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Hyperspectral anomaly detection with an improved approach: integration of go decomposition algorithm and laplacian matrix modifier

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Abstract

In this study, a hyperspectral anomaly detection method based on Laplacian matrix (HADLAP) is proposed. This paper addresses the problem of determining covariance matrix inversion in high-dimensional data and proposes a new approach for identifying anomalies in hyperspectral images (HSIs). The study's goals are to find anomalous locations in HSIs and to deal with the problem of calculating the inversion of the covariance matrix of high dimensional data. The method is centered on two main concepts. One of them is decomposition process. The other one is detection process. First, HSI data is decomposed as a low rank and sparse matrices. Second, the sparse component of the data is used to build Mahalanobis Distance (MD). In this study, go decomposition (GoDec) algorithm is employed to decompose the data. Then, the distance is calculated by obtained matrix with aim of detection of anomalous pixels in the HSIs. The method differs from previous studies that covariance matrix in the distance is computed with Laplacian matrix and MD. Experiments conducted on three hyperspectral datasets present the superiority and effectiveness of the proposed framework in terms of detection performance with respect to state-of-the-art methods.

Keywords: Anomaly detection; low rank matrix; laplacian matrix

1. Introduction

Hyperspectral imaging captures the reflected light from hundreds of narrow bands objects throughout the earth's surface. The information of high spectral resolution of the hyperspectral image with these bands allows for the differentiation of various ground objects. The imaging technology is applied in the field of image analysis, remote sensing, classification, and target detection [1]. Hyperspectral images (HSIs) hold abundant spectral information

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about characteristics of objects which enables to have an idea about the image scene [2]. Hyperspectral anomaly detection is an unsupervised approach that evaluates targets of interest against the background without any prior information about the target in advance [3]. By this methodology, abnormalities in HSIs are identified for usage with different purposes in applications such as camouflage detection, identification of minerals, fine agriculture, change detection etc. Hyperspectral anomaly detection has received substantial research in the literature. The most common methods for hyperspectral anomaly detection are statistical based models. Reed-Xiaoli (RX) detector is the most popular approach based on statistical model [4]. This approach relies on the idea of HSI the background follows a multivariate Gaussian distribution. Mahalanobis distance (MD) between the test pixels and the background is computed in order to identify the anomalous targets. Later, several expanded RX algorithms developed, including the subspace RX (SSRX) algorithm, which minimizes the effect of anomaly contamination on background estimation, and the local RX (LRX) algorithm, which models the local background using the inner and outer double windows approach [5, 6]. Support Vector Machines (SVM) and Random Forests (RF) techniques are also anomaly detection strategies based on advanced statistical model and get benefit from machine learning algorithms [7, 8].

Hyperspectral anomaly detection, known as unsupervised-based techniques, separates meaningful targets from the background without the need for prior information. Their aim is to distinguish outliers from background objects. Breaking down hyperspectral data into low rank and sparse components is a popular method that can be used to distinguish suspicious anomalies from background information. This process of decomposition promotes the identification of anomalous data by utilizing the data's inherent structure to differentiate context from potential anomalies.

A variety of low rank and sparse matrix decomposition-based techniques have been successfully used for hyperspectral anomaly detection. It is assumed that the anomalies are sparse, and background has low rank property. The advantage of this method is splitting HSIs as sparse and low rank matrices holding anomaly and background information respectively. Go Decomposition (GoDec) algorithm is one of the most used methods to decompose datasets can be either HSI or image or video [12]. The GoDec algorithm solves a convex optimization problem to separate datasets. As in the other datasets, it is used for background and foreground separations in HSIs. These separated matrices accurately detect the data's basic structure making it possible to explore anomalies. Low Rank and Sparse Matrix Decomposition (LRaSMD) model can be considered a novel strategy for finding hyperspectral anomalies as demonstrated in [13].

In addition to the previously discussed low rank and sparse matrix-based hyperspectral anomaly detection techniques, the robust subspace recovery algorithm via bi-sparsity pursuit is a further alternative decomposition strategy to GoDec. In [15], a robust subspace recovery algorithm is employed to isolate the data. MD is composed of the image's sparse components. Later, a different method based on LRaSMD was proposed in [17] that uses a Laplacian matrix to reconstruct MD. During the alteration of the distance function used to calculate the distance is employed as a weight function. In the context of this research, low rank and sparse matrices from the HSI dataset have been created using the GoDec algorithm. A map that detects anomalous behavior is created by executing MD on the sparse matrix. Notably, the Laplacian matrix is employed to invert the covariance matrix in MD, which increases the accuracy and effectiveness of anomalous behavior detection. Therefore, a new anomaly detection method for HSIs, HADLAP, is constructed and proposed.

This study is organized into various sections to present proposed method for hyperspectral anomaly detection. Section 2 provides information about the proposed method in detail. The datasets utilized for assessment are expressed in Section 3. Empirical findings are demonstrated in Section 4 where evaluations for proposed method with state-of-the-art methods is given. This section emphasizes how well the suggested strategy performs and how successful it is in finding anomalies in hyperspectral data. conclusion of the study is drawn in Section 5. Finally, Section 6 includes a discussion that highlights the importance of the sparse and low rank matrix decomposition based technique for hyperspectral anomaly identification.

2. Experimental design

In this section, the hyperspectral image is first separated aside GoDec method, which yields the low rank and sparse matrices. The sparse component is next substituted to MD. Cauchy function is used in place of explicitly inverting covariance matrix, allowing accurate estimates for anomalous pixels with comparable characteristics to be captured. The Laplacian matrix is then used to derive background statistics. This strategy has a number of beneficial aspects. First of all, only relevant bits of the data are inverted, which considerably lowers the cost of calculation. Additionally, it prevents the use of inaccurate statistics. As a result, the technique achieves high computing efficiency. By using the Laplacian matrix in MD after these stages, as opposed to computing the inverse of the complete dataset, the final anomaly map is created. This hybrid method, which combines matrix decomposition methods with modified MD computations, works well for locating hyperspectral anomalies.

Hyperspectral data in three dimensions is initially converted to a 2D matrix. Thus, the data $\mathbf{X} \in \mathrm{R}^{h\times w\times b}$ may be expressed by $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_b]$, $\mathbf{X} \in \mathrm{R}^{N\times b}$ in which N is the total number of pixels and b represents the quantity of bands in a spectrum. The data matrix X is written as in Eq. 1

$$X = L + E \tag{1}$$

where $L \in \mathrm{R}^{N\times b}$ is the background matrix and $E \in \mathrm{R}^{N\times b}$ is the sparse matrix that are two dimensional matrices with the size N×b. The optimization problem is then formulated as in Eq. 2.

$$min_{L,E} \parallel X - L - E \parallel_F^2, rank(L) \leq r, card(E \leq kN) \tag{2}$$

where r is the maximal rank of L and k is the cardinality of E. In order to solve the problem in Eq. 2 GoDec algorithm below is applied. After applying the algorithm, low rank L and sparse E matrices are extracted. By extracting E, MD distance in Eq. 3 is built.

Algorithm 1. GoDec Algorithm

Inputs:
$X \in \mathrm{R}^{h\times w\times b}$; data matrix, r; rank of L matrix, s; cardinality of E matrix, Iter; max iteration number
Outputs:
L; low rank matrix, E; sparse matrix
Initialize: $L_0 = X, S_0 = 0, i := 0, A_1 = \mathrm{randn}(L, r)$;
Repeat: Iter times
1) $i := i + 1$;
2) $Y_1 = (X - E_{i-1})A_1$, $A_2 = Y_1$; $Y_2 = (X - E_{i-1})^T A_2$;
3) If $\mathrm{rank}(A_2^T Y_1) < r$ then $r := \mathrm{rank}(A_2^T Y_1)$, go to step 2); end;
4) $L_i = Y_1(A_2^T Y_1)^{-1} Y_2^T$
5) $E_i = P_\Omega(X - L_{i-1})$ until $\|X - L - E\|_F^2 / \|X\|_F^2 < \varepsilon$

$$D(E) = (E - \mu)^T \Gamma^{-1} (E - \mu) \tag{3}$$

where μ is mean and Γ^{-1} is inverse of the covariance matrix of E. Each pixel in the E matrix is evaluated by considering itself and its neighboring pixels. In addition, Laplacian matrix and Cauchy function are implemented to calculate Γ^{-1} unlike other studies. Cauchy distance with spatial variant is applied. It returns the likelihood map of each pixel to be anomalous. Each pixel is evaluated by considering itself and its four-connected neighbors. Thus, the distance formula becames as in Eq. 4

$$D_{HADLAP}(E) = (E - \mu)^T L(E - \mu) \quad (4)$$

where the Laplacian matrix L is used for instead of Γ^{-1} by modifying MD. L matrix is computed by the following equations:

$$L = D - W \quad (5)$$

where D and W are degree and weight matrices respectively. W in Eq. 6 is Cauchy function by which pixels representing similar features are detected,

$$W_{xy} = \frac{1}{1+(\frac{\mu_x+\mu_y}{\beta})^2} \quad (6)$$

where μ_x and μ_y are means for band images x and y and β is a scaling parameter. Therefore, the anomaly map is extracted by computing the distance D. The proposed method's outcomes and performance results of detectors are given in the next section. The weights are normalized since it is more beneficial and preferred. Then, using Eq. 7,

$$L = D^{-\frac{1}{2}} L D^{-\frac{1}{2}} \quad (6)$$

the symmetric normalized Laplacian matrix is calculated. Finally, anomaly detection maps for each image are obtained.

3. Experimental data

The information about the data used for assessments is provided in this part. The following three hyperspectral datasets are employed: Airport Beach Urban (ABU) Airport_3, ABU Urban_1 and Salinas implemented 4 (Imp_4) datasets. Figure 1 shows the original band images. Their ground truth images are presented in Figure 2.

(a) (b) (c)

Fig. 1. Original band HSI. (a) Airport_3. (b) Urban_1. (c) Salinas Imp_4.

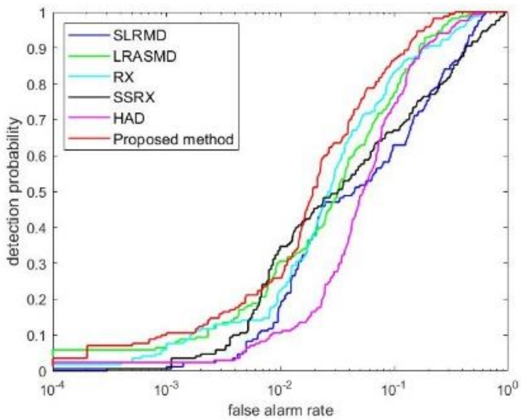
The first dataset in Figure 1 (a) has a resolution of 7.1m and 100 x 100 pixels with 205 spectral bands. The data is gathered by Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) in Los Angeles. The second dataset has a resolution of 17.2m and 100 x 100 pixels with 204 spectral bands is presented in Figure 1 (b). The data is collected by AVIRIS in Texas coast. Salinas Hyperspectral Dataset in Figure 1 (c) is lastly used hyperspectral dataset which is a popular data set used for several remote sensing applications. It obtains 224 bands which are also captured AVIRIS to provide the dataset for this investigation over an agricultural region close to Salinas Valley in California, USA. The original collection, which has a resolution of 512×217 pixels, includes images of 16 various kinds of vegetables, bare soils, and vineyard fields. The subset is sized as 126×150 with 204 spectral bands. Figure 2 presents ground truth images for all datasets. In Fig. 2a and 2b, planes are anomalies for ABU Airport_3 and ABU Urban_1. Anomalies are artificially implemented in Salinas. They are not real objects. The steps taken to create anomalies in the image are explained in depth in [18].

(a) (b) (c)

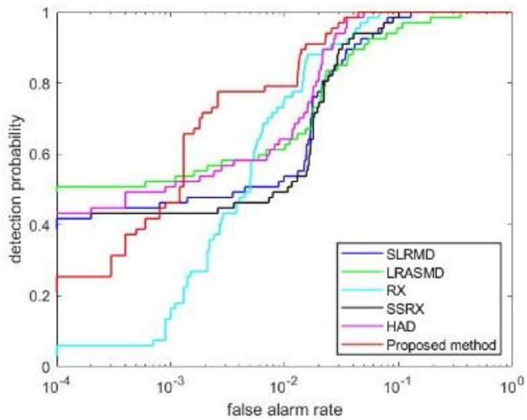
Fig. 2. Ground truth images. (a) Airport_3. (b) Urban_1. (c) Salinas Imp_4.

4. Experimental results

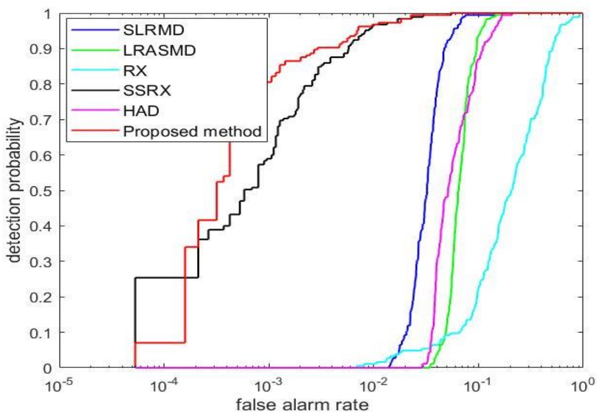
The experimental findings from an extensive research study were carried out to determine the effects of the proposed method compared with the state-of-the-art of the methods. Five anomaly detection techniques are compared with HADLAP: HADM, SLRMD, LRaSMD, RX, and SSRX. In order to examine the accuracy of various approaches, Receiver Operating Characteristic (ROC) curves and Area Under the ROC curve (AUC) measures are applied in this study. Figure 3 shows ROC curves of each datasets which includes six hyperspectral anomaly detection results false alarm rate (far) versus detection probability (pd).



(a)



(b)



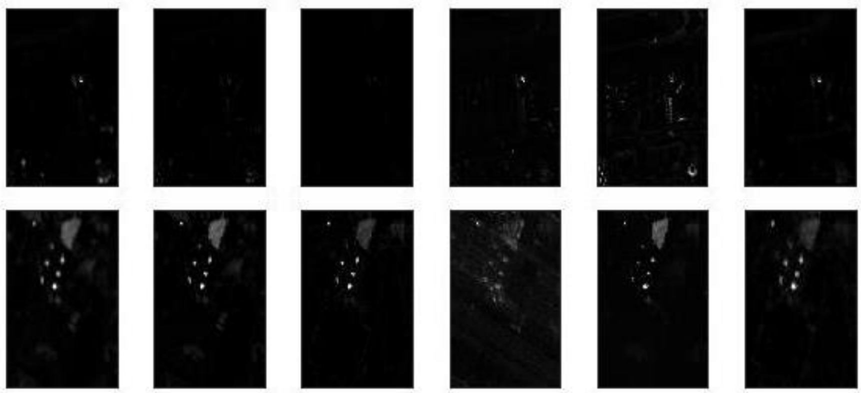
(c)

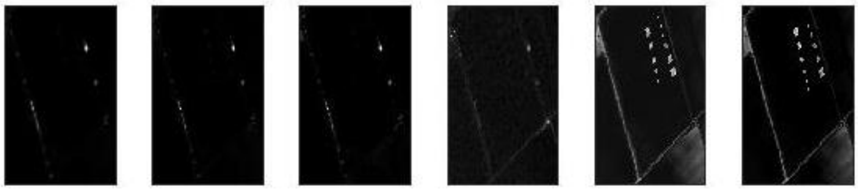
Fig. 3. ROC curves of (a) Airport_3. (b) Urban_1. (c) Salinas Imp_4.

The graph in Figure 3 (a) demonstrates the variety of methods used, from conventional methods to novel approaches. The suggested technique distinguishes out from the other methods since it has less far with higher pd. As may be observed from the figure that suggested technique is always above the other methods especially after 0.01far. LRASMD and RX come after suggested approach. LRASMD has higher pd than RX while its line approaching 100% pd. HADM and SLRMD is after them. It can be concluded that GoDec algorithm provides better detection rate than RoSuRe decomposition algorithm. While distance calculation is compared, modified MD gives better detection rate than original MD.

In the following Figure 3 (b) is ROC curve for ABU urban_1 datasets. The suggested technique performs better than the alternatives, producing better outcomes in terms of far and pd. It stands out for showing a distinct detection rate after a FAR of 0.001, while keeping a 100% PD rate from 50%. The suggested strategy is successful, as evidenced by the decline in the false alarm rate as the detection rate rises. In terms of performance, the HAD approach is quite similar to the suggested methodology.

In Figure 3 (c), the results for the Salinas imp_4 image are presented. The proposed method demonstrates a higher pd compared with the other methods. The SSRX approach, the second-most competitive technology, comes closely behind. The SLRMD, HADM, LRASMD, and RX techniques show increasingly greater detection rates in descending order. The figure highlights HADM superiority regarding Salinas imp_4 dataset detection performance.





(a) (b) (c) (d) (e) (f)

Fig. 4: 2D results for (a) HADM. (b) SLRMD. (c) LRaSMD. (d) RX. (e) SSRX. (f) HADLAP.

Figure 4 shows two dimensional results for each image obtained by the evaluated methods. The last column in Figure 4 (f) is the result for the proposed method named HADLAP. Analyzing the figure reveals that the proposed method regularly results in images with greater clarity, finer details, and overall visual representation, making it stand out as a top candidate among the five approaches examined. In addition, the suggested method's higher visual quality and distinctiveness are highlighted noticeably in the figure, underlining its promise as a sophisticated and cutting-edge way of hyperspectral image processing. By comparing the proposed approach, HADLAP, to existing modern techniques, the assessment of ROC findings and AUC values in Table 1 reveals that HADLAP performs better overall.

Table 1 presents the AUC findings for five different approaches that were used on three different HSIs. One of these techniques stands out as the suggested strategy. The accuracy and discrimination performance of each approach are shown by the AUC metric, which also acts as an assessment metric.

Table 1. AUC values for the methods.

HSI data	AUC VALUES					
	Proposed	HADM	SLRMD	LRaSMD	RX	SSRX
Airport_3	**0.9561**	0.9157	0.8712	*0.9346*	0.9300	0.8623
Urban_1	**0.9950**	*0.9914*	0.9849	0.9796	0.9907	0.9850
Salinas	**0.9984**	0.9358	0.9660	0.9313	0.7463	*0.9978*

We may evaluate the effectiveness of the suggested strategy in comparison to the alternatives by comparing the AUC scores. By comparing the proposed approach's AUC measures to the results attained using the other methodologies, the table highlights the worth of the suggested approach. The proposed method has the highest AUC values which are 0.956, 0.995, and 0.998 respectively. It is followed by RX for Aiport_3 data. For Urban_1 image, HADM has the second highest AUC value. Finally, the second-highest AUC value is held by SSRX in Salinas dataset.

5. Conclusion

In this study, low rank and sparse matrix decomposition-based method is proposed. GoDec algorithm is applied for low-in order to get background and foreground information. Modified MD is then adopted to get anomaly pixels. Different than previous studies MD is rebuilt by using Laplacian matrix. This is resulted with higher detection performance. There hyperspectral datasets are used for evaluations. Anomalies in Salinas data are artificially implemented. Five state-of-the-art approaches are compared with the suggested method. Two of these methods are the most known traditional anomaly detection methods; RX and SSRX. The others are LSDM based methods similar approaches with the proposed methods. ROC curves and AUC are used for evaluations. According to these performance analysis, proposed method outperforms the other methods with higher detection rate overall 90%.

6. Discussions

Here This section discusses five alternative approaches for finding hyperspectral anomalies using three different hyperspectral images. The five methods that are taken into account in this study are HADLAP, HADM, LRaSMD, RX, and SSRX. To find anomalies in the hyperspectral images, each strategy makes use of a different collection of algorithms, methods, or statistical techniques. Various approaches in an effort are compared to find the best successful strategy for hyperspectral anomaly detection. RX and SSRX are traditional anomaly detection methods. The others, however, employ strategy of low rank and sparse matrix decomposition. HADLAP and HADM use RoSuRe algorithm whereas LRaSMD uses GoDec algorithm to partition the data. When all findings considered decomposition based methods outperforms traditional ones in terms of anomaly detection. If the suggested approach and LRaSMD which both employ the GoDec algorithm for decomposition but employ various distances are compared, it may be concluded that the modified MD by the laplacian matrix performs better than MD. Overall, this thorough analysis of five approaches for detecting hyperspectral anomalies using three hyperspectral data gives researchers insightful information that will help them choose the best strategy for precisely identifying anomalies in subsequent applications.

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