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The Learning Trajectory Based on STEM of Elementary School Pupils' in Solving Proportion Material: Didactical Design-Research

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This study aims to determine the trajectory of students' thinking when solving proportion problems using STEM-based learning media. The participants were 27 fifth-grade students from SD Negeri 2 Pilangsari in Cirebon Regency. The students are divided into four groups using purposive sampling and receive the same treatment. The treatment involved a proportion study that utilized STEM media, and the student's learning trajectory was monitored based on their problem-solving patterns. Hypothetical Learning Trajectory (HLT) was used to develop the hypotheses. The HLT was used as a guide for the researchers' assumptions. The data were collected through observation by researchers, student work, and documentation. The results of the HLT were used to test the assumptions related to the student's thinking processes and their learning in completing proportion operations using STEM. Based on the results obtained during the practice, some findings exceeded the researcher's expectations and hypotheses, but some did not. These differences become a new finding expected to become a subject for further research, where several groups have different ways of thinking based on mathematical disposition. Through STEM media, the electrical engineering students' high enthusiasm and creativity can be known through the electric graph. In conclusion, proportional relationships are an important mathematical concept with practical applications in various fields. The use of STEM media for teaching materials can help students

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acquire a better understanding of mathematical concepts and skills.

Introduction

Mathematics learning involves understanding mathematical concepts, skills, and applications. Learning mathematics usually begins with understanding basic mathematical concepts and procedures. Furthermore, students learn to develop skills in applying concepts and procedures in various mathematical situations. Mathematics learning also involves using technology and mathematical aids to assist students in learning and understanding mathematical concepts better. Additionally, mathematics learning emphasizes the development of problem-solving and critical thinking skills to help students apply mathematical concepts in everyday life (Nishimura, 2023).

The characteristics of mathematics learning in classrooms often involve a focus on calculation, a teacher-centered approach, limited student engagement in reasoning and expressing opinions, inadequate conceptual understanding, a lack of problem-solving orientation, and disconnection from everyday life (Cazares, 2019). The application of mathematics in education is one of the tools or strategies to create meaningful and value-rich learning (Kaitera & Harmoinen, 2022). Therefore, it is not surprising that mathematics is related to our daily lives. Research shows that studying mathematics can provide a solid foundation for understanding the evolution of mathematical concepts, appreciating the effort and hard work behind its development, and cultivating students' interest and positive attitudes towards mathematics (Abramovich et al., 2019).

The importance of mathematics can be seen from its goals, as stated by the National Council of Teachers of Mathematics (NCTM), which are for students to: participating in mathematical problem-solving, employing mathematical reasoning, making connections between mathematical concepts and other subjects as well as daily life, effectively communicating mathematical ideas, building self-confidence in mathematical abilities, and recognizing the value and aesthetic appeal of mathematics (National Council of Teachers of Mathematics (NCTM)), 1989). These goals imply that students, in their learning process, build upon the experiences they have gained in their lives. According to Ausubel, meaningful learning is an active process that connects new material with the student's preexisting knowledge (Ausubel, 1962). To achieve these goals, Clements, Gravemeijer, and Simon propose conceptualizing a learning trajectory as a description of children's thinking and learning in a specific mathematical domain and the associated instructional tasks (Siemon et al., 2019; Yuliardi & Rosjanuardi, 2021). A learning trajectory, or hypothetical learning trajectory (HLT), is a way to explain the pedagogical thinking involved in understanding the teaching of mathematics (Ulfa & Wijaya, 2019).

Meanwhile, hypothetical learning trajectory (HLT) is a mathematical model that predicts students' ability to solve mathematical problems and build better mathematical understanding. HLT can help teachers plan more effective learning and provide more accurate feedback to students about their progress in learning mathematics (Ivars, 2018). The relationship between learning trajectory and hypothetical learning trajectory lies in their support in determining effective mathematics curricula and learning objectives. Learning trajectory can be used as a basis for developing HLT, while HLT can help inform the development of a better learning trajectory. Combined, both can help teachers design and implement more effective mathematics learning and help students build better mathematical understanding (Lestari, 2020).

The urgency of learning trajectory is that it provides clear and structured guidance for teachers in designing and implementing effective curricula. With learning trajectory, teachers can understand the sequence of learning that students should follow so that the learning process can be done gradually and structured. Learning trajectory can also help teachers to determine clear and specific learning objectives so that students can understand what is expected of them in each stage of learning (Dhuheir, 2023).

One of the main goals of mathematics learning is to help students solve mathematical problems using calculation and a good understanding of mathematical concepts. Using STEM-based learning media (Science, Technology, Engineering, and Mathematics) is appropriate for improving mathematical problem-solving skills. This learning media can provide students with a more interactive and enjoyable learning experience, increasing their motivation and interest in mathematics (Chen, 2023).

STEM-based learning media is also highly effective in improving problem-solving skills, especially in learning ratios in elementary school mathematics. The concept of ratios is one of the fundamental mathematical concepts that are important and commonly used in daily life. With the use of STEM-based learning media, students can easily understand and apply the concept of ratios in real-life situations (Ziyi, 2023). Overall, the use of STEM-based learning media in mathematics learning can help students acquire a better understanding of mathematical concepts and skills, as well as improve their problem-solving abilities effectively. Therefore, STEM-based learning media needs to be continuously developed and improved in mathematics teaching in elementary and secondary schools (Busch, 2023).

The concept of proportion in mathematics is not a new discovery but rather a longstanding field of study (Supply et al., 2023). From its origins in ancient geometry to its applications in algebra, statistics, and various real-world disciplines, the concept of proportion remains essential in mathematics education and problem-solving (Haggarty, 2002). Proportion in mathematics dates back to ancient times and has evolved with new perspectives and technological advances, making it essential in mathematics (Huh, 2023). The concept of proportionality is highly significant in many aspects of life, including mathematics, natural sciences, engineering, and so on (Zou, 2023). Ancient Egypt and Greek civilizations contributed to the development of proportionality, using it to measure and divide the land, construct buildings, and solve mathematical problems (Placido, 2023). Euclid, an ancient Greek mathematician, developed axioms that form the basis for applying proportionality in his book "Elements" (Simon, 2018).

Eudoxus developed the theory of ratios in the 4th century BCE, which is still used today to describe the relationship between two ratios and their values (Demetriou, 2023). Al-Khwarizmi, a mathematician and astronomer from the 9th century, was a famous figure in the development of proportionality in mathematics (Tykhonov, 2023). Muslim and European mathematicians, such as al-Khwarizmi and al-Haytham, contributed to the development of proportionality in the Middle Ages. (Wang, 2023). Proportionality is an important mathematical concept used in statistics, economics, and natural sciences (Broietti, 2022).

The development of proportion over time has undergone development starting from the 16th century, which was developed by Luca Pacioli (Kolaghassi, 2023), the 17th century by John Napier (Luo, 2023), the 18th century by Joseph-Louis Lagrange (Anwar, 2023), the 19th century by Leonhard Euler (Mattison, 2023), until the 20th century according to its intended use in various fields (Tordesillas, 2023). In proportion, the numerator and denominator of equivalent ratios can be reversed, as Taylor (2023) explained, which is useful in problem-solving. For example, reversing the equivalent ratio can help find the other number when only one number is given, as stated by Yuan (2023).

Mansouri (2023) also noted that reversing equivalent ratios is applicable in solving problems related to proportions or ratios. In an equivalent ratio, two numbers or quantities are expressed as a ratio or fraction and can be compared to determine their relationship (Baroody, 2022). A simple example of an equivalent ratio is comparing the prices of two different products (Sukestiyarno, 2023). Equivalent ratios can also be used to compare quantities or sizes in proportion (Zhan, 2023). Equivalent ratios and the reversal of equivalent ratios have various practical uses. As pointed out by Qian (2023), equivalent ratios are helpful in financial budgeting to compare income and expenses, while reversing the equivalent ratio can be used to compare two variables in natural or social sciences, providing valuable insights into their relative proportions or magnitudes. This is particularly useful in scientific research, data analysis, and decision-making processes. They are essential mathematical concepts useful for problem-solving in various fields (Ferreira and Silva, 2019).

This research will discuss proportion using a Hypothetical Learning Trajectory. The Hypothetical Learning Trajectory (HLT) bridges instructional theory and the implementation of a specific teaching experiment (Gravemeijer, 1994). It guides researchers and teachers on how to conduct the teaching experiment during the lesson. Following the teaching experiment, the HLT aids researchers in conducting a retrospective analysis, and the interaction between the HLT and empirical results serves as the foundation for theory development (Bakker & Van Eerde, 2015).

Hypothetical Learning Trajectory (HLT) is a valuable tool for developing effective mathematics teaching materials (Bahamonde et al., 2017). It offers a structured approach to understanding students' learning objectives and promoting gradual knowledge development. HLT emphasizes student thinking progression and differentiation of instruction based on prior knowledge (Feishi et al., 2017). It also encourages reflection and revision, promoting students' understanding and success in mathematics.

This study aims to determine the trajectory of students' thinking when solving proportion problems using STEM-based learning media. This research focuses on developing learning trajectories in mathematics education using a design research methodology. The sub-objectives of this study involve:

- Gaining an understanding of the principles underlying proportion, establishing ratios, learning about characteristics, and utilizing the concepts of direct and inverse proportion to address practical challenges encountered in daily life.
- Arriving at a point of reference to improve pre-service mathematics teachers' research and academic writing skills.

Methods

Research Design

This research utilizes a qualitative approach with the DDR (Didactical Design Research) method. DDR is a research methodology that emphasizes the significance of the design process and aims to design and create interventions to address intricate issues in the field of education (Lidinillah, 2011). Additionally, DDR seeks to generate knowledge regarding the qualities of implemented interventions and the process of designing and developing such interventions. DDR involves four main stages: analysis of learning needs, design of teaching materials, implementation of teaching materials, and evaluation of teaching materials (Angraini, 2021). DDR can assist in developing mathematics teaching materials that align with the theory of mathematics learning and didactics. Meanwhile, HLT can provide a clear framework for designing mathematics teaching materials that are continuous and appropriate to students' understanding development.

A teacher uses DDR to develop teaching materials to help students understand the concept of proportion. They analyze students' learning needs, identify specific objectives, and observe their struggles. They design teaching materials using visual aids, exercises, and activities to help students understand proportion. The teacher then implements these materials in the classroom, providing feedback and adjustments as needed. After evaluating the effectiveness of the materials, the teacher assesses students' performance on assessments related to proportion and gathers feedback. Based on the evaluation, the teacher makes the necessary adjustments to improve the materials' effectiveness. This systematic and structured approach is combined with using STEM media for teaching materials to help students understand mathematical concepts, such as proportion.

Participants

In this study, the participants were 27 fifth-grade students from SD Negeri 2 Pilangsari, Cirebon Regency. Fifth-grade students were chosen because it aligned with the topic of mathematics being discussed. Student selection is based on purposive sampling. Selecting students based on purposive sampling through teacher observation and adjusting to the teacher's perspective based on students' cognitive abilities can help create balanced and heterogeneous learning groups (Guarte & Barrios, 2006). This approach enables teachers to provide learning experiences that are suitable for individual students' needs, facilitate collaboration, and expand their understanding. Each group received the same treatment as we wanted to observe their learning trajectory. The treatment involved using STEM media in part of the research, and the students' learning paths were monitored based on their problem-solving patterns.

Hypothetical Learning Trajectory with STEM

The researchers utilized Hypothetical Learning Trajectory (HLT) in their study, which involved using STEM-based media. Using the media consisted of several stages, including gathering building materials, organizing the materials into a slide format, conducting experiments, recording the results, creating proportion charts, sketching light graphs, implementing the graphs into light charts, and evaluating the results with other students. The practice took place in the classroom and was demonstrated by the teacher.

The learning activities were separated into four lessons. The first lesson involved introducing the concept of proportion, determining ratios, and identifying types of proportion. The second lesson began with an inverse practice that involved graphs, real-life examples, and solving problems using STEM media. The Third lesson started with a diverse proportion practice that included graphs, real-life examples, and solutions using STEM media. The final lesson focused on a combination and value proportion and problem-solving using STEM media.

Table 1. Task descriptions based on STEAM of learning objectives along with HLT conjectures.

Sub Concept	Task Description based on STEM				Learning Objectives	HLT Conjectures
	Science	Technology	Engineering	Mathematics		
Understanding the concepts of direct proportion and inverse proportion	-	-	-	Understanding and introducing the concept of proportion includes not only basic material but also the properties of proportion	Conceptual understanding provides a more meaningful learning experience for the next stage of proportion learning.	In understanding all students are expected to understand the concept and properties of proportion.
Determining the ratios of direct proportion and inverse proportion	-	-	-	The next step after understanding the concept is to determine the ratio, which is also used to distinguish the type of proportion.	Determining the proportion ratio makes it easy for students to determine the proportion size.	Assumptions in determining the ratio are related to understanding, and it is expected that all students can calculate it.
Applying them in creating proportion graphs	-	-	-	Creating graphs based on the type of proportion has differences	Creating a proportion graph aims to evaluate students' understanding.	When drawing a proportion graph, there are assumptions with 3 types of graphs explained in the discussion.
Sketching a lamp graph	The lamp graph is related to the electrical circuit used in the lamp graph	Knowledge of electrical technology is necessary for implementing it into the lamp graph.	In sketching the picture, understanding the lamp circuit and its suitability in the calculation is needed.	In sketching the graph to be used on the lamp, it is necessary to understand the position of the lamp.	Sketching can enhance students' creativity in implementing proportion.	Sketching proportion is hypothesized to have compatibility with the proportion graph depicted.
Assembling a lamp graph	The lamp graph is related to the electrical circuit used in the lamp graph	Knowledge of electrical technology is necessary for implementing it into the lamp graph	The implementation of the graph sketch into the lamp graph	The suitability of the lamp graph will be evaluated by whether the lamp is turned on or off	Based on its design, implementing it on a light graph increases students' cognitive abilities.	In the implementation, it is assumed that some students have creativity in installing or engineering the light graph.

Data Collection and Analysis

This research begins with constructing the findings of students' initial skills, and then the results are developed using hypotheses based on the results obtained. Examine and evaluate the data obtained from all student work to improve the learning flow that has been implemented. After completing the explanation in the first lesson, it is improved by applying the learning flow using STEM media to understand the comparative settlement procedure. While the results of the review and their application will be carried out in class by utilizing the HLT as a guide for the researchers' assumptions, this study collected data through researcher observation, student work, and documentation. The documentation was used to test the assumptions about the student's thinking processes and learning in completing proportion operations using STEM. In this case, the steps taken by students during the research are documented so that they can review the appropriate learning process.

In this research, there are three stages in the Didactical Design Research data analysis. The first stage, namely didactical situation analysis, is the preparation and research design stage. The second stage is metapedadidactic analysis and retrospective analysis. Metapedadidactic analysis is the experimental stage of teaching. After that, the third stage is retrospective analysis, namely an analysis that links the results of the analysis of hypothetical didactic situations with the results of metapedadidactic analysis. From these three stages an Empirical Didactic Design will be obtained which is likely to continue to be refined through the three stages of DDR in this study.

Validity and Reliability

In the qualitative approach with DDR method, the concept of validity refers to the ability to present the researched subjects in an impartial and authentic manner (Kirk & Miller, 1988). Since the data is collected from its original source, researchers must verify its accuracy in both form and context through consistent comparisons, either independently or with colleagues (a form of triangulation) (George & Apter, 2004). To ensure research credibility, the involvement of language and field experts is necessary in developing data collection tools to guarantee the study's validity (Brinberg & McGrath, 1985). Conversely, in scientific studies, reliability is described as the reproducibility of results within the research scope (Merriam, 1998). (Silverman, 2009) suggested five approaches to enhance the reliability of the process and outcomes: critical analysis, constant comparative analysis, comprehensive utilization of data, inclusion of deviant cases, and the use of tables. To maintain research consistency, the research methodology should be described in detail as mentioned earlier, and the study should be open to review by seeking expert opinions when required.

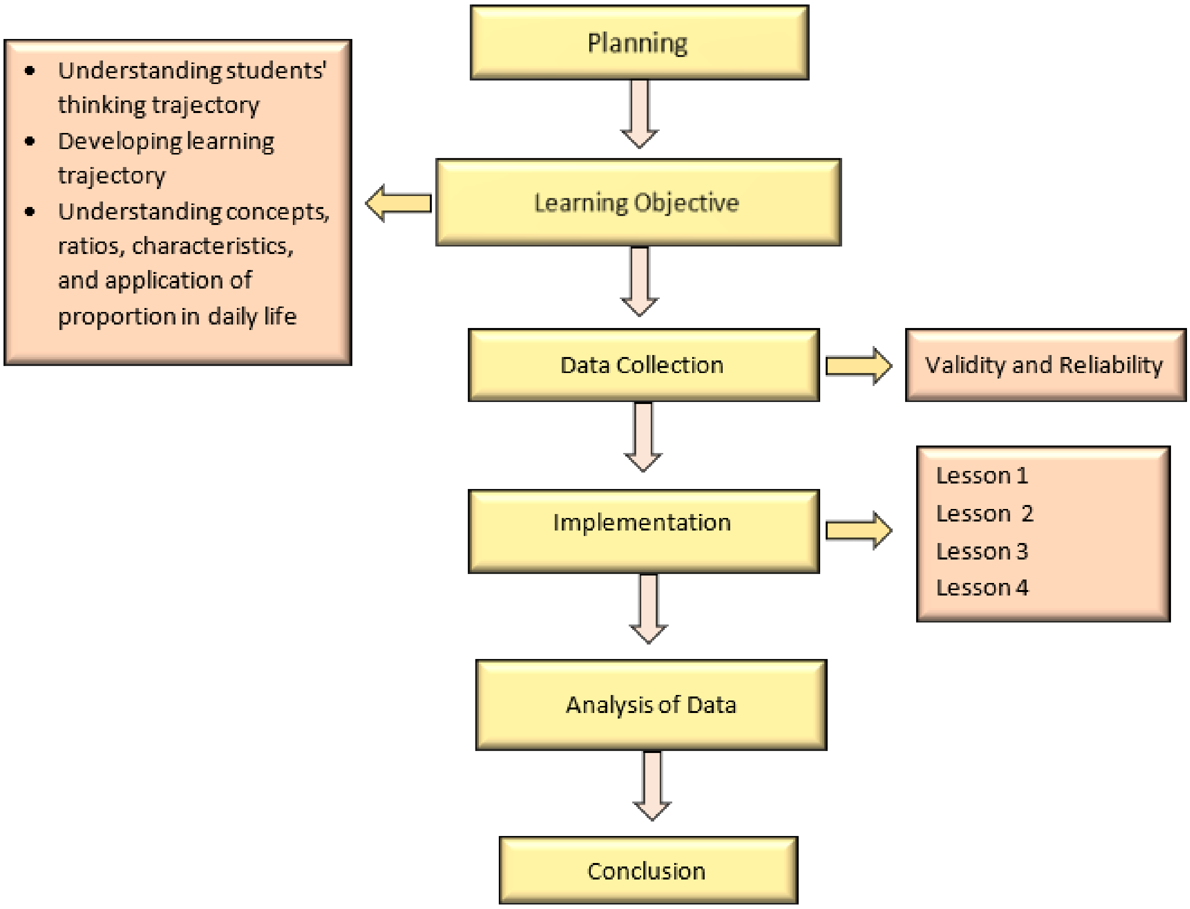

Figure 1. The Stages of Data Collection

Results

Lesson 1: Introducing the Concept of Proportion, Determining Ratios, and Identifying Types of Proportion

The first lesson begins with the introduction of the material concept, where students have not yet acquired the material at all before. Introduction to the material at the beginning of the study consists of understanding, simplified explanation of the concept, and presentation of concrete evidence of material comparison through everyday problems. In explaining the concept, simple ratios and how to determine them are also presented. After students understand the concept and ratio, the various types of proportion are also explained, namely diverse proportion and inverse proportion.

Figure 2. Introduction to Learning Trajectories

Direct proportion is comparing two ratios or fractions with the same or equivalent value. In direct proportion, the ratio or fraction can be simplified or enlarged by the same factor to obtain an equivalent form. An inverses proportion is a proportion between two ratios or fractions that have opposite or reverse values. In inverse proportion, if one fraction or ratio increases, the other fraction or ratio will decrease proportionally and vice versa. These differences become a new finding expected to become a subject for further research, where several groups have different ways of thinking based on mathematical disposition. Through STEM media, the electrical engineering students' high enthusiasm and creativity can be known through the electric graph. Based on the learning trajectory of students in four groups, both in direct and inverse proportion, some groups still make mistakes in implementing the results on the graph.

Lesson 2: Diverse Proportion Practice that Involved Graphs, Real-Life Examples, and Solving Problems Using STEM Media

The teacher's supervision determined the students' selection of group members. During the group formation process, the classroom environment was conducive and well-handled. The students were divided into several roles in groups, with tasks of notetaking, calculating, and conducting practice. Students who did not receive a role were assigned to observe their peers during the practice. The direct proportion practice was carried out effectively.

Figure 3. Process and Result of Practice Group 1

In the first group, a single divider was used, and no divider was used on the arranged media. The practice was conducted twice, and the results were obtained in the form of a table which will be used to calculate the ratio. Then, the students attempted to create a graph and represented it as a light graph. After that, a verification was performed to determine whether the light graph lit up. If the implementation on the graph was correct, the light would turn on, and vice versa. Based on the practice, the answer from the first group was correct, indicating the light was on. A rising graph indicates a direct proportion.

The results of practice Group 1 will be compared to those of other groups, such as Group 2. Group 2 was ready for task allocation and more prepared to conduct the practice than Group 1, as they paid attention to the researcher's instructions about things that needed to be considered.

Figure 4. Process and Result of Practice Group 2

In the second group, two dividers were used, and one divider was used on the arranged media. The practice was conducted twice, and the results were obtained in the form of a table which will be used to calculate the ratio. Then, the students attempted to create a graph and represented it as a light graph. After that, a verification was performed to determine whether the light graph lit up. If the implementation on the graph was correct, the light would turn on, and vice versa. Based on the practice, the second group's answer was wrong. Thus, the light did not turn on. A descending graph indicates that the graph is not a proportional relationship. Through discussions with the students, they explained that the graph they created was incorrect and realized mistakes in the placement and when drawing and representing it in the light graph. This caused the light on the light graph to not turn on.

Similar, to Group 2, the next group, Group 3, had better practice preparations as they evaluated the researcher's instructions given to Group 2. Each practice implementation was better prepared from Group 1 to subsequent groups. In this case, it can be concluded that the students always look at what the instructions are so as not to repeat mistakes.

Figure 5. Process and Result of Practice Group 3

In the third group, two dividers were used, and one divider was used on the arranged media. The practice was conducted three times, and the results were obtained in the form of a table which will be used to calculate the ratio. Then, the students attempted to create a graph and represented it as a light graph. After that, a verification was performed to determine whether the light graph lit up. If the implementation on the graph was correct, the light would turn on, and vice versa.

Based on the practice, the answer from the third group was wrong. Thus, the light did not turn on. A flat graph indicates that the graph is not a proportional relationship. Through discussions with the students, they explained that the graph they created was incorrect and realized mistakes in the placement and when drawing and representing it in the light graph. The third group understands that the concept of a proportional relationship means that two quantities are directly proportional if they increase or decrease together. This caused the light on the light graph to not turn on.

In Group 3, the preparation had improved, so the preparation for the next group was already at a maximum. Group 4 had better and more appropriate role-taking preparation, as seen from the trust among peers within the group. This made the researcher more interested in knowing the practice results of each group.

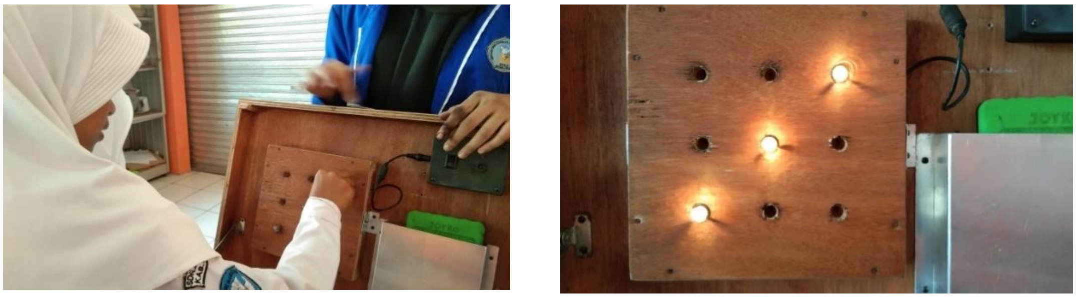

Figure 6. Process and Result of Practice Group 4

In the fourth group, two dividers were used, and no dividers were used on the arranged media. The practice was conducted twice, and the results were obtained in the form of a table which will be used to calculate the ratio. Then, the students attempted to create a graph and represented it as a light graph. After that, a verification was performed to determine whether the light graph lit up. If the implementation on the graph was correct, the light would turn on, and vice versa. Based on the practice conducted, the answer from the fourth group was correct, with the indication of the light being on. A rising graph indicates a proportional relationship.

Lesson 3: Inverse Proportion Practice that Included Graphs, Real-Life Examples, and Solving Problems Using STEM Media

Similar to the direct proportion practice, the inverse proportion practice also went very well. The roles and tasks of the students were the same as in the previous practice meeting, and technically the practice was the same. There were some differences in the discussion material and the implementation of inverse proportion. This resulted in different findings from the previous practice meeting.

Figure 7. Process and Result of Practice Group 1

Just like the previous procedure during the data collection process, on the third day of implementing the inverse proportion practice, group 1 did not answer the problem accurately. Based on the practice results, the mistake was in the graph sketch and its implementation on the light graph, causing the light not to turn on. Group 1 realized the mistake during the graph sketching and realized that it was a direct proportion.

Based on the learning experience of the previous meeting, Group 1 has improved significantly. They were able to practice independently. Not only that, but the other groups also showed improvements. In addition, the practice implementation became more conducive and effective, and the researcher also found new findings that can be further studied and will be presented in this article.

Figure 8. Process and Result of Practice Group 2

Unlike Group 1, Group 2 completed the task correctly, turning the light graph on. From the lab practice and calculation solution to the graph sketch and its implementation on the light graph, group 2 completed them accurately and correctly. This was based on their correct understanding of proportion and how to apply it to the graph. Among the other groups, group 2's answer was the one that met the expectations. Group 2 showed improvement and better understanding, resulting in correct answers, and the same goes for Group 3. Group 3 progressed in practicing based on its results, despite some errors that can still be corrected.

Figure 9. Process and Result of Practice Group 3

Group 3 is similar to Group 2 because their light graph turned on, indicating that the graph they made was correct. However, group 3 still made a mistake in calculating the ratio, which set them apart from Group 2. Based on the researcher's observation, group 3's calculation mistake was due to a lack of understanding of inverse proportion, as explained by the teacher.

Group 3 showed progress, unlike Group 4. This finding gave the researcher new insights beyond their assumptions. The understanding of the concept made the students portray it differently in their minds. However, the ratio calculation of this group was correct.

Figure 10. Process and Result of Practice Group 4

The most different thing beyond the researcher's hypothesis and expectation is the answer from group 4. This group already understood the concept of inverse proportion because they understood that if one variable increases, the other variable will decrease. However, their implementation of this concept on the graph was incorrect; the graph depicted goes up and down. Although their understanding of the concept and ratio calculation were correct, their implementation of the light graph was wrong. Thus, this can be one answer to the hypothesis related to didactical research design using Hypothetical Learning Trajectory (HLT) in learning using STEM-based media.

Lesson 4: Focused on a Combination of Direct Proportion and Inverse Proportion, Problem-Solving Using STEM Media, and Story Problems Related to Everyday Life

Proportional relationships are a fundamental concept in mathematics, and they play an important role in various fields such as engineering, physics, and economics. There are two types of proportional relationships: direct proportion and inverse proportion. Direct proportion is when two quantities increase or decrease at the same rate, while inverse proportion is when one quantity increases while the other decreases at a constant rate.

One practical application of proportional relationships is in problem-solving using STEM media. The practical application of proportional relationships is in story problems related to everyday life. Story problems are mathematical problems presented in a narrative form, often used to teach mathematical concepts in a more engaging and relatable way. Story problems related to everyday life allow students to see how mathematical concepts such as direct and inverse proportions are used in real-world situations.

For instance, a recipe for baking cookies can be used to teach direct proportion. The amount of ingredients needed to make a certain number of cookies is directly proportional to the number of cookies. If we want to make twice as many cookies, we must double the ingredients. This concept can be applied to other real-world situations, such as scaling up a recipe for a large gathering or increasing the production of a product in a factory.

In conclusion, proportional relationships are an important mathematical concept with practical applications in various fields. Understanding direct and inverse proportions is essential for problem-solving using STEM media and solving story problems related to everyday life. Students can develop critical thinking skills and apply their knowledge to real-world situations by mastering these concepts.

Table 2. The Result of Learning Trajectory based on STEM in Solving Proportion Material

Lesson	Description of Learning Trajectory based on STEM in Solving Proportion Material			
	Group 1	**Group 2**	**Group 3**	**Group 4**
Direct Proportion				
Concept	The understanding of the proportion concept in this group is already good.	The understanding of the proportion concept in this group is already good.	The understanding of the proportion concept in this group is lacking, thus affecting the next stage of learning.	The understanding of the proportion concept in this group is already good.
Ratio	The calculation of the proportion ratio is already correct and good.	The calculation of the proportion ratio is already correct and good.	The calculation of the proportion ratio is already correct and good.	The calculation of the proportion ratio is already correct and good.
Graph	The creation of the proportion graph, graph sketch, and implementation of the light graph is correct.	The creation of the graph in this group is incorrect because the proportion graph is not precise, resulting in the incorrect implementation of the light graph.	Creating the graph in this group is incorrect because the students understand the concept of making the graph is not precise.	The creation of the proportion graph, graph sketch, and implementation of the light graph is correct.
Evaluation	The light graph is on	The light graph is off	The light graph is off	The light graph is on
Inverse Proportion				
Concept	The understanding of the proportion concept in this group is already good.	The understanding of the proportion concept in this group is already good.	The understanding of the proportion concept in this group is already good.	The understanding of the proportion concept in this group is lacking, thus affecting the next stage of learning.
Ratio	The calculation of the proportion ratio is already correct and good.	The calculation of the proportion ratio is already correct and good.	The calculation of the proportion ratio in this group is incorrect because it has not been exchanged.	The calculation of the proportion ratio is already correct and good.
Graph	The graph creation in this group is incorrect because, in the sketch stage before implementation, the sketch is not precise even though the proportion graph calculation is correct.	The creation of the proportion graph, graph sketch, and implementation of the light graph is correct.	The creation of the proportion graph, graph sketch, and implementation of the light graph is correct.	The error in creating the graph is due to the immature understanding of the concept, resulting in a graph far beyond the researchers' expectations, even though the ratio calculation is correct.
Evaluation	The light graph is off	The light graph is on	The light graph is on	The light graph is off

Discussions

Based on the practice conducted by the research sample, which consisted of 4 groups, various answers, and findings were obtained, as discussed earlier. Regarding understanding the concept, the ratio calculation, graph creation, and evaluation of each group, there were differences, advantages, and disadvantages, as well as different trajectories of the students. These differences may be due to the student's previous knowledge and group dynamics. An unexpected finding regarding graph creation will be further investigated, along with its contributing factors. With this hypothetical learning trajectory-based research, the researcher not only observed the students' learning trajectories, discovered new findings, and analyzed them but also gained insights into the patterns emitted by the students through the researcher's observation process.

HLT can also be combined with reinforcement variables, as Fauziyah & Husniati (2023) discussed. Fauziyah's article findings show that problem-based learning with literacy and numeracy reinforcement can lead to a learning trajectory with modeling at level four: situational, referential, general, and formal levels. In addition to using Problem Based Learning (PBL), HLT can also be combined with Realistic Mathematic Education (RME). Rokhmawati et al. (2023) explain that HLT designed compared to actualization in the learning process can achieve a learning trajectory based on the RME approach that can help students understand addition and multiplication concepts in counting rules. Similarly, using RME, according to Febriyani, HLT can also bridge students' prior knowledge with the knowledge they need to master in the learning process. Not only that, Mutaqin et al. (2023) mention that Hypothetical Learning Trajectory is also used as a teaching guide for place value concepts in elementary school. Teachers can anticipate potential problems that students may face during the learning process. This can help obtain findings on the effectiveness of teaching strategies and students' conceptual understanding.

According to Kuncoro et al. (2021), by using the Hypothetical Learning Trajectory (HLT) designed in this study, the researcher identified potential learning obstacles that may arise during the learning process. The HLT developed in this study aims to overcome these obstacles and guide the creation of online teaching materials that foster problem-solving abilities, align with the curriculum, and meet students' special needs. Like Kuncoro's research, Irma's research also highlights that HLT is a promising solution to overcome students' difficulties in understanding the concepts of sets and values in the context of culture to improve students' abilities (Risdiyanti & Prahmana, 2021). Both researchers refer to Ariyadi's research, which explains that HLT helps students understand the concept of probability and the probability learning trajectory based on game-based learning viewed from the perspective of the emergent model (Wijaya et al., 2021).

The research conducted by Cuevas-Vallejo (2023) presents and evaluates a hypothetical learning trajectory where students bridge the transition from basic-level instruction to the university regarding the concept of vectors. The trajectory consists of five tasks and starts with a problem in context. Cuevas-Vallejo presents the hypothetical learning path that must be followed to complete the tasks and compares it in each case with the actual student learning path. Along with Cuevas-Vallejo, Mahammadovna (2023) also highlights the urgency of HLT in professionalism. Mohammed discusses the use of learning trajectories in forming specialist professional training. A specialist's professional competencies can be considered one of these constructions. His article does not explicitly state that findings were obtained using learning trajectories but emphasizes the importance of well-designed trajectories in achieving the desired level of professional competency.

Based on the discussions above regarding relevant studies on HLT, it can be concluded that its use is not only limited to elementary schools but also extends to universities. In addition, the use of HLT is not only focused on introducing concepts or evaluating students' thought patterns or learning trajectories. Still, it can also be used to overcome learning obstacles and analyze a person's professionalism. The use of HLT is expected to continue, and many other interesting findings are yet to be discussed in education.

The discoveries made by the researcher in proportion and STEM learning involve the students' use of a STEM teaching aid during comparative learning. The students, who are already divided into several groups, will execute the teaching aid. Each group member has their own tasks, such as recording the results and testing by moving a toy car within the STEM teaching aid to determine the distance traveled by the car in a minute.

After the students record the results, which include the car's speed and distance traveled per minute, each group is asked to create a light graph and calculate the ratios on a worksheet provided by the researcher. The researcher gives instructions to the students regarding the execution of the graph on the STEM teaching aid, such as parallel, ascending, and descending. The students then execute the instructions with their group members and engage in comparative learning using the STEM teaching aid and the provided worksheet. They discuss with their group members to obtain final results and arrange the lights within the STEM teaching aid.

The arrangement of the LED lights shows that Group 1 succeeded, with the lamp lights showing an upward trend on their graph. Group 2, on the other hand, had their lamp lights turned off, indicating an error in the arrangement showing a downward trend. The same situation occurred with Group 3, where their lamp lights were turned off, indicating an error in arranging the lamp lights in a parallel manner. Lastly, Group 4 managed to light up their lamp lights, indicating an upward trend.

From this, it can be concluded that each group had different final results in terms of lighting up the lamp lights within the STEM teaching aid. The researcher also tested the students in each group to connect the lamp lights to the STEM teaching aid. It can be inferred that each group had different outcomes, including in terms of lighting or turning off the lamp lights within the STEM teaching aid.

Conclusion

Mathematics learning that emphasizes concepts and demands concreteness based on students' understanding of the concept makes each student have their learning trajectory. This becomes a benchmark for research based on comparing the researcher's hypothesis. Based on the results obtained during the practice, some findings exceeded the researcher's expectations, but some did not. These differences become a new finding expected to become a subject for further research, where several groups have different ways of thinking based on mathematical disposition. Through STEM media, the electrical engineering students' high enthusiasm and creativity can be known through the electric graph. Based on the learning trajectory of students in four groups, both in direct and inverse proportion, some groups still make mistakes in implementing the results on the graph. Regarding concept understanding, almost all students have understood it, although there are still some mistakes in ratio calculations.

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