GAZİOSMANPAŞA BİLİMSEL ARAŞTIRMA DERGİSİ (GBAD)
Gaziosmanpasa Journal of Scientific Research
ISSN: 2146\8168
http://dergipark.gov.tr/gbad
Research Article (Araştırma Makalesi)

Cilt/Volume : 12
Sayı/Number: 2
Yıl/Year: 2023
Sayfa/Pages: 114-137

***Alınış tarihi (Received)*:** 27.06.2023
***Kabul tarihi (Accepted)*:** 19.07.2023

From a Different Aspect to Complementary Soft Binary Piecewise Difference and Lambda Operations

Aslıhan SEZGİN[1,*], **Hakan KÖKÇÜ**[2]

[1] *Department of Mathematics and Science Education, Faculty of Education, Amasya University, Amasya, Turkey.*
[2]*Department of Mathematics, Graduate School of Natural and Applied Sciences, Amasya University, Amasya, Turkey*
[*]*Corresponding author: aslihan.sezgin@amasya.edu.tr*

ABSTRACT: Soft set theory, introduced by Molodtsov in 1999, is a mathematical tool to deal with uncertainty. Since then, different kinds of soft set operations have been defined and used in various types. In this paper, it is aimed to contribute to the soft set literature by obtaining the distibutions of soft binary piecewise operations over complementary soft binary piecewise difference and lambda operations to present the connections between them.

Keywords **–** *Soft sets***,** *Soft Set Operations***,** *Conditional Complements*

1. Introduction

The existence of some types of uncertainty in the problems of many fields such as economics, environmental and health sciences, engineering prevents us from using classical methods to solve the problems successfully. There are three well-known basic theories that we can consider as mathematicals tool to deal with uncertainties, which are Probability Theory, Fuzzy Set Theory and Interval Mathematics. But since all these theories have their own shortcomings, Molodtsov (Molodtsov,1999) introduced Soft Set Theory as a mathematical tool to overcome these uncertainties. Since then, this theory has been applied to a variety of fields, including information systems, decision-making as in Özlü (2022a,2022b), optimization theory, game theory, operations research, measurement theory, and some algebraic structures (Özlü and Sezgin, 2021). First contributions as regards soft set operations are made in (Maji et. al., 2003; Pei and Miao, 2005). After then, several soft set operations (restricted and extended soft set operations) were introduced and examined in (Ali et. al., 2009). Basic properties of soft set operations were discussed and the interconnections of soft set operations with each other were illustrated in (Sezgin and Atagün, 2010). They also defined the notion of restricted symmetric difference of soft sets and investigate its properties. A new soft set operation called extended difference of soft sets was defined in (Sezgin et al., 2019) and extended symmetric difference of soft sets was defined and its properties were investigated in (Stojanovic, 2021). When the studies are examined, we see that the operations in soft set theory proceed under two main headings, as restricted soft set operations and extended soft set operations.

Two conditional complements of sets as a new concept of set theory, i.e., inclusive complement and exclusive complement were proposed and the relationships between them were explored in (Çağman, 2021). By the inspiration of this study, some new complements of sets were defined in (Sezgin et al., 2023c). They also transferred these complements to soft set theory, and some new restricted soft set operations and extended soft set operations were defined in (Aybek, 2023). Demirci, 2023; Sarıalioğlu, 2023; Akbulut, 2023 defined a new type of extended operation by changing the form of extended soft set operations using the complement at the first and second row of the piecewise function of extended soft set operations and studied the basic properties of them in detail. Moreover, a new type of soft difference operations was defined in (Eren and Çalışıcı, 2019) and by being inspired this study, Yavuz, 2023 defined some new soft set operations, which they call binary piecewise soft set operations and they studied their basic properties in detail, too. Also, Sezgin and Demirci, 2023; Sezgin and Sarıalioğlu, 2023; Sezgin and Yavuz, 2023; Sezgin and Aybek, 2023, Sezgin et al., 2023a and Sezgin et al., 2023b continued their work on soft set operations by defining a new type of binary piecewise soft set operation. They changed the form of soft binary piecewise operation by using the complement at the first row of the soft binary piecewise operations.

In Sezgin and Yavuz, 2023 and Sezgin and Çağman, 2023, complementary soft binary piecewise lambda and difference operation were defined, respectively. The algebraic properties of these new operations were examined in detail. Especially the distributions of these operations over extended soft set operations, complementary extended soft set operations, soft binary piecewise operations, complementary soft binary piecewise operations and restricted soft set operations were handled. In this study, we contribute to the literature of soft set theory by obtaining the distributions of soft binary piecewise operations over complementary soft binary piecewise difference and lambda operations in order to reveal the interrelations of them. The organization of the paper is as follows: In Section 1, literature survey is given with a conclusion paragraph summarizing what is obtained in the paper. In Section 2 the main definitions used throughout the paper is given. In Section 3, first of all the distributions of soft binary piecewise operations over complementary soft binary piecewise difference operation and then distributions of soft binary piecewise operations over complementary soft binary piecewise lambda operation are handled. This paper is a theoretical study of soft set.

2. Preliminaries

Definition 2.1. Let U be the universal set, E be the parameter set, $\mathrm{P(U)}$ be the power set of U and $\mathrm{A} \subseteq \mathrm{E}$. A pair (F, A) is called a soft set over U where F is a set-valued function such that $\mathrm{F}: \mathrm{A} \rightarrow \mathrm{P(U)}$. (Molodtsov, 1999)

The set of all the soft sets over U is designated by $\mathrm{S_E}$(U), and throughout this paper, all the soft sets are the elements of $\mathrm{S_E}$(U).

Çağman, 2021, defined two conditional complements of sets, for the ease of illustration, we show these complements as + and θ, respectively. These complements are defined as following: Let O and L be two subsets of U. L-inclusive complement of O is defined by, O+L= O'∪L and L-exlusive complement of is defined by OθL = O'∩L'. Here, U refers to a universe, O' is the complement of O over U. Sezgin et al., 2023c introduced such new three complements as binary operations of sets as following: Let O and Ö be two subsets of U. Then, O*L=O'∪L', OγL= O'∩L, O$\boldsymbol{\lambda}$L=O∪L' (Sezgin et al., 2023c). Aybek, 2023 conveyed these classical sets to soft sets, and they defined restricted and extended soft set operations and examined their properties.

As a summary for soft set operations, we can categorize all types of soft set operations as following: Let "∇" be used to represent the set operations (i.e., here ∇ can be ∩, ∪,\, Δ, +,θ, *, λ,γ), then restricted operations, extended operations, complementary extended operations, soft binary piecewise operations, complementary soft binary piecewise operations are defined in soft set theory as following:

Definition 2.2. Let (O, W) and (L, B) be soft sets over U. The restricted ∇ operation of (O, W) and (L, B) is the soft set (Y, S), denoted by $(\mathrm{O}, \mathrm{W}) \nabla_{\mathrm{R}} (\mathrm{L}, \mathrm{B}) = (\mathrm{Y}, \mathrm{S})$, where $\mathrm{S} = \mathrm{W} \cap \mathrm{B} \neq \emptyset$ and $\forall \tau \in \mathrm{S}$, $\mathrm{Y}(\tau) = \mathrm{O}(\tau) \nabla \mathrm{L}(\tau)$. (Ali et. al., 2009 (restricted intersection, union and difference), Sezgin and Atagün, 2011 (restricted symmetric difference), Aybek, 2023 (restricted plus,theta, star, theta and lambda)).

Definition 2.3. Let (O, W) and (L, B) be soft sets over U. The extended ∇ operation of (O, W) and (L, B) is the soft set (Y, S), denoted by $(\mathrm{O}, \mathrm{W}) \nabla_{\varepsilon} (\mathrm{L}, \mathrm{B}) = (\mathrm{Y}, \mathrm{S})$, where $\mathrm{S} = \mathrm{W} \cup \mathrm{B}$ and $\forall \tau \in \mathrm{S}$,

$$Y(\tau) = \begin{cases} O(\tau), & \tau \in W\backslash B, \\ L(\tau), & \tau \in B\backslash W, \\ O(\tau)\ \nabla L(\tau), & \tau \in W \cap B. \end{cases}$$

(Maji et.al., 2003 (extended union); Ali et. al., 2009 (extended intersection); Sezgin et. al., 2019 (extended difference); Stojanovic, 2021 (extended symmetric difference); Aybek, 2023 (extended plus, theta, theta, lambda and star))

Definition 2.4. Let (O, W) and (L, B) be soft sets over U. The complementary extended ∇ operation of (O, W) and (L, B) is the soft set (Y, S), denoted by $(O, W) \overset{*}{\nabla}_{\varepsilon} (L, B) = (Y, S)$, where $S = W \cup B$ and $\forall \tau \in S$,

$$Y(\tau) = \begin{cases} O'(\tau), & \tau \in W\backslash B \\ L'(\tau), & \tau \in B\backslash W, \\ O(\tau)\ \nabla L(\tau), & \tau \in B \cap W. \end{cases}$$

(Sarıalioğlu, 2023 (Complementary extended gamma, intersection, star); Demirci, 2023 (complementary extended plus, union and theta); Akbulut, 2023 (complementary extended difference and lambda)

Definition 2.5. Let (O, W) and (L, B) be soft sets over U. The soft binary piecewise ∇ operation of (O, W) and (L, B) is the soft set (Y, P), denoted by, $(O, W) \underset{\nabla}{\sim} (L, B) = (Y, P)$, where $\forall \tau \in W$,

$$Y(\tau) = \begin{cases} O(\tau), & \tau \in W\backslash B \\ O(\tau)\ \nabla L(\tau), & \tau \in W \cap B \end{cases}$$

(Eren and Çalışıcı, 2019 (soft binary piecewise difference); Yavuz, 2023 (soft binary piecewise intersection, union, plus, gamma, theta, lambda and star))

Definition 2.6. Let (O, W) and (L, B) be soft sets over U. The complementary soft binary piecewise ∇ operation of (O, W) and (L, B) is the soft set (Y, W), denoted by, $(O, W) \underset{\nabla}{\overset{*}{\sim}} (\ddot{O}, B) = (Y, W)$, where $\forall \tau \in W$;

$$Y(\tau) = \begin{cases} O'(\tau), & \tau \in W\backslash B \\ O(\tau)\ \nabla L(\tau), & \tau \in W \cap B \end{cases}$$

(Sezgin and Demirci, 2023 (complementary soft binary piecewise star operation); Sezgin and Sarıalioğlu, 2023 (complementary soft binary piecewise theta operation); Sezgin and Aybek, 2023 (complementary soft binary piecewise gamma operation); Sezgin et al., 2023a (complementary soft binary piecewise intersction operation); Sezgin et al., 2023b (complementary soft binary piecewise union operation); Sezgin and Yavuz, 2023 (complementary soft binary piecewise lambda operation); Sezgin and Çağman, 2023 (complementary soft binary piecewise difference operation))

Definition 2.7. Let (O, W) and (L, B) be soft sets over U. The complementary soft binary piecewise lambda (λ) operation of (O, W) and (L, B) is the soft set (Y, W), denoted by, $(O, W) \underset{\lambda}{\overset{*}{\sim}} (L, B) = (Y, W)$, where $\forall\tau \in W$,

$$Y(\tau) = \begin{cases} O'(\tau), & \tau \in W \setminus B \\ O(\tau) \cup L'(\tau), & \tau \in W \cap B \end{cases}$$

(Sezgin and Yavuz, 2023)

Definition 2.8. Let (O, W) and (L, B) be soft sets over U. The complementary soft binary piecewise difference $(\setminus)$ operation of Let (O, W) and (L, B) is the soft set (Y, W), denoted by, $(O, W) \underset{\theta}{\overset{*}{\sim}} (L, B) = (Y, W)$, where $\forall\tau \in W$,

$$Y(\tau) = \begin{cases} O'(\tau), & \tau \in W \setminus B \\ O(\tau) \cap L'(\tau), & \tau \in W \cap B \end{cases}$$

(Sezgin and Çağman, 2023)

3. Distribution Rules

In this section, distributions of soft binary piecewise operations over complementary soft binary piecewise difference and lambda operation are examined in detail and many interesting results are obtained.

3.1.1. Distribution of soft binary piecewise operations over complementary soft binary piecewise difference (\) operation

1) $(O,W)\,\tilde{\cup}[(L,B)\overset{*}{\underset{\setminus}{\sim}}(H,Z)]=[(O,W)\underset{\cup}{\sim}(L,B)]\,\tilde{\cap}\,[(O,W)\underset{\lambda}{\sim}(H,Z)]$, where $W\cap B\cap Z'=\emptyset$

Proof: Let first handle the left hand side of the equality and let $(L,B)\overset{*}{\underset{\setminus}{\sim}}(H,Z)=(M,B)$, where $\forall\tau\in B$;

$$M(\tau)=\begin{cases} L'(\tau), & \tau\in B\setminus Z \\ L(\tau)\cap H'(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(O,W)\,\tilde{\cup}(M,B)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cup M(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cup L'(\tau), & \tau\in W\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cup[(L(\tau)\cap H'(\tau)], & \tau\in W\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\underset{\cup}{\sim}(L,B)]\,\tilde{\cap}\,[(O,W)\underset{\lambda}{\sim}(H,Z)]$,

Let $(O,W)\underset{\cup}{\sim}(L,B)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cup L(\tau), & \tau\in W\cap B \end{cases}$$

Suppose that $(O,W)\,\tilde{\lambda}\,(H,Z)=(S,W)$, where $\forall\tau\in W$;

$$S(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cup H'(\tau), & \tau\in W\cap Z \end{cases}$$

Let $(V,W)\,\tilde{\cap}(S,W)=(T,W)$. Then for all $\forall\tau\in W$;

$$T(\tau)=\begin{cases} V(\tau), & \tau\in W\backslash W=\emptyset \\ V(\tau)\cap S(\tau), & \tau\in W\cap W \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau)\cap O(\tau), & \tau\in(W\backslash B)\cap(W\backslash Z)=W\cap B'\cap Z' \\ O(\tau)\cap[(O(\tau)\cup H'(\tau)], & \tau\in(W\backslash B)\cap(W\cap Z)=W\cap B'\cap Z \\ [(O(\tau)\cup L(\tau)]\cap O(\tau), & \tau\in(W\cap B)\cap(W\backslash Z)=W\cap B\cap Z' \\ [(O(\tau)\cup L(\tau)]\cap[(O(\tau)\cup H'(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau) & \tau\in(W\backslash B)\cap(W\backslash Z)=W\cap B'\cap Z' \\ O(\tau), & \tau\in(W\backslash B)\cap(W\cap Z)=W\cap B'\cap Z \\ O(\tau), & \tau\in(W\cap B)\cap(W\backslash Z)=W\cap B\cap Z' \\ [(O(\tau)\cup L(\tau)]\cap[(O(\tau)\cup H'(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Here let handle $\tau\in W\backslash B$ in the first equation. Since $W\backslash B= W\cap B'$, if $\tau\in B'$, then $\tau\in Z\backslash B$ or $\tau\in(B\cup Z)'$. Hence, if $\tau\in W\backslash B$, $\tau\in W\cap B'\cap Z'$ or $\tau\in W\cap B'\cap Z$. Thus, it is seen that N=T.

2) $[(O,W)\overset{*}{\underset{\backslash}{\sim}}(L,B)]\ \tilde{\cup}\ (H,Z)=[(O,W)\tilde{\cup}(H,Z)]\overset{*}{\underset{\cap}{\sim}}[(L,B)\overset{*}{\underset{+}{\sim}}(H,Z)]$, where $W\cap B'\cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W)\overset{*}{\underset{\backslash}{\sim}}(\ddot{O},B)=(M,W)$, where $\forall\tau\in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau\in W\backslash B \\ O(\tau)\cap L'(\tau), & \tau\in W\cap B \end{cases}$$

Let $(M,W)\ \tilde{\cup}\ (H,Z)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau\in W\backslash Z \\ M(\tau)\cup H(\tau), & \tau\in W\cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau\in(W\backslash B)\backslash Z=W\cap B'\cap Z' \\ O(\tau)\cap L'(\tau), & \tau\in(W\cap B)\backslash Z=W\cap B\cap Z' \\ O'(\tau)\cup H(\tau) & \tau\in(W\backslash B)\cap Z=W\cap B'\cap Z \\ [O(\tau)\cap L'(\tau)]\cup H(\tau) & \tau\in(W\cap B)\cap Z=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\ \tilde{\cup}\ (H,Z)]\ \overset{*}{\underset{\cap}{\sim}}\ [(L,B)\ \overset{*}{\underset{+}{\sim}}\ (H,Z)]$

Let $(O,W)\ \tilde{\cup}\ (H,Z)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cup H(\tau), & \tau\in W\cap Z \end{cases}$$

Suppose that $(L,B)\ \tilde{+}(H,Z)=(S,B)$, where $\forall\tau\in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau\in B\setminus Z \\ L'(\tau)\cup H(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(V,W)\ \overset{*}{\underset{\cap}{\sim}}\ (S,B)=(T,W)$, where $\forall\tau\in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau\in W\setminus B \\ V(\tau)\cap S(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus Z)\setminus B=W\cap B'\cap Z' \\ O'(\tau)\cap H'(\tau), & \tau\in(W\cap Z)\setminus B=W\cap B'\cap Z \\ O(\tau)\cap L'(\tau), & \tau\in(W\setminus Z)\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cap\ [L'(\tau)\cup H(\tau)], & \tau\in(W\setminus Z)\cap(B\cap Z)=\emptyset \\ [O(\tau)\cup H(\tau)]\ \cap L'(\tau), & \tau\in(W\cap Z)\cap(B\setminus Z)=\emptyset \\ [O(\tau)\cup H(\tau)]\ \cap\ [L'(\tau)\cup H(\tau)], & \tau\in(W\cap Z)\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

It is seen that N=T.

3) $(O,W)\ \tilde{\setminus}\ [(L,B)\ \overset{*}{\underset{\setminus}{\sim}}\ (H,Z)]\ =\ [(O,W)\ \tilde{\setminus}\ (L,B)\tilde{\cup}\ [(O,W)\ \tilde{\cap}\ (H,Z)]$, where $W\cap B\cap Z'=\emptyset$

Proof: Let first handle the left hand side of the equality and let $(L,B)\ \overset{*}{\underset{\setminus}{\sim}}\ (H,Z)=(M,B)$,

where $\forall\tau\in B$;

$$M(\tau)=\begin{cases} L'(\tau), & \tau\in B\setminus Z \\ L(\tau)\cap H'(\tau), & \tau\in B\cap Z \end{cases}$$

Let (O,W) $\tilde{\setminus}$ (M,B)=(N,W), where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cap M'(\tau), & \tau\in W\cap B \end{cases}$$

Thus

$$N(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cap L(\tau), & \tau\in W\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cap [(L'(\tau)\cup H(\tau)], & \tau\in W\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\,\tilde{\setminus}\,(L,B)]\ \tilde{\cup}\ [(O,W)\,\tilde{\cap}(H,Z)]$,

Let (O,W) $\tilde{\setminus}$ (L,B)=(V,W), where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cap L'(\tau), & \tau\in W\cap B \end{cases}$$

Suppose that (O,W) $\tilde{\cap}$(H,Z)=(S,W), where $\forall\tau\in W$;

$$S(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cap H(\tau), & \tau\in W\cap Z \end{cases}$$

Let (V,W) $\tilde{\cup}$(S,W)=(T,W) $\quad\forall\tau\in W$;

$$T(\tau)=\begin{cases} V(\tau), & \tau\in W\setminus W=\emptyset \\ V(\tau)\cup S(\tau), & \tau\in W\cap W \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau)\cup O(\tau), & \tau\in(W\setminus B)\cap(W\setminus Z)=W\cap B'\cap Z' \\ O(\tau)\cup[(O(\tau)\cap H(\tau)], & \tau\in(W\setminus B)\cap(W\cap Z)=W\cap B'\cap Z \\ [(O(\tau)\cap L'(\tau)]\cup O(\tau), & \tau\in(W\cap B)\cap(W\setminus Z)=W\cap B\cap Z' \\ [(O(\tau)\cap L'(\tau)]\cup[(O(\tau)\cap H(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau) & \tau\in(W\backslash B)\cap(W\backslash Z)=W\cap B'\cap Z' \\ O(\tau), & \tau\in(W\backslash B)\cap(W\cap Z)=W\cap B'\cap Z \\ O(\tau), & \tau\in(W\cap B)\cap(W\backslash Z)=W\cap B\cap Z' \\ [(O(\tau)\cap L'(\tau)]\cup[(O(\tau)\cap H(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Since $W\backslash B= W\cap B'$, if $\tau\in B'$, then $\tau\in Z\backslash B$ or $\tau\in(B\cup Z)'$. Hence, if $\tau\in W\backslash B$, $\tau\in W\cap B'\cap Z'$ or $\tau\in W\cap B'\cap Z$. Thus, it is seen that N=T.

4) $[(O,W)\underset{\backslash}{\overset{*}{\sim}}(L,B)]\,\tilde{\backslash}(H,Z)=[(O,W)\tilde{\backslash}(H,Z)]\underset{\cap}{\overset{*}{\sim}}[(L,B)\underset{\theta}{\overset{*}{\sim}}(H,Z)]$, where $W\cap B'\cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W)\underset{\backslash}{\overset{*}{\sim}}(Ö,B)=(M,W)$, where $\forall\tau\in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau\in W\backslash B \\ O(\tau)\cap L'(\tau), & \tau\in W\cap B \end{cases}$$

Let $(M,W)\,\tilde{\backslash}\,(H,Z)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau\in W\backslash Z \\ M(\tau)\cap H'(\tau), & \tau\in W\cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau\in(W\backslash B)\backslash Z=W\cap B'\cap Z' \\ O(\tau)\cap L'(\tau), & \tau\in(W\cap B)\backslash Z=W\cap B\cap Z' \\ O'(\tau)\cap H'(\tau) & \tau\in(W\backslash B)\cap Z=W\cap B'\cap Z \\ [O(\tau)\cap L'(\tau)]\cap H'(\tau) & \tau\in(W\cap B)\cap Z=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\,\tilde{\backslash}\,(H,Z)]\underset{\cap}{\overset{*}{\sim}}[(L,B)\underset{\theta}{\overset{*}{\sim}}(H,Z)]$

Let $(O,W)\,\tilde{\backslash}\,(H,Z)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\backslash Z \\ O(\tau)\cap H'(\tau), & \tau\in W\cap Z \end{cases}$$

Suppose that $(L,B) \underset{\theta}{\overset{*}{\sim}} (H,Z)=(S,B)$, where $\forall\tau\in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau\in B\backslash Z \\ L'(\tau)\cap H'(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(V,W) \underset{\cap}{\overset{*}{\sim}} (S,B)=(T,W)$, where $\forall\tau\in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau\in W\backslash B \\ V(\tau)\cap S(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau\in(W\backslash Z)\backslash B=W\cap B'\cap Z' \\ O'(\tau)\cup H(\tau), & \tau\in(W\cap Z)\backslash B=W\cap B'\cap Z \\ O(\tau)\cap L'(\tau), & \tau\in(W\backslash Z)\cap(B\backslash Z)=W\cap B\cap Z' \\ O(\tau)\cap[L'(\tau)\cap H'(\tau)], & \tau\in(W\backslash Z)\cap(B\cap Z)=\emptyset \\ [O(\tau)\cap H'(\tau)]\cap L'(\tau), & \tau\in(W\cap Z)\cap(B\backslash Z)=\emptyset \\ [O(\tau)\cap H'(\tau)]\cap[L'(\tau)\cap H'(\tau)], & \tau\in(W\cap Z)\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

It is seen that N=T.

5) $[(O,W) \underset{\backslash}{\overset{*}{\sim}} (L,B)]\, \tilde{\cap}(H,Z)= [(O,W)\tilde{\cap}(H,Z)] \underset{\cap}{\overset{*}{\sim}} [(L,B) \underset{\gamma}{\overset{*}{\sim}} (H,Z)]$, where $W\cap B'\cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W) \underset{\backslash}{\overset{*}{\sim}} (L,B)=(M,W)$, where $\forall\tau\in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau\in W\backslash B \\ O(\tau)\cap L'(\tau), & \tau\in W\cap B \end{cases}$$

Let $(M,W)\,\tilde{\cap}\,(H,Z)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau\in W\backslash Z \\ M(\tau)\cap H(\tau), & \tau\in W\cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus B)\setminus Z=W\cap B'\cap Z' \\ O(\tau)\cap L'(\tau), & \tau\in(W\cap B)\setminus Z=W\cap B\cap Z' \\ O'(\tau)\cap H(\tau) & \tau\in(W\setminus B)\cap Z=W\cap B'\cap Z \\ [O(\tau)\cap L'(\tau)]\cap H(\tau) & \tau\in(W\cap B)\cap Z=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\ \tilde{\cap}\ (H,Z)]\ \underset{\cap}{\overset{*}{\sim}}\ [(L,B)\ \underset{\gamma}{\overset{*}{\sim}}\ (H,Z)]$

Let $(O,W)\ \tilde{\cap}\ (H,Z)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cap H(\tau), & \tau\in W\cap Z \end{cases}$$

Suppose that $(L,B)\ \underset{\gamma}{\overset{*}{\sim}}\ (H,Z)=(S,B)$, where $\forall\tau\in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau\in B\setminus Z \\ L'(\tau)\cap H(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(V,W)\ \underset{\cap}{\overset{*}{\sim}}\ (S,B)=(T,W)$, where $\forall\tau\in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau\in W\setminus B \\ V(\tau)\cap S(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus Z)\setminus B=W\cap B'\cap Z' \\ O'(\tau)\cup H'(\tau), & \tau\in(W\cap Z)\setminus B=W\cap B'\cap Z \\ O(\tau)\cap L'(\tau), & \tau\in(W\setminus Z)\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cap\ [L'(\tau)\cap H(\tau)], & \tau\in(W\setminus Z)\cap(B\cap Z)=\emptyset \\ [O(\tau)\cap H(\tau)]\ \cap L'(\tau), & \tau\in(W\cap Z)\cap(B\setminus Z)=\emptyset \\ [O(\tau)\cap H(\tau)]\ \cap\ [L'(\tau)\cap H(\tau)], & \tau\in(W\cap Z)\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

It is seen that N=T.

6) $[(O,A) \overset{*}{\underset{\backslash}{\sim}} (L,B)] \underset{\lambda}{\sim} (H,Z)=[(O,W) \underset{\lambda}{\sim} (H,Z)] \overset{*}{\underset{\cap}{\sim}} [(L,B) \overset{*}{\underset{*}{\sim}} (H,Z))]$ where $W\cap B'\cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W) \overset{*}{\underset{\backslash}{\sim}} (L,B)=(M,W)$,

where $\forall\tau\in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau\in W\backslash B \\ O(\tau)\cap L'(\tau), & \tau\in W\cap B \end{cases}$$

Let $(M,W)\ \tilde{\lambda}\ (H,Z)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau\in W\backslash Z \\ M(\tau)\cup H'(\tau), & \tau\in W\cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau\in (W\backslash B)\backslash Z=W\cap B'\cap Z' \\ O(\tau)\cap L'(\tau), & \tau\in (W\cap B)\backslash Z=W\cap B\cap Z' \\ O'(\tau)\cup H'(\tau) & \tau\in (W\backslash B)\cap Z=W\cap B'\cap Z \\ [O(\tau)\cap L'(\tau)]\cup H'(\tau) & \tau\in (W\cap B)\cap Z=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\ \tilde{\lambda}\ (H,Z)] \overset{*}{\underset{\cap}{\sim}} [(L,B) \overset{*}{\underset{*}{\sim}} (H,C]$

Let $(O,W)\ \tilde{\lambda}\ (H,Z)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\backslash Z \\ O(\tau)\cup H'(\tau), & \tau\in W\cap Z \end{cases}$$

Suppose that $(L,B) \overset{*}{\underset{*}{\sim}} (H,Z)=(S,B)$, where $\forall\tau\in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau\in B\backslash Z \\ L'(\tau)\cup H'(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(V,W) \underset{\cap}{\overset{*}{\sim}} (S,B)=(T,W)$, where $\forall\tau\in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau\in W\setminus B \\ V(\tau)\cap S(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus Z)\setminus B=W\cap B'\cap Z' \\ O'(\tau)\cap H(\tau), & \tau\in(W\cap Z)\setminus B=W\cap B'\cap Z \\ O(\tau)\cap L'(\tau), & \tau\in(W\setminus Z)\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cap [L'(\tau)\cup H'(\tau)], & \tau\in(W\setminus Z)\cap(B\cap Z)=\emptyset \\ [O(\tau)\cup H'(\tau)]\cap L'(\tau), & \tau\in(W\cap Z)\cap(B\setminus Z)=\emptyset \\ [O(\tau)\cup H'(\tau)]\cap [L'(\tau)\cup H'(\tau)], & \tau\in(W\cap Z)\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

It is seen that N=T.

3.1.2. Distribution of soft binary piecewise operations over complementary soft binary piecewise lambda (λ) operation:

1) $(O,W)\,\tilde{\cap}[(L,B) \underset{\lambda}{\overset{*}{\sim}} (H,Z)]=[(O,W) \tilde{\cap} (L,B)]\,\tilde{\cup}\,[(O,W) \tilde{\setminus} (H,Z)]$, where $W\cap B\cap Z'=\emptyset$

Proof: Let first handle the left hand side of the equality and let $(L,B) \underset{\lambda}{\overset{*}{\sim}} (H,Z)=(M,B)$,

where $\forall\tau\in B$;

$$M(\tau)=\begin{cases} L'(\tau), & \tau\in B\setminus Z \\ L(\tau)\cup H'(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(O,W)\,\tilde{\cap}(M,B)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cap M(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cap L'(\tau), & \tau\in W\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cap [(L(\tau)\cup H'(\tau)], & \tau\in W\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\,\tilde{\cap}\,(L,B)]\ \tilde{\cup}\ [(O,W)\,\tilde{\setminus}\,(H,Z)]$,

Let $(O,W)\,\tilde{\cap}\,(L,B)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cap L(\tau), & \tau\in W\cap B \end{cases}$$

Suppose that $(O,W)\,\tilde{\setminus}\,(H,Z)=(S,W)$, where $\forall\tau\in W$;

$$S(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cap H'(\tau), & \tau\in W\cap Z \end{cases}$$

Let $(V,W)\,\tilde{\cup}(S,W)=(T,W)$ $\quad\forall\tau\in W$;

$$T(\tau)=\begin{cases} V(\tau), & \tau\in W\setminus W=\emptyset \\ V(\tau)\cup S(\tau), & \tau\in W\cap W \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau)\cup O(\tau), & \tau\in(W\setminus B)\cap(W\setminus Z)=W\cap B'\cap Z' \\ O(\tau)\cup[(O(\tau)\cap H'(\tau)], & \tau\in(W\setminus B)\cap(W\cap Z)=W\cap B'\cap Z \\ [(O(\tau)\cap L(\tau)]\cup O(\tau), & \tau\in(W\cap B)\cap(W\setminus Z)=W\cap B\cap Z' \\ [(O(\tau)\cap L(\tau)]\cup[(O(\tau)\cap H'(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau) & \tau\in(W\setminus B)\cap(W\setminus Z)=W\cap B'\cap Z' \\ O(\tau), & \tau\in(W\setminus B)\cap(W\cap Z)=W\cap B'\cap Z \\ O(\tau), & \tau\in(W\cap B)\cap(W\setminus Z)=W\cap B\cap Z' \\ [(O(\tau)\cap L(\tau)]\cup[(O(\tau)\cap H'(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Here let handle $\tau\in W\setminus B$ in the first equation. Since $W\setminus B=W\cap B'$, if $\tau\in B'$, then $\tau\in Z\setminus B$ or $\tau\in(B\cup Z)'$. Hence, if $\tau\in W\setminus B$, $\tau\in W\cap B'\cap Z'$ or $\tau\in W\cap B'\cap Z$. Thus, it is seen that N=T.

2) $[(O,W)\underset{\lambda}{\overset{*}{\sim}}(L,B)]\ \tilde{\cap}(H,Z)=[(O,W)\tilde{\cap}(H,Z)]\underset{\cup}{\overset{*}{\sim}}[(L,B)\underset{\gamma}{\overset{*}{\sim}}(H,Z)]$, where $W\cap B'\cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W)\underset{\lambda}{\overset{*}{\sim}}(L,B)=(M,W)$, where $\forall\tau\in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau\in W\setminus B \\ O(\tau)\cup L'(\tau), & \tau\in W\cap B \end{cases}$$

Let $(M,W)\ \tilde{\cap}\ (H,Z)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau\in W\setminus Z \\ M(\tau)\cap H(\tau), & \tau\in W\cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus B)\setminus Z=W\cap B'\cap Z' \\ O(\tau)\cup L'(\tau), & \tau\in(W\cap B)\setminus Z=W\cap B\cap Z' \\ O'(\tau)\cap H(\tau) & \tau\in(W\setminus B)\cap Z=W\cap B'\cap Z \\ [O(\tau)\cup L'(\tau)]\cap H(\tau) & \tau\in(W\cap B)\cap Z=W\cap B\cap Z \end{cases}$$

Now let hndle the left hand side of the equality: $[(O,W)\ \tilde{\cap}\ (H,Z)]\underset{\cup}{\overset{*}{\sim}}[(L,B)\underset{\gamma}{\overset{*}{\sim}}(H,Z)]$

Let $(O,W)\ \tilde{\cap}\ (H,Z)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cap H(\tau), & \tau\in W\cap Z \end{cases}$$

Suppose that $(L,B)\underset{\gamma}{\overset{*}{\sim}}(H,Z)=(S,B)$, where $\forall\tau\in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau\in B\setminus Z \\ L'(\tau)\cap H(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(V,W) \underset{\cup}{\overset{*}{\sim}} (S,B)=(T,W)$, where $\forall \tau \in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau \in W \setminus B \\ V(\tau) \cup S(\tau), & \tau \in W \cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau \in (W \setminus Z) \setminus B = W \cap B' \cap Z' \\ O'(\tau) \cup H'(\tau), & \tau \in (W \cap Z) \setminus B = W \cap B' \cap Z \\ O(\tau) \cup L'(\tau), & \tau \in (W \setminus Z) \cap (B \setminus Z) = W \cap B \cap Z' \\ O(\tau) \cup [L'(\tau) \cap H(\tau)], & \tau \in (W \setminus Z) \cap (B \cap Z) = \emptyset \\ [O(\tau) \cap H(\tau)] \cup L'(\tau), & \tau \in (W \cap Z) \cap (B \setminus Z) = \emptyset \\ [O(\tau) \cap H(\tau)] \cup [L'(\tau) \cap H(\tau)], & \tau \in (W \cap Z) \cap (B \cap Z) = W \cap B \cap Z \end{cases}$$

It is seen that N=T.

3) $(O,W) \tilde{\lambda} [(L,B) \underset{\lambda}{\overset{*}{\sim}} (H,Z)]=[(O,W) \tilde{\lambda} (L,B)] \tilde{\cap} [(O,W) \tilde{\cup} (H,Z)]$, where $W \cap B \cap Z' = \emptyset$

Proof: Let first handle the left hand side of the equality and let $(L,B) \underset{\lambda}{\overset{*}{\sim}} (H,Z)=(M,B)$, where $\forall \tau \in B$;

$$M(\tau)=\begin{cases} L'(\tau), & \tau \in B \setminus Z \\ L(\tau) \cup H'(\tau), & \tau \in B \cap Z \end{cases}$$

Let $(O,W) \tilde{\lambda} (M,B)=(N,W)$, where $\forall \tau \in W$;

$$N(\tau)=\begin{cases} O(\tau), & \tau \in W \setminus B \\ O(\tau) \cup M'(\tau), & \tau \in W \cap B \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O(\tau), & \tau \in W \setminus B \\ O(\tau) \cup L(\tau), & \tau \in W \cap (B \setminus Z) = W \cap B \cap Z' \\ O(\tau) \cup [(L'(\tau) \cap H(\tau)], & \tau \in W \cap (B \cap Z) = W \cap B \cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W) \tilde{\lambda} (L,B)] \tilde{\cap} [(O,W) \tilde{\cup} (H,Z)]$.

Let $(O,W)\ \tilde{\lambda}\ (L,B)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus B \\ O(\tau)\cup L'(\tau), & \tau\in W\cap B \end{cases}$$

Suppose that $(O,W)\ \tilde{\cup}(H,Z)=(S,W)$, where $\forall\tau\in W$;

$$S(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cup H(\tau), & \tau\in W\cap Z \end{cases}$$

Let $(V,W)\ \tilde{\cap}(S,W)=(T,W)$ $\quad\forall\tau\in W$;

$$T(\tau)=\begin{cases} V(\tau), & \tau\in W\setminus W=\emptyset \\ V(\tau)\cap S(\tau), & \tau\in W\cap W \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau)\cap O(\tau), & \tau\in(W\setminus B)\cap(W\setminus Z)=W\cap B'\cap Z' \\ O(\tau)\cap[(O(\tau)\cup H(\tau)], & \tau\in(W\setminus B)\cap(W\cap Z)=W\cap B'\cap Z \\ [(O(\tau)\cup L'(\tau)]\cap O(\tau), & \tau\in(W\cap B)\cap(W\setminus Z)=W\cap B\cap Z' \\ [(O(\tau)\cup L'(\tau)]\cap[(O(\tau)\cup H(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O(\tau) & \tau\in(W\setminus B)\cap(W\setminus Z)=W\cap B'\cap Z' \\ O(\tau), & \tau\in(W\setminus B)\cap(W\cap Z)=W\cap B'\cap Z \\ O(\tau), & \tau\in(W\cap B)\cap(W\setminus Z)=W\cap B\cap Z' \\ [(O(\tau)\cup L'(\tau)]\cap[(O(\tau)\cup H(\tau)], & \tau\in(W\cap B)\cap(W\cap Z)=W\cap B\cap Z \end{cases}$$

Since $W\setminus B= W\cap B'$, if $\tau\in B'$, then $\tau\in Z\setminus B$ or $\tau\in(B\cup Z)'$. Hence, if $\tau\in W\setminus B$, $\tau\in W\cap B'\cap Z'$ or $\tau\in W\cap B'\cap Z$. Thus, it is seen that N=T.

4) $[(O,W)\ \underset{\lambda}{\overset{*}{\sim}}(L,B)]\ \tilde{\lambda}\ (H,Z)=\ [(O,W)\ \tilde{\lambda}\ (H,Z)]\ \underset{\cup}{\overset{*}{\sim}}\ [(L,B)\ \underset{*}{\overset{*}{\sim}}\ (H,Z)]$ where $W\cap B'\cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W)\ \underset{\lambda}{\overset{*}{\sim}}\ (\ddot{O},B)=(M,W)$,

where $\forall\tau\in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau\in W\setminus B \\ O(\tau)\cup L'(\tau), & \tau\in W\cap B \end{cases}$$

Let $(M,W)\ \tilde{\lambda}\ (H,Z)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau\in W\backslash Z \\ M(\tau)\cup H'(\tau), & \tau\in W\cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau\in(W\backslash B)\backslash Z=W\cap B'\cap Z' \\ O(\tau)\cup L'(\tau), & \tau\in(W\cap B)\backslash Z=W\cap B\cap Z' \\ O'(\tau)\cup H'(\tau) & \tau\in(W\backslash B)\cap Z=W\cap B'\cap Z \\ [O(\tau)\cup L'(\tau)]\cup H'(\tau) & \tau\in(W\cap B)\cap Z=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\ \tilde{\lambda}\ (H,Z)]\ \overset{*}{\underset{\cup}{\sim}}\ [(L,B)\ \overset{*}{\underset{*}{\sim}}\ (H,Z)]$.

Let $(O,W)\ \tilde{\lambda}\ (H,Z)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\backslash Z \\ O(\tau)\cup H'(\tau), & \tau\in W\cap Z \end{cases}$$

Suppose that $(L,B)\ \overset{*}{\underset{*}{\sim}}\ (H,Z)=(S,B)$, where $\forall\tau\in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau\in B\backslash Z \\ L'(\tau)\cup H'(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(V,W)\ \overset{*}{\underset{\cup}{\sim}}\ (S,B)=(T,W)$, where $\forall\tau\in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau\in W\backslash B \\ V(\tau)\cup S(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau\in(W\backslash Z)\backslash B=W\cap B'\cap Z' \\ O'(\tau)\cap H(\tau), & \tau\in(W\cap Z)\backslash B=W\cap B'\cap Z \\ O(\tau)\cup L'(\tau), & \tau\in(W\backslash Z)\cap(B\backslash Z)=W\cap B\cap Z' \\ O(\tau)\cup\ [L'(\tau)\cup H'(\tau)], & \tau\in(W\backslash Z)\cap(B\cap Z)=\emptyset \\ [O(\tau)\cup H'(\tau)]\ \cup L'(\tau), & \tau\in(W\cap Z)\cap(B\backslash Z)=\emptyset \\ [O(\tau)\cup H'(\tau)]\ \cup\ [L'(\tau)\cup H'(\tau)], & \tau\in(W\cap Z)\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

It is seen that N=T.

5) $[(O,W) \underset{\lambda}{\overset{*}{\sim}} (L,B)] \tilde{\setminus}(H,Z)= [(O,W)\tilde{\setminus}(H,Z)] \underset{\cup}{\overset{*}{\sim}} [(L,B) \underset{\theta}{\overset{*}{\sim}}(H,Z)]$, where $W \cap B' \cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W) \underset{\lambda}{\overset{*}{\sim}} (\ddot{O},B)=(M,W)$,

where $\forall \tau \in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau \in W\setminus B \\ O(\tau) \cup L'(\tau), & \tau \in W \cap B \end{cases}$$

Let $(M,W) \tilde{\setminus} (H,Z)=(N,W)$, where $\forall \tau \in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau \in W\setminus Z \\ M(\tau) \cap H'(\tau), & \tau \in W \cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau \in (W\setminus B)\setminus Z=W \cap B' \cap Z' \\ O(\tau) \cup L'(\tau), & \tau \in (W \cap B)\setminus Z=W \cap B \cap Z' \\ O'(\tau) \cap H'(\tau) & \tau \in (W\setminus B) \cap Z=W \cap B' \cap Z \\ [O(\tau) \cup L'(\tau)] \cap H'(\tau) & \tau \in (W \cap B) \cap Z=W \cap B \cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W) \tilde{\setminus} (H,Z)] \underset{\cup}{\overset{*}{\sim}} [(L,B) \underset{\theta}{\overset{*}{\sim}}(H,Z)]$

Let $(O,W) \tilde{\setminus} (H,Z)=(V,W)$, where $\forall \tau \in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau \in W\setminus Z \\ O(\tau) \cap H'(\tau), & \tau \in W \cap Z \end{cases}$$

Suppose that $(L,B) \underset{\theta}{\overset{*}{\sim}}(H,Z)=(S,B)$, where $\forall \tau \in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau \in B\setminus Z \\ L'(\tau) \cap H'(\tau), & \tau \in B \cap Z \end{cases}$$

Let $(V,W) \underset{\cup}{\overset{*}{\sim}} (S,B)=(T,W)$, where $\forall \tau \in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau\in W\setminus B \\ V(\tau)\cup S(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus Z)\setminus B=W\cap B'\cap Z' \\ O'(\tau)\cup H(\tau), & \tau\in(W\cap Z)\setminus B=W\cap B'\cap Z \\ O(\tau)\cup L'(\tau), & \tau\in(W\setminus Z)\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cup[L'(\tau)\cap H'(\tau)], & \tau\in(W\setminus Z)\cap(B\cap Z)=\emptyset \\ [O(\tau)\cap H'(\tau)]\cup L'(\tau), & \tau\in(W\cap Z)\cap(B\setminus Z)=\emptyset \\ [O(\tau)\cap H'(\tau)]\cup[L'(\tau)\cap H'(\tau)], & \tau\in(W\cap Z)\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

It is seen that N=T.

6) $[(O,W)\overset{*}{\underset{\lambda}{\sim}}(L,B)]\,\tilde{\cup}\,(H,Z)=[(O,W)\,\tilde{\cup}\,(H,Z)]\overset{*}{\underset{\cup}{\sim}}[(L,B)\overset{*}{\underset{+}{\sim}}(H,Z)]$ where $W\cap B'\cap Z=\emptyset$.

Proof: Let first handle the left hand side of the equality and let $(O,W)\overset{*}{\underset{\cup}{\sim}}(\ddot{O},B)=(M,W)$,

where $\forall\tau\in W$;

$$M(\tau)=\begin{cases} O'(\tau), & \tau\in W\setminus B \\ O(\tau)\cup L'(\tau), & \tau\in W\cap B \end{cases}$$

Let $(M,W)\,\tilde{\cup}\,(H,Z)=(N,W)$, where $\forall\tau\in W$;

$$N(\tau)=\begin{cases} M(\tau), & \tau\in W\setminus Z \\ M(\tau)\cup H(\tau), & \tau\in W\cap Z \end{cases}$$

Thus,

$$N(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus B)\setminus Z=W\cap B'\cap Z' \\ O(\tau)\cup L'(\tau), & \tau\in(W\cap B)\setminus Z=W\cap B\cap Z' \\ O'(\tau)\cup H(\tau) & \tau\in(W\setminus B)\cap Z=W\cap B'\cap Z \\ [O(\tau)\cup L'(\tau)]\cup H(\tau) & \tau\in(W\cap B)\cap Z=W\cap B\cap Z \end{cases}$$

Now let handle the left hand side of the equality: $[(O,W)\,\tilde{\cup}\,(H,Z)]\overset{*}{\underset{\cup}{\sim}}[(L,B)\overset{*}{\underset{+}{\sim}}(H,Z)]$

Let $(O,W) \underset{\cup}{\sim} (H,Z)=(V,W)$, where $\forall\tau\in W$;

$$V(\tau)=\begin{cases} O(\tau), & \tau\in W\setminus Z \\ O(\tau)\cup H(\tau), & \tau\in W\cap Z \end{cases}$$

Suppose that $(L,B) \overset{*}{\underset{+}{\sim}} (H,Z)=(S,B)$, where $\forall\tau\in B$;

$$S(\tau)=\begin{cases} L'(\tau), & \tau\in B\setminus Z \\ L'(\tau)\cup H(\tau), & \tau\in B\cap Z \end{cases}$$

Let $(V,W) \overset{*}{\underset{\cup}{\sim}} (S,B)=(T,W)$, where $\forall\tau\in W$;

$$T(\tau)=\begin{cases} V'(\tau), & \tau\in W\setminus B \\ V(\tau)\cup S(\tau), & \tau\in W\cap B \end{cases}$$

Thus,

$$T(\tau)=\begin{cases} O'(\tau), & \tau\in(W\setminus Z)\setminus B=W\cap B'\cap Z' \\ O'(\tau)\cap H'(\tau), & \tau\in(W\cap Z)\setminus B=W\cap B'\cap Z \\ O(\tau)\cup L'(\tau), & \tau\in(W\setminus Z)\cap(B\setminus Z)=W\cap B\cap Z' \\ O(\tau)\cup[L'(\tau)\cup H(\tau)], & \tau\in(W\setminus Z)\cap(B\cap Z)=\emptyset \\ [O(\tau)\cup H(\tau)]\cup L'(\tau), & \tau\in(W\cap Z)\cap(B\setminus Z)=\emptyset \\ [O(\tau)\cup H(\tau)]\cup[L'(\tau)\cup H(\tau)], & \tau\in(W\cap Z)\cap(B\cap Z)=W\cap B\cap Z \end{cases}$$

It is seen that N=T.

4. Conclusion

In this paper, we explore more about complementary soft binary piecewise plus and lambda operation by examining the relationships between these soft set operation and soft binary piecewise operations. We contribute to the soft set literature by obtaining the distributions of soft binary piecewise operations over complementary soft binary piecewise difference and lambda operations. This is a theoretical study for soft sets and some future studies may continue by examining the distributions of soft binary piecewise operations over other complementary soft binary piecewise operations. Moreover, since soft sets are a powerful mathematical tool for detecting insecure objects, this work allows researchers to propose new encryption or decision methods based on soft sets. Also, the study of soft algebraic structures can be redone in terms of algebraic properties by the operations defined in this article.

5. References

Akbulut, E., 2023. New type of extended operations of soft set: Complementary extended lambda and difference operations. Amasya University, *The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya (in Turkish).*

Aybek, F., 2023. New restricted and extended soft set operations. Amasya University, *The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya (in Turkish).*

Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M., 2009. On some new operations in soft set theory. Computers and Mathematics with Applications, 57(9), 1547-1553.

Eren, Ö.F. and Çalışıcı, H., 2019. On some operations of soft sets, The Fourth International Conference on Computational Mathematics and Engineering Sciences, Antalya.

Çağman, N., 2021. Conditional complements of sets and their application to group theory. Journal of New Results in Science, 10 (3), 67-74.

Demirci, A.M., 2023. New type of extended operations of soft set: Complementary extended plus, union and theta operations, Amasya University, *The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya (in Turkish).*

Maji, P.K, Bismas. R., Roy, A.R., 2003. Soft set theory. Computers and Mathematics with Applications, 45 (1), 555-562.

Molodtsov, D., 1999. Soft set theory-first results. Computers and Mathematics with Applications, 37 (1), 19-31.

Özlü, Ş.,2022a. Interval Valued q- Rung Orthopair Hesitant Fuzzy Choquet Aggregating Operators in Multi-Criteria Decision Making Problems. Gazi University Journal of Science Part C: Design and Technology, 10 (4), 1006-1025.

Özlü, Ş.,2022b. Interval Valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging Operator Based On Multi-Criteria Decision Making Problems. Gazi University Journal of Science Part C: Design and Technology, 10 (4), 841-857.

Özlü, Ş. and Sezgin, A., 2020. Soft covered ideals in semigroups. Acta Universitatis Sapientiae, Mathematica. 12 (2), 317-346.

Pei, D. and Miao, D., 2005. From Soft Sets to Information Systems. In: Proceedings of Granular Computing. IEEE, (2), 617-621.

Sarıalioğlu, M., 2023. New type of extended operations of soft set: Complementary extended gamma, intersection and star operations. Amasya University. *The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya (in Turkish).*

Sezgin, A. and Atagün, A.O., 2011. On operations of soft sets. Computers and Mathematics with Applications. 61(5), 1457-1467.

Sezgin, A. and Aybek, F., 2023, New soft set operation: Complementary soft binary piecewise gamma operation. Matrix Science Mathematic. 7 (1), 27-45.

Sezgin, A, Aybek, F., Atagün, A.O. 2023a. New soft set operation: Complementary soft binary piecewise intersection operation. Black Sea Journal of Engineering and Science. (accepted/in press).

Sezgin, A, Aybek, F., Güngör, Bilgili N. 2023b. New soft set operation: Complementary soft binary piecewise union operation. Acta Informatica Malaysia. 7(1), 38-53.

Sezgin, A. and Çağman, N. 2023. New soft set operation: Complementary soft binary piecewise difference operation, Osmaniye Korkut Ata University Journal of the Institute of Science and Technology. (accepted/in press).

Sezgin, A., Çağman, N. Atagün, AO., Aybek, F., 2023c. Complemental binary operations of sets and their application to group theory. Matriks Sains Matematik. (accepted/in press).

Sezgin, A. and Demirci, A.M., 2023. New soft set operation: Complementary soft binary piecewise star operation. Ikonion Journal of Mathematics. 5(2), 24-52.

Sezgin, A, Shahzad, A., Mehmood, A., 2019. New Operation on Soft Sets: Extended Difference of Soft Sets. Journal of New Theory. (27), 33-42.

Sezgin, A. and Yavuz, E., 2023. New soft set operation: Complementary soft binary piecewise lambda operation. Sinop University Journal of Natural Sciences. (accepted/in press).

Stojanovic, N.S., 2021. A new operation on soft sets: extended symmetric difference of soft sets. Military Technical Courier. 69(4), 779-791.

Yavuz, E., 2023. Soft binary piecewise operations and their properties, Amasya University, Amasya University, *The Graduate School of Natural and Applied Sciences Master of Science in Mathematics Department, Amasya (in Turkish).*