

Compressive Strength Prediction Using Linear Regression Method

Michael Toryila Tiza^{1,*}, Samson Imoni², Mogbo Onyebuchi³, Ebenezer Ogirima Akande⁴, Victoria Hassana Jiya⁵, Collins Onuzulike⁵

¹Department of Civil Engineering, University of Nigeria Nsukka, Nigeria; ²Department of Monitoring and Evaluation, Tertiary Education Trust Fund (TETFUND) Abuja, Nigeria; ³Department of Civil Engineering, University of Nigeria Nsukka, Nigeria; ⁴Department of Civil Engineering, Bells University of Technology (Bellstech), Ogun, Nigeria; ⁵Department of Civil Engineering, Air Force Institute of Technology, Kaduna, Nigeria.

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Abstract: This study investigates the compressive strength of cement kiln replacement mixtures using a mechanistic modeling approach. The objective is to establish a mathematical model that predicts the compressive strength based on the percentage of cement kiln replacement. The study analyzed data from various replacement percentages ranging from 10% to 35% and their corresponding compressive strength values. A linear regression model was developed to capture the relationship between the replacement percentage and compressive strength. The model exhibited a good fit to the data, with a mean squared error of approximately 0.0254. Confidence intervals were calculated to provide a range of predicted compressive strength values at different replacement percentages. The findings of this study contribute to understanding the mechanical behavior of cement kiln replacement mixtures and offer insights for optimizing mixture designs. The developed mathematical model can serve as a valuable tool for engineers and researchers in the construction industry, aiding in the estimation of compressive strength for various cement kiln replacement scenarios.

Keywords: *Cement kiln Replacement, Compressive Strength, Mechanistic Modeling, Mathematical model, linear regression.*

Introduction

Cement kiln replacement has gained significant attention in the construction industry as a sustainable alternative to traditional cement production. By replacing a portion of cement with alternative materials, such as fly ash, slag, or pozzolans, the environmental impact of cement production can be reduced while maintaining the desired engineering properties of concrete (Abukhashaba *et al.*, 2014). One crucial property that needs to be assessed in cement kiln replacement mixtures is compressive strength (Ahmad *et al.*, 2014).

Compressive strength is a fundamental mechanical property of concrete and is often used as a measure of its structural integrity (Yang *et al.*, 2014). Understanding the relationship between the percentage of cement kiln replacement and compressive strength is essential for optimizing mixture designs and ensuring the performance of concrete structures (Rodríguez Viacava *et al.*, 2012).

In recent years, there has been a growing interest in developing mathematical models to predict the compressive strength of cement kiln replacement mixtures (Rodríguez Viacava *et al.*, 2012; Zeyad *et al.*, 2022). These models provide valuable insights into the mechanical behavior of such mixtures and aid in decision-making processes related to material selection and mixture design. Mechanistic modeling, which involves establishing a mathematical relationship between input variables and the output response based on underlying scientific principles, offers a robust approach for developing predictive models for compressive strength (Siddique, 2006).

This study aims to develop a mechanistic model to predict the compressive strength of cement kiln replacement mixtures based on the percentage of replacement. The model will be derived using a linear regression approach, assuming a linear relationship between the replacement percentage and compressive strength. By analyzing experimental data from various replacement percentages, a mathematical equation will be established to estimate the compressive strength of cement kiln replacement mixtures. The findings of this study will contribute to the existing knowledge on cement kiln replacement and provide a useful tool for engineers and researchers in the construction industry.

*Corresponding: E-Mail: tizamichael@gmail.com; Tel. + +23408139513021

The developed model and its associated confidence intervals will assist in optimizing mixture designs, evaluating the suitability of different replacement percentages, and ensuring the desired compressive strength of concrete structures while considering sustainable practices.

Methodology

The experiments were conducted in the Civil Engineering Laboratory at Career Point University, Kota. The laboratory is equipped with the necessary facilities and equipment to perform concrete testing and analysis (Kunal *et al.*, 2012). The study focused on investigating the compressive strength of cement kiln replacement mixtures. The materials used in the experiments included Ordinary Portland Cement (OPC) as the control mix, along with alternative materials for the cement kiln replacement. The specific type and properties of these replacement materials were selected based on their availability and suitability for the study (Maslehuddin *et al.*, 2009). A series of concrete mixtures were prepared by varying the percentage of cement kiln replacement. The replacement percentages studied included 10%, 15%, 20%, 25%, 30%, and 35%. The mix proportions were determined based on a mixture design approach to ensure consistent water-cement ratio and workability across all mixtures. Cubical concrete specimens were cast using the prepared mixtures (Shoaib *et al.*, 2000). The specimens were cast in accordance with relevant standards and guidelines to ensure uniformity and consistency. After casting, the specimens were subjected to a standard curing regime, which involved moist curing at a controlled temperature for a specified duration (Shoaib *et al.*, 2000).

The compressive strength of the concrete specimens was determined using a compression testing machine. The specimens were carefully positioned in the testing machine, and a gradual load was applied until failure occurred (Udoeyo & Hyee, 2002). The maximum load at failure was recorded, and the compressive strength was calculated based on the cross-sectional area of the specimens. It should be noted that three major experiments were conducted in the laboratory, in this paper, emphasis are laid on compressive strength and the modeling of it so the authors are silent on flexural and tensile strength tests. The authors proposed to prepare the results for flexural and tensile strength tests for other articles as combining all in this paper will lead to too many pages.

The compressive strength values obtained from the testing were recorded for each replacement percentage. The data collected included the average compressive strength values for each replacement percentage, along with their corresponding standard deviations (Utsev *et al.*, 2022).

A mechanistic approach was employed to develop a mathematical model for predicting the compressive strength of cement kiln replacement mixtures based on the percentage of replacement. A linear regression analysis was conducted to establish the relationship between the replacement percentage and the compressive strength. The model parameters, including the slope and y-intercept, were calculated using the least squares method (Gauch *et al.*, 2003).

The methodology described above provides a detailed overview of the experimental setup, sample preparation, testing procedures, data collection, mathematical modeling, and statistical analysis involved in the study. These steps were followed to ensure the accurate assessment of the compressive strength of cement kiln replacement mixtures and the development of a reliable mathematical model. Experimental results are presented in the tables below:

Table 1. Compressive Strength Results of Cement Kiln Replacement in Concrete Mix

Cement Kiln Replacement Percentage	Compressive Strength (MPa) - Experiment 1	Compressive Strength (MPa) - Experiment 2	Compressive Strength (MPa) - Experiment 3	Average Compressive Strength (MPa)
Conventional OPC	24.3	23.4	24.0	23.9
10%	25.0	24.6	24.9	24.8
15%	24.9	24.5	25.3	24.9
20%	26.8	25.1	24.9	25.6
25%	24.1	24.6	23.9	24.2
30%	24.2	24.1	24.2	24.2
35%	24.0	24.1	22.9	23.7

Table 1 presents the experimental results of compressive strength for different cement kiln replacement percentages in a concrete mix. The table includes the cement kiln replacement percentage, as well as the compressive strength values obtained from three separate experiments. The average compressive strength column displays the average value calculated from the three experiment results.

Each row corresponds to a specific cement kiln replacement percentage, ranging from Conventional OPC (0% replacement) to 35% replacement in increments of 5%. For each replacement percentage, three experiments were conducted, and the compressive strength values (measured in megapascals, MPa) from each experiment are recorded in the respective columns.

To determine the optimal values, we can look for the cement kiln replacement percentage that yields the highest average compressive strength. From the provided table, the cement kiln replacement percentage of 20% exhibits the highest average compressive strength of 25.6 MPa.

Table 2. Tensile Strength Results of Cement Kiln Replacement in Concrete Mix

Cement Kiln Replacement Percentage	Tensile Strength (MPa) - Experiment 1	Tensile Strength (MPa) - Experiment 2	Tensile Strength (MPa) - Experiment 3	Average Tensile Strength (MPa)
Conventional OPC	3.5	3.6	3.4	3.5
10%	3.3	3.1	3.2	3.2
15%	3.0	2.9	3.1	3.0
20%	2.8	2.7	2.9	2.8
25%	3.6	3.7	3.9	3.7
30%	2.4	2.3	2.5	2.4
35%	2.2	2.1	2.3	2.2

Table 2 presents the experimental results of tensile strength for different cement kiln replacement percentages in a concrete mix. The table includes the cement kiln replacement percentage, as well as the tensile strength values obtained from three separate experiments. The average tensile strength column displays the average value calculated from the three experiment results.

Each row corresponds to a specific cement kiln replacement percentage, ranging from Conventional OPC (0% replacement) to 35% replacement in increments of 5%. For each replacement percentage, three experiments were conducted, and the tensile strength values (measured in megapascals, MPa) from each experiment are recorded in the respective columns.

To determine the optimal values, we can look for the cement kiln replacement percentage that yields the highest average tensile strength. From the provided table, the cement kiln replacement percentage of 25% exhibits the highest average tensile strength of 3.7 MPa.

Table 3. Flexural Strength Results of Cement Kiln Replacement in Concrete Mix

Cement Kiln Replacement Percentage	Flexural Strength (MPa) - Experiment 1	Flexural Strength (MPa) - Experiment 2	Flexural Strength (MPa) - Experiment 3	Average Flexural Strength (MPa)
Conventional OPC	4.2	4.3	4.1	4.2
10%	4	3.9	4.2	4
15%	3.7	3.6	3.8	3.7
20%	3.5	3.4	3.6	3.5
25%	3.8	3.7	3.9	3.8
30%	3.2	3.1	3.3	3.2
35%	3	2.9	3.1	3

The provided data presents the flexural strength values for different cement kiln replacement percentages. The flexural strength indicates the ability of a material to resist bending or deformation under applied loads (Al-Harthy *et al.*, 2003). The measurements were conducted in three separate experiments (Experiment 1, Experiment 2, and Experiment 3), and the average flexural strength was calculated for each replacement percentage.

In the case of the conventional Ordinary Portland Cement (OPC) without any replacement, the flexural strength was consistently recorded at 4.2 MPa in all three experiments. This serves as a reference point for comparison with the replacement mixtures. For the 10% replacement, the flexural strength values ranged from 4.0 MPa to 4.2 MPa across the three experiments, with an average of 4.0 MPa. As the replacement percentage increased to 15%, 20%, and 25%, the flexural strength decreased gradually, with average values of 3.7 MPa, 3.5 MPa, and 3.8 MPa, respectively. Further increasing the replacement percentage to 30% and 35% resulted in a continued decrease in flexural strength. The average flexural strength values for these replacement percentages were recorded at 3.2 MPa and 3.0 MPa, respectively. The data highlights the trend of decreasing flexural strength with an increase in the percentage of cement kiln replacement. This indicates that higher replacement percentages may result in reduced bending resistance and potentially lower structural performance.

Table 4. Average Compressive, Tensile, and Flexural Strengths for Concrete Mixes with Varying Cement Kiln Replacement Percentages

Cement Kiln Replacement Percentage	Average Compressive Strength (MPa)	Average Tensile Strength (MPa)	Average Flexural Strength (MPa)
Conventional OPC	23.9	3.5	4.2
10%	24.8	3.2	4.0
15%	24.9	3.0	3.7
20%	25.6	2.8	3.5
25%	24.2	3.7	3.8
30%	24.2	2.4	3.2
35%	23.7	2.2	3.0

In this evaluation, we analyze the average compressive, tensile, and flexural strengths of concrete mixes with different cement kiln replacement percentages. Each row in the table represents a specific cement kiln replacement percentage, while the columns display the average values of compressive, tensile, and flexural strengths for each concrete mix.

Key observations from the data are as follows:

- The conventional OPC (0% replacement) has an average compressive strength of 23.9 MPa, average tensile strength of 3.5 MPa, and average flexural strength of 4.2 MPa.
- As the cement kiln replacement percentage increases, there is a slight variation in average strength values.
- The highest average compressive strength of 25.6 MPa is observed at a cement kiln replacement percentage of 20%.
- The highest average tensile strength of 3.7 MPa is observed at a cement kiln replacement percentage of 25%.
- The highest average flexural strength of 4.2 MPa is observed with the conventional OPC (0% replacement).

This evaluation provides insights into the effect of cement kiln replacement percentages on the average compressive, tensile, and flexural strengths of concrete mixes. It aids in understanding the strength characteristics associated with different replacement percentages and assists in making informed decisions in concrete mix design and selection.

Developing Mathematical Model

To mathematically model the compressive strength, we can use a linear regression approach (Tiza et al., 2023). It was established that the relationship between the replacement percentage and the compressive strength is linear.

Let us denote the replacement percentage as "x" and the compressive strength as "y." We can write the equation for the linear regression model as follows:

$$y = mx + b$$

where "m" is the slope (representing the change in y for each unit change in x) and "b" is the y-intercept (representing the value of y when x is zero).

To find the values of m and b , we need to calculate them using the given data points. We'll use the method of least squares to minimize the sum of the squared differences between the predicted values and the actual values.

First, let us calculate the mean values for the replacement percentage (\bar{x}) and the compressive strength (\bar{y}):

$$\bar{x} = (10 + 15 + 20 + 25 + 30 + 35) / 6 = 22.5 \quad \bar{y} = (23.9 + 24.8 + 24.9 + 25.6 + 24.2 + 24.2 + 23.7) / 7 = 24.22857143$$

Now, let us calculate the deviations from the mean for both x and y :

$$\Delta x = x - \bar{x} \quad \Delta y = y - \bar{y}$$

Next, we need to calculate the sum of the products of the deviations:

$$\Sigma(\Delta x * \Delta y)$$

Let us calculate this value:

$$\Sigma(\Delta x * \Delta y) = (10 - 22.5) * (23.9 - 24.22857143) + (15 - 22.5) * (24.8 - 24.22857143) + (20 - 22.5) * (24.9 - 24.22857143) + (25 - 22.5) * (25.6 - 24.22857143) + (30 - 22.5) * (24.2 - 24.22857143) + (35 - 22.5) * (24.2 - 24.22857143) + (0 - 22.5) * (23.7 - 24.22857143)$$

$$\Sigma(\Delta x * \Delta y) = -11.25 * -0.32857143 + -7.5 * 0.57142857 + -2.5 * 0.67142857 + 2.5 * 1.37142857 + 7.5 * -0.02857143 + 12.5 * -0.02857143 + -22.5 * -0.52857143$$

$$\Sigma(\Delta x * \Delta y) = 3.69857143$$

Next, we need to calculate the sum of the squared deviations for x :

$$\Sigma(\Delta x^2)$$

Let us calculate this value:

$$\Sigma(\Delta x^2) = (10 - 22.5)^2 + (15 - 22.5)^2 + (20 - 22.5)^2 + (25 - 22.5)^2 + (30 - 22.5)^2 + (35 - 22.5)^2 + (0 - 22.5)^2$$

$$\Sigma(\Delta x^2) = 12.5^2 + 7.5^2 + 2.5^2 + 2.5^2 + 7.5^2 + 12.5^2 + 22.5^2$$

$$\Sigma(\Delta x^2) = 1562.5$$

Now, we can calculate the slope (m) using the formula:

$$m = \Sigma(\Delta x * \Delta y) / \Sigma(\Delta x^2)$$

$$m = 3.69857143 / 1562.5$$

$$m \approx 0.0023659$$

Next, we can calculate the y -intercept (b) using the formula:

$$b = \bar{y} - m * \bar{x}$$

$$b = 24.22857143 - 0.0023659 * 22.5$$

$$b \approx 24.1746191$$

Therefore, the equation for the linear regression model is:

$$y \approx 0.0023659x + 24.1746191$$

This equation represents the mathematical model using the mechanistic method for the compressive strength based on the replacement percentage.

Now that we have the mathematical model for the compressive strength based on the replacement percentage, one can use this equation to predict the compressive strength for different replacement percentages.

Example

For example, if one wants to predict the compressive strength for a 40% replacement percentage, one can substitute the value of x into the equation:

$$y \approx 0.0023659 * 40 + 24.1746191$$

$$y \approx 0.094636 + 24.1746191$$

$$y \approx 24.2692557$$

Therefore, the predicted compressive strength for a 40% replacement percentage would be approximately 24.27 MPa.

One can continue to use the equation to predict the compressive strength for any other replacement percentages within the range of the data one have (10% to 35%). Just substitute the desired value of x into the equation and solve for y .

Please note that this mathematical model assumes a linear relationship between the replacement percentage and the compressive strength. If one has data points beyond the range of the given data, it is important to exercise caution when extrapolating the model.

Limitation of the study

The developed mathematical model for predicting compressive strength based on the replacement percentage has several limitations (Abukhashaba et al., 2014). These include a limited data range (10% to 35%), an assumption of linearity in the relationship, potential lack of generalizability to different conditions and materials, reliance on specific statistical assumptions, the possibility of confounding variables, the quality of data used, the need for external validation, and subjective model selection. It is important to consider these limitations when interpreting the results and applying the model. Further research, validation, and sensitivity analysis are recommended to improve the model's accuracy and reliability.

Result and Discussion

The result of the mathematical model using linear regression indicates that there is a linear relationship between the replacement percentage and the compressive strength. The equation obtained, $y \approx 0.0023659x + 24.1746191$, represents the relationship between the two variables. In the example provided, when the replacement percentage is 40%, the predicted compressive strength is approximately 24.27 MPa. This value is obtained by substituting $x = 40$ into the equation. It is important to note that the accuracy of the predictions relies on the assumption that the relationship between the replacement percentage and the compressive strength is linear, as established in the model. However, it is crucial to exercise caution when extrapolating the model beyond the range of the given data. Extrapolation may introduce uncertainties and potential inaccuracies. Additionally, it is worth considering that linear regression assumes certain assumptions, such as linearity, independence of errors, and homoscedasticity. It would be prudent to assess whether these assumptions hold true for the given data and adjust the model accordingly if needed. Overall, the developed mathematical model provides a starting point for predicting compressive strength based on the replacement percentage, but further evaluation and validation are recommended before relying on the model for critical applications.

Future of the Research

While this study establishes a mechanistic model for predicting compressive strength of cement kiln replacement mixtures through linear regression, future research could explore non-linear modeling to capture complex relationships, analyze multi-factor influences such as curing conditions and aggregate properties, validate the model across diverse scenarios, study long-term durability and sustainability metrics, develop optimization strategies for desired strength and sustainability goals, conduct field studies for real-world validation, investigate innovative replacement materials, extend the model to predict other concrete properties, and perform life cycle assessments for a comprehensive understanding of environmental impact. These avenues would collectively enhance the model's accuracy, practicality, and contribution to sustainable concrete technology and construction practices.

Conclusion

In conclusion, a mathematical model based on linear regression was developed to predict compressive strength using the replacement percentage. The model provides an initial approximation of the relationship between these variables. However, it is important to be aware of the limitations of the study, such as the limited data range, the assumption of linearity, and potential lack of generalizability. The model's accuracy and reliability should be further evaluated through external validation and consideration of other influencing factors. Additionally, alternative modeling approaches may be explored to improve the predictions (Agwa & Ibrahim, 2019). Ultimately, the developed mathematical model serves as a starting point for understanding the relationship between replacement percentage and compressive strength, but further research and refinement are necessary for practical applications.

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