) is a method that effectively combines science and mathematics in education. MM is generally defined as the process of expressing, testing and interpreting a mathematical or non-mathematical situation using mathematical language (Kertil, 2008; Kaiser, 2020; Kapur, 2023). It provides essential contributions by making sense of situations encountered in daily life and establishing a connection between mathematics and science (Kaiser, 2007; Başkan, 2011; Guerrero-Ortiz & Mena-Lorca, 2017; Deniz, 2018; Doğan & Gürbüz, 2019; Kaiser, 2020; Kapur, 2023). MM studies are applied in the fields of science and mathematics by associating with real life and help to connect these fields (Prins et al., 2009; Sağlam-Arslan & Arslan, 2010; Doruk, 2010; Başkan, 2011; Guerrero-Ortiz & Mena-Lorca, 2017; Deniz, 2018; Doğan & Gürbüz, 2019; Kapur, 2023).

The boundary between mathematics and science does not reflect modern science's interdisciplinary work, and an innovative curriculum integrating mathematics and science courses should be developed as an alternative to the traditional subject-oriented curriculum. (Michelsen, 2006; Başkan, 2011; Guerrero-Ortiz & Mena-Lorca, 2017; Deniz, 2018; Doğan & Gürbüz, 2019). Bridging the gap between the teachings of these disciplines, including MM in science courses, makes a significant contribution (Başkan, 2011).

The Significance of the Study

Studies show that students' Mathematical Modeling Competencies (MMC) are relatively low (Frejd & Ärlebäck, 2011; Gatabi & Abdolahpour, 2013; Kaiser, 2020; Kapur, 2023). Therefore, educational activities should be organized in this direction. Teachers and future teachers play essential roles in creating and organizing these environments. Teachers must understand MM to organize MM environments. However, studies show that teachers who will organize these environments and pre-service teachers who are future teachers do not even know about MM (Akgün et al., 2013; Anhalt & Cortez, 2016; Işık & Mercan, 2015; Urhan & Dost, 2016; Kaiser, 2020; Kapur, 2023). Teachers expected to use MM must gain modeling experience (Niss et al., 2007). It is impossible for teachers and pre-service teachers who have not received MM training, have never been involved in MM processes, and have not faced activities to acquire MMC (Doğan & Gürbüz, 2019). Studies on teachers' MMC are limited (Zbiek, 2016; Ferri, 2018).

When examining MM studies in the literature, it becomes apparent that there are very few studies in science education. Furthermore, studies on the MMC of science teachers are almost nonexistent (Yenilmez & Yıldız, 2019). A detailed search in the database of the National Thesis Center (https://tez.yok.gov.tr/accessed on 25.03.2023) reveals that there are 855 theses on 'mathematical modeling', and only Başkan (2011) and Güder (2019) have doctoral dissertations on MM in science education. Meta-analysis studies on MM determine that there are very few interdisciplinary and especially science education studies. It is emphasized that MM studies should increase, especially in different disciplines such as science education (Yıldız & Yenilmez, 2019; Koç, 2020; Koceva Lazarova, Stojkovic, & Stojanova, 2022). The starting point of this study is to fill these gaps in the literature and contribute to MM studies by providing an interdisciplinary perspective.

Science teachers should use MM because it enriches the learning experience, enhances problem-solving skills, integrates STEM disciplines, fosters critical thinking, prepares students for future careers, incorporates technology, aligns with inquiry-based learning, develops mathematical literacy, promotes creativity, and addresses complex global challenges (Koceva Lazarova, Stojkovic, & Stojanova, 2022). MM allows students to apply mathematical principles to solve practical problems in various scientific disciplines, such as physics, biology, chemistry, and engineering.

MMC plays a crucial role in science education. Science teachers with modeling competencies can bring real-world applications into the classroom, demonstrating to students how mathematical concepts are utilized to address complex scientific challenges. Teachers with MMC can enrich their teaching practices, provide students with a more comprehensive understanding of science, and better prepare them for future academic and professional pursuits in science and related fields. They can provide a more integrated and holistic approach to teaching scientific concepts. They can demonstrate the practical applications of mathematics in scientific inquiry, making the learning experience more engaging for students. As educational practices evolve, there is an increasing emphasis on interdisciplinary and practical learning experiences. Teachers with modeling competencies are better positioned to adapt to these trends and incorporate innovative teaching methods into their classrooms.

The Aim of the Study

This study aims to examine the mathematical modeling competencies of science and pre-service teachers by making them experience the MM process.

Research Question

How are the mathematical modeling competencies of pre-service science teachers, newly graduated science teachers and experienced science teachers who have experienced the mathematical modeling?

LITERATURE REVIEW

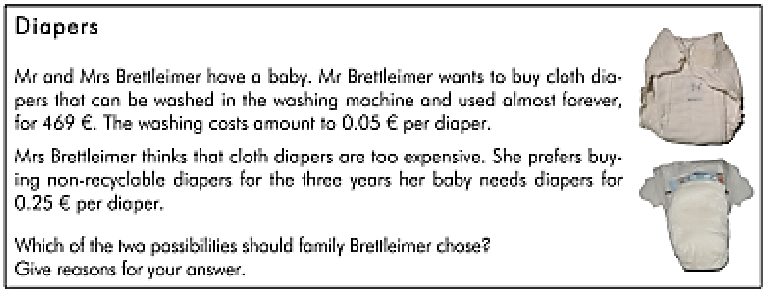
This section presents the contexts related to the research, forming the background of the research, the research bases in line with the literature, and similar studies in the literature.

Mathematical Model

Berry and Houston (1995), in one of the first studies on MM, define a mathematical model as a mathematical representation of the relationship between two or more variables related to a given situation. Lesh and Doerr (2003), prominent figures in MM studies, define the mathematical model as the forms of representation used in explaining, describing or structuring the behavior of specific systems. Çakmak (2019) defines a mathematical model as a generalizable and reusable mathematical representation created to represent a complex real-life situation.

Mathematical Modeling

Although there are many definitions in the literature about the concept of MM, it can be said that all definitions converge on a common denominator as the mathematical expression of a real situation (Doruk, 2010; Haines & Crouch, 2007; Lesh & Doerr, 2003). In addition, MM is the process of creating a model and is expressed as a bidirectional transformation process between the real world and the mathematical world (Blum & Borromeo Ferri, 2009). Sriraman (2006) explains the relationship between the mathematical model and MM as the process of creating a physical, symbolic, or abstract model of a situation, while the mathematical model is the product of this process. On the other hand, Lesh and Doerr (2003) use the concept of model-eliciting activity, which includes both the terms mathematical model and modeling in terms of meaning. An example of a modeling task and modeling steps is presented in Figure 1.



general description	example: the task "diapers"
1. First, the text and maybe a photo have to be read and the problem situation has to be understood by the problem solver, that is an *individual mental model of the real situation* has to be constructed (see section 2.1).	
2. For example by making assumptions or selecting given data the situation has to be simplified, structured and made more precise, leading to a *real model* of the situation.	- Which diaper sort causes less costs? - 469,-€ package 0,05 € per diaper - 0,25 € per diaper - 3 years (365 days) 5 diapers per day
3. Based on basic ideas (vom Hofe 1998) of different mathematical concepts, mathematisation transforms this real model into a *mathematical model*.	$y_1 = 469 + 0.05 \cdot x$ $y_2 = 0.25 \cdot x$ $x = 3 \cdot 365 \cdot 5$ $y = y_1 - y_2$
4. Then mathematical tools like rearranging a term or the rule of three are used, yielding a *mathematical result*.	$y = -626$
5. By activating basic ideas again, the mathematical result has to be interpreted in the real world as a *real result* for the given problem.	Difference in prices approx. 600 €
6. The next step is a *validation* of the real result: Is it reasonable? Is the accuracy appropriate? Are the assumptions/ simplifications adequate? Accordingly, one might go round the modelling loop several times.	- ...? - additional costs for the washing machine? - additional costs for a bigger garbage can? - 3, 4, 5, 6 diapers per day? - time needed for washing - ecological aspects ...
7. The process ends with an exposition of a final answer to the original problem.	If Mr. and Mrs. Brettleimer need 5 diapers per day for 3 years then cloth diapers are 600 € cheaper than non-recyclable diapers.

Figure 1. *An example of a modeling task and modeling steps (Lesh & Blum, 2010, p.122)*

Mathematical Modeling Cycles

The mathematical modeling process has multiple cycles, such as reaching a solution using the given data, comparing the solution with a real-life situation, and developing a different solution by improving the solution (Erbaş et al., 2013). It is also represented by a cycle designed according to different perspectives, such as realistic or applied modeling, contextual modeling, educational modeling, socio-critical modeling, and cognitive modeling (Greefrath & Vorhölter, 2016; Perrenet & Zwaneveld, 2012). In the literature, there are diagrammatic representations of MM as a process involving a repeatable cycle, which are similar but differ. When different MM cycles are examined, they include the real situation & problem (modeling situation), mathematical model, mathematical result, and real result steps. However, they differ according to the situation model and real model stages (Çakmak, 2019).

One of the modeling cycles frequently used in MM studies belongs to Lesh and Doerr (2003). In Lesh and Doerr's modeling process, unlike other studies, a complete circularity draws attention. In this study, it is seen that there is no hierarchy between the steps in the MM process, and it is not stated that all steps are interrelated. Doerr's MM cycle is presented in Figure 2. The modeling cycle consists of four fundamental steps: (a) Description: transferring the given real-life situation to the modeling world, (b) Manipulation: applying and calculating the model that emerges after transferring to the modeling world, (c) Prediction: transferring the obtained results back to the real world, and (d) Validation: verifying the usefulness of the predictions.

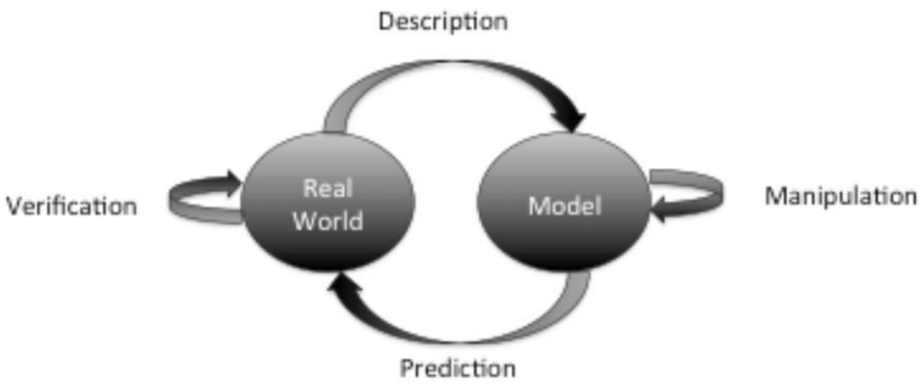


Figure 2. *The nodes of the modeling process (Lesh and Doerr, 2003)*

Unlike other studies, Borromeo-Ferri (2006) integrates the situation model, i.e., the mental representation of the situation, into the process in the MM cycle and presents the real situation, the situation model, and the real model separately. While this modeling cycle shown in Figure 3 consists of six steps, the presentation activity, which is the stage of returning from the mental representation of the situation to the real situation, is not included in the modeling process as a cognitive stage. Çakmak (2019), as a result of his detailed literature review, states that the situation model is not mentioned in the modeling cycles presented until Borromeo Ferri's cycle. This study, which examines the modeling cycle in more detail than others, focuses on the cognitive processes of individuals (Ural, 2018; Çakmak, 2019; Çoksöyler & Bozkurt, 2021).

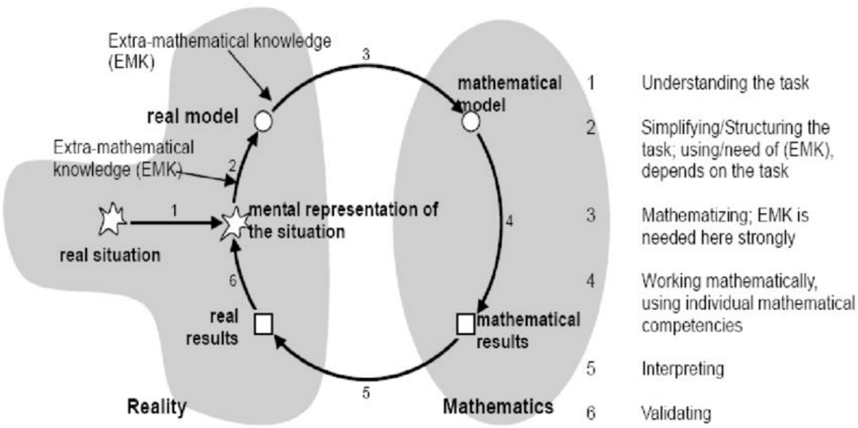


Figure 3. *Borromeo Ferri's (2006) Modeling Cycle*

Borromeo Ferri's (2006) modeling cycle, which deals with the cognitive aspects of the modeling process, seems to be a suitable tool for analyzing the cognitive processes of the participants because it is very detailed and prone to the use of different modeling types (Blum & Leiß, 2007; Tutak & Güder, 2014; Çakmak, 2019; Çoksöyler & Bozkurt, 2021). It was decided to use the MM cycle constructed by Borromeo Ferri (2006) in this study due to the importance of cognitive processes in science subjects and the fact that it allows for detailed and comprehensive analysis.

Mathematical Modeling Competencies (MMC)

According to Kaiser and Brand (2015), studies on modeling competencies are organized in line with the following general questions:

1) How can modeling competencies be conceptualized?
2) How can modeling competencies be tested?
3) How can modeling competencies be developed?
4) How can modeling competencies be fostered most effectively?
5) What assessment methods are appropriate in practice to assess modeling competence?

First of all, in order to understand the MMC of pre-service and in-service science teachers, the main objectives of this study, it is necessary to explain what mathematical modeling competency means in the literature. Competence and skill are two MM concepts frequently used in MM studies. Although these two concepts are used as alternatives to each other in many studies, there are semantic differences between these two concepts (Bukova Güzel, 2019). In TDK (2021) dictionary, *skill* is defined as a person's ability to accomplish a job depending on predisposition and learning and to finalize a process following the purpose, while *competence* is defined as exceptional knowledge, license, and sufficiency that provides the power to do a job. Tekin Dede (2018) defines skill as the organization of individuals' existing abilities to enable them to reach a goal depending on education, and competence as having the necessary equipment to achieve a goal. When MM studies are examined, it is seen that literature has recently emerged to define and develop modeling competencies, but a complete consensus has not yet been reached (Maaß, 2006; Çetinkaya, 2013; Kaiser & Brand, 2015). Maaβ (2006) describes modeling competencies as the goal-oriented skills and abilities that enable the modeling process and the willingness to exhibit these skills and abilities.

Bukova Güzel (2019, p.42) states that MMC in the literature is addressed from four different perspectives listed below.

1) Cognitive competencies

2) Metacognitive competencies

3) Affective competencies

4) Social competencies

Social Competencies: It is known that the MM process takes place in a learning environment as individual or group work. From this point of view, individuals participating in the process should have competencies such as being able to communicate, express themselves, work with the group, take responsibility, discuss, present their work, and share (Kaiser, 2007; Kaiser et al., 2010; Kaiser & Brand, 2015; Bukova Güzel; 2019, Çevikbaş, Kaiser, & Schukajlow, 2021).

Affective Competencies: While defining modeling competencies, Maaβ (2006) emphasizes the willingness of individuals towards the MM process, that is, the factors of voluntariness and motivation. Biccard and Wessels (2011) state that people involved in the MM process develop their beliefs and perceptions about the nature of the problem. In addition, as mentioned in the following stages of this study, one of the principles of mathematical modeling activities (MMA) is that MMA should make individuals feel the need to create a model, which can be said to be related to affective competencies.

Metacognitive Competencies: Metacognitive competencies include factors that support cognitive competencies such as knowing the MM process, planning, monitoring, controlling, verifying, judging the solution, reflecting, creating real-life problems, analyzing the task, and using the sense of orientation (Tanner & Jones, 1995; Maaβ, 2006; Kaiser, 2007; Ferri, 2011; Çakmak, 2019; Bukova Güzel, 2019).

Cognitive competencies: These include cognitive skills such as understanding the problem, simplification and structuring, mathematization, mathematical work, interpretation and verification, which cover the entire modeling process and occur during the process (Çakmak, 2019; Bukova Güzel, 2019).

Examining this study regarding cognitive competencies in the modeling process was deemed appropriate for determining the MMC of pre-service science teachers and teachers. In this context, the theoretical framework was advanced through cognitive competencies.

In the literature review, situations that may affect the development of MMC were identified (Anhalt & Cortez, 2016; Biccard & Wessels, 2011; Blomhøj & Jensen, 2003; Brand, 2014; Galbraith & Stillman, 2001; Ji, 2012; Kaiser & Grünewald, 2015; Kaiser et al, 2010; Kaiser & Stender, 2013; Maaß, 2006; Çakmak, 2019; Geefrath, 2020, English, 2006; Antonius, Haines, Jensen, M. Niss, & Burkhardt, 2007, Zawojewski, Lesh, & English, 2003). Some of these situations are as follows;

1) Information about mathematical model, modeling, and modeling cycle:

2) Group work

3) Content of modeling situations,

4) Preparation of teaching environment for holistic or atomistic approach

5) Technology use

6) The role of the teacher

7) Long-term studies

Include the background information and a review of the literature in this section. You may dedicate another separate section for the literature review. Include the background information and a review of the literature in this section. You may dedicate another separate section for the literature review. Include the background information and a review of the literature in this section. You may dedicate another separate section for the literature review. Include the background information and a review of the literature in this section. You may dedicate another separate section for the literature review. Include the background information and a review of the literature in this section. You may dedicate another separate section for the literature review.

METHOD

The method section includes the research design, the study group or participants of the study, data collection tools, and data analysis.

Research Design

Henning and Keune (2007) state that MMC cannot be directly observed, and that competencies can only be determined by observing the actions and behaviors of students or teachers while performing MM tasks. In this direction, it was decided to design the study according to the case study design, which is one of the qualitative research methods that enable detailed and in-depth investigations to examine the MMC of pre-service science teachers and teachers. This method enables close observation of the process and the actions and behaviors within the process. The case study is a design in which the researcher examines one or more situations limited in time in depth with data collection tools including multiple sources, investigates with a holistic approach, and defines situations and themes related to the situation (Creswell, 2013; Şimşek & Yıldırım, 2018).

In learning environments where the holistic approach is adopted, the aim is to enable individuals to go through the whole mathematical modelling process. In learning environments where the atomistic approach is adopted, the aim is for individuals to perform a sub-step or particular steps of the mathematical modeling process. In this study, a holistic approach is used due to evaluating the entire process by considering sub-competencies.

Study Group

The purposive sampling (non-probabilistic sampling) method was used in the study. The purposive sampling method aims to examine a subject in depth based on the research group selected for a specific purpose (Yıldırım & Şimşek, 2018). Three groups were formed to observe whether professional experience affects mathematical modeling competencies. The research group consisted of four pre-service teachers attending the fourth year of the Department of Science Education, four science teachers who have a bachelor's degree in science teaching but have not been appointed, and four experienced teachers who continue to teach science in public schools affiliated with the Ministry of National Education. This research group was formed by considering the criteria of completing physics, chemistry, biology, and mathematics courses in science education at the undergraduate level and their experience in the science teaching profession.

All participants signed the consent form that they voluntarily participated in the study. In the study, pre-service science teachers were coded as U1, U2, and U3; teachers who received a bachelor's degree in science teaching but were not appointed were coded as P1, P2, and P3; and teachers who continue to teach science in public schools affiliated to the Ministry of National Education were coded as T1, T2, and T3. Demographic information about the participants is shown in Table 1.

Table 1. Demographic information about the participants

Participant	Gender	Professional (Years)	Experience	Education Level	MM Experience
T1	Male	10		PhD Student	-
T2	Male	9		Master's degree graduate	-
T3	Male	9		PhD Student	-
T4	Male	11		Bachelor's degree	-
P1	Female	3		Graduate Student	-
P2	Female	-		Graduate Student	-
P3	Female	1		Graduate Student	-
P4	Male	-		PhD Student	-
U1	Female	-		4th Year Undergraduate	-

U2	Female	-	4th Year Undergraduate	-
U3	Female	-	4th Year Undergraduate	-
U4	Female	-	4th Year Undergraduate	-

Data Collection

In this study, the Activity Form, developed by Kenan and Polat (2022), was used as the data collection tool. The 'Activity Form' incorporates Borromeo Ferri's (2006) MM steps. The activity reflective diary form presented in Appendix-1 was used as another data collection tool. The form is a structured interview form that collects participants' feelings and thoughts during the activity.

Additionally, the observation technique was used to monitor the cognitive skills of pre-service science teachers and teachers in the learning environment created to examine their MMC. Observation is a technique used to describe in detail the behaviors occurring in any environment (Yıldırım & Şimşek, 2018). The most significant advantage of this technique is that it enables many behaviors to be objectively determined by observing the individuals in their natural environment (Karasar, 2016). The researcher and an expert with a PhD in MM took part as observers in the implementation process. For six weeks, two observers monitored the groups. The data transferred by the groups to the activity forms were continuously compared with the observation notes.

Process

The implementation of the research was designed by considering the situations that enhance MMC.

Information about model, modeling and modeling cycle: Experts in MM educated the pre-service science teachers and teachers who constituted the study group. The participants were received training in MM during these informative activities, which lasted an average of 2 hours and were video recorded.

Group work: As the literature suggests that group work is crucial in the development of MMC, the modeling environment of this study was organized by forming groups of four people.

The content of modeling activities: In the study, six MMA developed by Kenan and Polat (2022), considering Lesh et al.'s (2000) principles for developing thought provoking activities, were used. Care was taken to ensure that the contents of the mathematical situations were related to real life, at least at the undergraduate level, within the knowledge domain of teachers and pre-service teachers, involving more than one variable, and open to interpretation and elaboration.

Table 2. Mathematical modeling activities

Activity Name	Code	Learning Area	Learning Area Subject(s)
Melisa Project	MP	Biology	Photosynthesis, Respiration, Life Cycle
Reflux	RE	Chemistry	Acids and Bases, Neutralization Reactions
Houseboats	YE	Physics	Center of Gravity, Mass, Density, Buoyancy,
Plastic Bacteria	PB	Biology	Bacteria, Enzymes, Recycling
Ozone	OZ	Chemistry	Chemical Reactions, Environmental Pollution
Slide	KA	Physics	Kinetic and Potential Energy, Speed, Incline

Unlike the literature, the modeling activities developed by Kenan and Polat (2022) were presented in video format. This decision was based on the idea of engaging more senses, associating more efficiently with real life, including motivational elements, and integrating with technology. For example, the 'Reflux' activity, derived from the 'General Chemistry: Principles of Chemistry with a Molecular Approach -1 / Principles of Chemistry: A Molecular Approach' book published by Nobel Academic Publishing, was presented as a video interview using internet resources. Similarly, the Floating Houses activity, initially a verbal situation in a newspaper article (https://www.trthaber.com/haber/dunya/amsterdam-kanallari-uzerinde-yasam-438695.html), was presented using a promotional video of an actual floating houses project built on water in the city of Amsterdam, found through internet resource search.

''I noticed-curious'' activities were added to the process to make the modeling task more comprehensible. QR codes for accessing the videos of these six activities, which were deemed appropriate for use in this study, are given in Appendix 2.

Preparation of teaching environment according to holistic approach: In this study, we examined the competencies of science and pre-service teachers in the MM process and to tracked their development. We used a holistic approach, evaluating the entire process by considering sub-competencies.

Long-term studies: This study lasted eight weeks. The first week involved training in MM, followed by a sample mathematical modeling activity in the second week. For the remaining weeks, six mathematical modeling activities were conducted.

Teacher Role: Academics who were conducting Physics, Chemistry, and Biology courses at the undergraduate/graduate level in Science Education served as consultants in this study. These experts participated in the process alongside the students. This role, defined as an agent in the teaching experience methodology, was undertaken by academics who are experts in the subject area. Before each application, the expert in MM, the agent, and the conductor of the study exchanged ideas on potential questions related to the activity topic, what questions should be asked, what to pay attention to, and additional information and limits that could be shared with the participants. The team also met after each implementation to evaluate the process.

Use of Technology: During the study, which was conducted through the Zoom program, participants were encouraged to access internet information resources, use Office programs such as Excel, Word, PowerPoint, and similar Office programs, make calculations with the help of computers, and access activity content. The groups were able to monitor, share, and present their work through the use of digital documents.

Data Analysis

Descriptive analysis was used in the data analysis of the examination of the MMC of pre-service science teachers and teachers. Descriptive analysis involves interpreting the theoretical framework of the study, the research questions, and the dimensions in the interview or observation according to the performed themes (Yıldırım & Şimşek, 2018).

This study analyzed data according to Borromeo Ferri's (2006) MM cycle. Thus, it was determined at which stage pre-service science teachers and teachers were in the MM process. Ferri's 6-stage coding system consists of 'Understanding' (mental representation of the situation) stage 1, 'Simplification and Structuring (real model stage) 2, 'Mathematization' (mathematical model) 3, 'Mathematical Study' (mathematical result) stage 4, 'Interpretation' (real result) 5 and 'Verification' (validating) 6. The definitions and indicators of the levels of the MM cycle used in this study are presented in Table 3.

Table 3. Indicators of the levels of the mathematical modeling cycle

Stage	Competency	Cod	Indicators
Mental Representation of the Situation	Understanding	1.Stage	Understands the real-world situation and creates a mental representation of the situation, but cannot perform the skills of structuring, simplifying, making assumptions, and predicting.
Real Model	Simplification/Structuring	2. Stage	Makes assumptions about the modeling situation, simplifies and structures the situation, determines the variables and makes predictions about these variables. However, cannot mathematize.
Mathematical Model	Mathematization	3. Stage	Establishes the mathematical model and creates a mathematical problem. However, cannot solve the mathematical problem.
Mathematical Result	Mathematical Study	4. Stage	Solves the mathematical problem and gets the mathematical results. However, cannot make the transition to real results.
Real Result	Interpretation	5. Stage	Interprets mathematical results and get real-world results. However, cannot test their validity.
Validating	Validation	6. Stage	Tests the accuracy and validity of real results in the real world.

To determine the MMC of pre-service teachers and teacher groups in each MMA, the MMC and sub-competencies criteria developed by Borromeo Ferri (2006), adapted by Blum and Kaiser (2006) and revised by Çakmak (2019) were used. These competencies and sub-competencies are presented in Appendix-3.

The coding system and MMC and sub-competencies criteria determined which competency and MM stage the groups participating in the study could reach. Insufficient, partially sufficient, and sufficient dimensions were created for each stage and MMC. Groups observed to be insufficient were evaluated in a sub-stage and competency. For example, if a group identified the variables affecting the situation and made predictions about them, they performed sufficiently in the second stage (real model/simplification and structuring competency). However, if they identify some of the variables affecting the situation and make predictions only about them or if they identify only the variables and cannot make predictions, they show partially sufficient performance in the second stage. If the participants could not identify the variables directly affecting the situation, they were evaluated in the insufficient performance category. If they did not perform the competencies in question, it was accepted that they could not transition to this stage. In this case, the group was evaluated in the first stage, which is a sub-stage.

Validity

There are various strategies to ensure the validity of qualitative studies (Creswell & Miller, 2000). These strategies and their effects on the study are as follows;

Data triangulation: Multiple and different sources are used to ensure the validity of the findings of a study (Miles & Huberman, 1994). In this study, data diversity was ensured and validity was increased by using different data collection tools such as observation, interview, and document analysis.

Long-term interaction: Long-term observation in qualitative studies allows participants to build trust, recognize the culture, and check misinformation originating from the researcher (Glesne, 2016). The implementation of this study lasted eight weeks and is a long-term study. In addition, the data analysis was carried out over approximately one year; repeated analysis, evaluation of the data by different experts, and referring to the literature as a result of the emerging discrepancies are indicators that the field has been studied for a long time.

Expert review: The data related to the determination of mathematical modeling competencies were evaluated separately by the researcher and an expert with a doctorate in mathematical modeling. The subject area expert's approval was requested for the MMC evaluation in the relevant activity.

Reliability

Table 4 indicates the percentages of agreement, disagreement, and agreement between the analyses in each analysis. The coding was done by the researcher and an expert with a PhD in MM. The percentage of agreement between the analyses, i.e., the inter-rater reliability coefficient, was calculated using the formula [agreement/(agreement+disagreement)*100] proposed by Miles and Huberman (1994). The data that did not agree were subjected to re-analysis and a common decision was reached as a result of the discussions. It is stated that the reliability of coding for a qualitative study should be at least 80% agreement (Miles & Huberman, 1994). When the percentages of agreement in the study are examined, it is seen that the values are 85% and above.

Table 4. Agreement between analyses

Analyses	Coders	Agreement	Disagreement	Compliance Percentage (%)
Competencies	2 experts	34	2	94
Sub- competencies	2 experts	310	32	91

Two disagreements emerged when evaluating the stage of pre-service science teachers and teachers in MMC. One of these disagreements was expressed as the T group's 'Plastic Bacteria': "experienced teacher group determined the variables but could not make a transition to the second stage because these variables did not contribute to the problem situation they determined". As a result of the discussion between the coders, it was decided that they made a partial transition to the second stage.

Research Ethics

The pre-service science teachers and teachers participated in the study voluntarily. Accordingly, they read and signed a consent form. Each participant was given detailed and identical information about the research processes during the research. The personal information of the participants was kept confidential, and the codes assigned to them (T1, U1, P1) were used in reporting the data.

FINDINGS

This section presents the findings of the study, which examines the mathematical modeling competencies of pre-service teachers, newly graduated teachers and experienced teachers. The findings are presented in chronological order of the activities. First, the MMC of the groups related to the Melisa Project activity is examined, and finally, the MMC of the groups related to the Slide activity is examined. The findings obtained from the participants' reflective diaries recorded at the end of each activity support the analysis of MMC. The data obtained from the activity forms and video recordings of the participants are given as direct quotations.

Findings on Melisa Project Activity

Based on the data obtained, the general evaluation of the MMC of the groups in the Melisa Project activity is shown in Table 5. In the analysis of the groups' performances in the Melisa Project, it was concluded that the groups understood the problem situation but could not perform the skills of structuring, simplifying, making assumptions and predicting. Therefore, it was determined that the groups remained in Stage 1 and could not exhibit simplification and structuring competencies. In other words, the groups realized the mental representation of the situation but could not reach the real model (Stage 1).

Table 5. MMC of the groups in the Melisa Project activity

MMC	Insufficient	Partially sufficient	Sufficient
Understanding			T, P, U
Simplification and structuring	T, P, U		
Mathematization	T, P, U		
Mathematical work	T, P, U		
Interpretation	T, P, U		
Validation	T, P, U		

When analyzing the participants' reflective diaries of the activity, we find clues as to why the groups could not access the real model. For example, T3 from group T stated that the model they created was insufficient and that they could not reach essential variables and use the variables in the model as follows;

T3: The model we created is not a good model. We could make a much better model, but we need to come together again at different times to work on it. In addition, we need to reach variables that are very important for the model (for example, the amount of O2 consumed daily by an astronaut, the amount of H2O created).

Similarly, P2 from group P stated that he had difficulties in identifying the factors that significantly affected the problem situation;

P2: In this activity, we had great difficulty in determining the factors that would ensure the continuity of this cycle while dealing with the mouse-algae relationship for nutrient production.

P3, also from the P group, stated that they constantly encountered different variables and that this situation led to a complicated path;

P3: At first, I had difficulty determining the point we would focus on. After receiving the necessary feedback, we started to focus on the point we would focus on. We created the equation. While making comments on the equation, different variables were constantly appearing in front of us. While talking about these variables led us to a more complicated path, it would not be useful to go into too much detail. T1 from the T group implies that they tend to solve the problem directly.

T1: We built the model directly from the conservation of mass in chemical reactions based on the amount of carbon dioxide required for the amount of oxygen needed and neglected all external factors. The model is not inclusive enough in this respect.

The findings obtained from the activity forms, video recordings and reflective diaries show that the groups needed help in simplifying and structuring the problem in the Melisa Project activity, that is, at the real model stage. The groups tended to solve the problem directly. Participants had difficulties in the processes of identifying variables, simplifying the problem, and determining the relevant assumptions.

Findings on Reflux Activity

Based on the data obtained, the general evaluation of the MMC of the groups in the Reflux activity is shown in Table 6. It is observed that the groups understood the problem situation, made assumptions about the modeling situation, simplified and structured the situation, determined the variables and made predictions about these variables but could not mathematise. The T and P groups progressed to the real model step, which is the second step. It can be concluded that both groups understood the modeling situation by making a mental representation of the situation and obtained the real model based on this, but could not make a transition to the mathematical model. Therefore, the groups could not create a valid mathematical model and could not progress to the other steps correctly. While using the data related to the problem, they could not use a solution path that would lead to a solution to the problem.

Table 6. MMC of the groups in the Reflux activity

MMC	Insufficient	Partially Sufficient	Sufficient
Understanding			T, P, U
Simplification and structuring	U	P	T
Mathematization	T, P, U		
Mathematical work	T, P, U		

Interpretation	T, P, U		
Validation	T, P, U		

The reflective diaries provide clues about why the groups could not access the real model. For example, T3 and P3 stated the following;

T3: Again, we had to do logarithmic operations while designing this model. I did not have any experience with such operations. This situation made us very difficult.

P3: We determined the critical values for neutralization, which will form the basis of the model. We calculated the molar value of baking soda added to water based on the information we gained here. Similarly, we determined how many grams of 20 ml of stomach acid were required using its molarity. We had much difficulty doing these operations; although we did research, we still had difficulty. I realized that mathematical operations are also significant for such models.

In the reflux activity, participants from all groups reported deficiencies in chemistry subjects, difficulty remembering the subjects, and problems in the MM process. T3 stated that he had trouble remembering the chemistry knowledge he acquired during his undergraduate education because he did not use it in his professional life;

T3: I realized that I needed to remember the concepts such as molarity, mole, and molecular weight, which I learnt during my undergraduate education but rarely used in my professional life, and I could not use them sufficiently.

Findings on Houseboats Activity

The general evaluation of the MMC of the groups in the Houseboats activity is shown in Table 7. Upon analyzing Table 7, it is evident that the groups improved their competencies compared to the previous activities. While the T and P groups tested their mathematical working competencies for the first time, the U group tested their mathematization competencies for the first time. The U group remained in the real model stage, which is the 2nd stage, while the T and P groups progressed to the mathematization stage, which is the 3rd stage. Group U made assumptions about the modeling situation, structured it by simplifying it, identified the variables, and made predictions about them, but could not mathematise. The T and P groups, on the other hand, established the mathematical model and created a mathematical problem but could not solve the mathematical problem.

Table 7. MMC of the groups in the Houseboats activity

MMC	Insufficient	Partially Sufficient	Sufficient
Understanding			T, P, U
Simplification and structuring		U	T, P
Mathematization	U	T, P	
Mathematical work	T, P, U		
Interpretation	T, P, U		
Validation	T, P, U		

When the reflective diaries of the participants about the activity are examined, clues about why the groups were insufficient in mathematization and mathematical study competencies emerge. Group U, which progressed to the mathematization stage but could not create a mathematical model, could not create a model because they could not establish the relationship between concepts and variables. For example, group member U2 made the following statement;

U2: Actually, we found all the necessary information, the concepts of surface area or weight that pressure depends on, but we could not gather them in a formula due to lack of information. In this activity, we were weak in creating formulas.

T3 from the T group, who progressed to the mathematical results stage but was insufficient, emphasized that they had difficulty in unit conversions and attributed this to why they made incorrect operations. T3's statement is as follows;

T3: While calculating, we tried to develop the formula Weight=Lifting force. We had a lot of difficulty in unit conversions in the sinking volume we used to calculate the buoyancy force and made many incorrect operations. Using tools such as Excel for all modeling examples may be helpful.

The members of P, another group that progressed to the stage of mathematical work, similarly drew attention to the problems they experienced in determining and converting units. For example, group members P2 and P3 made the following statements.

P3: The part we have difficulty with is that we did not make data transformations while making assumptions.

P2: One of my difficulties was forgetting where the units used in the formula should be converted.

Findings on Plastic Bacteria Activity

The general evaluation of the MMC of the groups is shown in Table 8. When Table 8 is examined, all groups failed to demonstrate the mathematization competencies of the MM process. While there was a decrease in their competencies in the Houseboats activity, which was the previous activity, it is observed that there was an increase in their competencies compared to the Melisa Project, which is another activity related to the subject of biology.

Table 8. MMC of the groups in the Plastic Bacteria activity.

MMC	Insufficient	Partially Sufficient	Sufficient
Understanding			T, P, U
Simplification and structuring		T, U	P
Mathematization	T, P, U		
Mathematical work	T, P, U		
Interpretation	T, P, U		
Validation	T, P, U		

All the groups were able to progress to the real model stage, stage 2. They could understand the modeling situation and create a mental representation of it, but they needed assistance to transition to the mathematical model. They did not use a solution path that would solve the problem when using the related data. Therefore, they needed help to create a valid mathematical model or to correctly complete the other steps. Inadequacies in the first steps of modeling negatively affected the subsequent steps.

This activity provides clues as to why they could not reach the real model and solutions in the modeling process. For instance, group T could not present a real model be presented due to excessive simplification. T3' opinion on this situation is as follows;

T3: We prepared a model with the temperature values of the seasons as variables. However, we should have included many variables, such as the pH value of the environment, the amount of oxygen, and the type of bacteria. I was not comfortable making these omissions. I thought inwardly that these variables were crucial and should be addressed. However, we had no data on the neglected variables to test the model. Therefore, it took work to decide whether these variables were influential or not. Although we did not want to, we increased the number of omitted variables.

Group U had problems organizing and simplifying the variables they identified.

U2: We created a formula, but again there were missing things because our formula was working on the numbers, we determined ourselves, but we did not test how the formula works when there is any change, and we were again insufficient in this regard. Everything is what we know or what we can do, but when it comes to the formula, we did not know what and how to use it, so we had difficulties in creating a formula.

Findings on Ozone Activity

Table 9 shows that the teacher groups successfully performed in all MMC for the first time and completed the process. Group U, on the other hand, was evaluated as insufficient in the following process because they did not perform adequately in MMC. Group U was able to reach the simplification and structuring stage, Stage 2.

Table 9. MMC of the groups in the Ozone activity.

MMC	Insufficient	Partially Sufficient	Sufficient
Understanding			T, P, U
Simplification and structuring			T, P, U
Mathematization	U	T	P
Mathematical work	U		T, P
Interpretation	U		T, P
Validation	U		T, P

Group U could understand the modeling situation and create a mental representation of it, but they needed help to transition to the mathematical model. Groups T and P completed all stages of this activity. It is observed that the groups that can transition to the mathematical model can transition to mathematical results, and the groups that can transition to mathematical results can progress to real results.

When the reflective diaries of the participants regarding the Ozone activity are analyzed, three main clues emerge as to why the U group could not access the real model. Firstly, as a group, they needed more knowledge about the subject, and therefore, they spent their time on research and data collection. Secondly, they needed help in mathematising the relevant quantities and the relationships between quantities. Since the group could not mathematise the problem, they went for an immediate solution. Thirdly, the affective problems they experienced affected them in the modeling process. The opinions of Group U about their need for sufficient knowledge on the subject are as follows.

U2: … I had heard about CFC gas in chemistry class before. I had coded it in my mind as a gas coming out of refrigerators. I had no information about its harm or benefit. When we started the problem, we encountered a lot of CFC gas. I tried to find enough data, but my searches did not give me very clear data

U3: … In fact, at first glance, it seemed easy because I thought that if we calculate the cfc emission and proportion its effect on the Ozone layer, we could find the effect. Since we did not have much information about cfc, we did research from the meteorology page and various sites.

Some of the opinions about the affective problems that prevented the group from reaching the real model are as follows;

U1: I tried to find enough data, but my searches did not provide very clear data." I felt that "we will not be able to solve the problem." I felt very insufficient, and I was insufficient anyway.

U2: Our mistake was to be too hasty in searching for data and to give up immediately when we could not find any. Once we felt that "It will not happen, we cannot do it", we stopped and used to believe in ourselves. That is why we did not achieve any results.

When the opinions of the T and P groups, who reached the real model and results in the ozone activity, are analyzed, it is seen that the problems experienced by the participants are because what they learnt in chemistry subjects did not go through a meaningful and permanent learning process. In addition, their inability to make unit conversions correctly, which was also observed in previous activities, creates problems in the MM process.

P1: This activity was the most difficult, the longest, we could not find a common point, and we thought too much and even burned our brains. The reason was not knowing analytical chemistry, I think I was incomplete. Yes, we had an idea, we knew the way to go, we knew what we needed to do, but when we came to the model, we got stuck. We did a lot of research and finally made a modeling, was it a definite solution? No, it was not. When you think about it, it was a very broad subject, we had a chemical equation, what would we do now? I was thoroughly confused.

P2: Since we sometimes overlooked that the units should be the same in the formulas we used in the activity, errors occurred in the operations. This again caused our activity process to be prolonged.

T3: The most difficult point in this activity was to put the formulas we use in chemistry into practice. There needed to be more than just knowing the formula to be used to solve that operation. We initially thought that the solution would be solved with the concept of mole, but we had a lot of difficulty in the process time due to the fact that we did not record the operations in order.

T1: We realized that we used some data in the wrong places in the formulas to be used in calculating the law of multiple ratios and mole concepts in chemical reactions. I think this is because we forgot how to use the formulas. For example, in the formula n = m / ma, we should have taken the atomic weight for the ma of Cl, but we took a different value.

Findings on Slides Activity

The MMC of the groups in the slide activity are shown in Table 10. The groups performed sufficiently in all stages of the MM process. All groups understood the modeling situation and transitioned to the mathematical model. The groups that were able to make a transition to the mathematical model were able to make a transition to mathematical results, and the groups that transitioned to mathematical results were able to progress to real results. All stages in this activity were completed by the groups.

Table 10. MMC of the groups in the Slides activity.

MMC	Insufficient	Partially Sufficient	Sufficient
Understanding			T, P, U
Simplification and structuring			T, P, U
Mathematization		T, U	P
Mathematical work			T, P, U
Interpretation			T, P, U
Validation			T, P, U

Some of the participants' opinions about the overall process in the Slide activity are as follows;

P1: I think this activity was the easiest, the lack of numerical data made me nervous, but we created this model the easiest. I think the reason is that it was the last activity and we now know what needs to be done more easily. In the first activity, we were like fish out of water, we were thinking what will happen now why are we here, but now it was not like that, but we had a very good command of each step, our activities took a long time, but we had learnt the stages.

T1: In this problem, which I initially thought that I would not be able to solve, I saw that the solution could be reached by focusing on the right variables affecting the situation and using the theoretical knowledge correctly. As a result of this activity, I believe that even a very difficult problem that depends on many variables can be easily solved with mathematical modeling as a result of thorough and detailed thinking.

U3: Of course, we decided to move in this direction by thinking about what our teachers asked, we valued some things, we realized a little late that these assumptions of ours were not actually necessary. When we realized, we came to the conclusion that the formula we had was completely wrong. We had to redesign the whole thing from scratch. We started from the beginning with an inclined plane and formulas. This time we proceeded by finding a new formula over the formula for everything we did not know. Now everything started to settle down and we said and realized our mistake.

T3: The fact that the modeling was done in a certain process made our work much easier. It is obvious that if the stages of the processes are not separated, the assumptions will increase and more erroneous and general modeling will be created.

In examining the group members' opinions about the transition process to the real and mathematical models, it is understood that the model they obtained did not work because they could not determine the relevant variables and assumptions correctly at the beginning. They could not reach the real model since they tried to solve the problem directly by focusing on the mass variable. By increasing their knowledge of the subject, concentrating on the questions of the guide and spending extra time, they were able to create a real model this time. They succeeded in the MM when they identified the relevant variables and revealed the relationship between them. In addition, the fact that they frequently encountered MMA and mastered the process is a fundamental reason for their success. Since the subject is related to physics, the activity is initially perceived as accessible. The reason for this is the presence of fewer variables in Physics subjects than in Biology and Chemistry subjects. Some of the participants emphasized the inadequacy of their knowledge of mathematics content. In addition, making assumptions by focusing on the correct variables affecting the situation in the problem, that is, simplifying and structuring the problem situation correctly, is another reason for their success in this process. Finally, it is implied by the participants that there should be no time constraints in these high-level activities.

DISCUSSION & CONCLUSION

This study examines the mathematical modeling competencies of pre-service science teachers and teachers, discussing each competency separately.

Understanding competence

This study revealed that all groups successfully performed in the understanding competences. Similarly, in the literature, it is seen that individuals or groups involved in MMP do not have problems achieving understanding competence (Türker et al., 2010; Bukova-Güzel, 2011; Çiltaş, 2011; Ji, 2012; Gatabi & Abdolahpour, 2013; Kol, 2014; Çakmak, 2019; Derin & Aydın, 2020). In this study, it can be said that the activities of "I noticed-curious" under the guide's leadership and the discussion environment on MME at the beginning contributed positively to understanding the problem. However, the groups did not demonstrate the sub-competency of drawing the representation of the problem situation in any activity. The video-based modeling activities present mental representation. Because video-based modeling tasks are seen to have authentic features regardless of whether they contain real-world evidence or not, they provide the opportunity to explore the problem situation (Greefrath & Vos, 2021).

Simplification and structuring competence

The findings obtained from the activity forms, video recordings, and reflective diaries show that the groups struggled with simplifying and structuring the problem. The pre-service science teachers were unable to simplify and structure the problem situation in two activities (MP and RE), and the teacher groups struggled with the same issue in one activity (MP). Literature review reveals similar findings, emphasizing that participants make simple simplifications in the MM process, thus their simplification and structuring competencies are at low levels (Ikeda, 1997; Blum & Borromeo Ferri, 2009; Başkan, 2011; Şen Zeytun, 2013, Güç; 2015; Deniz & Yıldırım, 2018).

All groups failed at the real model stage in MP, the first activity in the study. In subsequent activities, it appears that the groups mostly passed this stage with sufficient performance. It is concluded that as participants become familiar with the MM process and gain experience, they improve in making assumptions, simplifying the situation, identifying and naming the quantities, qualities, and critical variables affecting the situation, establishing relationships between variables, finding existing knowledge, or making appropriate and accessible predictions. Other studies also report that pre-service teachers struggle to clarify the goal while solving non-routine MM problems (Kertil, 2008; Başkan, 2011; Derin & Aydın, 2020). This is because prospective teachers are not accustomed to MM problems related to daily life (Korkmaz, 2010; Başkan, 2011; Derin & Aydın, 2020).

The analysis of the reflective diaries reveals that the problems experienced stem from needing a better understanding of the subject and the need to establish relationships between related concepts. Similarly, Küçüközer (2010) and Başkan (2011) also discuss the problems arising from the inability to fully grasp the concepts and their interrelationships. Güç (2015) concludes that individuals' lack of detailed knowledge and experience about the subject causes them to struggle in determining the variables affecting the problem situation.

Mathematization competence

In this study, it was observed that pre-service science teachers and teachers had problems in all stages of MM, primarily in the transition from the real model to the mathematical model. The pre-service science teachers could not demonstrate MMC in three of the six activities (RE, PB, OZ), and the teacher groups could not create mathematical models in two (RE, PB). Many studies in the literature reveal problems in the transition phase from real model to mathematical model (Blum & Leiß, 2007; Kertil, 2008; Blum & Ferri, 2009; Borromeo Ferri, 2010; Türker et al., 2010; Frejd & Ärlebäck, 2011; Güç, 2015; Çakmak, 2019), emphasizing that the most problems in the MM process are seen in the transition from the real model to the mathematical model and in the mathematization competence (Stillman, 2006; Biccard & Wessels, 2011; Galbraith &; Gatabi & Abdolahpour, 2013; Çakmak, 2019) support the results of this study.

As seen in this study, it is concluded that the groups whose MM process experiences increased are exemplary in reaching the mathematical model after reaching the real model. Participants who determine the relevant variables and assumptions for the problem situation and reveal the relationship between them can mathematise the problem situation. It is seen that the pre-service science teachers successfully demonstrated their mathematization competencies in the last activity (YE) and the teacher groups in the last two activities (OZ, YE). In his study, Çakmak (2019) concluded that all pre-service teachers who made predictions by determining a variable performed the mathematisation competently. It is also seen in other studies in the literature that pre-service teachers who could not perform the mathematisation competence at the beginning were successful in this competence throughout the MMS (Kertil, 2008; Biccard & Wessels, 2011; Çiltaş, 2011; Ji, 2012; Güç, 2015; Kaiser & Brand, 2015). In line with the findings obtained from the observations and reflective diaries, it can be said that the reason for the inadequacy of the groups at the mathematization stage stems from the tendency to solve the problem directly.

Similarly, Başkan (2011) and Dede and Yılmaz (2016) found that students mostly estimated some numbers instead of making assumptions, while Çakmak (2019) found that pre-service teachers generally assigned a value to the variables affecting the situation. However, the problems experienced in the previous stage also affect this stage. When pre-service teachers or teachers cannot sufficiently reveal the relevant variables and the relationships between these variables, they fail in the mathematization stage (Başkan, 2011; Deniz & Yıldırım, 2018; Çakmak, 2019; Derin & Aydın, 2020).

Mathematical working competence

In evaluating mathematical working competence in this study, pre-service science teachers had no problems in one activity (KA), and teacher groups had no issues in two of the three activities where they passed the mathematization stage (OZ and KA). However, teacher groups failed in one activity (YE) after passing the mathematization stage. Generally, other studies have found that pre-service teachers and teachers successfully demonstrate mathematical working competence (Biccard & Wessels, 2011; & Kaiser & Brand, 2015; Çakmak, 2019).

Observations and examination of the reflective diaries showed that participants' failure in the mathematical work phase was due to carelessness in determining units and converting units. This result aligns with Obaidat and Malkawi's (2009) and Başkan's (2011) studies. This study also revealed that science teachers and pre-service teachers' insufficient command of logarithms, exponential functions, and similar mathematical contexts and deficiencies in mathematical operation skills caused problems in the mathematical operations phase.

Başkan (2011) found that they had gaps in subjects such as derivative, differential, and trigonometry for basic mathematics knowledge in using mathematical expressions and carrying out mathematical operations in physics courses. However, as seen in this study, using technological tools such as Excel, computer-based calculation programs, and similar technological tools helps to overcome deficiencies in the participants' mathematical operation skills. Similar studies also state that the use of technology significantly contributes to mathematical operations competence and, thus, to MMC (Stohlmann, 2012; Molina-Toro et al., 2019; Ortiz, 2020).

Interpretation competence

This study concluded that the groups who completed the mathematical study phase were exemplary in the interpretation phase. Due to the nature of science and the solid real-life context of problem situations, the familiarity of pre-service science teachers and teachers with real results can be considered the reason for this situation. In parallel with this result, Güç (2015) states that science teacher candidates and teachers who have enough experience with the real context tend to interpret the mathematical results they obtain in the real context. Although there are studies that emphasize that individuals who experience MM have difficulty in interpreting mathematical results in the real world (Maaß, 2006; Özer-Keskin, 2008; Türker et al., 2010; Bukova-Güzel, 2011; Çiltaş & Işık, 2013), the results obtained in this study coincide with the studies (Blum, 2011; Tekin-Dede & Yılmaz, 2013; Güç, 2015; Çakmak, 2019) that conclude that experience in MM is effective in questioning what the obtained results mean in real life.

Validation competence

Tekin-Dede and Yılmaz (2013) state that enabling pre-service teachers to work on MME by following the MM cycle reveals verification competence. In the literature, many studies show that the most challenging stage and competence of individuals or groups in MMS is verification (Özer-Keskin, 2008; Tipi, 2009; Bukova-Güzel & Uğurel, 2010; Başkan, 2011; Biccard & Wessels, 2011; Ji, 2012; Gatabi & Abdolahpour, 2013; Çakmak; 2019). In these studies, some reasons for the inadequacies in the verification phase are summarized as the design of the MME learning environment, the duration of the MME education, and the fact that MME is not in a structure that allows different solutions. Again, when the results of these studies are examined, it is emphasized that the reasons for the inadequacies in the verification stage are that pre-service teachers do not give this stage much importance, it is perceived as difficult and complex, it is kept short, the competence to think of different ways to solve the problem is not exhibited, and since it is challenging to create a mathematical model, there is no desire for a second model.

This study concluded that the groups who completed the mathematical study stage were exemplary in the verification stage, similar to the interpretation stage. It differs from the literature in terms of the results obtained. Güç (2015) found a partial improvement in the verification sub-competencies of pre-service teachers but stated that this development was not at the desired level. In the study, contrary to the literature, some factors come to the fore when we examine the reasons for the sufficient performance at the verification stage through the work of the groups and the opinions they conveyed in their reflective diaries. One can be said to be the environment's design by including experts in the field of activity as agents in the MM environment.

Mathematical modeling is a complex and long-term study. In this study, no time limit was set for the participants, and they were encouraged to conduct detailed research, use internet tools, and engage in long-term discussions when necessary. The fact that they were involved in a process that lasted for weeks and that there was no time limit for each activity can be considered to have turned the factors of the inadequacies in the verification phase, which are the duration of the MM education, the lack of competence to think of different ways to solve the problem, the short duration, and the lack of enthusiasm, into positive factors.

In this study, one of the reasons for the sufficient performance of pre-service science teachers and teachers is the structure of the activities. The fact that the real-life contexts of the activities designed under the MOP are strong, video-based, and unstructured without instructions plays a role in associating mathematical results with real results.

As a result, when all activities were considered, pre-service and in-service science teachers performed best in understanding competencies. Participants had difficulty simplifying and structuring competencies, i.e., identifying variables, simplifying the problem, and identifying relevant assumptions. In this study, it was determined that all of the pre-service teachers who were able to transition to mathematical results were able to transition to real results. Considering the performances in the last two phases, it is evident that pre-service science teachers and teachers are exemplary in making the transition from mathematical results to real results. Especially considering the progress made by the groups in the last two activities (OZ, KA). It is seen that the groups had no difficulty in understanding the problem situations, but they needed help in reaching the real model. When they could reach the real model, they could reach the real results easily. It is concluded that experience in MM effectively questions what the results mean in real life.

Considering the progress of the groups in the MM steps throughout the whole process, in the first activity, Melisa, all groups remained at the understanding stage. In contrast, in the last activity, Slide, all groups could progress to the verification stage. Considering the progress in the Ozone and Slide activities, it is seen that the groups that reached the mathematization stage also passed the following stages and completed the MM cycle. It is concluded that the groups that can transition to the mathematical model can transition to mathematical results, and those that can transition to mathematical results can progress to real results.

Another significant study result is that the pre-service teachers have different modeling cycles and solution processes in different modeling situations, and their modeling cycles are individually different. This was also found by Blum and Borromeo Ferri (2009), Blum and Leiß (2007) and Matsuzaki (2011). The reason for this difference was explained by Matsuzaki (2011) as "in the MM process since each individual's real and mathematical experiences are different from each other, their approaches to the modeling situation, solution processes, and results differ from each other" (Çakmak, 2019, p. 178).

When the process is evaluated in terms of groups, it is seen that teacher groups performed better than pre-service teachers. When we consider the groups of experienced and newly graduated teachers, it is revealed that both groups exhibited the same performances in all activities.

When the subject areas are considered, the groups exhibited the lowest performances in biology activities and the highest in physics activities. It is seen that this is because what they learned in chemistry subjects did not go through a meaningful and permanent learning process. It means that they need to encounter more problem situations related to daily life. U1 and U2 stated that they had deficiencies in chemistry subjects, they had difficulty remembering the subjects, and therefore, they had problems in the MM process. T3 stated that he had difficulty remembering the chemistry knowledge he acquired in his undergraduate education because he did not use it in his professional life; P2 stated that he did not use the concepts and formulas related to the subject at the undergraduate level later on; U1 and U2 from the U group said with similar expressions that they could not remember the chemistry subjects because they did not use them and therefore had problems in reaching the real model.

In line with the findings obtained from the observations and activity forms, it is concluded that the problems encountered in demonstrating competencies in the MM process stem from the following reasons;

- Groups' tendency to solve the problem directly.
- They need more knowledge about the subject.
- Too much simplification
- Failure to establish relationships between concepts
- Identifying and delimiting variables and establishing relationships between variables
- What they experience in affective terms

It is concluded that the improvement in the mathematical modeling competencies of pre-service science teachers and teachers during the research process is due to the following factors;

- Group Work
- Structure of MM task

- Guidance
- Detailed research on the subject
- Mathematical modeling process knowledge
- Long-term work
- Technology use
- No time limit

Limitations and Recommendations

1) Since pre-service science teachers and teachers have problems in the simplification and structuring competence and mathematization stage, it can be suggested that they improve their competencies by using a atomistic approach in line with this competence and stage.

2) It is seen that MM helps to reveal and eliminate misconceptions in science education. MM can be used to identify and eliminate misconceptions.

3) It is thought that teachers and pre-service teachers should be educated about mathematical modeling. Interdisciplinary and mathematical modeling studies be introduced in mathematics and other branches through in-service training seminars, and pre-service teachers can be informed about the interdisciplinary association and mathematical modeling activities through courses to be created in pre-service teacher education.

4) Mathematical modeling studies in the field of science are almost non-existent. More mathematical modeling studies should be conducted, and academicians should be encouraged to use them in their courses.

5) When the MM studies in the field of science are examined, it is seen that the studies conducted are in the subjects of physics, while almost no studies have been conducted in biology and chemistry at all levels. Therefore, conducting mathematical modeling studies in different science subject areas may be recommended.

6) This study shows that science teachers are better than pre-service teachers in demonstrating competencies in the mathematical modeling process. Since it is predicted that introducing and experiencing modeling studies to science teachers will lead to more positive results in practical terms, it can be recommended to provide mathematical modeling training through in-service applications.

7) It may be recommended to encourage mathematical modeling studies by bringing together groups from different disciplines, such as science and mathematics, science and classroom teachers, students and teachers.

8) It may be recommended to conduct different measurement and evaluation studies to determine the mathematical modeling competencies of pre-service science teachers and teachers. Developing a mathematical modeling competencies scale may be suggested, especially for researchers who want to work quantitatively.

9) It may be recommended to offer mathematical modeling education courses at the master's and doctoral levels since providing postgraduate mathematical modeling courses in science education will increase the number of teachers who have gained the knowledge and skills of interdisciplinary work in the field of education.

Statements of Publication Ethics

The approval of ethics committee for the present study was given by Erzincan Binali Yıldırım University Social and Humanities Ethics Committee with the issue number 66653 and authors declare that the principals of research and publication ethics were followed.

Researchers' Contribution Rate

The contribution rate of each author in the manuscript is equal.

Conflict of Interest

The authors declare that there is no conflict of interest.

REFERENCES

Akgün, L., Çiltaş, A., Deniz, D., Çiftçi, Z., & Ahmet, I. Ş. I. K. (2013). İlköğretim matematik öğretmenlerinin matematiksel modelleme ile ilgili farkındalıkları [Elementary mathematics teachers' awareness of mathematical modelling]. *Adıyaman Üniversitesi Sosyal Bilimler Enstitüsü Dergisi* (*12*), 1-34. https://doi.org/10.14520/adyusbd.410

Anhalt, C. O., & Cortez, R. (2016). Developing understanding of mathematical modeling in secondary teacher preparation. *Journal of Mathematics Teacher Education*, *19,* 523-545. https://doi.org/10.1007/s10857-015-9309-8

Antonius, S., Haines, C., Jensen, T. H., Niss, M., & Burkhardt, H. (2007). Classroom activities and the teacher. In *Modelling and applications in mathematics education* (pp. 295-308). Springer.

Başkan, Z. (2011). Doğrusal ve düzlemde hareket ünitelerinin matematiksel modelleme kullanılarak öğretiminin öğretmen adaylarının öğrenmelerine etkileri [The effectiveness of teaching one and two dimensional motion on prospective teachers' learning using mathematical modeling] (Doctoral dissertation). Karadeniz Technical University, Trabzon.

Berry, J. S., & Houston, S. K. (1995). *Mathematical modelling*. Edward Arnold.

Biccard, P., & Wessels, D. C. J. (2011). Documenting the development of modelling competencies of grade 7 mathematics students. In G. Kaiser, W. Blum, R. B. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 375-383). Springer.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. B. Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15-30). Springer. https://doi.org/10.1007/978-94-007-0910-2_3

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45-58.

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching Mathematics and Its Applications*, *22*(3), 123-139.

Blum, W., & Leiß, D. (2007). How do teachers deal with modeling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering and economics* (pp. 222-231). Horwood Publishing.

Blum, W., & Leiß, D. (2008). Investigating quality mathematics teaching – The DISUM project. In C. Bergsten, & B. Grevholm (Eds.), *Developing and researching quality in mathematics teaching and learning: Proceedings of MADIF 5* (pp. 3-16). Linköping University.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt für Didaktik der Mathematik, 38*(2), 86-95.

Borromeo Ferri, R. (2007). Modelling problems from a cognitive perspective. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering and economics* (pp. 260-270). Horwood Publishing.

Borromeo Ferri, R. (2010). On the influence of mathematical thinking styles on learners' modeling behavior. *Journal für Mathematik-Didaktik, 31*, 99-118.

Borromeo Ferri, R. (2011). Effective mathematical modelling without blockages- A commentary. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 181-185). Springer.

Borromeo Ferri, R. (2012, July). Mathematical thinking styles and their influence on teaching and learning mathematics. Paper presented at the meeting of the *12th International Congress on Mathematical Education*, Seoul, Korea.

Brand, S. (2014). Effects of a holistic versus an atomistic modelling approach on students' mathematical modelling competencies. In *Proceedings of the joint meeting of PME 38 and PME-NA 36* (pp. 185-191). PME.

Bukova Güzel, E. (2019). *Matematik eğitiminde matematiksel modelleme [Mathematical modeling in mathematics education]*. Pegem Academy.

Bukova Güzel, E., & Uğurel, I. (2010). Matematik öğretmen adaylarının analiz dersi akademik başarıları ile matematiksel modelleme yaklaşımları arasındaki ilişki [The relationship between pre-service mathematics teachers' academic achievement in analysis course and their mathematical modeling approaches]. *Ondokuz Mayıs University Journal of Education Faculty, 29*(1), 69-90.

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage.

Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into Practice, 39*, 124-130.

Çakmak, Z. (2019). Matematik öğretmeni adaylarının matematiksel modelleme süreçlerinin bilişsel açıdan incelenmesi [The investigation of mathematical modelling processes of pre-service mathematics teachers from a cognitive perspective] (Doctoral dissertation). Atatürk University, Erzurum.

Çiltaş, A. (2011). Dizi ve seriler konusunun matematiksel modelleme yoluyla öğretiminin ilköğretim matematik öğretmeni adaylarının öğrenme ve modelleme becerileri üzerine etkisi [The effect of teaching sequences and series subject through mathematical modeling on the learning and modeling skills of prospective elementary mathematics teachers] (Doctoral dissertation). Atatürk University, Erzurum.

Çiltaş, A., & Işık, A. (2013). The effect of instruction through mathematical modelling on modelling skills of prospective elementary mathematics teachers. *Educational Sciences: Theory & Practice, 13*(2), 1187-1192.

Doerr, H. M. (2007). What knowledge do teachers need for teaching mathematics through applications and modeling? In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education (ICMI 14)* (pp. 69-78). Springer.

Doruk, B. K. (2010). Matematiği günlük yaşama transfer etmede matematiksel modellemenin etkisi [The impact of mathematical modeling in transferring mathematics to daily life] (Doctoral dissertation). Hacettepe University, Ankara.

Erbaş, A. K., Çetinkaya, B., Çakıroğlu, E., & Alacası, C. (2013). Ortaöğretim matematik eğitiminde matematiksel modelleme: hizmet içi ve hizmet öncesi öğretmen eğitimi *[Mathematical modeling in secondary mathematics education: in-service and pre-service teacher training]* *https://app.trdizin.gov.tr/publication/project/detail/TVRReU5qRXg*

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics, 63,* 303-323.

Gatabi, A. R., & Abdolahpour, K. (2013). Investigating students' modeling competency through grade, gender, and location. In B. Ubuz, C. Haser, & M. A. Mariotti (Eds.), *Proceedings of the 8th congress of the European society for research in mathematics education (CERME 8)* (pp. 1070-1077). Middle East Technical University.

Glesne, C. (2016). *Becoming qualitative researchers: An introduction*. Pearson.

Güç, F. A. (2015). Matematiksel modelleme yeterliklerinin geliştirilmesine yönelik tasarlanan öğrenme ortamlarında öğretmen adaylarının matematiksel modelleme yeterliklerinin değerlendirilmesi [Evaluation of pre-service teachers' mathematical modeling competencies in learning environments designed to develop mathematical modeling competencies] (Doctoral dissertation). Karadeniz Technical University, Trabzon.

Greefrath, G. (2020). Mathematical modelling competence: Selected current research developments. *Avances de Investigación en Educación Matemática, 17*, 38-51.

Greefrath, G., & Vos, P. (2021). Video-based word problems or modelling projects—Classifying ICT-based modelling tasks. In *Mathematical modelling education in East and West* (pp. 489-499). Springer.

Güder, Y. (2019). Matematiksel modelleme yoluyla disiplinler arası geçiş [Interdisciplinary transition through mathematical modeling] (Doctoral dissertation). Adıyaman University Adıyaman.

Henning, H., & Keune, M. (2007). Levels of modelling competencies. In *Modelling and applications in mathematics education* (pp. 225-232). Springer.

Ji, X. (2012, July). A quasi-experimental study of high school students' mathematics modelling competence. Paper presented at the meeting of the 12th International Congress on Mathematical Education, Seoul, Korea.

Kaiser, G. (2007). Modelling and modelling competencies in school. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering and economics* (pp. 110-119). Horwood Publishing.

Kaiser, G. (2020). Mathematical modelling and applications in education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 553-561). Springer.

Kaiser, G., Blomhøj, M., & Sriraman, B. (2006). Towards a didactical theory for mathematical modelling. *Zentralblatt für Didaktik der Mathematik, 38*(2), 82-85. https://doi.org/10.1007/BF02655882

Kaiser, G., & Brand, S. (2015). Modelling competencies: Past development and further perspectives. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice* (pp. 129-149). Springer.

Kaiser, G., & Grünewald, S. (2015). Promotion of mathematical modelling competencies in the context of modelling projects. In N. H. Lee & N. K. E. Dawn (Eds.), *Mathematical modelling from theory to practice* (pp. 21-39). World Scientific Publishing.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *Zentralblatt für Didaktik der Mathematik, 38*(3), 302-310.

Kaiser, G., & Stender, P. (2013). Complex modelling problems in co-operative, self-directed learning environments. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 611-620). Springer.

Karasar, N. (2016). *Bilimsel araştırma yöntemi: Kavramlar, ilkeler, teknikler* (30th ed.). Anı Yayıncılık.

Kapur, J. N (2023). *Mathematical Modeling*. Berlin, Boston: Mercury Learning and Information (pp, 300). https://doi.org/10.1515/9781683928737

Kenan, A., & Polat, R. (2022). Matematik ve fen öğretmen eğitiminde kullanıma yönelik matematiksel modelleme etkinlikleri geliştirme çalışması [Developing mathematical modeling activities for use in mathematics and science teacher education]. *Academic Social Resources Journal, 7*(36), 472-483.

Kertil, M. (2008). Matematik öğretmen adaylarının problem çözme becerilerinin modelleme sürecinde incelenmesi [Investigation of prospective mathematics teachers' problem-solving skills in the modeling process] (Doctoral dissertation). Marmara University, Istanbul.

Koç, D. (2020). An investigation on thesis and dissertations on mathematical modeling in Turkey in the last two decades (Master's thesis). Middle East Technical University, Ankara.

Korkmaz, E. (2010). İlköğretim matematik ve sınıf öğretmeni adaylarının matematiksel modellemeye yönelik görüşleri ve matematiksel modelleme yeterlikleri [Elementary mathematics and classroom teacher candidates' views on mathematical modeling and their mathematical modeling competencies] (Doctoral dissertation). Balıkesir University, Balıkesir.

Koceva Lazarova, L., Stojkovic, N., & Stojanova, A. (2022). The importance of mathematical modeling in STEM education. *STEM Education Notes, 1*(1), 9-18.

Leiss, D., & Blum, W. (2010). The role of the situation model in mathematical modelling: Task analyses, student competencies, and teacher interventions. *Journal für Mathematik-Didaktik, 31*, 119-141.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modelling perspective on mathematics teaching, learning and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning and teaching* (pp. 3-33). Lawrence Erlbaum.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought revealing activities for students and teachers. In R. Lesh & A. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-645). Lawrence Erlbaum.

Maaß, K. (2006). What are modelling competencies? *Zentralblatt für Didaktik der Mathematik, 38*(2), 113-142.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Molina-Toro, J. F., Rendón-Mesa, P. A., & Villa-Ochoa, J. (2019). Research trends in digital technologies and modeling in mathematics education. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(8), 1-13.

Niss, M., Blum, W., & Galbraith, P. (2007). How to replace the word problems. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 3-32). Springer.

Sriraman, B. (2006). Conceptualizing the model-eliciting perspective of mathematical problem solving. In *Proceedings of the fourth congress of the European society for research in mathematics education (CERME 4)* (pp. 1686-1695). Universitat Ramon Llull.

Stohlmann, M. (2012). YouTube incorporated with mathematical modelling activities: Benefits, concerns, and future research opportunities. *International Journal for Technology in Mathematics Education, 19*(3).

Tekin, A., & Bukova Güzel, E. (2011). Ortaöğretim matematik öğretmenlerinin matematiksel modellemeye ilişkin görüşlerinin belirlenmesi [Determination of secondary mathematics teachers' views on mathematical modeling]. In *Proceedings of Mehmet Akif Ersoy University Faculty of Education*, 8-10 September, Burdur.

Tekin-Dede, A., & Bukova-Güzel, E. (2018). A rubric development study for the assessment of modeling skills. *The Mathematics Educator, 27*(2).

Yıldırım, A., & Şimşek, H. (2018). *Sosyal bilimlerde nitel araştırma yöntemleri* [Qualitative research methods in social sciences] (13th ed.). Seçkin Yayıncılık.

Yıldız, Ş., & Yenilmez, K. (2019). Matematiksel modelleme ile ilgili lisansüstü tezlerin tematik içerik analizi [Thematic content analysis of graduate theses on mathematical modeling]. *Eskişehir Osmangazi University Journal of Social Sciences, 20*, 1-22.

Zawojewski, J. S., Lesh, R., & English, L. (2003). A models and modelling perspective on the role of small group learning activities. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning and teaching* (pp. 337-358). Lawrence Erlbaum.