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Research Article

How do Middle School Students Use Their Knowledge of Geometric Area Measurement When Determining Fractions?

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Abstract – In this study, the objective was to assess students' proficiency in utilizing their knowledge of geometric area measurement and fractions, and to examine how they apply this knowledge in determining fractions. The research, structured as a case study, encompassed nine students from the 6th, 7th, and 8th grades. Data gathered through individual interviews were analyzed using content analysis. The findings revealed that participants predominantly exhibited a preliminary internalization profile in terms of their understanding of fractions and area measurement. This profile encompasses stages such as pre-internalization, internalization, condensation, and reification. It was observed that students employed diverse strategies and methods within various segmented fraction area models, thereby unveiling distinct mental schemes. The responses provided by students to the tasks illustrated their comprehension of fractions and area measurement. Based on the research results, it is recommended that mathematics teachers and classroom instructors, tasked with imparting crucial concepts like fractions, devise tasks that integrate fundamental mathematical principles and establish connections across different domains.

Key words: Fraction, Area Measurement, Middle School Students.

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Introduction

The concept of fractions is inherently developmental, and it is believed that students require significant experience and time to grasp this concept thoroughly (Van de Walle et al., 2013). It is underscored that fractions hold a crucial place in mathematics education, and as students comprehend the concept of fractions, they also lay the groundwork for mathematical success in later stages (Bailey et al., 2012). Siegler et al. (2013) asserted that fraction instruction should be enhanced, highlighting that the competencies associated with fractions constitute a fundamental skill that must be cultivated in students. The importance and difficulty of teaching fractions were emphasized, given that fractions lack a singular structure, consist of interconnected sub-structures, and possess various meanings such as part-whole, division, measurement, operator, and ratio meanings (Kieren, 1976). Despite these challenges, numerous studies suggest the effective use of visuals and models in the teaching of fractions, making the concept more accessible (Cramer & Henry, 2002; Siebert & Gaskin, 2006; Doğan Temur, 2011; Van de Walle et al., 2013). Cramer et al. (2010) argued that models serve as essential tools, allowing students to experience fractions in real-world contexts. Among the models employed for fraction representation is the area model (Van de Walle et al., 2013), illustrating how fractions constitute a part of an area or the relationship of a region to the whole. As the area model aligns with concepts of division and equal sharing, it is deemed a valuable starting point for introducing fractions (Van de Walle et al., 2013). However, some researchers have noted the oversight in emphasizing that fractional parts should be of the same size in field models, neglecting the recognition that parts do not necessarily need to be identical (Lee & Lee, 2020; Van de Walle et al., 2013), leading to a limited understanding of the area model (Van de Walle et al., 2013; Zeybek & Cross Francis, 2017). According to certain researchers (Lee & Lee, 2020; Lee & Hackenberg, 2014; Olive & Steffe, 2010), comprehending the area models of fractions not only facilitates a robust understanding of fractions but also enhances understanding of concepts in geometric measurement and their interconnectedness across various mathematical domains. In this regard, it can be asserted that grasping the area model of fractions is of paramount importance for students.

Similar to the concept of fractions, the principles and competencies associated with area measurement encompass fundamental knowledge and skills essential for students in their daily lives (Gürefe, 2018). Research has demonstrated that although students may be acquainted with the area formula, they encounter challenges in measuring area and struggle to apply the formula to given situations (Chappell & Thompson, 1999; Tan Şişman & Aksu, 2009). This difficulty arises because students are often taught formulas through procedural calculations without a thorough understanding of how and why these formulas are employed (Kordaki & Balomenou, 2006). Therefore, it is imperative to develop a robust comprehension of the concept of area measurement. Indeed, the concept of area measurement serves as a foundational prerequisite for numerous other subjects and significantly influences students' comprehension of various mathematical domains (Cavanagh, 2008). Given that the conceptual understanding of area measurement is believed to encompass knowledge of area measurement strategies, it is crucial to assess students' conceptual grasp of this concept, as it is indicative of their high-level thinking abilities (Lehrer et al., 2003).

Upon reviewing the literature, numerous researchers (Hackenberg & Lee, 2015; Lee, 2017; Lee & Hackenberg, 2014; Steffe & Olive, 2010; Van de Walle et al., 2013) have asserted that teaching fractions through a measurement-based approach contributes to a more meaningful understanding of fractions as quantities. For instance, the measurement interpretation of fractions offers a natural context for introducing complex fractions and aids students in quantitatively evaluating fractions (Hackenberg & Lee, 2015; Lee, 2017; Steffe & Olive, 2010; Van de Walle, 2013). Additionally, Mitchell (2011) established a relationship between measurement and fractions in his research, focusing on length and area models of fractions. He found that students utilize fractional knowledge and geometric area measurement knowledge concurrently when solving problems related to area models. While Mitchell's (2011) research illuminates how the interpretation of measurement can be beneficial in teaching fraction concepts, it also underscores the limitations of a part-whole interpretation in fractions. In this study, students were tasked with solving problems involving atypically partitioned area models and expressing the strategies and reasoning they employed in this process. The aim was to scrutinize students' knowledge and reasoning regarding the area model of fractions by altering the typical representations of these models—specifically, by questioning how many fractions of different shapes in the area models could be identified when presented in an unconventional manner. Some studies (Kavuncu, 2019; Toptaş et al., 2017; Yakar, 2019; Yavuz Mumcu, 2018) have been conducted on how to utilize various models, such as length, cluster, and area models, when representing fractions. In contrast to those studies, this study reveals how the area model is employed in determining fractions, the level of knowledge related to fractions and area measurements, and the students' learning status regarding the subject by considering these knowledge levels collectively.

Conceptual Framework

Sfard (1991) presents a theoretical framework for acquiring mathematical concepts and asserts that the capability to perceive a number or function as both a process and an object is essential for achieving a profound understanding of mathematics, regardless of the concept's definition. Sfard distinguishes between operational and structural understandings of a mathematical concept. A student with operational understanding comprehends how a concept operates through algorithms, processes, and actions. On the other hand, structural understanding involves recognizing the concept as a mathematical object. While structural understanding is immediate, static, and integrative, operational understanding is dynamic, sequential, and detailed (Sfard, 1991). Sfard suggests that operational understanding should precede structural understanding, proposing a progression from working with physical models to using pictorial representations and eventually dealing solely with symbols. According to Sfard (1991), the transition from operational to structural understanding occurs in three stages: internalization, condensation, and reification. In the internalization phase, students engage with familiar objects and processes that lead to a new concept, internalizing the process through mental representations. At this stage, the process no longer requires physical execution but can be considered, analyzed, and compared mentally. An example is a student becoming adept at subtracting negative numbers. The condensation phase involves compacting long sequences of transactions into more manageable units, enabling students to think about a particular process as a whole without delving into details. Progress in condensation facilitates the ease of switching between different representations of the concept. For instance, in the context of negative numbers, this stage is where a student becomes proficient in arithmetic operations involving negative and positive numbers. The reification stage allows students to perceive the concept as a complete object, solve problems containing all examples meeting a certain condition, and initiate the internalization of higher-level concepts. For example, at this stage, in the case of negative numbers, a student can treat them as a subset of the ring of integers. As students' progress from internalization to condensation, the redoing phase marks a significant advancement. Repetition, defined as a change, is the ability to see something familiar in a completely new light (Dougherty & Slovin, 2004). The continuous nature of Sfard's concept formation process provides a framework for observing students' ability to connect geometric measurement knowledge and fractional knowledge (Lee & Lee, 2020). When students calculate the area of each piece, they consider crucial features of the fractional area model, such as equal partitioning, the whole, and the piece-whole relationship. Therefore, utilizing these understandings across multiple fields of mathematics and coordinating them is expected to yield rich data.

Lee and Lee (2020) have extended Sfard's (1991) theory by introducing a pre-internalization stage and applied this model to assess students' understanding of geometric measurement in problems involving area models of fractions. In the pre-internalization stage, as described by Lee and Lee (2020), students do not recognize the geometric measurement unit and become aware that the area or length to be measured is a quantity. They may lack awareness of the relationship between co-partition and part-whole in fractions, or if they express it, it is with limited understanding. Lee and Lee (2020) have outlined four profiles representing different stages in this model, incorporating area measurement from geometric measurement:

Profile 1 (Pre-internalization): This profile indicates that students struggle to understand both fractions and area measurement. The levels in this profile are as follows:

- area measurement level 0 and fraction knowledge level 0,

- area measurement level 1 and fraction knowledge level 1 or

-area measurement level 0 and fraction knowledge level 1

Profile 2 (Internalization): This profile illustrates students' limited understanding of both area measurement and fractions. The levels in this profile are as follows:

-area measurement level 2 and fraction knowledge level 2

Profile 3 (Condensation): This profile demonstrates that students have a solid understanding of fractions but a limited understanding of area measurement knowledge. The levels in this profile are as follows:

-area measurement level 2 and fraction level 3

Profile 4 (Reification): This profile signifies that students possess a solid understanding of both area measurement and fractions. The levels in this profile are as follows:

-area measurement level 3 and fraction level 3

The explanations for the levels specified in the profiles, regarding fractions and area measurement, are as follows:

Four levels have been determined for fraction knowledge. At Level 0, the student is unaware of the concept of the whole and equal parts. In other words, they do not recognize that fractions involve equal division, and they cannot represent the whole. At Level 1, the student is aware of equal partitioning but with limited understanding. They can express equal partitioning but with constraints. For instance, they might require shapes to be congruent, insist on an equal number of points in the three parts, or stipulate that two parts must appear larger than the other part. At Level 2, the student understands equal partitioning without referencing the part-whole relationship. They compare the number of unit squares without considering the area of the whole and its relationship to each part of the area. At Level 3, the student comprehensively understands the amount of the whole, equal division, and the relationship between the parts and the whole. They have a holistic understanding of fractions, encompassing the entire quantity, the equal division of that quantity, and the interconnection between the parts and the whole.

The four levels for field knowledge in area measurement have been delineated. At Level 0, the student does not recognize the area unit. For instance, a student at this stage can only count the number of dots within a given shape on a grid sheet without specifying the unit of area. At Level 1, the student is aware that area is the amount of a surface, but they do not specify the unit of area. At Level 2, the student recognizes the area unit, but they count partial units based on guesswork, showcasing uncomplicated spatial reasoning. The student is aware of area conservation; for example, they calculate the area measure by counting the number of unit squares in each piece, considering the whole square, and then combining the piece squares based on estimation. At Level 3, the student comprehensively understands that applied geometric features are associated with the numerical structure. For instance, the student can count unit squares by combining partial squares using methods such as the cut-out method or the take-away method.

By utilizing these profiles, the aim is to unveil students' ability to employ their knowledge of geometric area measurement and fraction knowledge, as well as to observe how they apply their knowledge of area measurement in determining fractions.

Method

Research Model

This study was structured as a case study, aiming to investigate students' proficiency in utilizing geometric area measurement knowledge and fraction knowledge. In case studies, the researcher delves into one or more situations in detail over a specific period (Creswell, 2007). The focus lies in exploring the understanding derived from the situation rather than making broad generalizations (Denzin & Lincoln, 2011). In this research, the responses provided by students to questions regarding the area models of fractions were considered as the situation, and these responses were further elucidated through additional questions during the interviews. As the study involved an in-depth examination of more than one sub-case and unit, a nested single case design was employed (Yıldırım & Şimşek, 2006). Within the research framework, student answers to questions prepared with reference to Lee and Lee's (2020) categories and Sfard's (1991) theory were assessed as the situation.

Participants

The research participants consist of a total of 9 students, with 3 students each from the 6th, 7th, and 8th grades attending a public school in the fall semester of the 2021-2022 academic year. The students' achievement levels were categorized as good, medium, and low for each grade level, considering factors such as school report grades and teachers' opinions to assess their success. Participants were assigned code names such as G-S1, M-S2, L-S3, where G represents good level, M represents medium level, L represents low level, and S denotes student. Among the participants, students coded G-S1, M-S2, L-S3 are in the 8th grade, students with code G-S4, M-S5, L-S6 are in the 7th grade, and students with code G-S7, M-S8, L-S9 are in the 6th grade. Teacher opinions played a crucial role in the selection of participants, considering students who had a background in the subject under investigation, either through classroom instruction or private lessons. The selection process prioritized students with prior exposure to the relevant topic. All participating students signed a consent form indicating their willingness to participate in the study.

Data Collection Tool and Process

The research data were gathered through one-to-one interviews and document analysis. Shapes and operations drawn by the students on paper during the research constituted the documents examined. These shapes and processes were documented by taking photos. To design questions that assess students' ability to connect geometric measurement knowledge and fraction knowledge, a literature review was conducted.

Questions were formulated, drawing inspiration from Lee and Lee's (2020) study, specifically focusing on area model questions that were typically unpartitioned. These questions contained parts with identical areas but were not compatible with each other. The intent was to explore the part-whole relationship for fractions and equal partitioning, while also integrating fundamental concepts such as attribute, addictiveness, and units in measurement. For instance, questions were created by presenting rectangular shapes divided into pieces to be expressed as the compound fraction 4/3 and asking students to identify the fraction represented by the shape as well as expressing each piece as part of the whole fraction.

While formulating questions, attention was given to using different geometric shapes, such as area models of a triangle (Question 1-Q1), square (Question 2-Q2), rectangle (Question 3-Q3), and nested squares (Question 4-Q4), which were not typically segmented. Additionally, various types of fractions, including simple fractions, compound fractions, and integer fractions, were incorporated into the questions. To ensure content validity, the questions were reviewed by three mathematics education experts, and adjustments were made accordingly. Some questions from the initial pool were excluded as they were deemed inappropriate for the students' level or the study's objectives. The final questionnaire used in the study consisted of 4 questions. To determine the reliability of the test, a pilot study was conducted with 4 students before the main application. The prepared questions were administered to the students in two sessions through one-on-one interviews, each lasting 30 minutes. To address potential boredom and distraction, the interviews were kept within a reasonable time frame. The sessions were recorded using a video camera for subsequent analysis.

Data analysis

During the data analysis process, video recordings of the interviews with students were transcribed, and the students' responses were systematically documented in a Word file along with accompanying photographs. The analysis was conducted by considering the levels established by Lee and Lee (2020) according to Sfard's theory. The answers were categorized into profiles that align with these levels. In light of the students' responses, the levels for both the area and fraction questions (Level 0, Level 1, Level 2, and Level 3) were determined, and based on these levels, the students' profiles (Profile 1, Profile 2, Profile 3, and Profile 4) were identified.

To ensure the reliability of the coding during data analysis, the data were independently reviewed by a different expert than the researchers. The consistency of coding between a researcher and the expert was established at 90%. In instances where discrepancies arose, discussions were held to reach a consensus, achieving 100% agreement on the data. This process helped enhance the overall reliability of the analysis.

Ethics Committee Decision

Ethics committee approval was obtained for this research with the decision of the Social and Human Scientific Research and Publication Ethics Committee of Uşak University, dated 09/09/21 and numbered 2021-168.

Results

In the analysis of data pertaining to students' proficiency in utilizing geometric area measurement knowledge and fraction knowledge, four main themes were identified. These themes corresponded to the profiles established in the study: Profile 1 (Pre-internalization), Profile 2 (Internalization), Profile 3 (Condensation), and Profile 4 (Reconstruction). Furthermore, the table includes information on the specific profiles each student exhibited in response to particular questions. The participants' profiles are detailed in Table 1.

Table 1 Information about Profiles of the Participants

Profiles	Questions	Students
Profile 1 (pre-internalization)	Q1, Q2, Q3, Q4	L-S3 (8th grade), boy
	Q1, Q2, Q3, Q4	M-S5 (7th grade), boy
	Q1, Q2, Q3, Q4	L-S6 (7th grade), boy
	Q1, Q2	M-S8 (6th grade), girl
	Q1, Q2, Q3, Q4	L-S9 (6th grade), boy
Profile 2 (internalization)	Q3	M-S8 (6th grade), girl
Profile 3 (condensation)	Q1, Q2, Q3	G-S7 (6th grade), boy
Profile 4 (reification)	Q1, Q2, Q3, Q4	G-S1 (8th grade), boy
	Q1, Q2, Q3, Q4	M-S2 (8th grade), boy
	Q1, Q2, Q3, Q4	G-S4 (7th grade), girl
	Q4	G-S7 (6th grade), boy
	Q4	M-S8 (6th grade), girl

As indicated in Table 1, the responses provided by students to the area model questions exhibited variations based on the question type or the students' subject knowledge. Consequently, the same student may have been classified into different categories across distinct questions. Notably, students with lower success levels tended to consistently fall within Profile 1 and 2 across all questions. Conversely, students with medium and high success levels predominantly demonstrated proficiency at Profile 3 and 4 across all four questions. Below are detailed presentations of students' fraction knowledge and area measurement knowledge levels within each category.

Findings Related to Profile 1 (Pre-internalization)

Students in Profile 1 (Pre-internalization) exhibited fraction knowledge levels and area measurement knowledge levels ranging between 0 and 1. At this level, it is evident that these students lack an understanding of the part-whole relationship and co-dividing in fraction knowledge. Moreover, they are aware of area as a quantity in area measurement, but they do not possess knowledge of the area measurement unit. These characteristics indicate that students in this category face challenges in grasping fractions and measuring area. During data analysis, several subcategories emerged under the theme of Profile 1 (Pre-internalization), including Counting Intervals, Counting Points, Counting the Number of Sides of the Shape, and Making Decisions Based on the Appearance of the Shape. These subcategories further elucidate the specific challenges and limitations observed in students classified under Profile 1.

Counting Dot

In this category, it was observed that students did not recognize the area unit and resorted to counting points within the figure, including points on the edges, inner regions, or line segments. The area measurement knowledge of these students, who relied on a point-counting approach, was categorized as level 0, and their fraction knowledge varied between level 0 and level 1. Specifically, among the seventh and eighth-grade students coded as L-S3, M-S5, and L-S6, a tendency to count points was noted. For L-S3, the student counted points on the edges in rectangular and square area model questions, while focusing on points in the inner region for triangular area model questions. Contrary to L-S3, M-S5 counted points on the side lengths of the parts in all shapes presented in the area model questions. On the other hand, L-S6 counted points on the longest line identified in some questions and points on the line segments outside the parts within the figures in other questions. Additionally, L-S6 tended to count points above the side lengths while determining the area. In expressing fractions, this student used the statement "When two of the four parts are equal, it means 2/4, and when all three are equal, it means 3/4," without considering the concepts of the whole and equal division. The drawings made by L-S6 and their corresponding dialogue are provided in Figure 1.

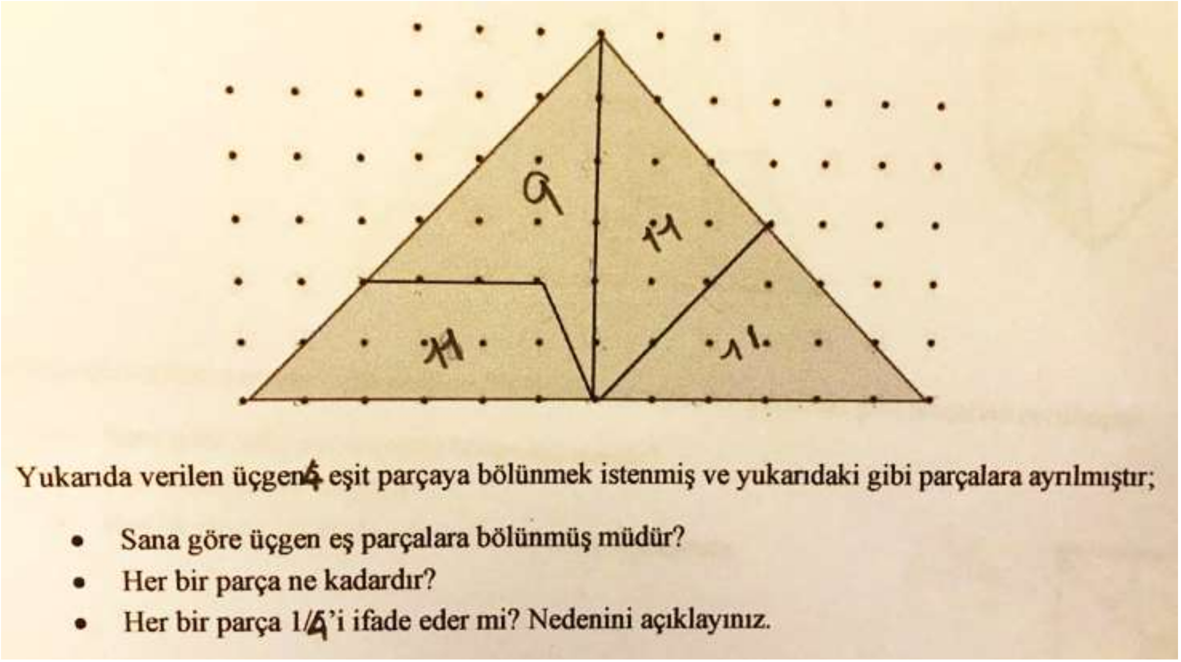

Figure 1 The Answer Of L-S6 Coded Student

L-S6: 1,2,3,4,5,6,7,8.

Researcher: What did you count?

L-S6: I count the sides. (Again, counting the dots on the outer edges of the parts)

L-S6: None 11.

Researcher: Why did you change what happened? What are you counting now?

L-S6: I counted the sides. (This time he counted the points on all side lengths with perimeter logic)

L-S6: Here is 9, here is 11, here is 11.

L-S6: Is it divided into equal parts? It is not divided. 3 out of 4 equal parts.

Researcher: You said 3 out of 4 are equal parts. Is it true?

L-S6: Yes, we say 3/4 for this fraction.

Researcher: Why did you say that?

L-S6: When 3 of them are equal, it becomes 3/4.

Researcher: Well, do these pieces represent 1/4?

L-S6: No, it's not 1/4. It is 3/4. Three pieces are equal.

It was observed that the area measurement level of the student coded as L-S6 in this question was determined to be 0. In this particular question, the student did not recognize the unit of area measurement and instead counted the number of points on the side lengths of the shape. Furthermore, the student struggled to articulate how the whole and congruent division actually occurred, only stating that three of the four parts in Figure 1 were equal to each other and the fraction was thus 3/4. Consequently, it was evident that the student's fraction knowledge level was 0.

Counting the Number of Sides of a Shape

In the category, it was observed that students counted the number of sides of the shape without recognizing the area unit. The area measurement knowledge of students utilizing this counting process was categorized as level 0, and their fraction knowledge varied between level 0 and level 1. While only the student coded as L-S3 was categorized under counting points in other questions, they appeared in the category of counting the number of sides in the integer fraction question (See Figure 2). This student asserted that since the shape is not equal, it would not represent a fraction. The following dialogue occurred with the student coded as L-S3.

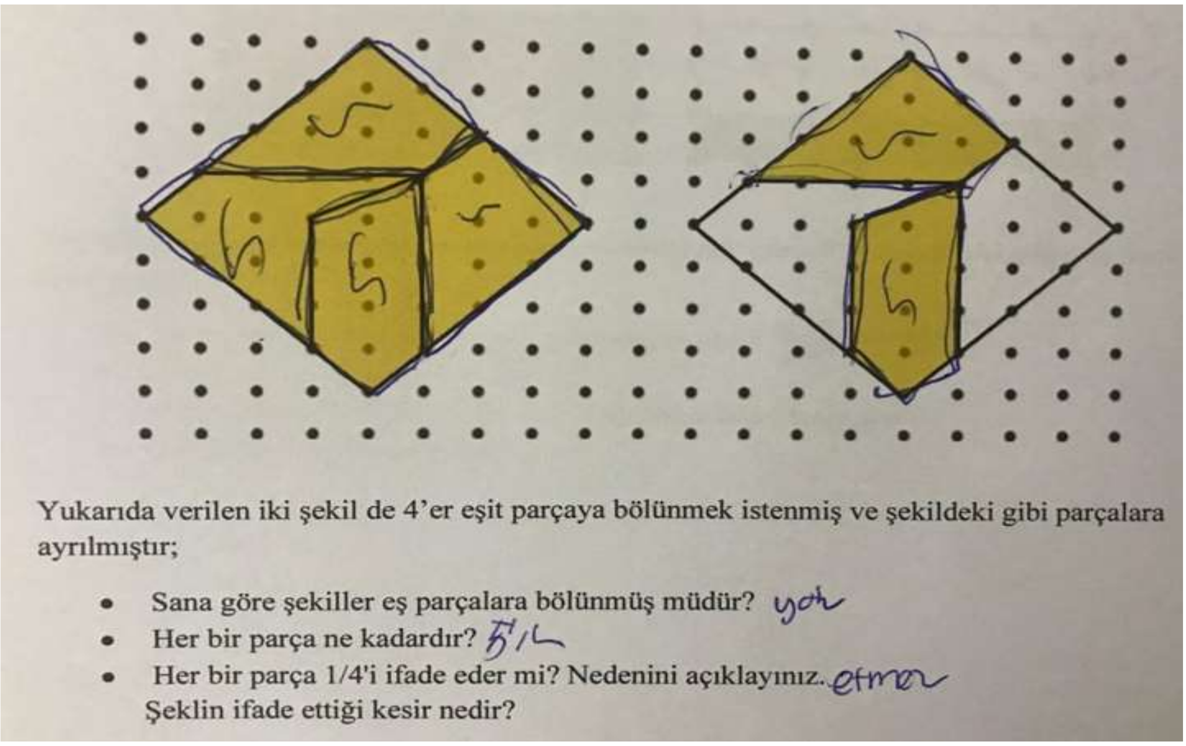

Figure 2 The Answer of L-S3 Coded Student

L-S3: Let's count the lines.

Researcher: Why did you count the lines?

L-S3: We find the inside of the figure.

Researcher: Are you counting the lines while you find the inside of the shape?

L-S3: We find the area.

Researcher: Is this how we calculate the area of a shape?

L-S3: Yes.

Researcher: Okay, let's see.

L-S3: 1, 2, 3, 4, 5;1, 2, 3, 4, 5;1, 2, 3, 4;1, 2, 3, 4, 5 (counting how many sides the pieces have) Is it broken into pieces? No, it is not divided.

Researcher: How much is each piece?

L-S3: 5 and 4.

Researcher: Does each piece represent ¼?

L-S3: Not equal.

Researcher: What fraction does the figure represent?

L-S3: It does not mean fraction.

Researcher: You don't answer the last question because it doesn't express a fraction?

L-S3: Yes.

For the student coded as L-S3, it was found that their area measurement level was 0, and their fraction knowledge level was 1. While the student did not recognize the unit of area and counted points in all questions except this one, they adopted a different method by counting the number of sides of the figure in the integer fraction question. The area measurement level of the student, who claimed to calculate the area by counting the number of sides, was determined to be 0. Furthermore, it was observed that the student's knowledge of fractions was at level 1, indicating an awareness of equal division with limited understanding. The student stated that the number of sides of the given four pieces should be equal but struggled to express the fraction. In this context, it can be inferred that the student has a limited understanding when articulating the division of fractions.

Interval Counting

In this category, students failed to recognize the area unit and attempted to determine the area by counting the distances between points on the edges of the shape. It was observed that the geometric area measurement knowledge level of students employing this interval counting process was 0, and their fraction knowledge level was 0 or 1.

Students coded as M-S8 and L-S9 exemplified this behavior by counting intervals on the edges of the area. For instance, L-S9 determined the side lengths of the shapes provided in the figure (See Figure 3) and endeavored to identify equal parts with precisely the same side lengths. The dialogue with the student coded as L-S9 is presented below:

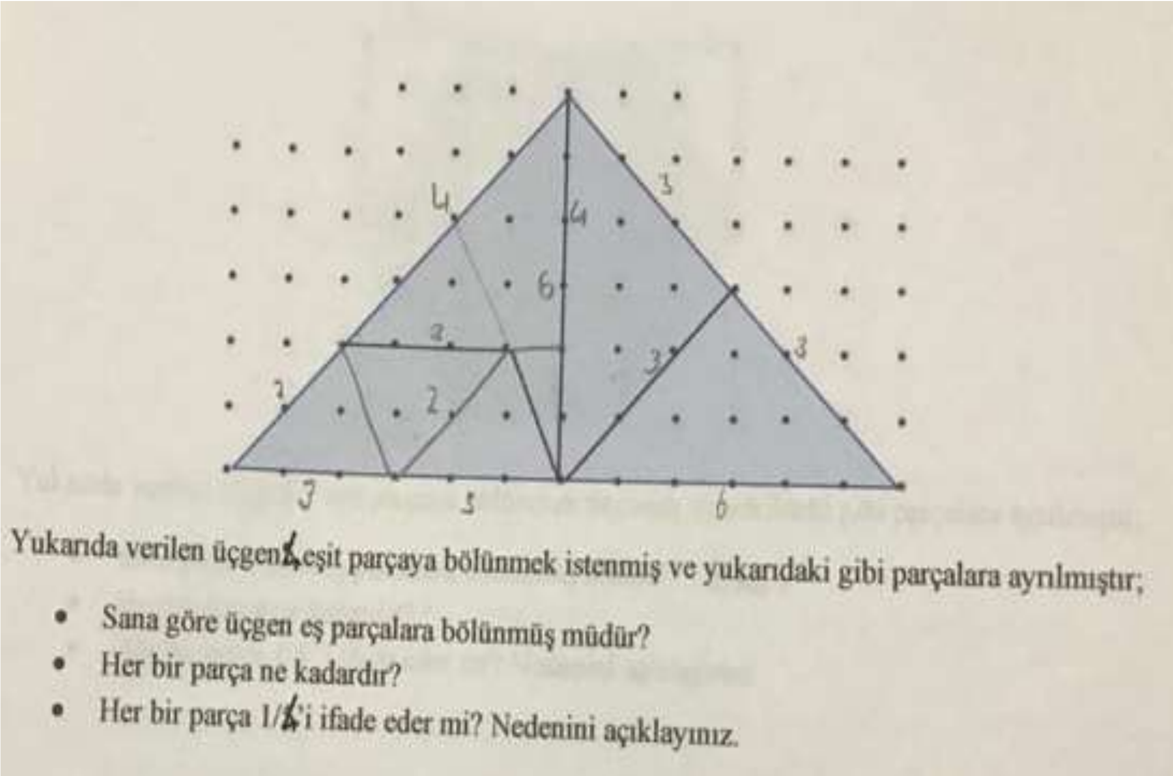

Figure 3 The Answer of the Student Coded L-S9

L-S9: He said, "Is the triangle divided into equal parts according to you?" First we need to add the following sides of a triangle. If we count them as one unit, the long place at the bottom is 6 units and each side is 3 units. This is a complete triangle (meaning the rightmost triangle)

Researcher: What do you say you need to collect?

L-S9:…(Goes to the other triangle-shaped piece), the long side of this is 6 units and the short sides are 3.3. Then these are equal triangles. Now what makes the most sense to me is to make them a whole piece. We draw a line through (dividing the quadrilaterals into triangles)

L-S9: We made a triangle with 4 sides and 4 sides. If my calculation is correct, this is a triangle with all sides 4. If we make a triangle here, there remains a piece, how can we handle it?

Researcher: Well, what are we calculating here, I don't understand, what use will these triangles you divide serve us?

L-S9: These triangles were not equal.

Researcher: Should it be in the form of 3.3.6 units to be equal?

L-S9: It doesn't need to be equal.

L-S9: Actually, if we draw a line from here, we will complete a triangle here as well, and its two down sides will be 3 units.

L-S9: We divided it into 4 equal parts, but we still have these parts.

Researcher: Did you divide the quadrilateral pieces equally?

L-S9: As a result, the parts are not equal and do not represent ¼.

Researcher: How do you decide with the triangles you separate inside the quadrilaterals that they are not equal?

L-S9: It should have been 3,3,6, it didn't happen, the triangles are not equal, that's why.

L-S9, coded as a student, was observed not to recognize the unit of measurement for the area. In an attempt to achieve triangles with the same side lengths for fraction equality, and failing to obtain these triangles, the student asserted that each part was not equal. While there was some awareness of the concept of equality in fractions, it appeared to be limited, and the student's knowledge of fractions was categorized at level 1, indicating a lack of understanding of which parts should be congruent and how.

Deciding Based on the Appearance of the Shape

In this category, the students did not recognize the area unit and opted to calculate the area based on the shape's appearance (whether it appeared large or small, etc.). Students with the codes M-S5 and L-S6, who were in the seventh grade, provided responses falling within this category. These students attempted to determine the area solely based on the appearance of the figure, avoiding calculations for shapes they deemed difficult or different. In the frame question presented (Figure 4), both students stated that the frame decreases in size from the outside to the inside, implying that its area also diminishes. Student M-S5 asserted that the parts in fractions cannot be equal because they decrease in size, demonstrating limited awareness of division and participating in the knowledge of fractions at level 1. Conversely, student L-S6 claimed that each part would represent 1/3 because the figure is divided into three parts. Their explanation suggested a lack of awareness of fraction division, placing them at level 0 in terms of fractions.

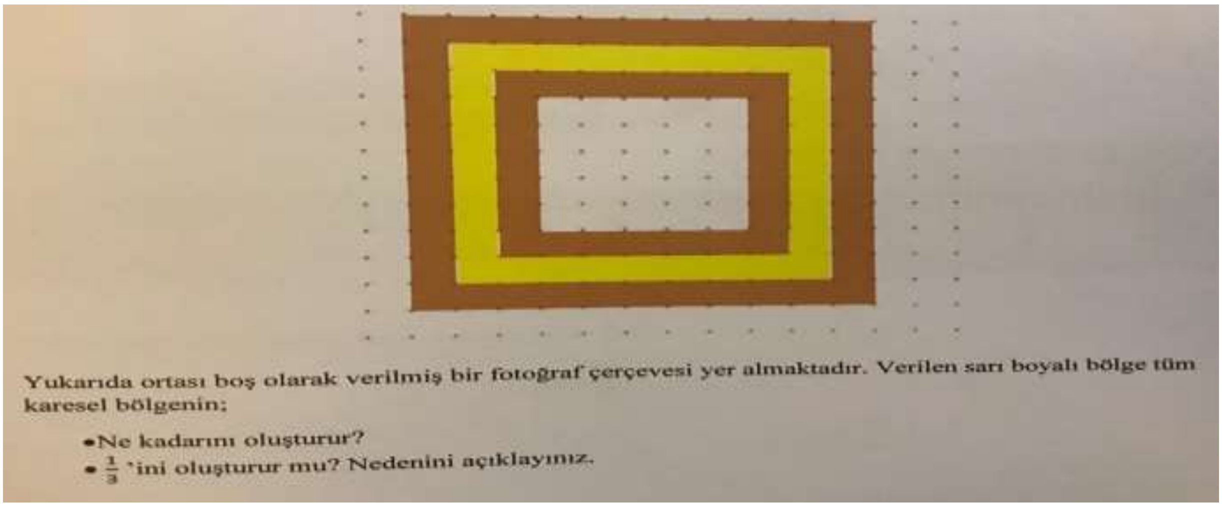

Figure 4 The Question Asked to the L-S6 Coded Student

Findings Regarding Profile 2 (Internalization)

Students in this category recognized the area unit, counted the unit squares in the figure, and combined the partial squares as an approximation. However, in fractions, they only compared the number of unit squares without paying attention to the relationship between the part and the whole. It was determined that the knowledge of fractions and geometric area measurement of the students in this category were at level 2. Only M-S8, who was in the sixth grade, provided answers in this category. While this student was in the point counting category of Profile 1 in all other questions, she expressed the area of the rectangle in the area model questions containing rectangular pieces as the vertical and horizontal area (in terms of width and height). In the area model questions with triangular parts, she created a rectangular shape by combining the triangles and expressed the area covered by the rectangle as the area it occupies vertically and horizontally (in terms of width and height). She stated that these rectangular pieces she found were equal to each other and represented fractions, but she referred to equipartition without explaining the part-whole relationship in fractions. An example dialogue of M-S8, who gave answers in this category, is presented below:

M-S8:1 right here. This is 1, 2, 3, 4, 5, 6, 7. Seven times 7 is here.

Researcher: Why did you multiply there?

M-S8: I found the area of the rectangle.

Researcher: You didn't calculate the area in the previous question, but never. You counted the range. Why did you calculate the area in this one?

M-S8: Because I know the area of the rectangle, I do not know the others. I didn't calculate because I couldn't find the area of those shapes.

Researcher: So how do you calculate the area of these top pieces?

M-T8: Except for that, this is actually the rectangle. But I don't know how to find this triangle either. Although here (showing the sides of the triangle he drew) he went two units up and one unit to the side. Here, too (he shows the congruent triangle next to the triangle he drew), two units go up and one unit to the side, and they are equal to each other.

Researcher: So how much is each piece?

M-S8: 7 units

Researcher: You said the triangles must be equal, but did you find out how many parts are 7?

M-S8: Hmm, all of this (showing the white part) is 7 times 2 out of 14, since they are equal, each of them becomes 7.

M-T8: 3 means one and the value of 1 is exactly 1/3.

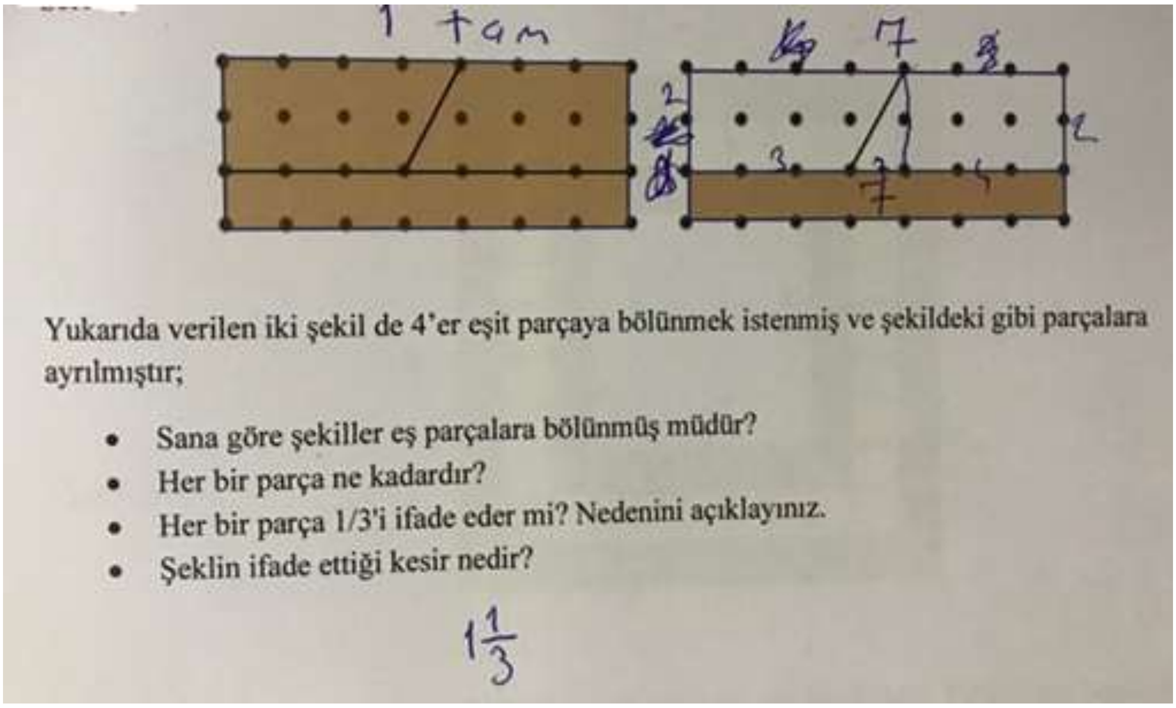

Figure 5 The Answer of the Student Coded Profile 2-M-S8

The student coded as M-S8 expressed the area of the unshaded rectangle at the top as "7 times 2 equals 14". The area of the shaded rectangle below was calculated with the formula "7 times 1 equals 7". However, she stated that she did not know how to calculate the area of the triangle, and by looking at the side lengths of the triangular regions, she decided that the parts were the same and determined that all parts were equal by saying "half of 14 is 7". Therefore, it was seen that the student's knowledge of geometric area measurement was at level 2. In addition, she determined that the parts were equal without referring to the part-whole relationship in the knowledge of fractions, and she provided information showing that the fraction was at level 2 by saying "1 represents 1/3 of the whole".

Findings Regarding Profile 3 (Condensation)

Students in this category recognized the area unit, counted the number of square units in each piece to calculate the area, and combined the partial units as an approximation. In fractions, students determined equal partitioning, the amount of the whole, and the part-whole relationship. Geometric area measurement knowledge of the students in this category is expressed as level 2, and fraction knowledge level is 3. This category shows that students understand fractions but have limited understanding of measuring area. Only students with the code G-S7 were included in this profile. Except for the G-S7 coded student frame question, all other area questions took place at profile 3 level. The student with the code G-S7 counted the unit squares in most of the questions and combined the partial units. After dividing the parts into unit squares (See Figure 6.), he counted the complete unit squares, and combined the incomplete unit squares as larger than half and smaller than half. The drawing and dialogues of the student with the code G-S7 were as follows:

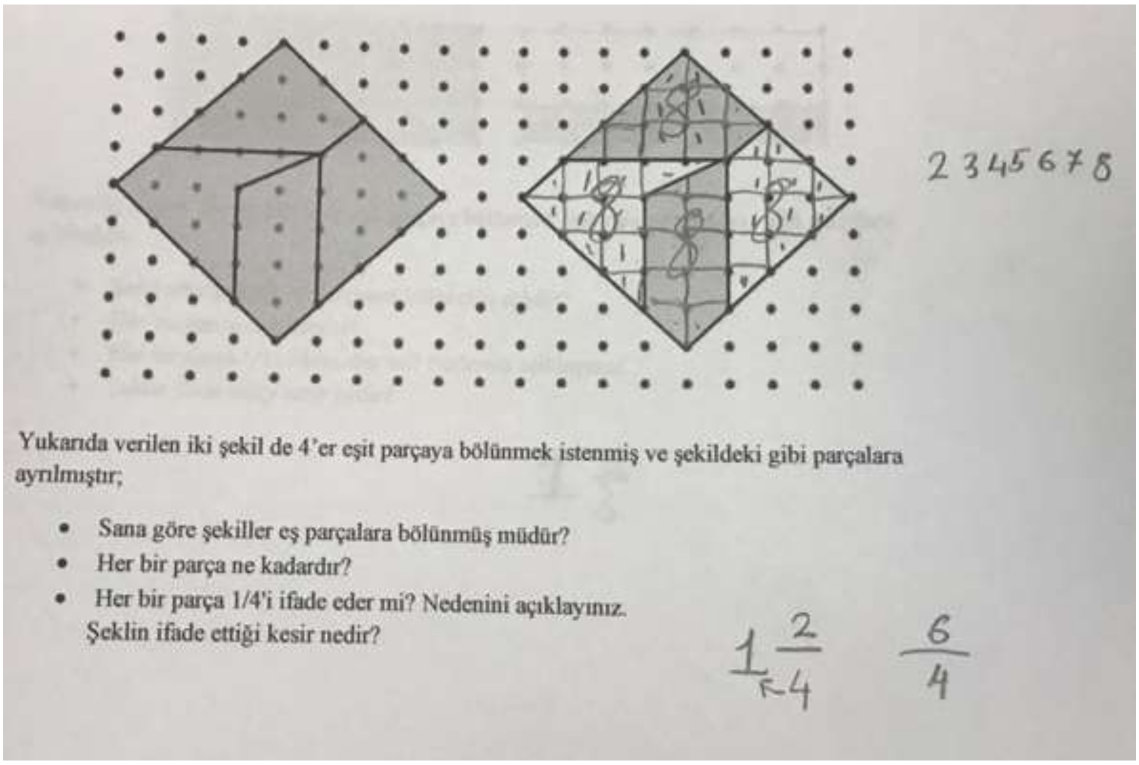

Figure 6 The Answer of The Student With The Code G-S7

G-S7: What is the area of this place? We need to find it. 1,2,3,4,5 here was half 6. It was 7 and 8.

G-S7: This place is also 8.

Researcher: How did it become 8?

G-S7: One of these two pieces is understood to be larger than half, and the other is smaller than half. When combined, it becomes one.

G-S7: This place is 1,2,3,4,5,6,7 and when combined with this, it became 8 units.

G-S7: 1,2,3,4,5,6 came out here and here, and this came out as 8.

Researcher: And is the figure divided into equal parts?

G-S7: Yes, each of them was 8 square units.

Researcher: Do the parts mean ¼?

G-S7: Yes, it will.

Researcher: So what does this figure represent as a fraction?

G-S7: 1 full 2 quarters (1 full 2/4)

Researcher: How else can we express this fraction?

G-S7: Compound fraction is also possible. It happens at 6/4.

The student with the code G-S7 stated, *"One of these two pieces is perceived as larger than half, and the other is smaller than half. When you combine them, it forms a whole."*. He inclined to aggregate partial units using expressions as they are. He counted the number of unit squares in each piece, considering the entire square, and combined the piece squares based on estimation. Asserting that each piece consisted of 8 unit squares and was equal, he conveyed an understanding of equal division and the relationship between the parts and the whole.

Findings Regarding Profile 4 (Reification)

The students in this category exhibit a comprehensive understanding, recognizing the unit of area, associating applied geometric features with numerical structures, and demonstrating proficiency in determining the whole amount in fractions, equal division, and the relationship between parts and the whole. Both area measurement knowledge and fraction knowledge levels of students in this category are expressed as 3, signifying a thorough comprehension of both fractions and area measurement. Within this category, students showcase diverse approaches such as parsing the shape and employing formulas, combining parts, counting unit squares, and determining area by completing the shape. Students with the codes G-S1, M-S2, G-S4, G-S7, and M-S8 provided responses that align with this category.

For instance, the student coded G-S1 consistently demonstrated a Profile 4 level in all area-related questions. During area calculations, this student effectively decomposed geometric shapes, calculated the area of the created parts using formulas, and demonstrated the ability to determine the whole amount, establish equal partitioning, and articulate the relationship between the part and the whole.

The student with the code M-S2 consistently demonstrated a Profile 4 level in all questions related to the area model. In the process of area calculation, the student effectively divided the shape into parts and applied area calculation methods using formulas. In instances where challenges arose in determining the area for specific parts, the student employed innovative methods. For example, in one question, the student drew a rectangle around a part that couldn't be calculated using the formula (refer to Figure 7) and calculated the area of this surrounding rectangle. Subsequently, the student attempted to find the area of the remaining piece by subtracting the area of the small triangles on the side of the figure from the overall area. While the student initially faced difficulties in unfamiliar methods, particularly in the initial stages, it is noteworthy that the student gradually overcame these challenges and successfully calculated the areas of those pieces with the techniques they developed. The answer and dialogue of the student coded M-S2 are presented below:

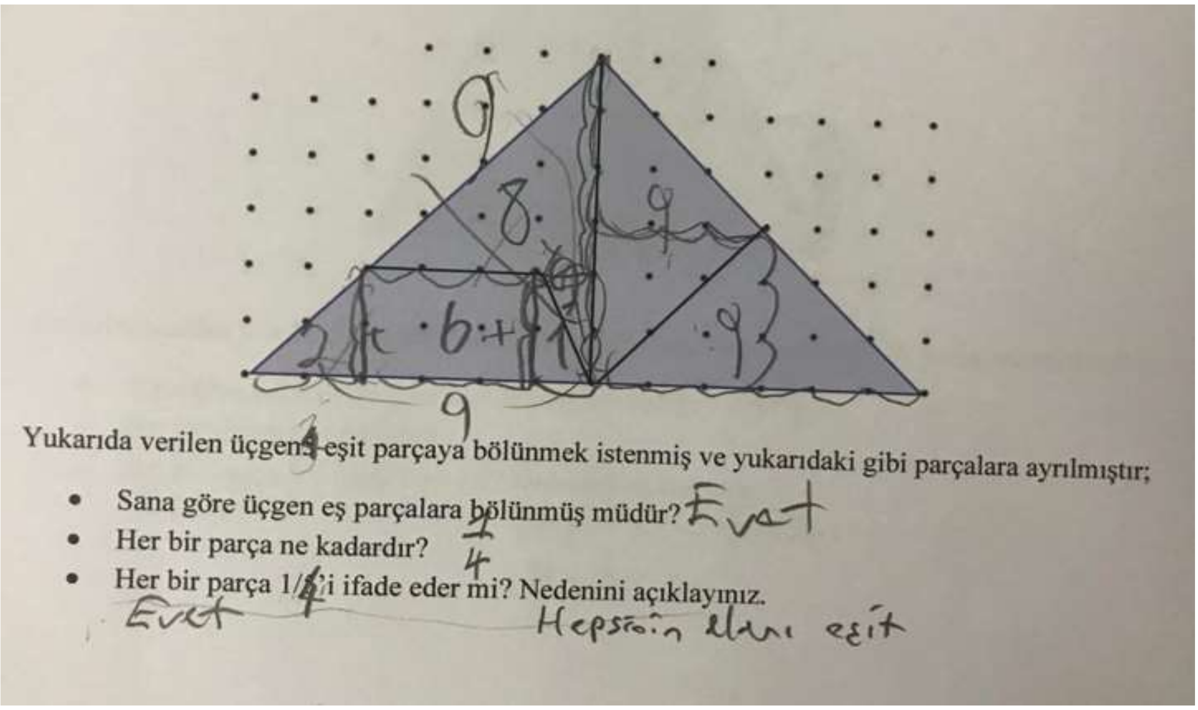

Figure 7 The Answer of Student Coded M-S2

M-S2: I will do the same thing again.

M-S2:1,2,3,4,5,6. that is also 3. 18 divided by 2 is 9. This is the 9th.

M-S2: This is 6, this is 2, and 1 is 9 in total.

M-S2: If we divide it like this, this will also be 8 and 9 from 1.

M-S2: Yes, all the pieces were equal. It means ¼.

Researcher: What if the areas were not equal?

M-S2: Then we couldn't say ¼.

The student coded M-S2 consistently displayed a tendency to decompose the shape and utilize formulas for area calculations. When faced with challenges in calculating specific areas, the student employed a strategy of transforming the part into a rectangle by drawing lines. This approach involved determining the area of the entire shape, subsequently calculating the shaded area by subtracting the areas of the smaller parts. Additionally, the student accurately determined that each piece comprised 9 unit squares, expressing the corresponding fraction as 1/4. This capability demonstrates the student's proficiency in creating equal parts and articulating the relationship between the part and the whole.

Conclusion, Discussion and Recommendations

The study aimed to assess students' proficiency in utilizing geometric area measurement knowledge and fraction understanding, particularly in determining fractions related to area models. Student responses to fraction area model questions were analyzed based on the identified profiles: Profile 1 (pre-internalization), Profile 2 (internalization), Profile 3 (condensation), and Profile 4 (reconstruction).

Upon examining the findings related to the area model, it was observed that students were predominantly distributed across the pre-internalization (Profile 1) and reconstruction (Profile 4) stages. Additionally, students were found in each profile category, showcasing a diverse range of responses. Notably, 6th, 7th, and 8th-grade students participated in both Profile 1 and Profile 4. Specifically, students coded as L-S3 and M-S2 were at the Profile 1 level in the 8th grade, M-S5 and L-S6 in the 7th grade, and M-S8 and L-S9 in the 6th grade. On the other hand, students with codes G-S1 and M-S2 at the Profile 4 level were in the 8th grade, G-S4 in the 7th grade, and G-S7 and M-S8 in the 6th grade. Furthermore, specific students were categorized into different levels: L-S3, M-S2, M-S5, L-S6, M-S8, and L-S9 were at the pre-internalization (Profile 1) level. Meanwhile, M-S8 demonstrated proficiency at the internalization (Profile 2) level, G-S7 exhibited answers aligning with the condensation (Profile 3) level, and students coded G-S1, M-S2, G-S4, G-S7, M-S8 provided responses at the reconstruction (Profile 4) level.

From the study's findings, it was inferred that students' distribution across profiles was related to their success levels. Upon examining students at Profile 1 level (L-S3, M-S2, M-S5, L-S6, M-S8, L-S9), it was noted that students at Profile 4 level (G-S1, M-S2, G-S4, G-S7, M-S8) generally demonstrated moderate to good achievement levels. The profiles revealed that students with moderate and good success could effectively articulate the concept of fractions and establish connections with area measurement. Conversely, students with low achievement levels struggled to express the concept of fractions and link it to area measurement. Interestingly, students at the good achievement level, coded as G-S1 (8th grade), G-S4 (7th grade), and G-S7 (6th grade), predominantly provided answers at Profile 4 and Profile 3 levels. Those at the intermediate level, such as M-S2 (8th grade), M-S5 (7th grade), and M-S8 (6th grade), exhibited diverse responses across all levels. Students at the low achievement level, including L-S3 (8th grade), L-S6 (7th grade), and L-S9 (6th grade), generally answered at the Profile 1 level, occasionally reaching higher levels in familiar questions but predominantly remaining at Profile 1. The study revealed that there was no straightforward relationship between profiles and age or grade levels. For instance, a 6th-grade student was found in both Profile 1 and Profile 4, while an 8th-grade student exhibited responses in profiles 1 and 4. Consequently, grade levels did not dictate the determined levels of geometric area measurement knowledge and fraction knowledge. The findings suggested that as students progressed through higher grade levels, there was no consistent increase in their understanding of fraction content knowledge. Notably, 6th-grade students could provide answers at Profile 4 level, while 8th-grade students might respond in line with Profile 1. This conclusion aligns with Ciosek and Samborska's (2016) study, which found misconceptions about fractions prevalent across various educational levels. Similarly, in Lee and Lee's (2020) research involving pre-service teachers, many demonstrated reasoning at Profile 1 level when solving fraction-related problems. While these studies support the current findings, it's important to note that Ozansak Topçu (2019) argued that mental schemas related to fractions develop rapidly with increasing grade levels.

The study's conclusion highlights the variability in students' performance based on the specific question and geometric shape presented. For instance, the student coded M-S8 exhibited different profiles across the four area model questions, being at Profile 4 level in one question, Profile 2 level in another, and Profile 1 level in two questions. This variability is evident when examining the questions individually; the student demonstrated Profile 1 level in the area model question featuring a triangle, Profile 2 level in the rectangular area model question, and Profile 4 level in the square-shaped area model question. This observation aligns with previous studies by Tan (1998) and Gürefe (2018), which suggested that students often resort to using formulas when calculating the areas of basic shapes like triangles and rectangles. Additionally, Tabak, Ahi, Bozdemir, and Sarı (2010) found that while students were successful in expressing fractions in the area model, their interpretation faced challenges when applied to geometric shapes they were less familiar with, such as triangles and trapezoids. These findings underscore the influence of both question structure and geometric shape on students' ability to integrate fraction knowledge with geometric area measurement.

The study revealed variations in area calculation methods based on students' success levels. Students with low success levels were observed to have errors related to area calculation, such as point counting and interval counting. Conversely, students with a good level of success tended to employ calculations with formulas, applying the rules provided in the questions. Interestingly, students at the intermediate level sought alternative methods, possibly due to difficulty recalling specific rules. An illustrative example is the case of the student coded M-S8, who, when faced with uncertainty in calculating the area of a triangle, opted to combine triangles to form a rectangle. Similarly, in the frame question, this student calculated the area painted in yellow to express the part-whole relationship. Subsequently, the student subtracted this area twice from the overall area and concluded that it represented 1/3. This diverse range of approaches showcases how students adapt their strategies based on their understanding and recall of geometric and fractional concepts.

The study noted that students who responded at Profile 2 and Profile 3 levels predominantly employed the unit square counting strategy for area measurement. During this process, students counted complete unit squares as one and sometimes completed incomplete ones to reach a unit count. However, errors occurred, such as combining inappropriate parts and counting them as a whole. Rectangular and square-shaped pieces facilitated accurate unit counting, while errors were more common when dealing with triangles, where counting and joining pieces proved challenging. Previous research by Gürefe (2018) and Torbens et al. (2004) supports the idea that students may easily make mistakes using the counting strategy. In contrast, Battista (2003) emphasized the significance of students' conceptual understanding of the area measurement formula rather than relying solely on the counting strategy. The current study observed that students frequently utilized incorrect expressions when calculating area measurement using formulas. Furthermore, students tended to perform operations based on memorized information rather than expressing the conceptual understanding of the area formula. This highlights a potential gap in the students' grasp of the underlying concepts, emphasizing the importance of promoting conceptual understanding alongside procedural knowledge.

The study observed that students faced challenges in establishing the part-whole relationship in fractions, struggling to understand how the whole is divided into equal parts. Some students incorrectly expressed fractions as 2/5 when two out of five parts were deemed equal, and 3/5 when three parts were perceived as equal. Despite assertions by various researchers (Acar, 2010; Akbaba-Dağ, 2014; Baştürk, 2016; Şen, 2021) that teachers and pre-service teachers often focus on the part-whole meaning of fractions, the current findings suggest that students may encounter difficulties in grasping this concept. In a study conducted by Yavuz Mumcu (2018) with pre-service teachers, it was noted that they struggled to establish the relationship between the part and the whole. Karaağaç and Köse (2015) reported that students tended to think fractions with the same symbols represented the same amount, overlooking the reference whole. Similarly, Olkun and Toluk-Uçar (2014) highlighted the challenges associated with the part-whole relationship in fractions due to its incorporation of spatial relations and content knowledge. As emphasized by Cramer and Whitney (2010), understanding the part-whole meaning is crucial for constructing the meaning of fractions, suggesting that greater importance should be placed on reinforcing this concept. The study also concluded that students faced difficulties in calculating area and determining fractions in questions related to compound fractions. In these questions, students often described an exact part as a whole, struggling to identify the fraction part accurately. This finding aligns with research by Kurt (2006), who identified common mistakes in questions related to the area model of compound fractions among 6th, 7th, and 8th-grade students. Topçu (2020) similarly noted deficiencies in compound fractions among students in his study. Uslu (2006) found that a significant percentage of 5th, 8th, and 10th-grade students had problems expressing compound fractions verbally and representing them with a model. Ma (1999) suggested that these challenges might stem from a neglect of the conceptual structure during fraction teaching.

The findings not only revealed the correctness of the students' problem-solving approaches but also demonstrated their competence in integrating and explaining fundamental mathematical concepts. For instance, students coded M-S8 and G-S7 articulated that the fraction was 1/3 by subtracting the painted area from the whole area twice in order to express the fraction in the frame question. Similarly, students coded M-S2 and G-S4 attempted to establish the relationship between the area and the fraction by employing various strategies, such as finding and subtracting the area of the whole or calculating the area by dividing the shape into parts, to determine the areas of the small parts in the given area model questions. These examples underscore the significance of not only the accuracy of the approach but also the integration of basic mathematical concepts in solving such problems.

The results of this study suggest that students in Grades 6 to 8 exhibit varying levels of understanding in commonly used models and mathematical language. The research provides insights into ways to elevate students to higher levels of understanding and highlights common mistakes made by students in both geometric measurement and fractions. Observations in the study indicate that students struggle with clarifying units, distinguishing between the knowledge of measuring perimeter and area, and correctly determining fractions during counting and other estimations. The findings also illustrate the diverse strategies and methods employed by middle school students in different segmented area models, reflecting distinct mental schemas. Encouraging students to discover connections between concepts, surpass the limits of their previous knowledge, and enhance their understanding of intuitive thinking, strategy creation, and the representation of units, quantities, and fractions emerges as a valuable and meaningful exercise.

The findings not only affirmed the correctness of students' problem-solving approaches but also gauged their success in explaining and integrating fundamental mathematical concepts. As a result, mathematics teachers are encouraged to design tasks that integrate various areas in mathematics, fostering connections between different mathematical concepts within their classroom practices.

The study revealed that students encountered less difficulty with rectangular fractional area models compared to triangular area models. To enhance fraction teaching, it is suggested that educators incorporate various types of area models, including different segmented area models, both in classroom applications and textbooks.

Moreover, the research indicated that students faced challenges in calculating area and determining fractions in questions related to compound fractions. In these instances, students often misinterpreted an exact part as a whole being painted, struggling to identify the fractional part. It is recommended to conduct similar studies for pre-service teachers to raise awareness and expand their content knowledge.

This study focused on sixth, seventh, and eighth-grade students. However, it can be expanded to include teachers and teacher candidates. Area model questions, which are usually presented without segmentation, can also be addressed using conventionally provided area model questions. This research is qualitative in nature, and the participants were restricted to nine students within a single school. Consequently, conducting additional studies with a larger sample size is possible. Furthermore, beyond qualitative research, quantitative research can be explored.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

There was no conflict of interest in this study.

Funding

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CRediT author statement

This study is part of the master's thesis of the first author under the supervision of the second author.

Research involving Human Participants and/or Animals

This research was conducted with the permission obtained with the decision of the Social and Human Scientific Research and Publication Ethics Committee of Uşak University, dated 09/09/21 and numbered 2021-168.

Ortaokul Öğrencilerinin Kesir ve Geometrik Alan Ölçme Bilgisini Birbirine Bağlama Becerisinin Belirlenmesi

Özet:

Bu çalışmada ortaokul öğrencilerinin kesir bilgisi ve geometrik alan ölçme Bilgisi ortaya çıkarılarak kesri belirlerken alan ölçme bilgisini nasıl kullandığı belirlenmiştir. Durum çalışması şeklinde tasarlanan bu çalışma 6, 7 ve 8. sınıflardan belirlenen dokuz öğrenci ile yürütülmüştür. Birebir görüşmeler yoluyla toplanan veriler betimsel analiz kullanılarak analiz edilmiştir. Bulgular sonucunda katılımcıların kesir ve alan ölçme bilgisinin ön içselleştirme, içselleştirme, yoğunlaştırma ve yenidenleştirme profillerinden çoğunlukla ön içselleştirme profilinde olduğu belirlenmiştir. Öğrencilerin farklı bölümlendirilmiş kesir alan modellerinde, değişik stratejiler ve yöntemler kullandıklarını, farklı zihinsel şemalar ortaya çıkardıklarını ve öğrencilerin görevlere verdiği yanıtların kesir bilgisi ve alan ölçme bilgisini ortaya çıkardığı görülmüştür. Araştırmanın sonucunda kesir kavramı gibi önemli bir kavramı öğreten matematik öğretmenlerinin ve sınıf öğretmenlerinin temel matematiksel kavramları bütünleştirebilen görevler tasarlamaları ve farklı alanlar arasında bağ kurmaları konusunda öneride bulunulabilir.

Anahtar kelimeler: Kesir, alan ölçme, ortaokul öğrencileri

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