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Dynamical Analysis and Electronic Circuit Implementation of Fractional-order Chen System

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ABSTRACT In the last decade, there has been a notable increase in research focus on fractional calculus and its applications. Fractional-order analysis shows promise in enriching the dynamic behavior of chaotic systems. This paper focuses on the dynamic analysis of the Chen system with low fractional-order values and its fractional-order electronic circuit. Notably, there is a lack of studies about chaotic electronic circuits in the literature with a fractional-order parameter value equal to 0.8, which makes this study pioneering in this regard. Moreover, necessary numerical analyses are presented to investigate the system's dynamic characteristics and complexity, such as chaotic phase planes, Lyapunov spectra, and bifurcation diagrams. As expected, oscilloscope views of the electronic circuit realization align with the numerical analysis and PSpice simulation results.

KEYWORDS
Chaos
Chen system
Fractional-order system
Bifurcation
Electronic circuit implementation

INTRODUCTION

Fractional calculus offers greater dynamic richness for chaotic systems. Even a small change in the fractional order of a chaotic system can lead to entirely new bifurcation diagrams. Therefore, in recent years, researchers have studied numerous implementations of chaotic systems in both digital and analog domains, considering different fractional-order values (Yang and Wang 2021; Wang *et al.* 2021; Li *et al.* 2020; Gokyildirim *et al.* 2023; Liu *et al.* 2021; Chen *et al.* 2013; Pham *et al.* 2017). Gokyildirim presented an electronic circuit for the Sprott K system using discrete circuit elements with a fractional-order value of 0.8 (Gokyildirim 2023).

Altun presented research that involved studying numerical computations of fractional-order Rössler and Sprott H systems, as well as their hardware implementations using field-programmable analog array (FPAA) technology (Altun 2021a). In reference (Silva-Juárez *et al.* 2020), FPAA-based applications of fractional-order chaotic systems were realized with active filters, particularly for a fractional-order parameter q value equal to 0.9. Moreover, the fractional-order Sprott H system was utilized to generate a multi-scroll attractor exhibiting hyperchaotic behavior, and its implementation utilizing FPAA was illustrated in (Altun 2022). In another study (Altun 2021b), a field-programmable gate array (FPGA) is used for the implementation of a fractional-order system. The works of Dang focused on studying the fractional-order designs of E (Dang 2014b) and N (Dang 2014a) systems presented by Julien Sprott in 1994 (Sprott 1994). Digital designs of chaotic systems present various benefits in terms of high performance and cost-effectiveness. However, when integrating fractional-order chaotic systems, the limited memory capacity of microcontrollers can potentially impact their overall performance. This limitation arises because the parameter of fractional-order serves as an indicator of memory (Du *et al.* 2013).

Some researchers have focused their studies on the fractional-order analysis of the Chen system and its engineering applications (Li and Peng 2004; Lu and Chen 2006). In their research, Nuñez-Perez et al. introduced the use of different optimization algorithms to amplify the chaotic behavior of the fractional-order chaotic Chen system (FOCHEN) (Nuñez-Perez *et al.* 2021). The outcomes demonstrate that the optimized FOCHEN systems exhibit higher maximum Lyapunov exponents compared to the non-optimized system. Ozkaynak et al. designed a new since substitution box (S-box) using the Fractional-order Chen system with a predictor-corrector scheme (Özkaynak *et al.* 2017). The study indicates that utilizing the FOCHEN system can enhance the performance of the S-box. Zouad et al. designed a secure communication electronic circuit using the delayed FOCHEN system with the Multisim simulation program (Zouad *et al.* 2019). Wang et al. present the development of a nonstandard finite discretization scheme for the FOCHEN system's numerical solutions (Wang *et al.* 2020). All the

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studies mentioned above have successfully achieved analog or digital implementations of chaotic systems with fractional orders. However, a common characteristic observed in analog implementation studies is their focus on fractional-order parameters that are greater than 0.8. The primary contribution of this paper is the construction of the FOCHEN system's electronic circuit for using standard components. Notably, this study focuses on achieving the lowest feasible value ($q = 0.8$) of fractional-order, which has only had a few examples in the literature. For this purpose, the fractional-order values of the FOCHEN system that exhibit chaotic behavior are decided through bifurcation analyses.

The organization of this study is as follows: Section 2 presents the dynamical equations of the FOCHEN system and provides a concise introduction to fractional calculus. In Section 3, some dynamics of the fractional-order system are presented, such as phase planes, Lyapunov spectra, and bifurcation diagrams. Section 4 presents the construction of an electronic circuit for the fractional-order system on a breadboard, along with a comparison between oscilloscope outputs and PSpice simulation results. The final section contains the conclusion.

CHEN CHAOTIC SYSTEM WITH LINEAR SCALING AND FRACTIONAL CALCULUS

In 1999, Chen and Ueta presented a chaotic attractor that is a special case of the Lorenz system (Chen and Ueta 1999). The system has seven terms and three constant parameters, as shown in the following equation:

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (c-a)x - xz + cy \\ \dot{z} &= xy - bz \end{aligned} \tag{1}$$

To enable the implementation of an electronic circuit, linear scaling is required in the original Chen system, as the output values of state variables x, y, and z exceed the necessary limitations. If the system is linearly scaled to maintain the output voltages of the electronic circuit between -5V and +5V, the differential equations of the system (1) are rewritten as follows:

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (c-a)x - 10xz + cy \\ \dot{z} &= 10xy - bz \end{aligned} \tag{2}$$

In this form, the variables are rescaled as $x = 10v_x/\mathrm{V}$, $y = 10v_y/\mathrm{V}$, and $z = 10v_z/\mathrm{V}$. The system (2) produces chaotic signals when a, b and c are 35, 3 and, 28, respectively, with initial conditions $x(0) = 0$, $y(0) = 1$, and $z(0) = 0$.

In fractional calculus, the concept of non-integer differentiation and integration is introduced, allowing us to analyse and model complex phenomena with non-integer dynamics. The fractional-order derivatives and integrals are represented using The fractional-order elementary operator ${}_aD_t^q$, where t and a are the limits of the operation, and q is a real number representing the fractional-order. Depending on the value of q, these operators can act as fractional-order differentiators (fractional derivatives) or fractional-order integrators (fractional integrals). Fractional calculus provides a powerful mathematical tool to describe complex processes that cannot be fully captured by classical integer-order calculus. The continuous-time fractional-order operator is expressed as follows:

$${}_aD_t^q = \begin{cases} \frac{d^q}{dt^q}; & \mathrm{Re}(q) > 0, \\ 1; & \mathrm{Re}(q) = 0, \\ \int_a^t (d\tau)^{-q}; & \mathrm{Re}(q) < 0. \end{cases} \tag{3}$$

Equivalent definitions for the fractional operator ${}_aD_t^q$ are various mathematical representations used to describe the behavior of fractional calculus. Some of these definitions include Grünwald-Letnikov, Riemann-Liouville, Caputo, Grünwald-Letnikov Matrix, Marchaud, and Weyl definitions. Among these, the initial conditions of the Caputo fractional definition resemble those of differential equations with integer order. As a result, Caputo's definition is selected for the fractional derivative calculations of bifurcation diagrams and phase portraits in this study. The Caputo method is defined as follows:

$${}_aD_t^q f(t) = \left\{ \frac{1}{\Gamma(n-q)} \left(\frac{d}{dt}\right)^n \int_a^t (t-\tau)^{n-q-1} f^n(\tau) d\tau \right. \tag{4}$$

where $n - 1 < q < 1$. The Laplace transform of the Caputo definition is represented as follows:

$$H(s) = L\left\{\frac{d^q f(t)}{dt^q}\right\} = s^q L\{f(t)\} \tag{5}$$

Under the assumption of zero initial conditions, the transfer function $H(s)$ is established as a linear fractional-order integrator with $H(s) = 1/s^q$. Moreover, eq. (6) provides fractional derivatives' generalized Laplace transform with order $q > 0$.

$$L\left\{{}_0D_t^q f(t)\right\} = s^q F(s) \tag{6}$$

Thus, the differential equations of the FOCHEN system are written as follows:

$$\begin{aligned} D_t^{q_1} x &= a(y-x) \\ D_t^{q_2} y &= (c-a)x - 10xz + cy \\ D_t^{q^3} z &= 10xy - bz \end{aligned} \tag{7}$$

THE FOCHEN SYSTEM'S DYNAMICAL ANALYSES

In this Section, the required dynamical analyses of the FOCHEN system, including Lyapunov spectra, bifurcation diagrams, and phase planes, are thoroughly investigated. In this manner, the chaotic behavior and dynamic properties of the system (7) can be observed. However, solving a nonlinear fractional-order system analytically presents challenges. As a consequence, various methods have emerged to address these systems, including the utilization of MATLAB-based tools such as FOMCON (Tepljakov and Tepljakov 2017), fde12 (Garrappa 2018), and ninteger (Valerio and Da Costa 2004). In this section, the fde12 toolbox is used to perform all dynamical analyses and simulations, excluding the Lyapunov spectra analysis.

Bifurcation diagrams are used to understand and analyze the behaviors of complex systems. Especially in chaotic systems, bifurcation diagrams are essential tools to explore and analyze the system's different behaviors. On the other hand, Lyapunov exponents are valuable analysis tools used to understand and predict the nature of chaotic systems and the transitions between order and disorder. Figures 1, 2, and 3 illustrate the Lyapunov Exponents and corresponding bifurcation diagrams for both the fractional-order and integer-order versions of Chen system for $b = 3$ and

$c = 28$. In the figures, initial conditions are $x(0) = 0$, $y(0) = 1$, and $z(0) = 0$. Additionally, Figure 4 displays the phase planes of the Chen system (2) and the FOCHEN system (7) based on the bifurcation diagrams. In contrast to other numerical analyses in this study, the Lyapunov exponents are calculated using the Grünwald-Letnikow method (Li *et al.* 2023; Hosny *et al.* 2022).

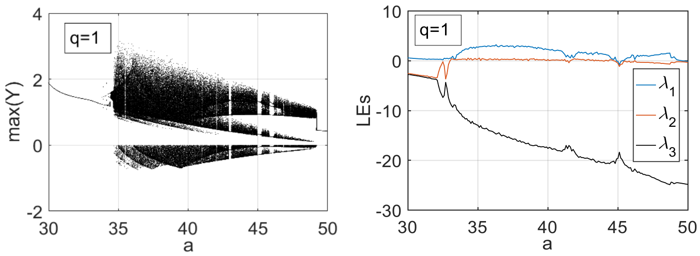


Figure 1 Integer-order Chen system's Bifurcation diagram and Lyapunov spectra.

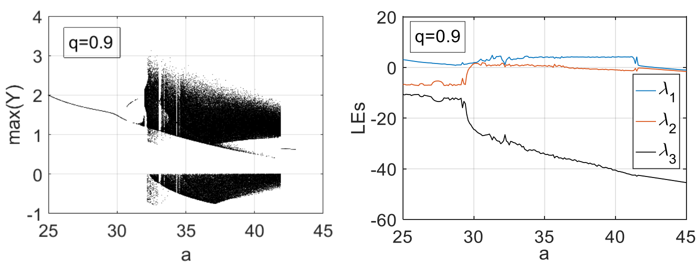


Figure 2 Bifurcation diagram and Lyapunov spectra of FOCHEN system for $q = 0.9$.

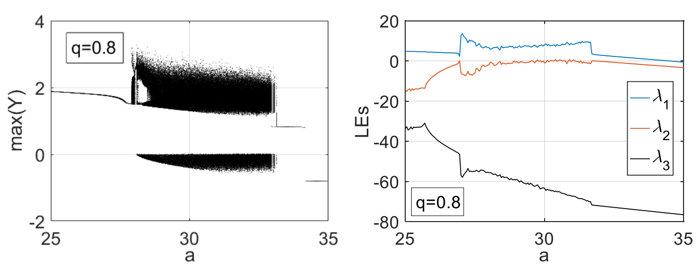


Figure 3 Bifurcation diagram and Lyapunov spectra of FOCHEN system for $q = 0.8$.

ELECTRONIC CIRCUIT IMPLEMENTATION OF THE FOCHEN SYSTEM

The implementation of fractional-order chaotic systems in electronic circuits is important for analyzing system behaviors and controlling complex dynamics. The electronic circuits of fractional-order chaotic systems refer to the electronic implementations of systems with complex dynamics represented by differential equations with fractional degrees (q). These systems offer more versatility and diversity compared to traditional integer-order differential equations. The realization of electronic circuits for fractional-order chaotic systems provides significant advantages in various engineering applications. These systems exhibit nonlinear and random behaviors, making them suitable for randomization and security-based applications. Additionally, fractional-order chaotic systems

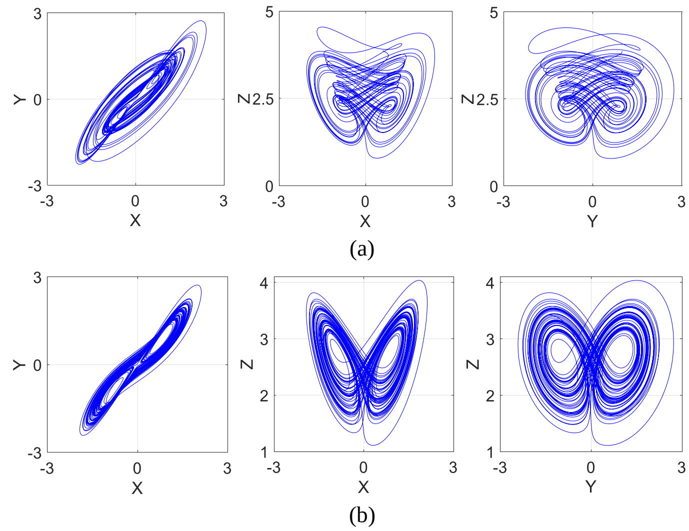


Figure 4 Phase planes of integer-order and fractional-order chaotic systems for t(s) $\in$ [0.1, 20]: (a) $q = 1$, a=35, $b = 3$, $c = 28$, and t(s) $\in$ [0.6, 20], (b) $q = 0.8$, $a = 30$, $b = 3$, $c = 28$, and t(s) $\in$ [0.1, 20].

can be used as functions that randomly mix signals and increase entropy.

In electronic circuits, the basic elements of fractional-order chaotic systems are fractional-order circuit components. These components have different mathematical properties compared to traditional resistors, capacitors, and inductors and are expressed by the fractional degree (q). Fractional-order circuit elements are used for the electronic implementations of fractional-order differential equations.

An electronic circuit of the FOCHEN for (q)=0.8 is implemented with standard components, in this section. According to circuit theory (Podlubny 1999), an electronic circuit that exhibits dynamics of non-integer order is referred to as a "fractance". To realize a chaotic system's electronic circuit implementation, resistor-capacitor (RC) circuits obtained from the approximate transfer function are utilized. Researchers commonly use three approaches, namely chain fractance, domino ladder, and binary tree in their studies. In this research, the chain fractance approach is employed for fractional-order circuits. In this approach, there are N serial RC pairs, where N denotes the number of layers. The transfer function of the chain fractance in the Laplace domain is expressed as following equation, based on the two-port network theory (Yao *et al.* 2020; Ahmad and Sprott 2003):

$$H^{RC}(s) = \frac{1}{C_1 s + \frac{1}{R_1}} + \frac{1}{C_2 s + \frac{1}{R_2}} + \ldots + \frac{1}{C_N s + \frac{1}{R_N}} \tag{8}$$

By utilizing eq. (5), the transfer function of the chain fractance for $q = 0.8$ is written as follows:

$$\frac{1}{s^{0.8}} \approx \frac{5.3088(s + 0.1333)(s + 2.371)(s + 42.17)(s + 750)}{(s + 0.01333)(s + 0.2371)(s + 4.217)(s + 75)(s + 1333)} \tag{9}$$

Considering eq. (9), Table 1 depicts the values of passive circuit elements for the fractional-order module with $q = 0.8$

Table 1 The values of passive circuit elements required for the fractional-order module of the FOCHEN system

Component	Value
R_a	17.9 kΩ
R_b	17.075 kΩ
R_c	170.6 kΩ
R_d	1.756 MΩ
R_e	37.865 MΩ
C_a	418.83 pF
C_b	780.955 pF
C_c	1.39 nF
C_d	2.4 nF
C_e	1.98 nF

Taking into account Table 1, the fractional-order module's electronic circuit is constructed as shown in Figure 5.

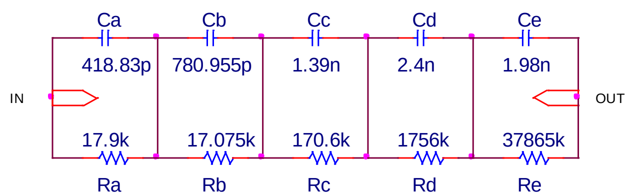


Figure 5 The electronic circuit of integrator for $q = 0.8$.

The circuit schematic of the integer-order Chen system for $a = 35$ is illustrated in Figure 6, with initial conditions $x(0) = 0$, $y(0) = 1$, and $z(0) = 0$.

Referring to Figure 6, the dimensionless equations of the system (2) can be expressed as follows:

$$
\begin{aligned}
RC_1\frac{dv_x}{dt} &= \frac{Rv_y}{10R_2} - \frac{Rv_x}{R_1},\\
RC_2\frac{dv_y}{dt} &= \frac{Rv_x}{R_4} - \frac{Rv_xv_z}{10R_5} + \frac{Rv_y}{R_3},\\
RC_3\frac{dv_z}{dt} &= \frac{Rv_xv_y}{10R_6} - \frac{Rv_z}{R_7},
\end{aligned} \tag{10}
$$

where the component values are $C_{1,2,3}$ = 2.5nF, $R_{1,2}$ = 11.4kΩ, R_3 = 14.286kΩ, R_4 = 51.14kΩ, $R_{5,6}$ = 4kΩ, R_7 = 133kΩ, and $R_{8,9}$ = 10kΩ. RC is the time scale factor and is set to 1ms. DC voltage sources are also $V_P = -V_N = 15$V. The plot in Figure 7 displays the voltage values on the X, Y, and Z terminals in relation to one another.

As shown in Figure 8, the electronic circuit realization of the integer-order Chen system is constructed on a breadboard. When considered together with Figure 7, the oscilloscope outputs of the PSpice simulation and electronic circuit realization of the integer-order Chen system are similar.

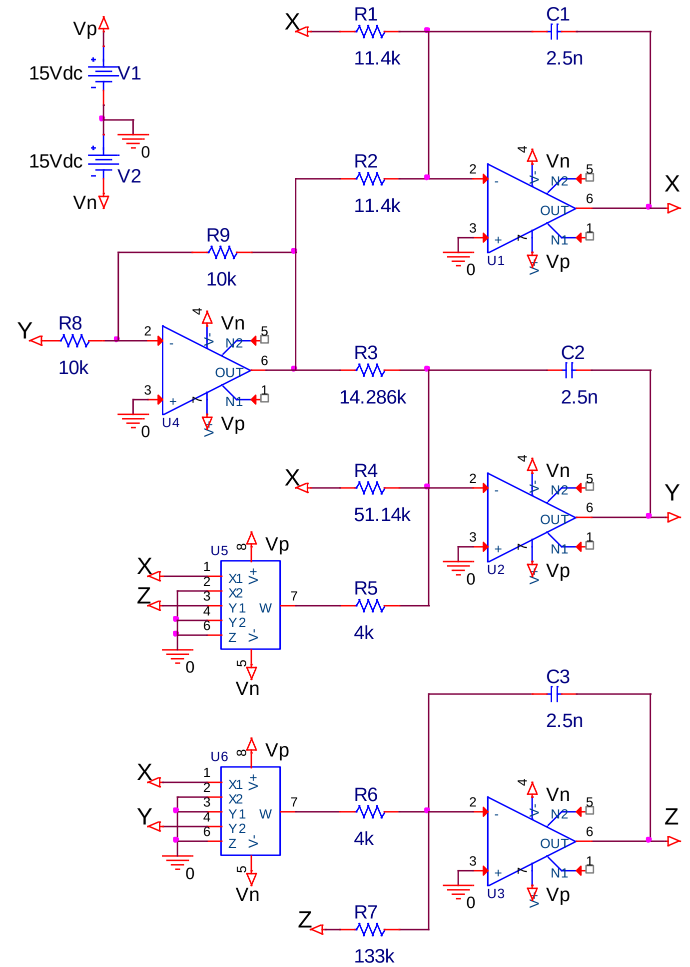


Figure 6 The circuit of the original Chen system (2) with linear scaling in PSpice program.

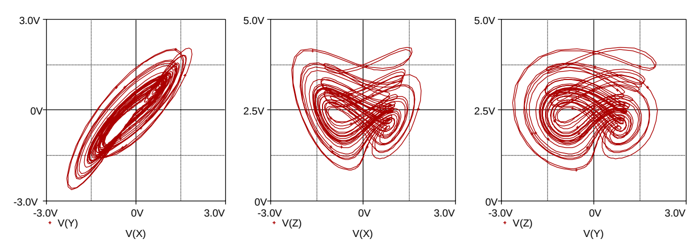
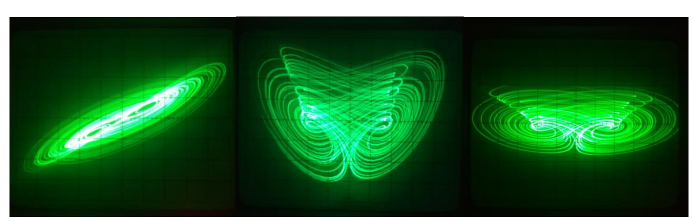


Figure 7 Phase portraits of integer-order Chen chaotic system in PSpice simulation.



(a) (b) (c)

Figure 8 Integer-order Chen chaotic system's oscilloscope views: (a)v_x(0.5V/div)-v_y(1V/div), (b) v_x(0.5V/div)-v_z(0.5V/div),(c) v_y(0.5V/div)-v_z(1V/div).

The primary challenge in implementing an electronic circuit lies in modeling a fractional-order system using standard components. Considering Figure 3, it is observed that the suitable parameter values for $q = 0.8$ are $a = 30$, $b = 3$, and $c = 28$. The electronic circuit implementation of the FOCHEN system employing the chain fractances for $q = 0.8$ is shown in Figure 9.

The initial conditions are chosen as $x(0) = 0$, $y(0) = 1$, and $z(0) = 0$. Note that the fractional integral operator is transformed into a chain fractance with N=5. The circuits depicted in Figures 6 and 9 consist of passive and active circuit elements, including TL081 (operational amplifiers) and AD633 (multipliers), which are readily available in the market. Component values of the fractional-order electronic circuit are as follows: $C_{1,6,11} = 418.83$pF, $C_{2,7,12} = 780,955$pF, $C_{3,8,13} = 1.39$nF, $C_{4,9,14} = 2.4$nF, $C_{5,10,15} = 1.98$nF, $R_1 = 13.3\text{k}\Omega$, $R_2 = 13.3\text{k}\Omega$, $R_3 = 14.286\text{k}\Omega$ $R_{5,6} = 4\text{k}\Omega$, $R_7 = 133\text{k}\Omega$, $R_{8,9} = 10\text{k}\Omega$, $R_{10,15,20} = 17.9\text{k}\Omega$, $R_{11,16,21} = 17.075\text{k}\Omega$, $R_{12,17,22} = 170.6\text{k}\Omega$, $R_{13,18,23} = 1.756\text{M}\Omega$, $R_{14,19,24} = 37.865\text{M}\Omega$. The DC voltage sources are $V_P = -V_N = 15$V. The oscilloscope views of the voltages on the terminals (X, Y, and Z) of the FOCHEN system's electronic circuit, plotted against each other, are shown in Figure 10.

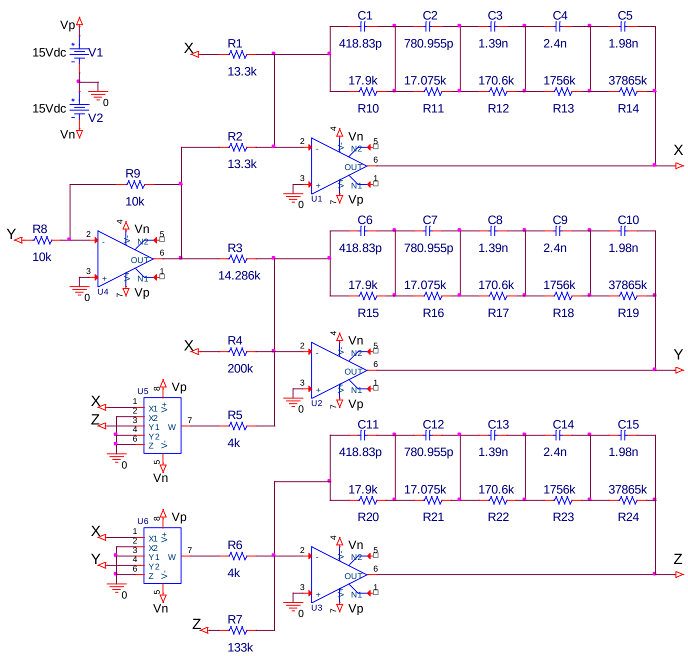


Figure 9 Circuit schematic of FOCHEN system for $q = 0.8$.

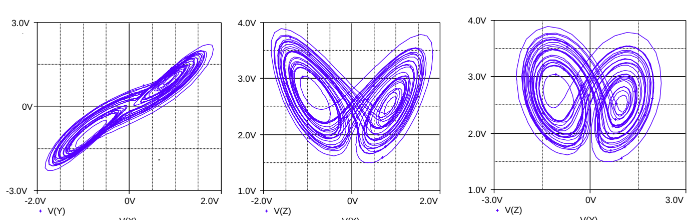
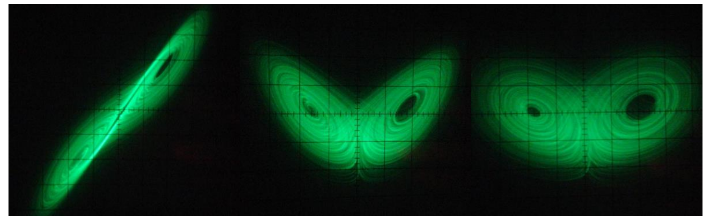


Figure 10 Phase planes of FOCHEN system in PSpice simulation for $q = 0.8$.

As presented in Figure 11, the electronic circuit realization of the FOCHEN system is constructed on a breadboard. When considered together with Figure 10, the oscilloscope views of PSpice simulation and electronic circuit realization of the fractional-order system are very similar.



(a) (b) (c)

Figure 11 The FOCHEN system's oscilloscope views: (a) v_x(0.5V/div)-v_y(0.5V/div), (b) v_x(0.5V/div)-v_z(0.5V/div),(c) v_y(0.5V/div)-v_z(0.5V/div).

Finally, it is observed that the results of PSpice simulations and electronic circuit implementations are consistent with the numerical analysis results conducted in the previous section. This confirms the applicability and consistency of the fractional-order modules. The electronic circuit of the FOCHEN system, along with the fractional-order modules, constructed on a breadboard, is shown in Figure 12.

Figure 12 The electronic circuit of the FOCHEN system with the fractional-order modules.

CONCLUSION

Fractional-order analysis offers a means to enhance the diversity of dynamics in chaotic systems. This study presents an electronic circuit realization for the Chen system, incorporating a low-value fractional order and utilizing standard electronic components. The dynamic characteristics of the FOCHEN system are examined by conducting various analyses, such as phase portraits, Lyapunov spectra, and calculations of bifurcation diagrams. Additionally, the system's chaotic behavior for different fractional-order values is revealed through bifurcation diagrams and Lyapunov exponents analyses. Based on the numerical analyses and PSpice simulations, the minimum applicable fractional-order value (q) for the electronic circuit implementation of the FOCHEN system is found to be 0.8. The electronic circuit of the fractional-order system is constructed on a breadboard using discrete circuit elements, which are easily available in the market. The electronic circuit realization's voltage outputs, as observed in oscilloscope images, align with numerical analyses and PSpice simulation program results. As a result, through fractional-order calculus, the dynamic diversity of the Chen system is enhanced. Thus, the FOCHEN system is a potentially chaotic system for use in data security applications where applicability and complexity are crucial. As expected, oscilloscope views of the electronic circuit realization align with the numerical analysis and PSpice simulation results.

Availability of data and material

Not applicable.

Conflicts of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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