CHAOS
Theory and Applications
in Applied Sciences and Engineering

e-ISSN: 2687-4539
RESEARCH ARTICLE
Vol.5 / No.3 / 2023 / pp.233-241
https://doi.org/10.51537/chaos.1320492

Chaos and Control of COVID-19 Dynamical System

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ABSTRACT Chaos, which is found in many dynamical systems, due to the presence of chaos, systems behave erratically. Due to its erratic behaviour, the chaotic behaviour of the system needs to be controlled. Severe acute respiratory syndrome Coronavirus 2 (Covid-19), which has spread all over the world as a pandemic. Many dynamical systems have been proposed to understand the spreading behaviour of the disease. This paper investigates the chaos in the outbreak of COVID-19 via an epidemic model. Chaos is observed in the proposed SIR model. The controller is designed based on the fractional-order Routh Hurwitz criteria for fractional-order derivatives. The chaotic behaviour of the model is controlled by feedback control techniques, and the stability of the system is discussed.

KEYWORDS
Chaos
Feedback control
Fractional order
Routh Hurwitz criteria
Chaotic attractor
Stability

INTRODUCTION

Mathematical modelling is one of the best ways to understand the dynamics of physical phenomena. Some dynamical systems, whether they are linear or nonlinear, show unpredictable behaviour which is termed "chaos." Chaos is a very active area of research for researchers who are working particularly in the nonlinear dynamical system. Chaos does not have a unified definition, yet this phenomenon is observed and studied in different branches of science and technology, whether it is science, population dynamics, telecommunication engineering, etc.

The COVID-19 epidemic first broke out in December 2019, when its danger and impact were not known. The conditions under which this disease will propagate are also unknown to the world. It is necessary to control the spread of any disease. To control the spread of the disease, we must understand its behaviour particularly the virus's speed of infection and the duration of its symptoms. All the governments and world health organisations are trying to control and prevent the spread of COVID-19. One of the important steps to controlling the spread of COVID-19 is the mathematical modelling of this disease and its analysis. Various techniques have been developed to model infectious diseases.

Manuscript received: 28 June 2023,
Revised: 15 October 2023,
Accepted: 27 October 2023.

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One of the popular methods is the compartmental method. In this method, the entire population is segregated into different compartments, and the interplay between these compartments is represented in the form of equations to represent the model. (Kermack and McKendrick 1927) have proposed for the first time the mathematical model of an epidemic where they have separated the entire population into three compartments: (i) people who are prone to the disease; (ii) people who are already infected and can spread the infection; (iii) people who are already recovered and have developed the immune system; or (iv) people who have left the study area. Many mathematical models (Alsadat *et al.* 2023; Debbouche *et al.* 2021; Giordano *et al.* 2020; Haq *et al.* 2022; Javeed *et al.* 2021; Babu *et al.* 2021; Mandal *et al.* 2020) are proposed for the study of COVID-19. (Xie 2020; Maltezos and Georgakopoulou 2021; Farshi 2020) have used Monte Carlo simulation models to determine the development of COVID spread.

Chaos in the dynamical system of COVID-19 was analysed by (Mangiarotti *et al.* 2020) in 2020, where he worked on the data of the national health commission of the People's Republic of China. In this work, (Mangiarotti *et al.* 2020) have proposed a model based on three variables: (i) the cumulated number of daily confirmed cases; (ii) the daily number of serious cases and those who are under intensive care at present; (iii) the daily cumulated number of deaths. From these parameters, the daily number of new cases, the daily number of additional severe cases, and the daily number of new deaths are derived. The chaos in this model has been observed with 11 parameters. (Debbouche *et al.* 2021)have conceived the dynamical system model proposed by (Mangiarotti *et al.* 2020) of COVID-19 with fractional order differentiation in the

Caputo sense. The fractional order derivative with commensurate and incommensurate order has been analyzed, and the chaotic behaviour of it has been observed. (Postavaru *et al.* 2021) in 2021 studied the Covid-19 pandemic and chaos.

The fractional order derivative is considered for the consideration of memory concepts in the dynamical system. Although it is quite difficult to formulate a complete model of any novel epidemic, many parameters may still not be known. (Higazy 2020) has used the fractional-order SIDARTHE model and proposed the control strategy. (Ahmad *et al.* 2022) proposed the fractional order model considering five classes of the population. (Borah *et al.* 2022) have investigated the memory effect by introducing the fractional derivative and chaos. They used different methods for controlling the chaos. (Xu and Tang 2021) proposed an integrated epidemic modelling framework for the real time forecast of COVID-19.(Xu *et al.* 2020) proposed a generalised fractional order SEIR model for forecast analysis of the epidemic trends in the USA.(Chandra and Bajpai 2022) have proposed the fractional order model with the consideration of social distancing as one parameter to make the model mimic real-time data.

These proposed COVID-19 models do not address how to control the chaos present in the dynamical system. There are numerous methods to achieve chaos control. Due to their ease of design, the first two primary methods for managing chaos are feedback control and non-feedback control, which are particularly appealing and have been widely used in actual implementation. (Bai and Lonngren 2000) put forth the Active Control Method, which, due to its ease of use and simplicity in applications, has drawn the attention of many researchers working in the field of nonlinear dynamics. (Srivastava *et al.* 2014) have controlled the chaos of the fractional-order Rabinovich-Fabrikant system. (Borah *et al.* 2021) have controlled and anti-controlled fractional order models of diabetes, HIV, dengue, migraine, Parkinson's, and Ebola-virous diseases.

The present article is further divided in the following sections: (i) Section 2 explains the preliminary concepts of fractional differentiations and the stability of fractional order Routh-Hurwitz criterion, it has the basic information about the proposed model (ii). Section 3 contains the stability analysis of the system (iii). Section 4 contains the analysis of the chaos controls, and the parameters required for the control are presented in this section. (iv) Section 5 talks about the results (v). Section 6 is the conclusion. It is to the author's knowledge that no author has tried to control the chaos of a dynamical system of the kind proposed in the current article.

PRELIMINARIES

Definition: The Riemann-Liouville (Podlubnv 1999) type fractional derivative of order $\alpha \geq 0$ of function f(0,∞)↦ R. is defined by

$$D^{\alpha} f(t) = \frac{d^n}{dt^n} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f(\tau)\, d\tau \tag{1}$$

where n=[α]+1 and [α] is the integer part of α.

Definition: The Caputo type (Podlubnv 1999) fractional derivative of order α>0 of the function f (0,∞)→ R is defined by

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{\alpha-1} f^{(n)}(\tau)\, d\tau \tag{2}$$

where n= [α]+1 and [α] is the integer part of α.

Theorem: (Matignon 1996) an autonomous system of type (3)

$$\begin{aligned} D_t^{\alpha} x(t) &= f_1(x,y,z) \\ D_t^{\alpha} y(t) &= f_2(x,y,z) \\ D_t^{\alpha} z(t) &= f_3(x,y,z) \end{aligned} \tag{3}$$

is said to be asymptotically stable by if and only if all its eigenvalues of the Jacobian matrix.

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix} \tag{4}$$

at its equilibrium point meets specific requirements of.$|arg(\lambda)| > \frac{\alpha\pi}{2}$.This result is derived by (Matignon 1996) for a linear dynamical system. Since local linearization is a technique used to test the local stability of equilibrium points in nonlinear systems, the theorem can be used in this context (Srivastava *et al.* 2014).
The characteristic equation of the Jacobian matrix at the equilibrium is

$$TP(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 \tag{5}$$

The discriminant is
$D(P) = 18a_1a_2a_3 + (a_1a_2)^2 - 4a_3a_1{}^3 - 4a_2^3 - 27a_3^2$
The fractional order Routh-Hurwitz criterion (Ahmed *et al.* 2006; Srivastava *et al.* 2014) is as follows for the system to be stable.:
(i) The equilibrium point meets the necessary and sufficient conditions in order to be locally asymptotically stable, and if $D(P) > 0$, these conditions are $a_1 > 0, a_3 > 0, a_1a_2 - a_3 > 0$.
(ii)If $D(P) < 0, a_1 \geq 0, a_2 \geq 0, a_3 > 0$ then the equilibrium point is locally asymptotically stable for $\alpha < \frac{2}{3}$. However, if $D(P) < 0, a_1 < 0, a_2 < 0, \alpha > \frac{2}{3}$,then all the roots of the characteristic equation satisfy the condition
(iii)If $D(P) < 0, a_1 > 0, a_2 > 0,\ a_1a_2 - a_3 = 0$then all individuals $0 \leq \alpha < 1$ are locally asymptotically stable at the equilibrium point.
(iv) A need for equilibrium points to be locally stable asymptotically is $a_3 > 0$.

Proposed Model: The considered model in this article is. As proposed in (Mangiarotti *et al.* 2020) The three decision variable x (Number of daily cases) ,y (Number of daily serious cases reported) and Z(Number of daily deaths) along with 11 parameters $\alpha_1,\alpha_2,\ \alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7,\alpha_8,\alpha_9,\alpha_{10},\alpha_{11}$.

$$\begin{aligned} \frac{d^{\alpha}x}{dt^{\alpha}} &= \alpha_1 z^2 + \alpha_2 x^2 + \alpha_3 y(z + \alpha_4 x) \\ \frac{d^{\alpha}y}{dt^{\alpha}} &= \alpha_5 x + \alpha_6 y + \alpha_7 z^2 \\ \frac{d^{\alpha}z}{dt^{\alpha}} &= \alpha_8 xz + \alpha_9 xy + \alpha_{10} z + \alpha_{11} x^2 \end{aligned} \tag{6}$$

Values of the parameters are considered as The above model shows the chaotic behaviour with initial values x=180, y=30, z=8 for the time variation of t=100, for order of derivatives α=0.97.

Table 1 Values of the parameter in the model (6)

α_1 = -0.10530723	α_2 = $2.343X10^-5$	α_3=0.15204	α_4=-0.01451520 .
α_5=-0.20517824	α_6=0.44040714	α_7=0.16060376	α_8=-0.00011493.
α_9=$-1.215X10^-5$	α_{10}=0.2844499	α_{11}=$2.38X10^-6$	.

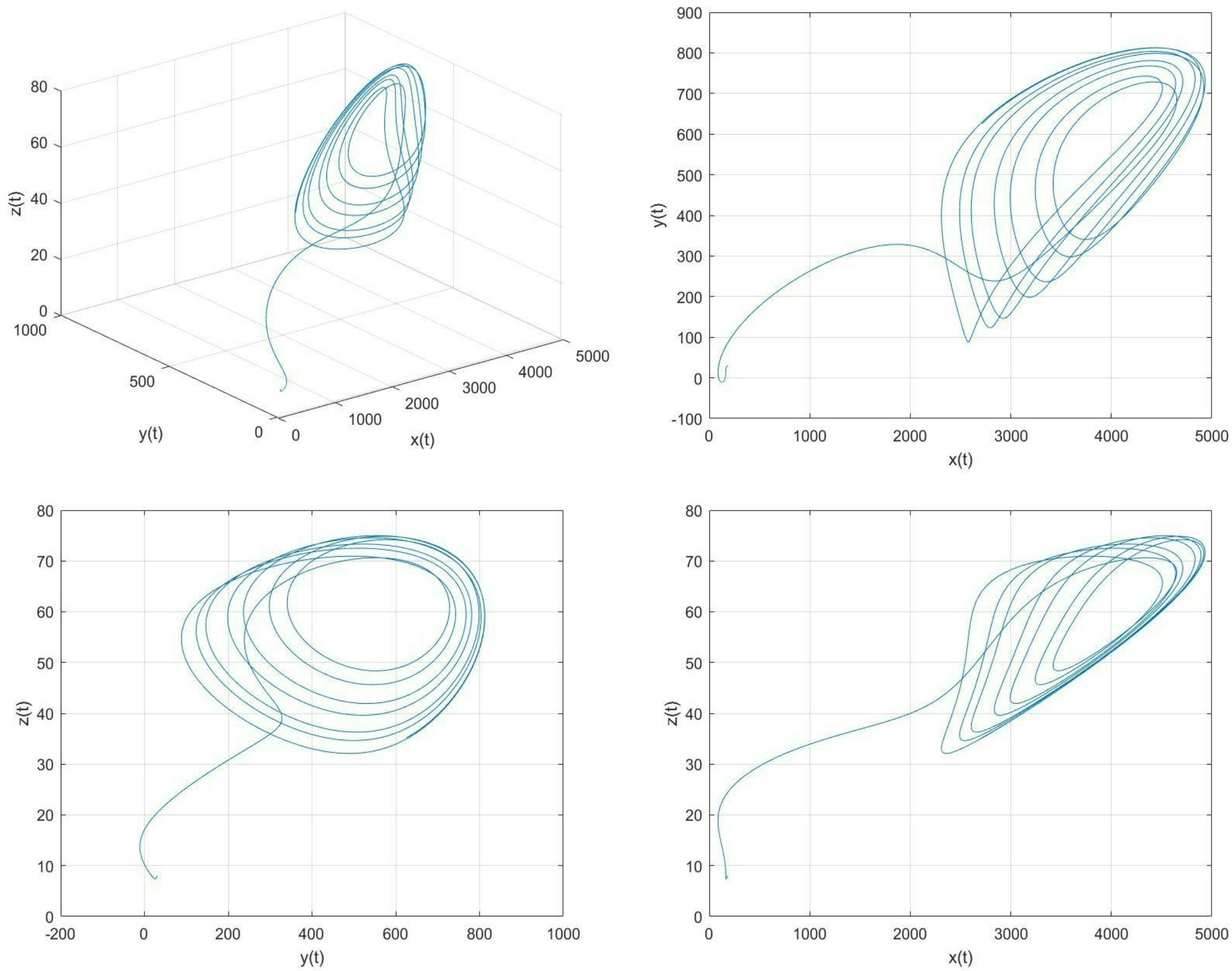

Figure 1 Chaotic behaviour of the system in (6) with initial values x=180, y=30, z=8 for the time variation of t=100, for order of derivatives $\alpha = 0.97$

STABILITY OF THE SYSTEM

To analyse the stability of the system we have

$$\begin{aligned} \alpha_1 z^2 + \alpha_2 x^2 + \alpha_3 y\,(z + \alpha_4 x) &= 0 \\ \alpha_5 x + \alpha_6 y + \alpha_7\, z^2 &= 0 \\ \alpha_8 xz + \alpha_9 xy + \alpha_{10} z + \alpha_{11} x^2 &= 0 \end{aligned} \tag{7}$$

On solving this nonlinear system of equations with the given parameters as in proposed model is given, we have. The Jacobian matrix (J) of the above system is

$$J = \begin{bmatrix} 2\alpha_2 x + \alpha_3 \alpha_4 y & \alpha_3 z & 2\alpha_1 z + \alpha_3 y \\ \alpha_5 & \alpha_6 & 2\alpha_7 z \\ \alpha_8 z + \alpha_9 y + 2\alpha_{11} x & \alpha_9 x & \alpha_8 x + \alpha_{10} \end{bmatrix} \tag{8}$$

The equilibrium point is calculated on solving the equations in (7) and we get 4 equilibrium points which are $E_1(0,0,0)$,$E_2(-1149.44, -590.097, 12.2352)$,$E_3(-619.232, -6075.71, 125.975)$, $E_4(25638.5, 6103.77, -126.557)$.

The eigen values of the Jacobian matrix at these points are $0.2202 + 18.7956i, 0.2202 - 18.7956i, 0.2844 + 0.000i$, on E_1, on E_2 the eigen values are $0.0007 + 9.4981i, 0.0007 - 9.4981i, 0.0004 + 0.000i$ .The eigen vales of the Jacobian matric on E_3 is $0.0007 + 2.0519i, 0.0007 - 2.0519i, 0.001 + 0.000i$ on the last point E_4 the eigen values are $-3.9423, 3.9415, -0.0001$ which shows that all the equilibrium points are unstable.

CHAOS CONTROL

To control the chaos of the covid dynamical system as proposed in (6) let us construct the feedback controller such as

$$\begin{aligned}\frac{d^\alpha x}{dt^\alpha} &= \alpha_1 z^2 + \alpha_2 x^2 + \alpha_3 y\,(z + \alpha_4 x) - k_1\,(x - \bar{x})\\ \frac{d^\alpha y}{dt^\alpha} &= \alpha_5 x + \alpha_6 y + \alpha_7\, z^2 - k_2\,(y - \bar{y})\\ \frac{d^\alpha z}{dt^\alpha} &= \alpha_8 xz + \alpha_9 xy + \alpha_{10} z + \alpha_{11} x^2 - k_3\,(z - \bar{z})\end{aligned} \tag{9}$$

Where k_1, k_2, k_3 are control parameters and $\bar{x}$, $\bar{y}$, $\bar{z}$ are Equilibrium points of the system. At equilibrium point the Jacobian of this system is

$$\begin{bmatrix} 2\alpha_2\bar{x} + \alpha_3\alpha_4\bar{y} - k_1 & \alpha_3\,(\bar{z} + \alpha_4\bar{x}) & 2\alpha_1\bar{z} + \alpha_3\bar{y} \\ \alpha_5 & \alpha_6 - k_2 & 2\alpha_7\bar{z} \\ \alpha_8\bar{z} + \alpha_9\bar{y} + 2\alpha_{11}\bar{x} & \alpha_9\bar{x} & \alpha_8\bar{x} + \alpha_{10} - k_3 \end{bmatrix} \tag{10}$$

characteristic polynomial of the above Jacobian matrix with the parameters as in Table 1 is

$$\begin{aligned}P(t) =& t^3 + (k_1 + k_2 + k_3 - 0.72485704 + 0.00007\bar{x} + 0.00221\bar{y})\ t^2\\ &+ (-0.44040714\ k_3 + 0.125273767 - 0.72485704 k_1 +\\ &0.2844499 k_2 + k_1k_2 + k_1k_3 + k_2k_3 - 0.00047\ \bar{x} + 0.00012\ k_1\ \bar{x}\\ &+ 0.00006806999999999999\ k_2\ \bar{x} - 0.00004686\ k_3\ \bar{x} -\\ &5.3856198 \times 10^{-9}\ \bar{x}^2 - 0.00158\ \bar{y} + 0.00221\ k_2\ \bar{y} + 0.00221\\ &- 4.700724164505601 \times 10^{-7} \bar{x}\ \bar{y} + 0.031171093689712204\ \bar{z} -\\ &0.000002558965689\ \bar{y}\ \bar{z} + 0.0000049\ \bar{x}\ \bar{z} + 0.00000185\ \bar{y}^2)\ t\\ &- 0.2844499\ k_1\ k_2 + 0.12527376693228598\ k_1 - 0.44040714\ k_1k_3\\ &+ k_1k_2k_3 + 0.00012293029636844125\bar{x} - 0.00005\ k_1\bar{x}\\ &+ 0.000013329322313999999\ k_2\bar{x} + 0.00011493\ k_1\ k_2\ \bar{x} -\\ &0.0004321685343128659\ k_3\ \bar{x} - 0.00004686\ k_2\ k_3\ \bar{x} + 0.00027\bar{y}\\ &- 0.0006102759693364992\ k_2\ \bar{y} - 0.000971930557124997\ k_3\ \bar{y} +\\ &0.002206891008\ k_2\ k_3\ \bar{y}\ - 1.719996417347599 \times 10^{-7} \bar{x}\ \bar{y}\\ &- 4.700724164505601 \times 10^{-7}\ k_2\ \bar{x}\ \bar{y} - 0.008862839394471904\ \bar{z}\\ &- 0.0000242059198878\ k_2\bar{z} + 0.03119529961\ k_3\ \bar{z} +\\ &0.000003587\ \bar{x}\ \bar{z} - 4.966912964857768 \times 10^{-8}\ \bar{x}^2 -\\ &5.3856198 \times 10^{-9}\ k_2\ \bar{x}^2 - 8.13557944 \times 10^{-7} \bar{y}^2 + 0.00000185\ k_2\ \bar{y}^2\\ &+ 0.000003902671368\ k_1\ \bar{x}\ \bar{z} + 0.0000010025248296\ k_2\ \bar{x}\ \bar{z} +\\ &3.191341960623825 \times 10^{-9}\ \bar{x}^2\ \bar{z} + 0.0000011269867604506l9\ \bar{y}\ \bar{z}\\ &- 0.000002558965689\ k_2\ \bar{y}\ \bar{z} + 0.0000056128\ \bar{z}^2 -\\ &- 0.00000255897\ k_2\ \bar{y}\ \bar{z} + 0.000005618\ \bar{z}^2 -\\ &2.32461222782208 \times 10^{-7} \bar{x}\ \bar{z}^2 + 5.933621547907201 \times 10^{-7} \bar{y}\bar{z}^2\end{aligned} \tag{11}$$

For Routh Hurwitz criteria for fractional order, we have

$$a_1 = k_1 + k_2 + k_3 - 0.72485704 + 0.00006807\bar{x} + 0.00220689101\bar{y} \tag{12}$$

$$\begin{aligned}a_2 =& - 0.44040714\ k_3 + 0.1252737669 - 0.72485704\ k_1 + 0.2844499 k_2\\ &+ k_1k_2 + k_1k_3 + k_2k_3 - 0.0004694552045990659\ \ \bar{x} + 0.00011493\ k_1\ \bar{x}\\ &+ 0.00006806999999999999\ k_2\ \bar{x} - 0.00004686\ k_3\ \bar{x} + 0.00011493\ k_1\ \bar{x}\\ &+ 0.00006807\ k_2\ \bar{x} - 0.00004686\ k_3\ \bar{x} - 5.3856198 \times 10^{-9}\ \bar{x}^2\\ &- 0.0015822065264615\ \bar{y} + 0.002206891008\ k_2\ \bar{y} + 0.0022069\ k_3\ \bar{y}\\ &- 4.700724164505601 \times 10^{-7} \bar{x}\ \bar{y} + 0.031171094\ \bar{z} - 0.00000256\ \bar{y}\ \bar{z}\\ &+ 0.0000049051962\ \bar{x}\ \bar{z} + 0.0000018473\ \bar{y}^2,\end{aligned} \tag{13}$$

$$\begin{aligned}a_3 =& - 0.2844499\ k_1\ k_2 + 0.12527376693228598\ k_1 - 0.44040714\ k_1k_3\\ &+ k_1k_2k_3 + 0.0001229303\bar{x} - 0.000050615992600199994\ k_1\bar{x}\\ &+ 0.00001333\ k_2\bar{x} + 0.000115\ k_1\ k_2\ \bar{x} - 0.0004322\ k_3\ \bar{x}\\ &- 0.0000479\ k_2\ k_3\ \bar{x} + 0.00027\ \bar{y} - 0.0006103\ k_2\ \bar{y} - 0.00097\ k_3\ \bar{y}\\ &+ 0.0022069\ k_2\ k_3\ \bar{y}\ \ - 1.71999642 \times 10^{-7} \bar{x}\ \bar{y}\\ &- 4.700724165 \times 10^{-7}\ k_2\ \bar{x}\ \bar{y} - 0.00886284\ \bar{z} - 0.00002421\ k_2\bar{z}\\ &+ 0.0311953\ k_3\ \bar{z} + 0.000003587330339763896\ \bar{x}\ \bar{z}\\ &- 4.966912964857768 \times 10^{-8}\ \bar{x}^2 - 5.3856198 \times 10^{-9}\ k_2\ \bar{x}^2\\ &- 8.1355794402204 \times 10^{-7} \bar{y}^2 + 0.00000185\ k_2\ \bar{y}^2\\ &+ 0.00000390267\ k_1\ \bar{x}\ \bar{z} + 0.0000010025248296\ k_2\ \bar{x}\ \bar{z}\\ &+ 3.191341960624 \times 10^{-9}\ \bar{x}^2\ \bar{z} + 0.000001126986760451\ \bar{y}\ \bar{z}\\ &- 0.00000255897\ k_2\ \bar{y}\ \bar{z} + 0.0000056127665\ \bar{z}^2\\ &- 2.32461222782208 \times 10^{-7} \bar{x}\ \bar{z}^2 + 5.93362154791 \times 10^{-7} \bar{y}\bar{z}^2\end{aligned} \tag{14}$$

$$D(P) = 18a_1a_2a_3 + (a_1a_2)^2 - 4a_3a_1^3 - 4a_2^3 - 27a_3^2 \tag{15}$$

RESULTS AND DISCUSSION

In the control analysis of the above problem, we observe that the system is getting controlled at every equilibrium point with the feedback controller. At first equilibrium point $E_1(0, 0, 0)$ the stability is achieved at $k_1 = 1, k_2 = 2, k_3 = 5$ for the values of $\alpha = 0.97$. when we increase the values of k_3, the first eigenvalue of the Jacobian matrix increases in negative direction very fast so that system goes towards the equilibrium point with fast rate.

It shows that if we subtract from the first equation in the model (6) the daily cases one time, from the second equation twice the rate of change of the daily number of critical cases, and from the third equation five times the daily deaths, then the system is under control. The other case that is possible is that instead of controlling too many death cases, we could reduce the 6 times daily critical cases and control the chaos in the system. If we could control the 3 times daily critical cases, then the system would also be under control.

The second equilibrium point E_2 shoes the stability with $k_1 = 8$, which means that at this juncture the system will not be chaotic if 8 times we could reduce the daily cases or 5 times we reduce the daily critical cases, or if the daily cases are reduced by 10 times and daily deaths are reduced by 12 times or more, the system is under control.

Table 2 : Stability using Routh Hurwitz criteria at the first equilibrium point $E_1(0,0,0)$ after putting these points in the equation $12, 13, 14, 15$ is as follows.

Sr. No	k_1	k_2	k_3	a_1	a_3	$a_1a_2 - a_3$	D(P)	Eigen values of Jacobian Matrix of Controlled system	Stable / Unstable
1	1	2	5	7.2751	7.3543	100.08	94.97	-4.7156, -1.5596, -1.0000	Stable for $0 < \alpha < 1$.
2	1	2	10	12.2751	15.1523	323.2150	$4.6960x10^3$	-9.7156 -1.5596 -1.0000	Stable for $0 < \alpha < 1$
3	1	2	≥ 5	+ive	+ive	+ive	+ive	-ive	Stable for $0 < \alpha < 1$.
4	1	6	1	7.2751	3.9782	95.4491	240.2804	-0.7156, -5.5596, -1.0000	Stable for $0 < \alpha < 1$..
5	1	≥ 6	1	+ive	+ive	+ive	+ive	-ive	Stable for $0 < \alpha < 1$.
6	3	1	1	4.2751	1.2013	19.2970	8.0779	-0.7156, -0.5596, -3.0000	Stable for $0 < \alpha < 1$.
7	≥ 3	1	1	$+ive$	$+ive$	$+ive$	$+ive$	$-ive$	Stable for $0 < \alpha < 1$.

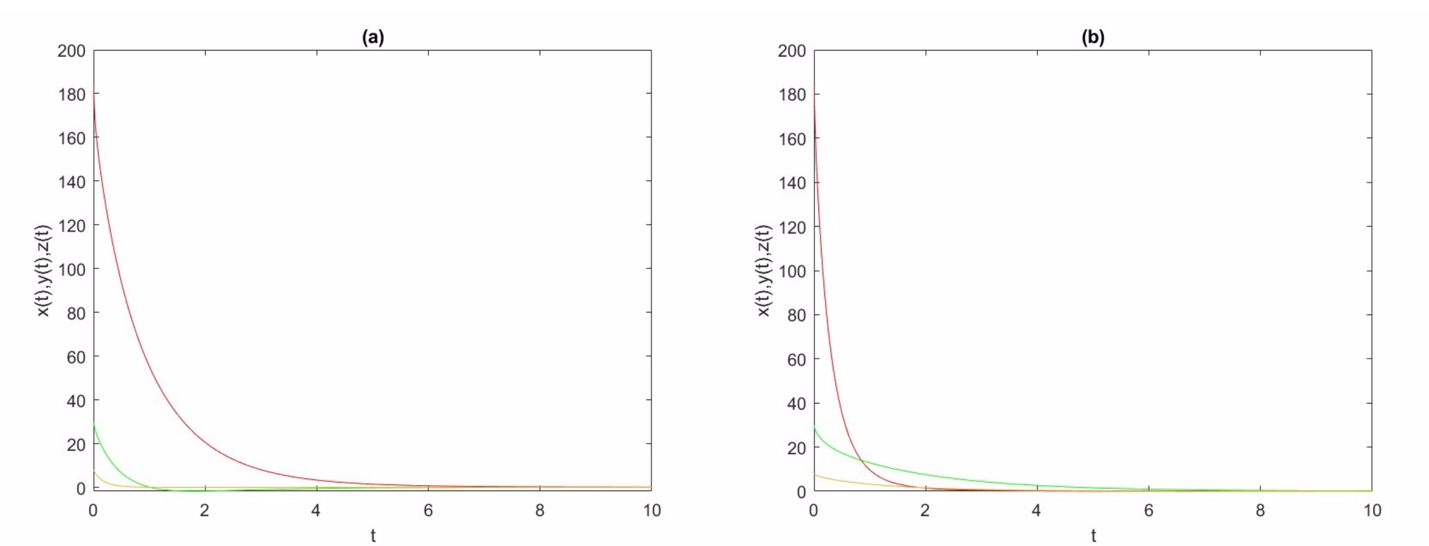

Figure 2 Plot x,y,z at (a) $k_1 = 1, k_2 = 2, k_3 = 5$ at α=0.97 at E_1 (b) Plot at $k_1 = 3, k_2 = 1, k_3 = 1$ at α=0.97 at E_1

The third equilibrium point, E_3 is such that we need to reduce the critical cases by $12 - 15$ times and the daily deaths by 21 times to control the system. we need to reduce the daily critical cases by 3 times, or more than system is under control. Similarly, at the fourth equilibrium point E_4 the system is under control if 9 times daily cases are reduced and 3 times daily critical cases are reduced,

Table 3 : Stability using Routh Hurwitz criteria at the second equilibrium point $E_2(1149.44, 590.097, 12.2352)$ after putting these points in the equation 12, 13, 14, 15 is as follows.

Sr. No	k_1	k_2	k_3	a_1	a_3	$a_1a_2 - a_3$	D(P)	Eigen values of Jacobian Matrix of Controlled system	Stable / Unstable
1	8	1	1	7.8946	0.0762	75.7604	$2.1594x10^3$	-6.5686, -0.9980, -0.3280	Stable for $0 <\alpha< 1$.
2	≥ 8	1	1	+ive	+ive	+ive	+ive	-ive	Stable for $0 <\alpha< 1$
3	1	5	1	4.8946	0.4466	25.1061	74.3602	-4.3634 -0.1542 -0.3770	Stable for $0 < \alpha < 1$
4	1	≥ 5	1	+ive	+ive	+ive	+ive	-ive	Stable for $0 <\alpha< 1$
5	10	1	12	20.8946	54.6713	$2.3339x10^3$	$4.7597x10^3$	-0.6626, -8.7041, -11.5279	Stable for $0 <\alpha< 1$
6	10	1	≥ 12	$+ive$	$+ive$	+ive	+ive	-ive	Stable for $0 <\alpha< 1$

Table 4 : Stability using Routh Hurwitz criteria at the Third equilibrium point $E_3(-619.232, -6075.71, 125.975)$ after putting these points in the equation 12, 13, 14, 15 is as follows.

Sr. No	k_1	k_2	k_3	a_1	a_3	$a_1a_2 - a_3$	D(P)	Eigen values of Jacobian Matrix of Controlled system	Stable / Unstable
1	12	3	21	21.8246	24.3566	$2.0203x10^3$	7.5909x105	-16.7962 + 0.0000i, -2.5142 + 1.0957i, -2.5142 - 1.0957i	Stable for 0<α<1
2	12	≥ 3	21	+ive	+ive	+ive	+ive	-ive	Stable for 0<α<1
3	12	3	21-27	+ive	+ive	+ive	+ive	-ive	Stable for 0<α<1
4	12-15	3	21	+ive	+ive	+ive	+ive	-ive	Stable for $0 <\alpha< 1$

whereas 13 times daily deaths are reduced. The system is under control, and it will not generate chaos.

On observing all the cases at the equilibrium points we observe that system is under control if we could reduce the daily cases by 12 times and daily critical cases by 3 times and daily deaths by 21 times then system is under control. These changes in the system

Table 5 : Stability Using Routh Hurwitz criteria at the first equilibrium point $E_4(25638.5, 6103.77, -126.557)$ after putting these points in the equation $12, 13, 14, 15$ is as follows.

Sr. No	k_1	k_2	k_3	a_1	a_3	$a_1a_2 - a_3$	D(P)	Eigen values of Jacobian Matrix of Controlled system	Stable / Unstable
1	9	3	13	39.4907	211.1828	$1.3829x10^4$	$1.7508x10^7$	-0.0417, -25.7969, -13.6521	Stable for 0<α<1
2	≥ 9	3	13	+ive	+ive	+ive	+ive	-ive	Stable for 0<α<1
3	9	≥ 3	13	+ive	+ive	+ive	+ive	-ive	Stable for 0<α<1
4	9	3	≥ 13	+ive	+ive	+ive	+ive	-ive	Stable for $0 < \alpha < 1$

Table 6 : Stability analysis with the control parameters values as $k_1 = 12, k_2 = 3, k_3 = 21$

Equilibrium Point	a_1	a_3	$a_1a_2 - a_3$	D(P)	Eigen values of Jacobian Matrix of Controlled system	Stable / Unstable
E1	35.2751	636.2805	$1.1147x10^4$	$2.0627x10^6$	-20.7156, -2.5596, -12.0000	stable
E2	33.8946	565.7037	$9.7454x10^3$	$1.9480x10^6$	-2.6660, -10.6586, -20.5700	stable
E3	21.8246	24.3566	$2.0203x10^3$	$7.5909x10^5$	-16.7962 + 0.0000i, -2.5142 + 1.0957i, -2.5142 - 1.0957i	stable
E4	50.4907	788.8388	$3.0759x10^4$	$4.4526x10^7$	-0.8051, -30.6777, -19.0079	stable

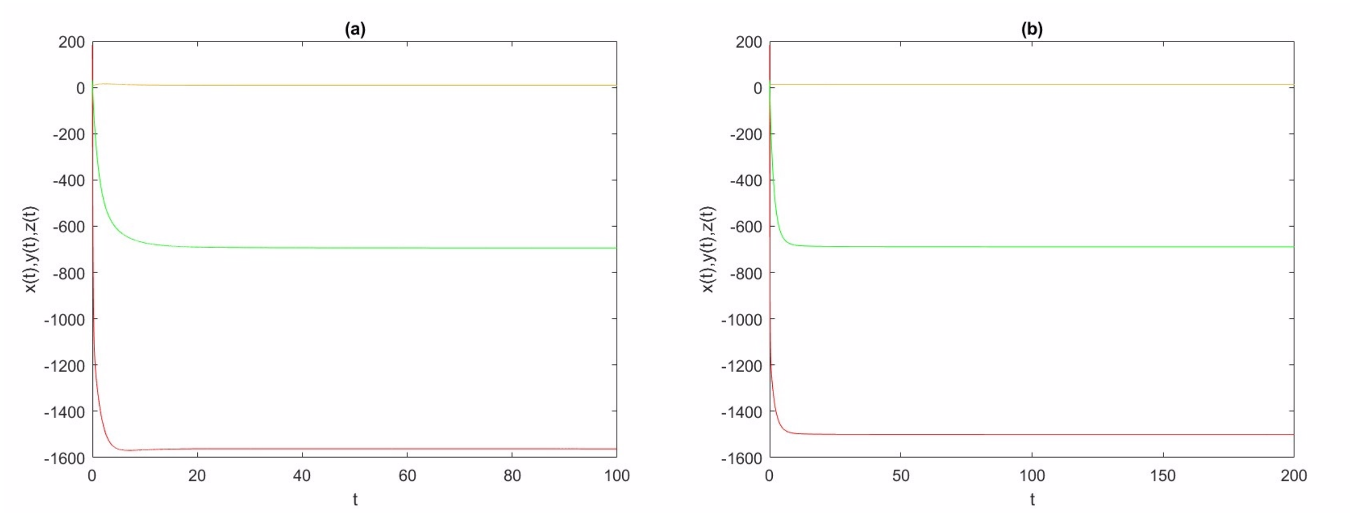

Figure 3 Plot x,y,z at (a) $k_1 = 8, k_2 = 1, k_3 = 1$ at α=0.97 at E_2 (b) $k_1 = 10, k_2 = 1, k_3 = 12$ at α= 0.97 at E_2

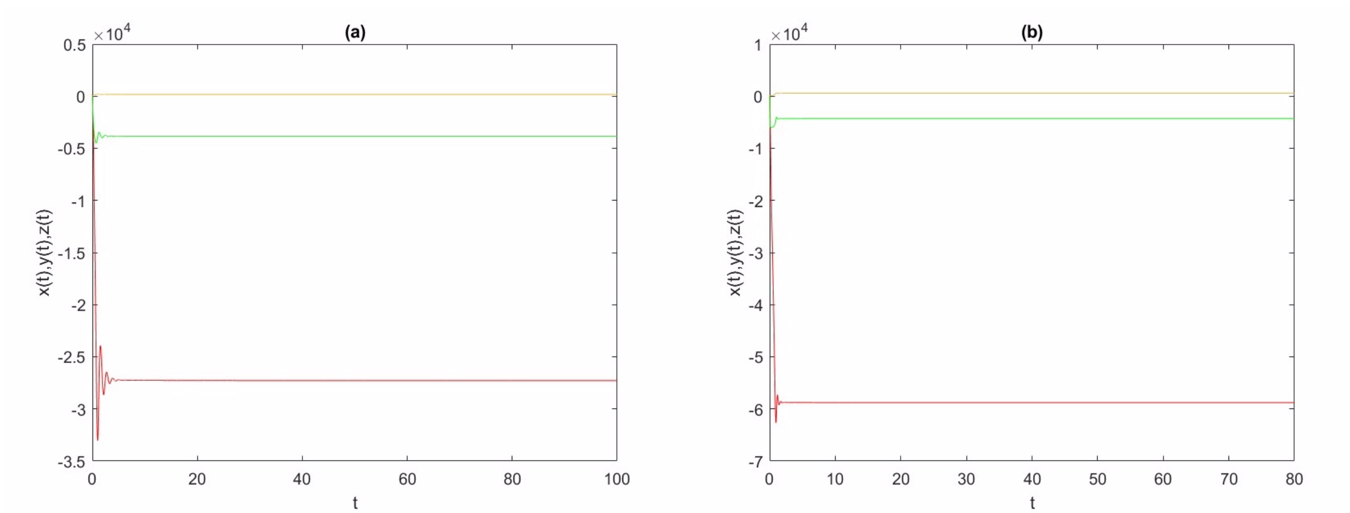

Figure 4 Plot x,y,z at (a) $k_1 = 12, k_2 = 3, k_3 = 21$ at α=0.97 at E_3 (b) $k_1 = 15, k_2 = 31, k_3 = 21$ at α= 0.97 at E_3

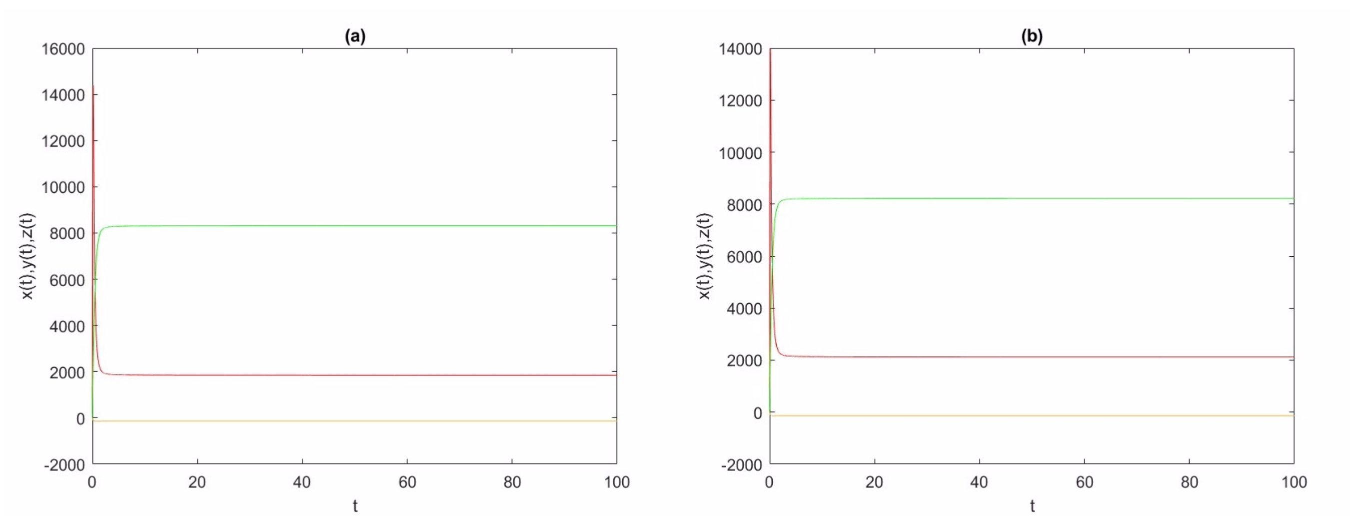

Figure 5 Plot x,y,z at (a) $k_1 = 9, k_2 = 3, k_3 = 13$ at α=0.97 at E_4 (b) $k_1 = 9, k_2 = 3, k_3 = 20$ at α= 0.97 at E_4

can be achieved by the social distancing which could reduce the daily cases and daily critical cases and preventing deaths by proper treatment on time.

CONCLUSION

In the present article, the feedback control method has been applied to control the chaos in the dynamical system of COVID-19, as proposed by (Mangiarotti *et al.* 2020) , which has been studied by (Debbouche *et al.* 2021). In the present article, the fractional order Routh- Hurwitz stability criteria have been utilized, and to solve the fractional-order system, Adams-Bashforth-Molton methods are used. The control of chaos is obtained under different equilibrium points and parameters. In this article, chaos is studied in the dynamical system that is proposed for representing the spread of COVID-19. In the present article, it is shown under what conditions the control parameters of daily infected cases, daily critical cases, and daily deaths should be controlled so that chaos can be controlled in the dynamical system.

Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

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How to cite this article: Mishra, V., Maitra, S., Dash, M., Agrawal, S. K., and Agarwal, P. Chaos and Control of COVID-19 Dynamical System. *Chaos Theory and Applications*, 5(3), 233-241, 2023.