Journal of Theoretical Educational Science, 17(3), 563-591, July 2024
Kuramsal Eğitimbilim Dergisi, 17(3), 563-591, Temmuz 2024
[Online]: http://dergipark.org.tr/akukeg
DOI number: http://doi.org/10.30831/akukeg.1323295

Examining the Process of Middle School Math Teachers Diagnosing and Eliminating Student Misconceptions in Algebra*

Ortaokul Matematik Öğretmenlerinin Öğrencilerin Cebirdeki Kavram Yanılgılarını Tespit Etme ve Giderme Süreçlerinin İncelenmesi

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Received: 05 July 2023 **Research Article** **Accepted:** 15 April 2024

ABSTRACT: The aim of this study is to examine how middle school mathematics teachers diagnose and attempt to eliminate students' misconceptions in algebra. The study employed a case study method and embedded single-case design. The research was conducted with three mathematics teachers working in different state schools and having different professional experiences, as well as ten students from the eighth-grade classes of the same schools. The data collection instruments used in the study included the Diagnostic Test developed by researchers to identify students with misconceptions, the Assessment Framework prepared for the evaluation of teacher performances in diagnosing and eliminating student misconceptions, and Semi-Structured Interviews conducted between students and teachers, which were recorded and made available in audio/video format. The study revealed that teachers generally resorted to conventional methods in the process of diagnosing and eliminating students' misconceptions. In most cases, teachers superficially addressed students' errors and did not fully focus on students' thinking. Regarding the processes aimed at eliminating student misconceptions, teachers preferred to directly inform students that their answers were incorrect, rather than facilitating students in recognizing their own mistakes. The findings highlight the need to increase teachers' awareness and student knowledge regarding misconceptions in algebra.

Keywords: Misconceptions in algebra, middle school math teachers, knowledge of student thinking, pedagogical content knowledge.

ÖZ: Bu çalışmanın amacı ortaokul matematik öğretmenlerinin öğrencilerin cebirdeki kavram yanılgılarını tespit etme ve giderme süreçlerinin incelenmesidir. Çalışma kapsamında durum çalışması yöntemi ve iç içe geçmiş tek durum deseni kullanılmıştır. Çalışma farklı devlet okullarında görev yapan ve farklı mesleki deneyimlere sahip üç matematik öğretmeni ve aynı okulların sekizinci sınıf şubelerinde öğrenim gören on öğrenci ile yürütülmüştür. Veri toplama aracı olarak kavram yanılgısına sahip öğrencilerin belirlenmesine yönelik araştırmacılar tarafından geliştirilmiş olan Teşhis Testi, öğretmenlerin kavram yanılgılarını tespit etme ve giderme süreçlerinin değerlendirilmesine yönelik olarak hazırlanmış olan Değerlendirme Çerçevesi ile öğrenci ve öğretmenler arasında yürütülmüş olan ve ekran/ ses kaydı alınarak saklanabilir hale getirilmiş olan Yarı Yapılandırılmış Görüşmeler kullanılmıştır. Çalışma sonucunda öğretmenlerin öğrencilerin kavram yanılgılarını tespit etme ve giderme süreçlerinde geleneksel yöntemlere başvurdukları görülmüştür. Öğretmenler çoğu durumda öğrencilerin hatalarını yüzeysel olarak ele almış ve öğrenci düşüncesine tam olarak odaklanamamışlardır. Öğrenci yanılgısının giderilmesine yönelik olarak yürütülen süreçlerde ise öğretmenlerin, öğrencilerin kendi hatalarını fark etmelerini sağlamak yerine, onlara cevaplarının yanlış olduğunu doğrudan söylemeyi tercih ettikleri gözlenmiştir. Bu çalışmanın sonuçları, cebirdeki kavram yanılgıları konusunda öğretmenlerin farkındalıklarının ve öğrenci bilgilerinin arttırılması gerektiğini ortaya koymaktadır.

Anahtar kelimeler: Cebirdeki kavram yanılgıları, ortaokul matematik öğretmenleri, öğrenci düşüncesi bilgisi, pedagojik alan bilgisi.

* This study was produced from the first author's master thesis titled 'Investigation of Secondary School Mathematics Teachers' Prediction and Elimination Processes of Students' Possible Misconceptions in Algebra' under the supervision of the second author.

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Citation Information
Kabadaş, H., Yavuz Mumcu, H. (202?). Examining the process of middle school math teachers diagnosing and eliminating student misconceptions in algebra. *Kuramsal Eğitimbilim Dergisi [Journal of Theoretical Educational Science], 17*(3), 563-591.

ISSN: 1308-1659

From past to present, mathematics has generally been perceived as a difficult and complex subject, and one of the reasons for this perception is its abstract nature (Baykul, 2014; Martino et al., 2023; O'Leary et al., 2017; Reyes et al., 2019; Yang et al., 2020). Students often have difficulties in abstract thinking processes when trying to understand mathematical concepts without having the opportunity to observe them concretely. In particular, algebra stands out as one of the areas in school mathematics that students have difficulty in understanding due to its abstract nature (NCTM, 2000).

The field of algebra employs functions as a means of illustrating mathematical relationships, analyzing change, and highlighting connections between quantities. These functional relationships are conveyed and abstracted through the use of mathematical symbols. Algebra, often described as generalized arithmetic, was defined by Kieran (1992) as a discipline that represents numerical relations and mathematical operations. It utilizes symbols and generalized numbers to solve equations, examine functional relationships, and establish a system of expressions and relations (Lew, 2004). As an integral component of the secondary school mathematics curriculum, algebra is functionally employed across all areas of mathematical learning, imparting a comprehensive structure to the subject. According to Moses (2000) and Strong and Cobb (2000), algebra should be included in the education of all students, as it serves as a critical gateway to advanced mathematics and to many respected professions.

In the realm of school mathematics, algebra is often perceived as the application of symbols, solving complex equations, and simplifying algebraic expressions. However, algebra encompasses much more than these aspects. It is crucial that students develop an in-depth understanding of the mathematical structures and algorithms underlying the use of algebraic concepts and symbols and gain insight into their application in a variety of situations (NCTM, 2000). In the absence of such an understanding, concepts are poorly understood and cannot be related to each other, symbols lose their meaning and algorithms are simply memorized without being understood. As a result, various learning difficulties and misconceptions may arise. Similarly, Suparno (2005) argues that when students have difficulty in understanding a new concept and cannot assimilate it into their mental schema, they develop their own incoherent conceptual framework that is different from the actual concept.

In mathematics learning processes, it is seen that students develop many misconceptions of different types, especially in algebra. Research on algebra teaching has been conducted over the years, and some of the problems have been solved in these studies, but many problems still persist (Baş et al., 2011; Dede & Peker, 2007). Among these studies, Barbieri et al., (2019), Demirören (2019), Lucariello et al., (2019) and Ralston and Li (2022) show that students still lack basic competencies to effectively handle variables, algebraic expressions and equations. Current studies conducted in Turkey (Birgin & Demirören, 2020; Çakmak Gürel & Okur, 2017; Demirören, 2019; Örnekçi, 2019; Şahiner, 2018; Şahin & Soylu, 2011) also report that there are still problems in teaching algebra. Accordingly, in Demirören's (2019) study, it was determined that some students had various misconceptions in visual and geometric representations of algebraic expressions, selection and priority of operations, and setting up and solving equations. In this study, different types of students' misconceptions about understanding algebraic expressions, writing them in different forms, performing operations with algebraic expressions and solving equations were reported. Birgin and

Demirören (2020) found that students who could not establish the algebra-geometry relationship had difficulty in understanding algebraic expressions and could not move from pattern to algebraic equation. It was also observed that students' errors in simple visual and algebraic expressions were caused by choosing the wrong algebraic operation, ignoring parenthesis in algebraic expressions, misinterpreting the shape pattern and arithmetic operation errors. Çakmak Gürel and Okur (2018) examined misconceptions about equality and equation in their study. In terms of misconceptions, it was determined that students had the misconception that 'variables are always different from each other' at the highest level and 'not considering the importance of parentheses in algebra' at the lowest level. In his thesis study, Örnekçi (2019) mentions different types of misconceptions that eighth grade students have about slope. According to the results of Şahiner (2018), it was determined that students had various misconceptions about algebraic expressions. It was determined that students could not mathematically structure identity expressions and had difficulty in modeling. In addition, it was found that they had misconceptions about factoring and simplifying rational algebraic expressions. Şahin and Soylu (2011) reported the misconceptions regarding the variable as overlooking the variables, processing the different units under the same unit, focusing on 'x', 'y' variables, not being able to find the connection between the verbal expressions and the variables, reducing the variables to constants, attributing digits to the variable in multiplication, confusing the 'x' unknown with the multiplication sign and not using parenthesis. In their study, Ralston and Li (2022) reported that students focused on the operational meaning of the equals sign rather than its relational meaning and saw the equals sign as a symbol expressing a result. Lucariello et al. (2019) revealed students' misconceptions about the concept of variable. These misconceptions include ignoring the variable, perceiving the variable as a label of an object, and seeing the variable as an unknown value.

Therefore, in order to solve this problem in algebra teaching, it is necessary to seek answers to two basic questions: How can students' algebra learning process be improved and how can misconceptions be corrected? Although there are many studies in the literature on improving algebra learning processes, the persistence of the existing problem led this study to focus on the answers to the second question. Accordingly, this study focuses on exploring strategies to eliminate students' misconceptions in algebra.

Knowledge of Student Thinking in Eliminating Misconceptions

Misconceptions are caused by students' constructing concepts in their minds in line with their own understanding and are generally defined as a phenomenon that is not scientifically correct but can be explained by students in their own way (Ebenezer & Fraser, 2001). According to Baki (1999) and Driver and Easley (1978), misconceptions arise as a result of individuals' experiences and false beliefs. If students have a misconception in their prior learning, it is highly likely that new concepts will also contain misconceptions because mathematics is learned in a relational way. However, since these misconceptions arise from students' incorrect coding of new information in their minds and are supported by the individual's experiences, these are constantly resistant to change (Tafara, 2015). In this context, Minstrell (1982) see misconceptions as permanent barriers to conceptual understanding. For this reason, as educators, we need to know the underlying causes of these conceptions and take measures to create more efficient learning environments (Ojose, 2015).

In this context, teachers should be aware of their students' ways of thinking, anticipate scenarios in which students may form misconceptions, and organize their teaching accordingly. All these processes are associated with knowledge of student thinking (K-ST), as an important component of pedagogical content knowledge (An et al., 2004; Shulman, 1987). Pedagogical content knowledge, which is defined as the knowledge of how to teach a certain subject (An et al., 2004), is expressed in most studies with the components of subject knowledge, pedagogical knowledge, student knowledge, and curriculum knowledge (An et al., 2004; Morine-Dershimer & Kent 1999). One of these components, K-ST, is defined as knowing the characteristics of a particular group of students and planning the teaching accordingly by creating a classroom environment that meets the needs of these students (Fennema & Franke 1992). An et al. (2004) mention four components of K-ST. These are i) *building new knowledge on the student's existing ideas,* ii) *identifying students' misconceptions*, iii) *involving students in mathematics learning processes,* and iv) *encouraging students' thinking about mathematics*. In the same study, the authors stated that teachers should link students' prior knowledge with new ones through various representations, examples, and manipulatives, and focus on students' conceptual understanding rather than procedures or rules. Teachers also need to accurately identify students' misconceptions and eliminate such misconceptions by using appropriate questions or tasks. In this context, strong student knowledge enables teachers to measure how well students understand mathematical concepts, to understand possible misconceptions and their causes, and to develop clear strategies to correct these misconceptions (An & Wu, 2012; Even & Tirosh, 1995).

Purpose

Since it is not possible to completely prevent students' misconceptions (Ünlü, 2015), instructors in mathematics education will always encounter students with misconceptions. Based on this fact, it is important how to behave in these situations. Considering the misconceptions that students have in algebra, it is necessary for teachers to be aware of these misconceptions and to be able to use appropriate teaching methods and strategies to overcome these misconceptions. Therefore, in this study, knowledge of student thinking was used in the context of pedagogical content knowledge.

However, when the studies about student misconceptions in algebra are examined in the literature, most of them (Aydın-Güç & Aygün, 2021; Akhtar et al., 2020; Bush & Karp, 2013; Erdem & Aktaş, 2018; Rathnayake & Jayakody, 2022; Sarımanoğlu, 2019; Welder, 2012; Yasseen et al., 2020) focused on the current situation but practical studies on how to diagnose and eliminate existing misconceptions (Bingölbali, 2010; Chick & Baker, 2005; Erdem & Sarpkaya-Aktaş, 2018; Kılıç, 2011) were found to be quite limited. In one of these studies, Bingölbali (2010) investigated how mathematics teachers deal with student difficulties in their lessons. In this study, five primary school teachers' mathematics lessons were observed in order to reveal how they intervene in students' errors and misconceptions. Algebra activities were utilized in the study. Click and Baker's (2005) study with nine middle school teachers was based on interview processes conducted through open-ended questions similar to this study. Teachers were compared with students' answers containing misconceptions and were asked the question ‘What would you say to a student with such a misconception?’. The questions used in this research are related to different subject areas of mathematics. Erdem and Sarpkaya Aktaş (2018) investigated the effectiveness of activity-based instruction in eliminating misconceptions in algebra. Kılıç (2011) examined student knowledge in the context of pedagogical content knowledge of the participants in his study with six pre-service teachers. In the study in which she participated as a participant observer, the author observed the pre-service teachers during their undergraduate course and used different sources such as interviews, observations, questionnaires and written documents to collect data.

Although the studies mentioned here so far differ in terms of purpose and methodology, it can be said that they generally try to observe how teachers or pre-service teachers intervene in students' errors or misconceptions in different learning areas. This study differs from other studies in that it focuses on mathematics teachers rather than pre-service teachers, focuses only on pre-defined possible misconceptions about algebra learning, and offers the opportunity to observe all the processes carried out by mathematics teachers and their students in detail. Within the scope of the study, it is aimed to obtain richer and more useful data on specific situations. In this context, the aim of the study is to examine how mathematics teachers use knowledge of student thinking in the process of identifying and eliminating students' misconceptions in algebra. The research problem can be expressed as ‘How do mathematics teachers use the knowledge of student thinking in the process of identifying and eliminating students' misconceptions about algebra?’.

Method

This study has a qualitative design, and the case study method was used to examine a specific phenomenon or situation in detail during the study process. Within the scope of the study, an embedded single case design from different types of case study was used. This type is often used to understand a complex event or situation, identify cause-effect relationships, or develop a theoretical framework. Besides, in this type of studies, there is the existence of more than one analysis unit in a single situation (Yin, 2018).

Each teacher involved in this study has been considered as a different unit of analysis, and a detailed analysis of these different cases has been conducted in an attempt to develop a theoretical framework for teachers' process of addressing student misconceptions. The results obtained from this study were interpreted on this axis. So, the relevant type appears to be suitable for this study.

Participants

The participants in the study consisted of three mathematics teachers and ten secondary school students who were selected from the classes taught by these teachers. In the determination of the participants, convenient and criterion sampling methods were used together. The teachers involved in the study were selected from individuals accessible to the researcher, and they volunteered to participate in the study. The criteria for selecting teachers included having more than 10 years of professional experience and working in schools with an average level of academic performance in the city center. Gender diversity was also considered in the selection of teachers. Therefore, the teacher coded as T1 in the study was a male with 13 years of professional experience. The teacher coded as T2 was a female with 15 years of professional experience. Lastly, the teacher coded as T3 was also a female with 21 years of professional experience.

Additionally, ten secondary school students were selected from the classes taught by these teachers. In the selection of these students, criteria such as providing expected answers to questions in the diagnostic test prepared by the researchers and voluntary participation in the study have been taken into account. At this stage, the students who were predicted to have misconceptions were included in the study with the help of the answers they gave to the questions in the diagnostic test and the explanations including the reasons for their answers.

Ethical Procedures

This research was approved by University Social and Human Sciences Research Ethics Committee with its decision dated 27.01.2021 and numbered 2020-12. In addition, necessary permissions with the 20982064 and 20981990 numbered, were obtained from the provincial directorate of national education to carry out the study and the institutions of the participating teachers were informed about the study. However, parental consent was obtained from the students participating in the study, the participation of the students was completely voluntary, and it was stated that any participant could leave the study whenever they wanted. Confidentiality principles were also complied with within the framework of ethical rules.

Data Collection Procedures and Instruments

In the study process, a diagnostic test was first developed for the possible misconceptions that students may have and applied to a total of 94 eighth grade students selected from the classes of the participating teachers. Based on the answers they provided to the questions in the test, students who were likely to have misconceptions were identified. Since the students were asked to explain the reasons for their answers in writing in the diagnostic test, student expressions were used in this process. At the end of the process, students who were predicted to have misconceptions were paired with their teachers. Accordingly, three teachers were matched with a total of ten students and semi-structured interview processes were planned between teachers and students. At the end of the process, it was tried to reveal how the teachers used the knowledge of student thinking in eliminating misconceptions by utilizing the interview processes carried out by the teachers.

Diagnostic Test (DT)

In the preparation of the diagnostic test, a comprehensive content was created by conducting a literature review in the field, focusing on different types of misconceptions in algebra. The categories found in the studies of Sarpkaya-Aktaş (2019), Güler (2014), and Baki (2008) have been referenced in the final version of the diagnostic test. Accordingly, there are 14 questions in seven different categories in the test. These categories are i) *Misconceptions about the concept of algebraic expression and variable*, ii) *Misconceptions about the concept of equality*, iii) *Misconceptions about the* *concept of identity*, iv) *Misconceptions about the concept of equation*, v) *Misconceptions about the concept of inequality*, vi) *Misconceptions about the concept of pattern*, vii) *Misconceptions about linear relations and equations*. Expert opinions were used for the validity of the test, and a pilot study was conducted for reliability. For the pilot study, the test was administered to 27 students from a different school in the same school district, and the questions were finalised accordingly. Each of the questions in the diagnostic test may contain misconceptions belonging to more than one category. Accordingly, the categories to which the 14 questions in the diagnostic test belong and questions 4, 5, 7, 9, 10 and 14 in the findings section are given in Appendix.

Semi-Structured Interviews (SSI)

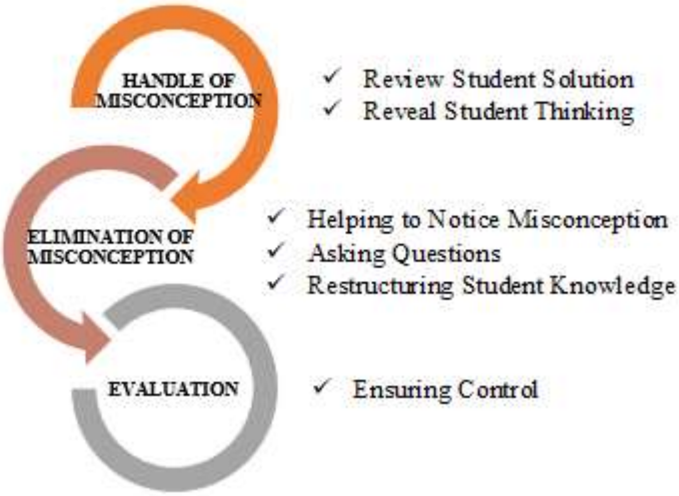
In this study, SSIs were conducted between students with misconceptions and teachers. Since this study was conducted during the pandemic period, the interview process was conducted online and recorded using screen and audio recordings to protect the data. No time constraints were imposed on the teachers during the interview sessions. Some of the questions in the semi-structured interview form can be exemplified by ‘Can you explain to me what you did in this question?’, ‘Why do you think this answer is correct?’, ‘Can there be a different solution to this question?’.

In order to analyze the data obtained within the scope of the study and evaluate teacher performances, researchers conducted inductive coding processes. All of the data obtained from the interview processes were analysed inductively, and an attempt was made to develop a framework for how teachers deal with student misconceptions. In this process, teacher behaviours were associated with the indicators of K-ST by making use of the relevant literature (An et al., 2004; Özaltun, 2014) and an evaluation framework was created to outline the interview processes and to analyse teacher performances.

In the formation of the framework, the researchers acted together and created certain categories and stages for teacher behaviours, and then the views of a faculty member who had different studies in the field were consulted for the relevant framework. Accordingly, the components of the relevant framework are in Figure 1. At the stage of handle of misconception, which is the first stage of the process, the teacher examines the student's solution. At this stage, the teacher tries to understand student's thinking by using appropriate questions. In the second stage of the process, the teacher tries to help the student recognize the misconception and determine the source of his mistake. Then, the teacher tries to ensure that the student replaces the wrong knowledge with the correct one/ones by using appropriate methods and teaching strategies. During the evaluation stage, the teacher implements techniques to determine whether the student's misconception persists. Although this process has a hierarchical structure in basic stages, in fact all processes are intertwined. For example, in the stage of elimination of misconception, the teacher may ask the student to explain his/her solution (sub-solutions) with own sentences, in accordance with the indicators belong to the step of handle of misconception. Similarly, a teacher who tries to give the correct information to the student in the process of restructuring student knowledge, may apply the processes related to the evaluation stage and try to control whether the student understands the outputs of the pedagogical methods used in the process or not.

Figure 1

Components (Stages) of the Evaluation Framework



Therefore, although it is not possible to separate this intertwined process with definite boundaries, the relevant framework has been formed in general terms to carry out the data analysis process in the study and relevant indicators have been created for each step. Based on this structure, the indicators created for each stage of the process are as follows (Table 1).

Table 1

Assessment Framework and Relevant Indicators

	Inadequate (IA)	**Partially Adequate (PA)**	**Adequate (A)**
Handle of Misconception			
Review Student Solution	*Prevents the student from explaining their solution process in their own words, as the solution is read out to them without allowing them to express it in their own terms.*	*Provides the opportunity for the student to explain their solution but makes inducements during the process.*	*Allows the student to explain their solution process in their own words.*
Reveal Student Thinking	*Asks more information-based questions that require a single answer, where the answer is evident from the teacher's speed and tone of voice or the nature of the question itself.*	*The questions asked, provide partially evidence for the student's thinking.*	*Asks questions that allow the student to provide evidence for their thinking and elaborate on the given information.*
Elimination of Misconception			
Helping to Notice Misconception	*Does not provide any feedback to the student regarding the validity of their solution. Gives incorrect/incomplete feedback regarding*	*Provides clues for the student to recognize why their solution is incorrect/invalid and gives some guidance, but these clues and guidance mostly*	*Provides the student with clues to recognize why their solution is incorrect/invalid and enables them to identify the source of*

	whether the solution is valid or not. Does not facilitate any process for the student to recognize their misconception.	*remain at the procedural or rote knowledge level rather than addressing conceptual understanding.*	*their misconception/error.*
Asking Questions*			
Restructuring Student Knowledge	i) The teacher prompts the student to think and develop their ideas based on evidence. ii) The teacher utilizes different representations and models to facilitate conceptual understanding. iii) The teacher breaks down/simplifies the concept into sub-concepts in order to address student misconceptions. iv) The teacher aims to facilitate the construction of accurate knowledge by creating a discussion environment through a question/answer method.		
	The teacher does not use any of the above strategies and methods in the process of knowledge construction.	*The teacher partially utilizes the above strategies and methods in the process of knowledge construction.*	*The teacher appropriately utilizes the above strategies and methods in the process of knowledge construction.*
Evaluation			
Ensuring Control	*The teacher does not conduct any control process regarding the continuation of the misconception.*	*The teacher partially addresses the control of the misconception by asking questions like "Do you understand?"*	*The teacher ensures the continuation of the misconception by checking through similar situations.*

**Asking questions- Reveal student thinking stages have the same indicators.*

The framework and related indicators in Table 1 were used to evaluate teacher performances in interview processes. The encoder reliability for the coding processes of teacher behaviours was calculated in data analysis processes and in case of conflict, the relevant records were listened again, and the researcher codings were finalized.

Findings

In this study, the performance of three different teachers in diagnosing and eliminating student misconceptions can be summarized based on the processes included in the evaluation framework as follows. The data in the Table 2 are generated based on the high-frequency performances of the teachers.

Upon examining the data presented in Table 2, it can be observed that the teachers generally exhibited similar and inadequate performances throughout the entire process. However, it is evident that T3 coded teacher demonstrated significantly better performance compared to the other teachers. However, when considering different stages of the process, it can be said that the teachers exhibited better performances in ensuring control compared to other stages.

Table 2

Teacher Performances for Diagnosing and Eliminating Misconceptions

	Teacher Performances		
	T1	**T2**	**T3**
Handle of Misconception			
➢ *Review Student Solution*	IA	IA	IA
➢ *Reveal Student Thinking*	IA	IA	PA
Elimination of Misconception			
➢ *Helping to Notice Misconception*	IA	IA	PA
➢ *Asking Questions*	IA	IA	PA
➢ *Restructuring Student Knowledge*	IA	IA	PA
Evaluation			
➢ *Ensuring Control*	IA	PA	PA

In order to present the findings obtained from this research in more detail, in this section, the performances of the participating teachers will be exemplified with selected direct quotations. The exemplification aims to highlight crucial aspects of their performance. The quoted questions in this section are provided in appendix.

Findings Obtained from Teacher Coded T1

The teacher with the code T1, showed inadequate performance in the majority of the interview processes. This teacher read and solved all the questions himself throughout the process and did not pay much attention to the students' solutions. This teacher, who used totally traditional methods, received feedback from his students solely to validate himself, and did not include questions that attempted to disclose how they thought. It was observed that the teacher in question attempted to explain the correct answers to the students (without revealing the reasons) rather than correct the students' misconceptions. The interview process conducted by the T1 coded teacher regarding the 5th, 9th and 10th questions in the diagnostic test is given below respectively.

Example Case 1: T1 for the 5th question. Some excerpts from the interview between the T1 and the student who stated that the given problem solution is correct, are provided below. This student accepted the solution with 96-69/(7-a) written as 27/ (7-a).

T1: Any idea how to solve this question? (The teacher writes the question again as)

Student: …. (She cannot give an answer).

T1: Because the unknown is the denominator of this expression, we can answer this question as follows. What number is subtracted from 96 to yield 9?

Student: 87

T1: So, we can say that this expression is equal to 87, right? Can we now imagine that the number 1 is under 87 here and perform the cross multiplication?

Student: Yes.

> T1: (The teacher equated the given expression to 87 and discovered the value of 'a' as 180/29 by conducting the whole process herself). Equalizing the denominator was a bit of a challenge here because it was in the unknown denominator. So, whatever I subtract from 96 equals 9, we found the answer. Ok?
>
> Student: Yes.

The teacher does not implement any method to reveal the reasons of the student's wrong answer and addressing the student's mistake, instead, he solves the supplied problem himself. However, this student thinks that the equation of 96-69/(7-a) =9 can be written as 27/(7-a) =9. It is likely that the learner did not grasp the division operation in the given expression, or that the operation priority rule was violated. The teacher was content to show the student the correct solution for the question instead of focusing on the wrong solution.

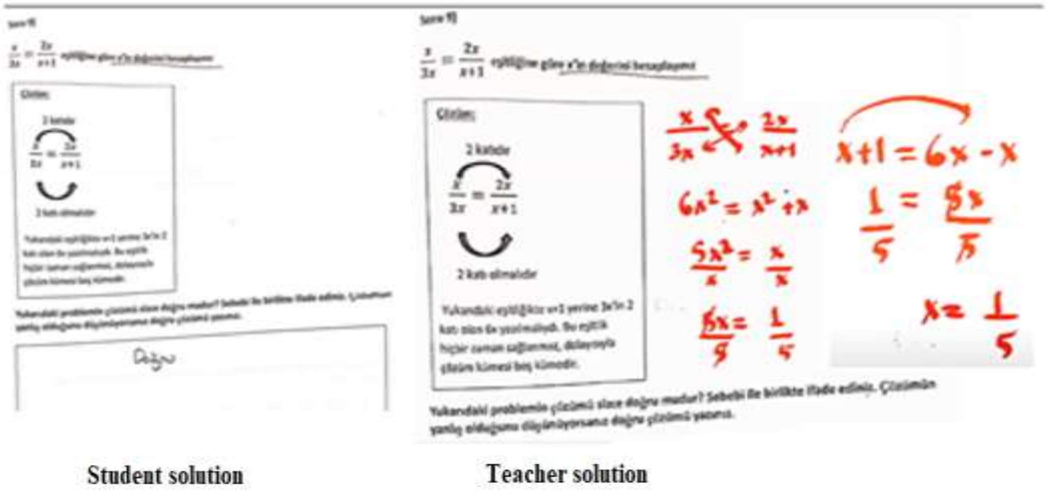
During the phase of 'review student solution' for the 'handle of misconception process', this process was coded as inadequate because the teacher read the student's solution himself and did not give the student the opportunity to explain her solution. Since the teacher did not take any action to elicit student thinking, the process of 'reveal student thinking' was also coded as inadequate. For the process of 'elimination of misconception', the process was coded as inadequate because the teacher did not take any action to make the student realize that the solution was wrong during the phase of 'helping to notice misconception'. Regarding the 'asking questions' phase, it was accepted that the teacher performed inadequately because he asked such questions that the answer was generally given by him and only wanted to be confirmed. In the process of 'restructuring student knowledge', the teacher's performance was coded as inadequate because he did not use any strategies or methods in this process. Finally, since the teacher did not carry out any process to determine whether the student's misconception continued during the 'ensuring control' phase of the 'evaluation' process, his performance was coded as inadequate.

Example Case 2: T1 for the 9th question. In question 9, the student stated that the solution given was correct. Some parts of the interview process between this student and the teacher coded T1 are given in Figure 2. This student believes that the equation has no solution. The reason for the student's thinking may be due to a misconception designed by the researchers or it may be because the student has a different perspective. If the student has a misconception, she may claim that since the numerator is doubled in the given expression, the denominator should also be doubled and therefore the expression should be written as '6x' instead of 'x+1'. Although this claim is true, the fact that the student thinks that 'x+1' and '6x' are different from each other constitutes the reason for the student's thinking that the given equation does not have a solution. Therefore, the process and questions that the teacher will ask in order to reveal the student's thinking are of great importance.

In this process, the teacher solved the equation himself and reached the solution x=1/5. Here, the operations performed by the teacher without taking into account the student's thoughts are noteworthy. The teacher ignores the student's mistake and accepts that 'x+1' is equal to '6x' and does not question the meaning of the result x=1/5. It is also noteworthy that at the end of the process, the teacher did not carry out any process to determine whether the student understood the solution of the problem or whether she had any questions.

Figure 2

Student and teacher solutions for the 9th question



T1: What is the first solution that comes to your mind to solve this question?

Student: …to equalise the denominators.

T1: Yes, it's possible, it can be done, and what about doing the cross multiplication here?

Student: Yes, maybe.

After that, the teacher reached the solution by performing the necessary mathematical operations for the solution of the problem and found the x value as 1/5. Then, the teacher handled the question in a different way.

T1: When you look at the solution to this question, what happened to x? Did it become 2x, doubling? So, what happens if I double 3x as well?

Student: 6x

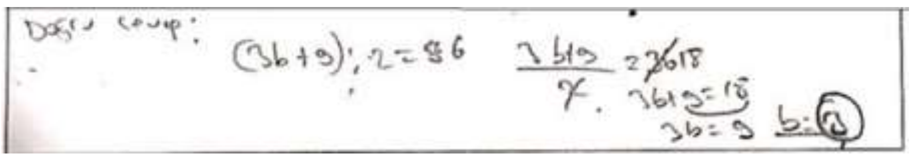
T1: It should be 6x. So, we can write x+1=6x.

Therefore, in this question, since the teacher coded T1 did not carry out any process to reveal the student thinking, to make her notice and eliminate her misconception, and to determine whether the student's misconception continued or not, all stages for the processes of 'handle of misconception', 'elimination of misconception' and 'evaluation' were coded as inadequate.

Example Case 3: T1 for the 10th question

Figure 3

Student solution for the 10th question



Below is the interview process conducted by the T1 coded teacher with the student who made the mistake in Figure 3 for the 10th question.

T1: Your answer is wrong, now let's look at your mistake. (3b + 9) /2 =36, now what did you do here? You simplified 36 and 2, but what were we doing in the rational equations?

Student: Don't know.

T1: (Teacher solves the question by equating the denominator). In rational equations, we solve the equation by equating the denominator, not by simplification.

In this question, it is seen that the teacher only uses rote knowledge to correct the student's mistake and continues the process without questioning the reason or source of the student's wrong answer. For this reason, the teacher's performances in all processes were coded as inadequate in this question as in the other questions.

Findings Obtained from Teacher Coded T2

When the interview processes conducted by the T2 coded teacher were examined, it was seen that this teacher read the student solutions throughout the process and conducted the process in a teacher-centered manner. This teacher generally used a rule-based approach when addressing students' misconceptions and asked them to remember the rules they had memorized while providing correct information. It was observed that the teacher did not focus much on why the student's solution was wrong. In addition, she did not check whether the students' misconceptions persisted, but only asked questions such as ‘Do you understand?’ and ‘Okay?’. Throughout the process, it was observed that the teacher displayed partially adequate and mostly inadequate performance. The interview process conducted by the teacher coded T2 regarding the 4th and 14th questions in the diagnostic test is given below, respectively.

Example case 1: T2 for the 4th question. Some parts of the interview process between the T2 and the student who said that the given solution of the problem is correct, are given below. This student considered the solution in which ‘13-7x’ is written as ‘6x’ to be correct.

T2: You answered this question wrong; would you like to have a look again?

Student: (he doesn’t say anything)

T2: What were we doing in such questions, were we grouping the known (variables) to one side and the unknown (variables) to the other in such questions?

Student: Yes.

T2: Well, you said that this solution is right, how did we get 6x?

Student: I don’t know.

T2: So, let's try it again. You tell me what you want, and I'll write it. Known variables on one side and unknowns on the other, correct?

Following that, the teacher solved the problem with the help of the student and arrived at the conclusion $x=1$.

T2: So, the answer given in the question is correct ($x=1$) but is the solution correct?

Student: No.

T2: In other words, the result is correct by chance, but this procedure was wrong from the beginning. You know what she did here, she subtracted 7x from 13 and said 6x, then I can subtract 3x from 9 as well. It's a bit of ridiculous solution. The answer is correct, but the solution is wrong, understand?

Student: Yes.

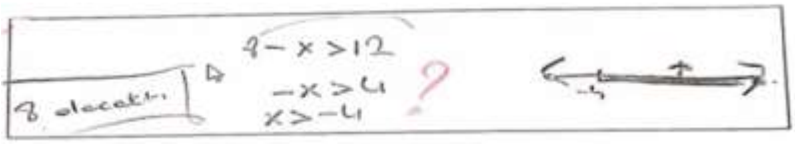
Here, it is seen that the teacher partially involved the student in the process but did not explain the reasons for the incorrect mathematical knowledge to the student. Therefore, it can be said that the teacher did not give the student enough opportunity to question the current situation.

In the process of 'handle of misconception', the stages of ‘review student solution’ and ‘reveal student thinking’ were coded as inadequate because the teacher directly told the student that she answered this question incorrectly and did not give the student the opportunity to explain her thinking. Since there was no teacher intervention in the ‘helping to notice misconception’ and ‘asking questions’ stages of ‘elimination of misconception’ stage, these stages were also coded as inadequate. However, since the teacher partially involved the student in the process at the ‘restructuring student knowledge’ and used her ideas in the solution process, this stage was coded as partially adequate. During the ‘evaluation phase’, although the teacher did not carry out a process to determine whether the student's misconceptions persisted or not, his performance at this stage was coded as partially adequate since he used questions such as ‘did you understand’ during the interview process.

Example Case 2: T2 for the 14th question

Figure 4

Student solution for the 14th question



The relevant part of the interview process conducted by the T2 coded teacher, regarding the student mistake (given above) is given below.

T2: Can our unknown be negative? (The teacher says for -x)

Student: It can’t.

T2: What do we need to divide each side in order to make it (x) positive?

Student: -1.

T2: So, this side is x, and this one is -4? Ok? And do you remember inequalities, how we multiply or divide an inequality by a negative number? There was a rule, remember? Inequality was shifting. Do you remember this?

Student: I couldn't remember.

T2: If we divide or multiply by a negative number, the direction of our inequality changes.

In the interview process above, it is seen that the teacher tried to correct the student's misconception with rule-based knowledge. However, it can be said that the mathematical expressions used by the teacher in the process have the potential to lead the student to different misconceptions. Expressions such as 'Can our unknown be negative' and ‘What do we need to divide each side in order to make it positive' may cause misconceptions in students because the minus sign in front of the variable ‘x’ does not mean that it is negative.

Although the above process was inadequate for all stages of the process, the stage of ‘reconstructing student knowledge’ was coded as partially adequate because the teacher included the student in the process, even if partially, and used her answers in the process.

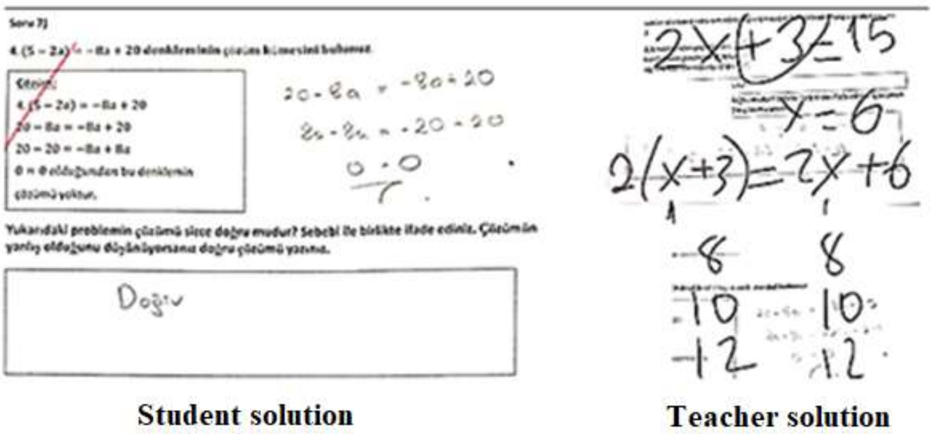
Findings Obtained from Teacher Coded T3

The teacher coded T3 focused on the student's thinking throughout the interview processes and tried to progress by relating new knowledge with the student's existing knowledge in most cases. Although this teacher often allowed the student to express his own thoughts, question and explore the reasons for the student's misconception, it was observed that in some questions she made explanations such as 'this is not true', 'it cannot be written this way'. It was observed that the teacher who tried to explain the reasons for mathematical rules showed adequate and partially adequate performances in most of the interview processes.

Example Case 1: T3 for the 7th question. Some parts of the interview process between the T3 and the student who said that the given solution of the problem is correct, are given below. Based on the information given in Figure 5, it can be observed that the student, using the distributive property, arrives at the equation 0=0 and concludes that the equation has no solution.

Figure 5

Student solution for the 7th question



T3: What if I told you that I found a=1, how would you check if this is the right solution?

Student: Do we assign value of 1 to ‘a’, and proceed accordingly?

T3: Yes, set a=1 and see what happens.

Student: Ok, 12=12.

T3: So, is the value of 1 for ‘a’ which I found, is wrong?

Student: Yes.

T3: Why, 12=12 is not a wrong situation?

Student: Oh, yes. (The student thinks a little here but cannot be sure). Is it correct?

After this conversation, the teacher changed the course of the interview and tried to clarify the underlying concepts to the student.

T3: Have you ever heard of something called ‘identity’ as a mathematical subject?

Student: Yes.

T3: Well, can you write an identity that you know for me?

Student: I saw the topic, but I can't remember right now.

T3: Ok, have you ever heard of something called ‘equation’ as a mathematical subject?

Student: Yes.

T3: Well, can you write an identity that you know for me?

Student: Ok, 2x+3=15.

T3: Well, if we look at the equation you wrote, 2x=12 and x=6 so the solution of the equation you wrote, is 6. So let me write you an expression like this, 2(x+3) =2x+6. Is this an equation or not, let's talk about it with you.

Student: That's an identity.

T3: Oh, why?

Student: Because they are equal.

T3: Huh okay because the left side and the right side are equal. There is a 'identity' when the left and right sides are equal. Now you can substitute any value for 'x' in the expression we call identity, because the left side and the right side are equal. Now if we say x=1, we get 8=8. If we say x=2, we get 10=10. If we say x=3, we get 12=12. In other words, equality has already been attained and is known as 'identity' regardless of what we write in place of x (we don't say there is no solution). So, every number is a solution for identities. If the right and left sides are equal, we say that this equation is provided for each number.

Here, it is seen that the teacher tried to explain the concepts of 'equation' and 'identity' to the students. It was accepted that the teacher's explanations here were partially adequate in terms of reconstructing knowledge. The reason for this situation is that although the teacher tries to present the information with reasons, she cannot give up the approach of giving memorized knowledge and does not allow the student to construct the knowledge herself.

At the stage of 'review student solution' in the process of 'handle of misconception', it was observed that the teacher did not give the student the opportunity to explain her solution and focused directly on the process. Therefore, the related stage was coded as inadequate. However, since the teacher predicted how the student thought and gave the wrong answer to this question, the stage of 'reveal student thinking' was coded as adequate since the teacher asked questions which enabled the student to elaborate on the knowledge she had and provided evidence for her thinking. The 'helping to notice misconception' stage of the 'elimination of misconception' process was coded as partially adequate because even though the teacher provides guidance on why the student's thinking is wrong, these remain at the procedural level rather than the conceptual level due to the rote definitions used by the teacher regarding the concept of identity. The stage of 'asking questions' was coded as partially adequate because the questions asked by the teacher in this phase of the 'elimination of misconception' process partially provide sufficient evidence about the student's thinking. The stage of 'restructuring student knowledge' was coded as partially adequate because the teacher was able to partially develop it based on student ideas. Finally, since the teacher did not carry out a process to determine whether the student's misconception persisted or not, the 'ensuring control' phase of the 'evaluation' process was coded as inadequate.

Discussion and Conclusion

This study aimes to observe how mathematics teachers diagnose and attempt to eliminate the misconceptions held by their students and examined the pedagogical methods and strategies used by the teachers throughout the process. Based on the theoretical foundation of the framework, the study aimed to reveal the extent to which teachers can benefit from student thinking in identifying and eliminating students' misconceptions.

As a general comment, it was observed that all teachers in this study focused on the result rather than the cause in student solutions and tried to explain only the correct solution to the students. Unfortunately, there was no teacher who questioned the reasons behind the students' answers sufficiently during the study. Although all of the teachers participating in the study had at least ten years of professional experience, it was observed that all of them preferred to explain or retell the subject during the interview process. In general, these teachers tried to ensure that students were able to answer the questions correctly without focusing too much on students' mistakes or misconceptions. In this process, it was observed that teachers adhered to traditional methods and did not generally utilize different pedagogical methods. Related to this situation, studies conducted in the literature show that teachers believe that they will have more permanent learning and prevent time loss by lecturing themselves (Akpınar & Ergin, 2005; Erdem & Ersoy, 2009; Keser, 2003, as cited in Gür & Kobak Demir, 2019). Önen et al. (2008) show that the reason why teachers do not use teaching methods and techniques that allow students to construct their knowledge is that they do not have sufficient knowledge on this subject.

In traditional teaching practices, assessment is done around the axis of true-false, pass-fail and there is no opportunity to correct students' mistakes. In teaching practices based on the constructivist approach, on the other hand, assessment is carried out to reveal students' inadequacies and the reasons for their misunderstandings. The aim is to make a diagnosis. Students are given feedback about their deficiencies and misunderstandings. Thus, students have the opportunity to complete their deficiencies and correct their mistakes within the system (Baki, 2008). In this context, Gelbal and Kelecioğlu (2007) concluded that teachers saw themselves as more competent and preferred traditional measurement-evaluation methods. The results of the study show that teachers have problems using measurement methods due to the crowding of the classroom, lack of time and difficulty in preparation. Another reason for the failure to implement alternative assessment and evaluation techniques is the resistance of teachers to change stereotyped traditional assessment and evaluation approaches (Lambdin, 1993). Teachers' inability to strike a balance between alternative assessment and evaluation approaches and the expectations of traditional education and the incompatibility of alternative assessment approaches and traditional methods cause a dilemma (Suurtamm, 2004, as cited in Gür & Kobak Demir, 2019).

When the performances of different teachers are compared, it can be said that the teacher with the highest professional experience (T3) had the best performance in identifying the causes of students' misconceptions and eliminating them, while the teacher with the lowest experience (T1) had the lowest performance. It was observed that the teacher coded T3 allowed the students to express their answers in their own words during the process of handle of misconceptions and asked questions which provide evidence to reveal the students' thinking. During this process, it was discovered that the T1 coded teacher generally read the students’ solutions without letting them to re-express their thoughts, and even began the process by ignoring the student's incorrect solution, and he asked more knowledge-based and single-answer questions. The T2 coded teacher, on the other hand, followed a rule-based method by taking a more behavioural approach than the T3 coded teacher, emphasising on memorised knowledge, even if he carried out processes to allow students to express themselves and disclose their thinking. It can be said that the T2 coded teacher has a performance between the other two teachers' performances. In the last step of the interview processes, it was observed that T3 coded teacher asked his students ‘Did you understand?’ ‘Okay?’ questions to control student learnings. Furthermore, it was discovered that this teacher (T3) controlled whether or not the misconceptions continues by having the student develop a solution again using similar examples in some cases. While the T2 coded teacher is trying to provide the control of student learning by asking questions such as 'Do you understand?', 'Ok?', it was observed that the teacher with the lowest professional experience (T1) did not carry out any control process to determine whether the mistakes/misconceptions continued or not in most cases, and after solving the subject problem, he went directly to answering other questions. So, it can be stated that as teachers' professional seniority grows, they apply their knowledge of student thinking more effectively in the context of pedagogical content knowledge for this study. Although there are not many studies examining the processes of eliminating students' misconceptions in the literature, there are studies examining teachers' pedagogical content knowledge in the context of student thinking knowledge. According to these studies' findings (Carpenter et al. 1988; Feiman-Nemser & Parker, 1990; Shulman, 1987; as cited in Cochran et al., 1993), novice teachers exhibit inadequate pedagogical content knowledge, which is consistent with the findings of the present study. A beginner teacher may also lack a logical framework for delivering information, according to Cochran et al., (1993), who also noted that novice teachers frequently rely on unmodified subject knowledge that is taken directly from the text or curriculum materials. According to Brown and Borko (1992), beginner teachers are not always ready to take on the tasks that are required of them as developmentally competent mathematics teachers. According to Grouws and Schultz (1996), teachers should be aware of their students' thinking styles so that they may address their students' present mathematical knowledge and misconceptions in the classroom.

When the findings obtained from this study are considered in terms of teacher performances, it is seen that there are different studies with similar results in the literature. Mulungye (2016) found in his research with fifteen mathematics teachers that, while teachers are aware of student mistakes and misconceptions, they are unable to apply their knowledge to eliminate these misconceptions. It was reported that the teachers in the study used teacher-centered instruction, so the weak students were identified and supported during classroom discussions. The analyses based on the statistical methods used in the study, revealed that the student errors did not occur by chance, but rather as a result of the teacher's methods. Therefore, teacher practices are very important in teaching algebra. Kimii and Declark (1985) suggests that teachers' focus should be on students' thinking rather than correct answers. We can say that this proposal, which was made years ago, still maintains its importance and up-to-dateness today. Because the teachers in this study generally focused on taking the students to the right solution rather than correcting their misconceptions.

In this part of the research, the discussion on teacher performances will continue to be carried out through the pedagogical methods used by teachers regarding the subject. As a result of this research, it was seen that the methods used by teachers in the process of eliminating misconceptions were traditional and limited. When the results of the studies in the literature are examined, it can be said that in parallel with the results obtained from this study, teachers generally prefer similar methods in the process of eliminating misconceptions, and they do not generally interfere with the student to recognise his mistake. In these processes, it is seen that the teachers directly tell the student their mistake or explain/tell the concepts/subjects again. Different studies in the literature (Bingölbali, 2010; Bursalı & Gökkurt-Özdemir, 2019; Chick & Baker, 2005;

Scleppenbach et al., 2007; Şahin et al., 2016; Şahin, 2011) state that teachers prefer to tell the students the correct answer, give the rule directly, or ignore the mistake. In this context, it's revealed that the instructional explanations of teachers and teacher candidates in eliminating misconceptions are inadequate. Although instructional explanations in which the knowledge of student thinking is used extensively, are one of the most important dimensions of pedagogical content knowledge and studies in the literature (Gökkurt-Özdemir & Soylu, 2017; Kılcan, 2006; Kinach, 2002a, 2002b; Şahin et al., 2016) show that the instructional explanations used by teachers and pre-service teachers are generally rote-based and rule-process-oriented rather than understanding. These situations, which were revealed in the literature, were also frequently observed on the data obtained from this study. Related to this, Borko and Putnam (1996) and Thompson (1992) stated that the explanations of a teacher without adequate conceptual knowledge, would not be at the conceptual level and that the explanations of a teacher who sees mathematics as a set of rules would be rule-based. According to Ersoy and Erbaş (2005), when it comes to teaching algebra, teachers frequently overlook the conceptual side of the idea of variable and emphasize its practical aspect. Therefore, as expressed in these research findings, the teachers in this study mostly focused on the correct response rather than student misconceptions, and on rule knowledge rather than concept knowledge. So, it is seen that the situations expressed in the literature are still valid today.

This research deals with misconceptions in algebra and shows that students still have a wide range of misconceptions in algebra. While this research focuses on the elimination of these misconceptions, there is no doubt that taking instructional measures to prevent the emergence of these misconceptions will contribute to the field. The teaching methods to be used in overcoming the difficulties experienced in the field of algebra learning are of great importance. Kaya (2015) states that different teaching methods used in the lessons provide meaningful and lifelong development of students' algebraic thinking skills. The transition from arithmetic to algebra can be facilitated when students have physical experiences in order to comprehend abstract algebraic knowledge (Tunç et al., 2012). Baykul (2014) suggests using models to concretize abstract concepts in algebra teaching and that these models help in the comprehension of algebraic expressions as well as the ability to perform operations with algebraic expressions and the concept of identity. Similarly, Bukova Güzel (2016) states that it is important to make use of real-life visuals or visualization, which we can use as concrete models in the construction of algebraic expressions, to conceptualize the subject and to increase student motivation in order to realize qualified understandings. Therefore, it can be said that the studies in the literature suggest using visualization to reduce student difficulties in algebra teaching.

All mathematical concepts are related to one another, and hence the teaching of any concept in teaching processes is dependent on other (premise) notions that are necessary for this concept. As a result, learning a concept wrongly, creates a barrier for all future concepts to be taught. Considering that one of the areas where misconceptions are observed most in mathematics education is algebra, it will not be possible to completely prevent these misconceptions, so ways to eliminate them should be found. In this context, it is important to present the results of this research and making suggestions for the future in the light of these results. According to Tafara (2015), the literature indicates that student misconceptions are difficult to resolve. Even if the student's misconception is eliminated, it is common for the same misconception to resurface after a period. As a result, active participation by students in the process of overcoming misunderstandings is a key need in removing these misconceptions. However, the findings of this study demonstrate that teachers still utilise traditional ways to eliminate student misconceptions and focus on the outcome rather than the process. So, it appears to be a serious problem in mathematics instruction today.

Limitations and Recommendations

In order for teachers to be aware of student misconceptions and to acquire more effective methods to eliminate them, courses or activities to create awareness on the subject can be held in teacher training institutions. Teachers might attend in-service programmes to learn about misconceptions and how to overcome them. Collaborations on this topic might be created between schools and educational institutions. Academicians or professionals can create materials for teachers to use in identifying and correcting student misconceptions in this setting. It is thought that teachers who frequently encounter student difficulties and misconceptions in the classroom and having difficulties in teaching processes, will show great interest in these materials and resources.

This research was conducted with three mathematics teachers, so this can be considered as a limitation for the study. In addition, the time allocated to the interview processes for the teachers involved in this research is a limitation for them. Although there is no time limit for the relevant interviews, the number of questions can be considered as a limitation that prevents teachers from acting more flexible in the interview process. In different studies to be conducted on the subject, more participants can be studied by focusing more on the quantitative dimension of the subject.

Acknowledgement

We would like to thank all the teachers and students who participated in this study.

Statement of Responsibility

The authors of this study are responsible from all parts of the study such as conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing-original draft, writing-review&editing together.

Conflicts of Interest

There is no conflict of interest in the research.

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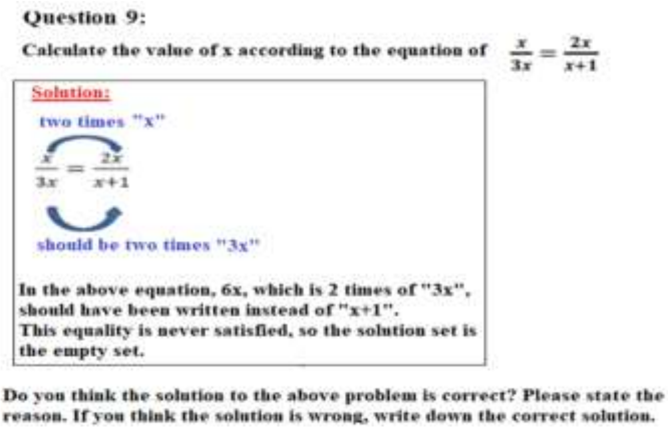
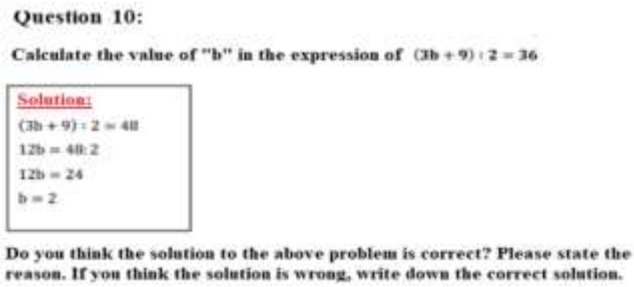
Appendix

The Questions in DT

Question	Type of misconception
1	❖ Misconceptions about algebraic expression and variable concept • Failure to distinguish the purposes of letter symbols in arithmetic and algebra ❖ Misconceptions about the concept of equation • İnversion error
2	❖ Misconceptions about algebraic expression and variable concept • Inability to associate coefficient with terms
3	❖ Misconceptions about the concept of equality • Thinking that there should always be the same expressions on different sides of equality • Considering terms close to the equality sign
4	❖ Misconceptions about algebraic expression and variable concept • Always start on the left
	Question 4: **Find the value of x in the equation of** $13 - 7x = 9 - 3x$ **Solution:** $13 - 7x = 9 - 3x$ $6x = 9 - 3x$ $6x + 3x = 9$ $9x = 9$ $x = 1$ **Do you think the solution to the above problem is correct? Please state the reason. If you think the solution is wrong, write down the correct solution.**
5	❖ Misconceptions about algebraic expression and variable concept • Always start on the left
	Question 5: **Find the value of "a" in the equation of** $96 - 69/(7 - a) = 9$ **Solution:** $96 - 69/(7 - a) = 9$ $27/(7 - a) = 9$ **Since when we divide 27 by 3 it will equal 9** $7 - a = 3$ $a = 4$ **Do you think the solution to the above problem is correct? Please state the reason. If you think the solution is wrong, write down the correct solution.**
6	❖ Misconceptions about linear relationships and linear equations • Misconceptions related to interpreting the graph of linear equations
7	❖ Misconceptions about the concept of identity • Thinking that a given algebraic expression is either an identity or an equation
	Question 7: **Find the solution of the equation** $4.(5 - 2a) = -8a + 20$ **Solution:** $4.(5 - 2a) = -8a + 20$ $20 - 8a = -8a + 20$ $20 - 20 = -8a + 8a$ $0 = 0$ **so, this equation has no solution.** **Do you think the solution to the above problem is correct? Please state the reason. If you think the solution is wrong, write down the correct solution.**

8	❖ Misconceptions about the concept of pattern • Misconceptions about the concept of nth digit
9	❖ Misconceptions about the concept of equality • Thinking that there should always be the same expressions on different sides of equality
	 **Question 9:** Calculate the value of x according to the equation of $\frac{x}{3x}=\frac{2x}{x+1}$ Solution: two times "x" $\frac{x}{3x}=\frac{2x}{x+1}$ should be two times "3x" In the above equation, 6x, which is 2 times of "3x", should have been written instead of "x+1". This equality is never satisfied, so the solution set is the empty set. Do you think the solution to the above problem is correct? Please state the reason. If you think the solution is wrong, write down the correct solution.
10	❖ Misconceptions about the concept of equation • Limited application of the reverse transaction ❖ Misconceptions about algebraic expression and variable concept • Using the constant term instead of the coefficient of the variable
	 **Question 10:** Calculate the value of "b" in the expression of $(3b+9):2=36$ Solution: $(3b+9):2=48$ $12b=48:2$ $12b=24$ $b=2$ Do you think the solution to the above problem is correct? Please state the reason. If you think the solution is wrong, write down the correct solution.
11	❖ Misconceptions about the concept of inequality • Not being able to determine the solution set while solving the inequality correctly
12	❖ Misconceptions about the concept of equation • Errors due to lack of understanding of the transfer method
13	❖ Misconceptions about the concept of identity • Misconceptions about the identity of a perfect square
14	❖ Misconceptions about the concept of equation • Inversion error ❖ Misconceptions about the concept of inequality • Not changing the direction of the inequality when the inequality is multiplied by a negative number

Question 14:

$4-\frac{x}{2}>6$

Find the solution set of the inequality and show it on the number line.

Solution:

$4-\frac{x}{2}>6$

$4-x>12$

$-x>12-4$

$-x>8$ If we edit this expression

$-8<x$ The solution set is shown on the number line as follows.

Do you think the solution to the above problem is correct? Please state the reason. If you think the solution is wrong, write down the correct solution.