

Novel Traveling Wave Solutions of Jaulent-Miodek Equations and Coupled Konno-Oono Systems and Their Dynamics

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ABSTRACT This research article deals with analytical solutions to two problems. The first is the (1+1)-coupled Jaulent-Miodek system of equations, which is associated with the energy-dependent Schrödinger potential, whereas the second problem, the system of coupled Konno-Oono equations relates to complexity and chaos in electromagnetic fields. Similarity reductions via Lie-symmetry analysis is performed for the systems to derive their analytical solutions. Since Lie symmetry involves arbitrary constants in the infinitesimals, this opens up more possibilities for getting a rich variety of analytical solutions for both real-life problems. The analytical solutions are supplemented graphically to understand them in a better way. Traveling wave profiles are obtained eventually. Solution for CKOEs are different from the earlier research (Kumar and Kumar 2022a; Kumar et al. 2022) as far as the authors are aware.

KEYWORDS
CKOEs
JMEs
Lie-symmetry analysis
Traveling wave solutions
Chaos

INTRODUCTION

During the last few decades, interest in solving nonlinear systems of partial differential equations has increased. Nonlinear partial differential equations (NPDEs) are used in many different scientific disciplines to describe the motion of specific waveforms. In physics, for example, NPDEs can be used to study complexity in electromagnetic fields, chaos theory (Karaca and Baleanu 2022; Karaca 2023) shallow-water wave propagation, oceanic research and engineering, material science, optics, and many other fields. The appearance of NPDEs is cause for serious concern when research numerical results are physically defined. The energy-dependent Schrödinger potential and electromagnetic fields have a connection to this occurrence.

An adequate literature review (Jaulent and Miodek 1976; Zhou 1997; Özer and Salihoğlu 2007; Xu et al. 2014) that includes historical context, various sorts of solutions, and employed methodologies is presented for the following form of Jaulent-Miodek equations (JMEs).

tions (JMEs).

$$3u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} - 2 \frac{\partial v}{\partial x} = 0, \text{ and } 4v \frac{\partial u}{\partial x} - 2u \frac{\partial v}{\partial x} - \frac{\partial^3 u}{\partial x^3} + 2 \frac{\partial v}{\partial t} = 0, \quad (1)$$

where the wave components in space are denoted by the variables $u(x, t)$ and $v(x, t)$, and both of these variables depend on time t as well. Another form of JMEs is as follows:

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + \frac{3}{2}v \frac{\partial^3 v}{\partial x^3} + \frac{9}{2} \frac{\partial v}{\partial x} \frac{\partial^2 v}{\partial x^2} - 6u \frac{\partial u}{\partial x} - 6uv \frac{\partial v}{\partial x} - \frac{3}{2}v^2 \frac{\partial u}{\partial x} = 0, \text{ and } \frac{\partial v}{\partial t} + \frac{\partial^3 v}{\partial x^3} - 6v \frac{\partial u}{\partial x} - 6u \frac{\partial v}{\partial x} - \frac{15}{2}v^2 \frac{\partial v}{\partial x} = 0. \quad (2)$$

This form of JMEs was also tried by the authors to be solved analytically by using Lie symmetry, but its reduction could not be further solved. It opens the door for further research in this field.

Some more literature review for the JMEs (1) is also presented. The JMEs (1) were first introduced by (Jaulent and Miodek 1976) using the inverse scattering transform, which associates with the energy-dependent Schrödinger potential (Özer and Salihoğlu 2007). The finite-band solution of the JMEs (1) can be obtained through nonlinearization of the Lax pair (Zhou 1997). Darboux transformation (Xu et al. 2014) for the JMEs (1) yields some accurate solutions like a kink- and bell-type solitons. These results are based on the Lax pair of the JMEs spectral issue.

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In this research, second problem *i.e.* coupled Konno-Oono Equations (CKOEs) is also solved by employing Lie-symmetry analysis (Konno and Oono 1994). The general form of the CKOEs is represented by

$$\begin{aligned} \frac{\partial^2 u}{\partial x \partial t} - (2\alpha u - \gamma w) \frac{\partial v}{\partial x} - (2\beta u - \gamma v) \frac{\partial w}{\partial x} &= 0, \\ \frac{\partial^2 v}{\partial x \partial t} + 2(2\beta u - \gamma w) \frac{\partial u}{\partial x} - 2(\alpha v - \beta w) \frac{\partial v}{\partial x} &= 0, \text{ and} \\ \frac{\partial^2 w}{\partial x \partial t} + 2(2\alpha u - \gamma w) \frac{\partial u}{\partial x} + 2(\alpha v - \beta w) \frac{\partial w}{\partial x} &= 0. \end{aligned} \quad (3)$$

where the wave components in space are denoted by the variables $u(x, t)$, $v(x, t)$ and $w(x, t)$, all of these variables depend on x and time t , while α , β and γ are parameters. The CKOEs are classified as Coupled Integrable Dispersionless (CID) equations (Pan and Yan 2010; Souleymanou *et al.* 2012). The CKOEs describe how a string moves in a three-dimensional space when interacting with a magnetic field surrounding it. Each point on the curve along the time direction appears to be transiting in parallel in a magnetic-field (Konno and Oono 1994; Konno and Kakuata 1995; Souleymanou *et al.* 2012).

Another particular forms of the CKOEs (3) are discussed in (Pan and Yan 2010; Souleymanou *et al.* 2012; Konno and Kakuata 1995) and derived assuming particular values of α and β as zero while $\gamma = 1$ and recasts as:

$$\begin{aligned} \frac{\partial^2 u}{\partial x \partial t} + w \frac{\partial v}{\partial x} + v \frac{\partial w}{\partial x} = 0, \quad \frac{\partial^2 v}{\partial x \partial t} - 2v \frac{\partial u}{\partial x} = 0, \text{ and} \\ \frac{\partial^2 w}{\partial x \partial t} - 2\gamma w \frac{\partial u}{\partial x} = 0. \end{aligned} \quad (4)$$

Under the conditions u takes value u_0 when v tends to 0 and $|x|$ tends to ∞ , this kind of CKOEs can be resolved by the inverse scattering method (ISM) and satisfies the conservation law (Konno and Oono 1994; Kakuata and Konno 1996; Konno and Kakuata 1996). As these requirements are satisfied with the proper conversion in (5), the solution of CKOEs gradually resembles as Sine-Gordon's solutions and Pohlmeyer-Lund-Regge equations (Pan and Yan 2010; Konno and Kakuata 1996; Hirota and Tsujimoto 1994).

Furthermore, a form of CKOEs in which only u and v appears can be discussed as

$$\frac{\partial u}{\partial t} + 2v \frac{\partial v}{\partial x} = 0, \text{ and } \frac{\partial^2 v}{\partial x \partial t} - 2uv = 0, \quad (5)$$

Konno and Oono (1994) derived the system (5) after replacing R_x , S , T , α , β and γ by $-iu$, iv , iv , 0 , 0 , 1 respectively into the following integrable PDEs

$$\begin{aligned} i \frac{\partial^2 R}{\partial x \partial t} + (2\alpha R + \gamma T) \frac{\partial S}{\partial x} + (2\beta + \gamma) S \frac{\partial T}{\partial x} &= 0, \\ i \frac{\partial^2 S}{\partial x \partial t} + 2(2\beta R + \gamma B) \frac{\partial R}{\partial x} - 2(\alpha S - \beta T) \frac{\partial S}{\partial x} &= 0, \text{ and} \\ i \frac{\partial^2 T}{\partial x \partial t} + 2(2\beta R + \gamma T) \frac{\partial R}{\partial x} - 2(\alpha T - \beta S) \frac{\partial T}{\partial x} &= 0. \end{aligned} \quad (6)$$

Furthermore, the stochastic form of CKOEs (Mohammed *et al.* 2021) is given by

$$\frac{\partial u}{\partial t} + 2v \frac{\partial v}{\partial x} = 0, \text{ and } \frac{\partial^2 v}{\partial x \partial t} - 2uv = \sigma F(v), \quad (7)$$

where the noise term, $F(v)$ is a function of strength (σ) of noise. CKOEs (5) can be obtained by Eq. (7) if σ vanishes.

Some tools/methods such as ISM (Konno and Oono 1994; Kakuata and Konno 1996), classical Lie-symmetry (Khalique 2012), G'/G -expansion and tanh (Abdullah *et al.* 2023), and some others (Khan and Akbar 2013; Alam and Belgacem 2016; Yel *et al.* 2017; Mohammed *et al.* 2021) are used for solving CKOEs (3) in their various forms.

Khalique (2012) used similarity reduction to solve the CKOEs (3), and obtained kink type solutions. Abdullah *et al.* (2023) derived rational, trigonometric and hyperbolic solutions. Zahran and Bekir (2023) got W-shaped, singular dark solitons type solutions. Mohammed *et al.* (2021) and Wang and Liu (2022) obtained solitary wave type solutions, whereas in our previous contributions (Kumar and Kumar 2022a; Kumar *et al.* 2022) optimal sub-algebra utilizing killing form is exploited and derived some initial traveling wave solutions for the same form of the CKOEs (3). Besides the work of (Khalique 2012), and authors (Kumar *et al.* 2022) a group of researchers (Bashar *et al.* 2016; Khan and Akbar 2013) has solved some specific forms of the CKOEs (5) and obtained more traveling wave solutions.

The research conducted by Torvattanabun *et al.* (2018) derived both trigonometric and hyperbolic solutions. The hyperbolic, trigonometric, and rational types were obtained by Alam and Belgacem (2016), whereas Khater *et al.* (2018) derived travelling and solitary wave type solutions. The hyperbolic and trigonometric wave types were derived by Mirhosseini-Alizamini *et al.* (2020). The results of Manafian *et al.* (2018) were hyperbolic, elliptic, and solitons. Solitons were obtained by Yel *et al.* (2017), whilst solutions in the form of travelling waves were obtained by Koçak *et al.* (2016). In addition, CKOEs (5) are solved by Abdelrahman and Alkhidhr (2020) that contain solitary type solutions.

Above reviews motivate to derive some novel variety of solutions for JMEs (1), and CKOEs (3).

INFINITESIMALS VIA LIE-SYMMETRY ANALYSIS

In this section, infinitesimals of the JMEs (1) and CKOEs (3) are derived by using one parameter Lie group similarity transformations method (STM). Such transformations can be treated as:

$$\begin{aligned} x^* &\rightarrow x + \epsilon \zeta(\Xi) + o(\epsilon^2), \quad t^* \rightarrow t + \epsilon \tau(\Xi) + o(\epsilon^2), \\ u^* &\rightarrow u + \epsilon \eta^{(u)}(\Xi) + o(\epsilon^2), \quad v^* \rightarrow v + \epsilon \eta^{(v)}(\Xi) + o(\epsilon^2), \text{ and} \\ \frac{\partial u^*}{\partial x^*} &\rightarrow \frac{\partial u}{\partial x} + \epsilon [\eta_x^{(u)}] + o(\epsilon^2) \text{ etc.} \end{aligned} \quad (8)$$

where ζ , τ , $\eta^{(u)}$ and $\eta^{(v)}$ are the infinitesimals for x , t , u and v respectively and $(\Xi) \equiv (u, v, x, t)$.

Let $u = \theta^{(u)}(x, t)$, and $v = \theta^{(v)}(x, t)$ be the solutions for JMEs (1), then its invariance conditions are

$$\begin{aligned} [\eta_t^{(u)}] + 3\theta_x^{(u)} \theta^{(u)} + 3u [\eta_x^{(u)}] - 2[\eta_x^{(v)}] &= 0, \text{ and} \\ 2[\eta_t^{(v)}] + 4\theta_x^{(u)} \theta^{(v)} + 4v [\eta_x^{(u)}] - 2\theta_x^{(v)} \theta^{(u)} - 2u [\eta_x^{(v)}] - [\eta_{xxx}^{(u)}] &= 0. \end{aligned} \quad (9)$$

One can follow the textbooks (Bluman and Cole 1974; Olver 1993) and research articles (Kumar and Kumar 2022a,b; Kumar *et al.* 2023) for getting the values of the extensions $[\eta_x^{(1)}]$, $[\eta_{xx}^{(1)}]$, and $[\eta_{xxx}^{(1)}]$ etc. Making use of Eq. (1) into Eq. (9) which gives

$$\begin{aligned} \tau_u = \tau_v = \tau_x = 0, \quad \tau_{tt} = 0, \quad \zeta_u = \zeta_v = \zeta_t = 0, \quad 2\zeta_x = \tau_t, \quad \zeta_{xx} = 0, \\ 2\eta_u^{(1)} = -u\tau_t, \text{ and } \eta_v^{(2)} = -v\tau_t. \end{aligned} \quad (10)$$

The resulting infinitesimal generators for Eq. (1) can be derived by solving the above determining Eqs. (10).

$$\xi = a_1x + a_2, \tau = 2a_1t + a_3, \eta^{(u)} = -a_1u, \text{ and } \eta^{(v)} = -2a_1v. \quad (11)$$

where all a_i 's are arbitrary constants.

Similarly for CKOEs (3), the infinitesimals are as follows:

$$\xi = 2c_1x + c_2, \tau = -c_1t + c_3, \eta^{(u)} = c_1u, \eta^{(v)} = c_1v, \text{ and } \eta^{(w)} = c_1w. \quad (12)$$

where c_i 's are arbitrary constants.

SIMILARITY REDUCTIONS AND INVARIANT SOLUTIONS

Jaulent-Miodek equations

Case 1: For $a_1 \neq 0$, the Eq. (11) provides,

$$\frac{dx}{x + A_1} = \frac{dt}{2t + A_2} = -\frac{du}{u} = -\frac{dv}{2v}. \quad (13)$$

On solving above, similarity variable is given as $X_1 = (x + A_1)(2t + A_2)^{-\frac{1}{2}}$ and similarity functions $u = (2t + A_2)^{-\frac{1}{2}}F_1(X_1)$, and $v = (2t + A_2)^{-1}G_1(X_1)$, where $A_1 = \frac{a_2}{a_1}$, and $A_2 = \frac{a_3}{a_1}$. Treating $G_1 = F_1$, similarity reduction of JMEs (1) yield the following ODE:

$$\bar{F}_1 - 6F_1^2\bar{F}_1 + 9X_1F_1\bar{F}_1 + F_1^2 - 2X_1^2\bar{F}_1 - 2X_1F_1 - 4C_1\bar{F}_1 = 4C_1, \quad (14)$$

where C_1 is an integration constant.

On solving Eq. (14), one can find

$$F_1 = X_1 \pm \sqrt{X_1^2 + 4C_1}. \quad (15)$$

So, the first solution of JMEs (1) is

$$u_1 = \frac{1}{(2t + A_2)} \left[(x + A_1) \pm \sqrt{(x + A_1)^2 + 4C_1(2t + A_2)} \right], \text{ and } v_1 = \frac{(x + A_1)}{2(2t + A_2)^2} \left[(x + A_1) + \sqrt{(x + A_1)^2 + 4C_1(2t + A_2)} \right] + \frac{4C_1}{(2t + A_2)}. \quad (16)$$

Case 2: For $a_1 = 0$ and $a_3 \neq 0$, Lagrange's characteristic equations for the Eq. (11) recasts as

$$\frac{dx}{B} = \frac{dt}{1} = \frac{du}{0} = \frac{dv}{0}, \quad (17)$$

where $B = \frac{a_2}{a_3}$. On integrating, one can get similarity forms as

$$X_2 = \frac{1}{B}(x - t) \text{ and } u = F_2(X_2), \text{ and } v = G_2(X_2).$$

Hence, similarity reduction of JMEs (1) is represented by the following ODE (for $F_2 = G_2$).

$$\bar{F}_2 - B^4\bar{F}_2 + 6B^3F_2\bar{F}_2 - 6B^2F_2^2\bar{F}_2 - 2B^2C_2F_2 = 0, \quad (18)$$

where integration constant is C_2 .

Eq. (18) is satisfied by

$$F_2 = \frac{1}{2}B \pm \frac{1}{2}\sqrt{B^2 - 4C_2} \tanh\left(\pm C_3 + \frac{1}{2}\sqrt{B^2 - 4C_2}BX_2\right). \quad (19)$$

So, another solution of JMEs (1) is given by

$$u_2 = \frac{1}{2}B \pm \frac{1}{2}\sqrt{B^2 - 4C_2} \tanh\left(\pm C_3 + \frac{1}{2}\sqrt{B^2 - 4C_2}BX_2\right) \text{ and } v_2 = -\frac{9}{16}B^2 + \frac{3}{16}(B^2 - 4C_2) \tanh^2\left(\pm C_3 + \frac{1}{2}\sqrt{B^2 - 4C_2}BX_2\right) - \frac{3}{8}B\sqrt{B^2 - 4C_2} \tanh\left(\pm C_3 + \frac{1}{2}\sqrt{B^2 - 4C_2}BX_2\right) + C_3. \quad (20)$$

where C_3 is an integration constant.

Coupled Konno-Oono equations

The Lie algebra L^3 can be generated by L_i s ($1 \leq i \leq 3$) in which $L_1 = \frac{\partial}{\partial x}$, $L_2 = \frac{\partial}{\partial t}$, and $L_3 = 2x\frac{\partial}{\partial x} - t\frac{\partial}{\partial t} + u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v} + w\frac{\partial}{\partial w}$. Thus symmetry reductions of the CKOEs (3) are as follows.

Case 1: The symmetry generator $\mu_1L_1 + L_2$ gives rise to the group-invariant solution $u = U(X)$, $v = V(X)$, and $w = W(X)$; in which $X = x - \mu_1t$ is an invariant of the symmetry $\mu_1L_1 + L_2$, where μ_1 is an arbitrary constant. Substituting these values into (3) yields the system of ODEs as

$$\begin{aligned} \mu_1U'' + 2\alpha V'U + 2\beta W'U - \gamma V'W - \gamma W'V &= 0, \\ \mu_1V'' + 2\alpha V'V - 4\beta U'U - 2\beta V'W + 2\gamma U'V &= 0, \text{ and} \\ \mu_1W'' - 4\alpha U'U - 2\alpha W'V + 2\beta W'W + 2\gamma U'W &= 0. \end{aligned} \quad (21)$$

After solving the above system of reduction, following variety of solutions can be obtained

$$u_1 = 0, v_1 = \tanh C_1\alpha(x - \mu_1t), w_1 = 0. \quad (22)$$

$$u_2 = \frac{\gamma}{2\alpha} \left(C_1(x - \mu_1t) + C_2 \right), v_2 = \frac{\beta}{\alpha} \left(C_1(x - \mu_1t) + C_2 \right), w_2 = C_1(x - \mu_1t) + C_2. \quad (23)$$

$$u_3 = \frac{\gamma}{\mu_1\alpha^2(\gamma^2 + 4\alpha\beta)} \tanh\left(\frac{1}{2}C_3(x - \mu_1t) + C_2\right)C_3,$$

$$v_3 = -\frac{1}{2} \frac{\gamma^2}{\mu_1\alpha^2(\gamma^2 + 4\alpha\beta)} \tanh\left(\frac{1}{2}C_3(x - \mu_1t) + C_2\right)C_3,$$

$$w_3 = \frac{2}{\mu_1\alpha(\gamma^2 + 4\alpha\beta)} \tanh\left(\frac{1}{2}C_3(x - \mu_1t) + C_2\right)C_3. \quad (24)$$

Case 2: The symmetry operator $\mu_2L_1 + L_2 + \mu_3L_3$ (where μ_2 and μ_3 are arbitrary constants) provides group-invariant solution of the form $u = U(X) + \mu_3\gamma t$, $v = V(X) + 2\mu_3\beta t$, $w = W(X) + 2\mu_3\alpha t$ where $X = x - \mu_2t$ is an invariant of $\mu_2H_1 + H_2 + \mu_3H_3$ and the functions U , V and W satisfy the same reductions as given in Eq. (21), but solutions are different due to different similarity forms.

So, solution is given by

$$u_4 = 0, v_4 = \tanh(x - \mu_2t)C_1\alpha + 2\mu_2\gamma t, w_4 = 0. \quad (25)$$

$$u_5 = \frac{\gamma C_1}{2\alpha} + \mu_t\gamma t, v_5 = C_1 + 2\mu_2\beta t, w_5 = C_1 + 2\mu_2\alpha t. \quad (26)$$

$$u_6 = \frac{\gamma}{2\alpha} \{ C_1(x - \mu_2t) + C_2 \} + 2\mu_2\alpha t,$$

$$v_6 = \frac{\beta}{\alpha} \{ C_1(x - \mu_2t) + C_2 \} + 2\mu_2\beta t,$$

$$w_6 = C_1(x - \mu_2t) + C_2 + \mu_2\gamma t. \quad (27)$$

$$u_7 = \frac{\gamma}{\mu_2\alpha^2(\gamma^2 + 4\alpha\beta)} \tanh\left(\frac{1}{2}C_3(x - \mu_2t) + C_2\right)C_3 + \mu_2\gamma t,$$

$$v_7 = -\frac{\gamma^2}{2\mu_2\alpha^2(\gamma^2 + 4\alpha\beta)} \tanh\left(\frac{1}{2}C_3(x - \mu_2t) + C_2\right)C_3 + 2\mu_2\beta t,$$

$$w_7 = \frac{2}{\mu_2\alpha(\gamma^2 + 4\alpha\beta)} \tanh\left(\frac{1}{2}C_3(x - \mu_2t) + C_2\right)C_3 + 2\mu_2\alpha t. \quad (28)$$

PHYSICAL ANALYSIS AND DISCUSSION

Two analytic solutions for the JMEs (1) are represented by the expressions (16) and (20) while seven analytic solutions for the CKOEs (3) are expressed by Eqs. (22)–(28). The solution profiles are shown which have different animation behaviour. The stationary-progressive profile of the JMEs is shown via the Figs. (1) and progressive, traveling and stationary-progressive profiles for the CKOEs (3) are given by Figs. (2)–(4). A progressive wave is a wave that conveys energy and momentum from one region of space to another. The numerical simulation is performed with the help of MATLAB. The space and the time ranges are taken as $-25 \geq x \leq 25$ and $0 < t \leq 50$ respectively for each profile. For all profiles of the CKOEs (3), the arbitrary constants involved in the solutions are chosen as $C = C_1 = C_2 = C_3 = \mu_1 = \mu_2 = 1.25$ and $\alpha = 0.95$, and $\beta = \gamma = 1.50$. For all the JMEs profiles, the arbitrary constants are taken as $a_1 = 0.4387, a_2 = 0.3816, a_3 = 0.7655$, and $A_1 = 0.3804, A_2 = 0.4217$ and $A_3 = 0.7537$.

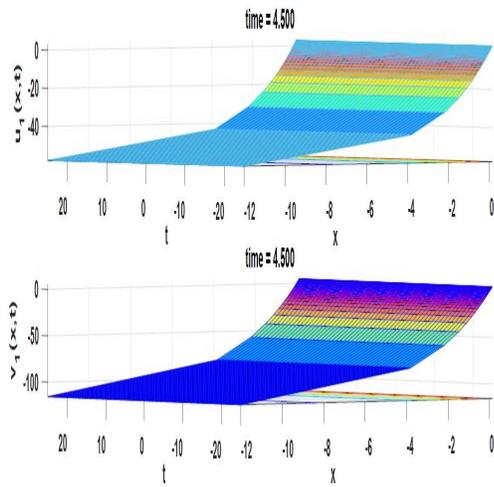


Figure 1 Stationary-progressive profile for the solution (16)

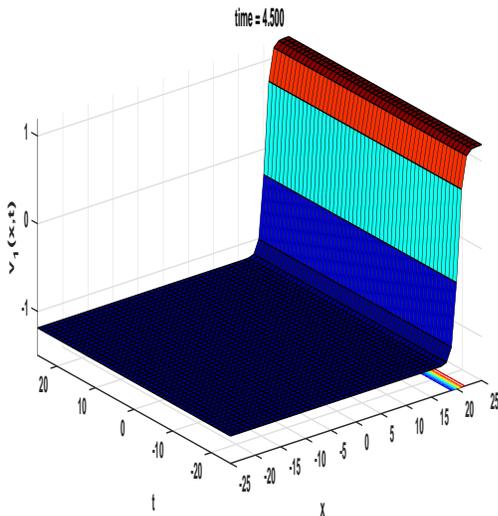


Figure 2 Progressive profile for the solution (22)

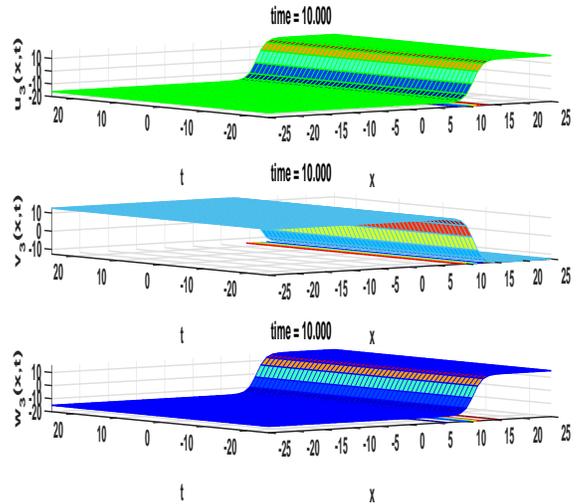


Figure 3 Traveling wave profile for the solution (25)

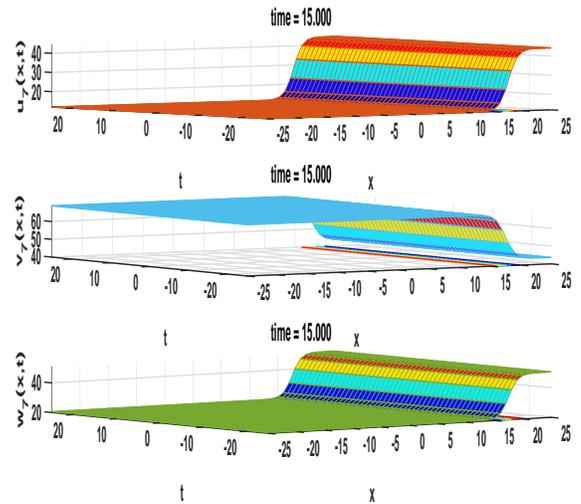


Figure 4 Stationary-progressive profile for the solution (28)

CONCLUSION

The Lie-symmetry analysis is successfully explored to obtain seven analytic solutions for the CKOEs (3) and two analytic solutions for the JMEs (1). Solutions of JMEs are given by Eqs. (16) and (20), while solutions to CKOEs (3) are represented by Eqs. (22)–(28). Mathematical expressions are explored physically as stationary-progressive and progressive profiles for the solutions (16) and (20) are depicted in Figure 1 and Figure 2 respectively. The progressive, traveling and stationary-progressive profiles for the CKOEs (3) are given by Figures (2)–(4). The solutions established here can explore some more applications in the fields of chaos and the complexities of the magnetic field since they provide a realistic perspective.

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Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there are no competing interests in the publication of this research.

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