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Examining Turkish Graduate Theses on Mathematics Education for Hearing Impaired Individuals

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ABSTRACT

The aim of this research was to conduct a systematic review to examine the theses written in Türkiye on mathematics education among individuals with hearing impairment. Studies were identified through searches of the electronic database of the Higher Education Council Publication and Documentation Department Thesis Center. Theses were the full text of which can be accessed until 30 September 2022. The inclusion criteria were met by 9 theses. In this research, theses were classified in terms of aim, study design, the sample definition of hearing impairment, the main results, and recommendations related to the learning and teaching process of mathematics education. The studies primarily focused on the impacts, due diligence, relationship identification, and perspectives concerning the teaching of mathematics to students with hearing impairments. A wide range of teaching methods, tools, activities, and resources were found to influence learning outcomes. The analysis of the theses underscored the significance of factors beyond hearing loss, such as teaching methods, students' experiences, and the ability to connect mathematical concepts to real-life situations. Sign language and gestures emerged as crucial for mathematics education. Despite some highlighted challenges in problem-solving and geometry performance, a plethora of recommendations was provided to enhance mathematics instruction for students with hearing impairments, ranging from classroom utilization to supporting and evaluating students. This comprehensive review underscores the need for diversified and tailored strategies to enhance mathematics education for students with hearing impairments.

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INTRODUCTION

Education is a fundamental aspect that plays a crucial role in the development of individuals. It is widely accepted that all individuals, regardless of their differences, should acquire social qualities and acquire affective, cognitive, or psychomotor skills through education to effectively participate in their respective societies. However, the acquisition of these skills is contingent upon effective communication, with hearing playing a central role in this regard (Diefendorf, 2015). Indeed, hearing is a critical sense in the process of learning to communicate effectively (Chen et al., 2014). Hearing impairment (HI), a condition characterized by a decrease in hearing sensitivity due to congenital or subsequent issues, is the result of the loss of this sense (Ministry of National Education [MoNE], 2008). Simply, the term "hearing impaired" refers to anyone who has experienced some degree of hearing loss, whether in one or both ears or who relies on hearing aid technology (Turkish Statistical Institute [TUİK], 2010). HI can be better comprehended and approached holistically by acknowledging the diverse range of definitions associated with it (Woods, 2022).

According to the World Health Organization [WHO] (2021), there exists a population of approximately 430 million individuals who necessitate hearing loss rehabilitation services. This translates to a staggering 1.5 billion people globally who experience hearing loss. The Center for Barrier-Free Access, Population and Housing Research of the Ministry of Family and Social Services has reported that 836 thousand individuals, equivalent to 1.1% of population of Türkiye, have some degree of hearing loss. The Turkish National Disability Data System (General Directorate of Services for Persons with Disabilities and the Elderly [GDSPDE], 2021) indicates that around 228,589 living persons have been identified as hearing impaired.

Distinct learning styles are inherent to individuals and are most appropriate for their specific circumstances. Similarly, individuals with HI who require special education have diverse educational needs. The varying degrees of HI – from mild to extremely severe – along with different influencing factors like the cause of the impairment and duration of hearing aid use, underscore the need for customized teaching methods that cater to the unique needs of each student (Shields & Lennox, 2017). Moreover, it is recognized that such individuals employ sign language for their interaction needs (Sandler & Lillo-Martin, 2006; Valli & Lucas, 2000). Consequently, lesson plans and classroom settings should be tailored to address these unique instructive necessities. Sign language, particularly when accompanied by gestures, is emphasized as a vital tool for effectively teaching mathematics to students with HI (Heslinga & Nevenglosky, 2012; Krause, 2019). This is because every individual has the right to an education on an equitable basis with others, and these rights are safeguarded by decisions made both within Türkiye and on the international stage. For example, several documents, including the Individuals with Disabilities Education Act (IDEA, 2007), the Salamanca Statement (UNESCO, 1994), the Principles and Standards for School Mathematics (NCTM, 2000), and the No Children Left Behind Act [NCLB] (2001), emphasize the significance of education for all individuals. Furthermore, in Türkiye, the Ministry of National Education (MoNE, 2020) has established specific goals for special education services. These objectives correspond with the overall aim and core principles of Turkish National Education. This alignment serves to facilitate individuals with special educational needs to fully utilize their capacities, in harmony with their educational requirements, qualifications, interests, and abilities. Furthermore, these objectives also aim to prepare these individuals for higher education, professional life, and active participation in society, reflecting the comprehensive vision of Turkish National Education. In light of this, it is imperative that individuals with hearing loss who require special education receive equal educational opportunities and participate in social life, acquire knowledge, and have equivalent educational prospects as individuals with normal development. Therefore, it is crucial to create accessible learning environments for individuals with HI, promote educational opportunities, and fully integrate them into society (MoNE, 2016).

Owing to differences in learning characteristics and contexts, students who are hard of hearing may have trouble in acquiring the same amount of information as their hearing peers over a given period. Consequently, individuals with HI face prejudice and bias irrespective of their socioeconomic status or gender. This is primarily attributable to the fact that hearing loss not only affects learning but also growth in other areas. Furthermore, the inability to comprehend and process auditory cues from the environment negatively impacts the cognitive, social, and linguistic capacities of hearing-impaired individuals. Studies have shown that when individuals with HI are placed in environments where sign language is used, they tend to naturally pick up and learn this form of communication (Goldin-Meadow, 1993). Many challenges encountered by these persons in the educational setting, as well as performance issues, can be traced to communication difficulties arising from hearing loss. In such instances, hearing-impaired individuals may not grasp everything that the teacher says and may have to make guesses at certain points (Reuterskiold et al., 2010). Nonetheless, hearing deficiency does not affect the cognitive abilities required for learning. With the right support mechanisms in place, hearing-impaired individuals can catch up to their peers in cognitive abilities, even if there is a two or three-year age gap (Reuterskiold et al., 2010). Unfortunately, the learning environment for those who are hard of hearing or deaf is significantly more restricted than that of their typically developing peers (Wilkens & Hehir, 2008). In this context, it's vital to draw upon a variety of methodologies to provide a comprehensive evaluation of complex educational practices (Lahdenperä et al., 2023; Miller-Young & Poth, 2022; Ocean & Hicks, 2021). Due to the necessity of using individualized education plans that consider student characteristics such as age at which HI occurs, prior learning, preference for learning channels, parental interest, and ability to use hearing, education and training for individuals with HI are more challenging than for their typically developing counterparts. Employing diverse data collection tools is instrumental in these situations. It can enable teachers and researchers to acquire a more detailed understanding of the mathematical skills and attitudes of students with HI, as well as to gain better insights into their experiences and perspectives (Rishaelly, 2017; Sarı & Pürsün, 2018). By gaining insight into students' learning perspectives, teachers are empowered to design learning environments that better cater to their students' needs (Nurhasanah & Suryaman, 2022). The challenges in the educational process for students with HI primarily stem from the unique characteristics of both the teachers and the students. One of the most significant challenges that teachers in schools for individuals with HI face is their inability to choose the best teaching strategy for a given lesson (Sarıkaya & Börekçi, 2016). Undoubtedly, teachers hold a central position in the education of students with HI, thereby underlining the importance of offering them professional support (Li & Miloň, 2022). Given its abstract nature and dependence on mathematical terminology, mathematics is one subject where instruction and training for hearing-impaired individuals may encounter difficulties (Traxler, 2000).

Hearing Impaired Students and Teaching Mathematics

Due to the enhancement of people's logical and intellectual capacities through the study and appreciation of mathematics, those who possess such skills will have a significant advantage in today's rapidly evolving world (National Council of Teachers of Mathematics [NCTM], 2006). As such, the acquisition of mathematical knowledge is essential for the success, effectiveness, and independence of both hearing and deaf or hard of hearing individuals. However, studies have revealed that individuals who are deaf or hard of hearing face particular difficulties in acquiring mathematical skills (Mousley & Kelly, 1998; Nunes & Moreno, 2002; Pau, 1995; Swanwick et al., 2005). Most notably, it has been emphasized by various studies that students with hearing loss encounter greater difficulties in learning mathematics than their typically developing peers, and that this difficulty increases with the severity of their hearing loss, age, school environment, and the type of hearing aids they use (Davis & Kelly, 2003; Gottardis et al., 2011; Hyde et al., 2003; Kritzer, 2008; Traxler, 2000; Vosganoff et al., 2011). Moreover, studies have shown that individuals who are deaf or hard of hearing require a longer period to learn new content (Drigas et al., 2005; Epstein et al., 1994), which may negatively impact their progress in school and their ability to learn (Frostad, 1996). Attempts have been made to identify the causes for the difficulties experienced by hearing-impaired individuals in mathematics, and several explanations have been posited regarding their performance in such lessons (Epstein et al., 1994; Hitch et al., 1983; Kelly et al., 2003; Nunes & Moreno, 2002; Pau, 1995; Swanwick et al., 2005; Traxler, 2000). These include a lack of experience in learning during early childhood, insufficiencies in language, the nature of mathematical language, insufficiencies related to mathematical concepts, language as an essential component of learning mathematical concepts, difficulties with reading comprehension, difficulties in solving verbal mathematical problems, difficulties with learning speed, difficulties with short-term memory performance, and difficulties with attention skills. While these are the most common challenges, other issues may be related to instructors and technological advancements. Some of these drawbacks include the teachers' inability to choose appropriate teaching methods (Sarıkaya & Börekçi, 2016) or use the wrong teaching method (Arnold, 1996), their inability to create suitable learning environments (Kelly et al., 2003), the low perception of teachers' expectations from students (Arnold, 1996; Kelly et al., 2003), and a lack of materials for hearing-impaired students (Frostad, 1996). Despite these challenges, it is recognized that hearing-impaired individuals can learn mathematics just like their peers. Meadow-Orlans (1980) posited that there exists no discernible difference in mathematical thinking between individuals with HI and their hearing counterparts. In a study conducted by Hitch, Arnold, and Phillips (1983), it was observed that typically developing and hearing-impaired individuals displayed remarkable similarities in their reaction strategies. To expand the learning opportunities and enhance the arithmetic abilities of the hearing-impaired, researchers (Nunes & Moreno, 2002; Tanrıdiler, 2012) have experimented with novel instructional designs. The researchers reported that the hearing-impaired individuals exhibited significant improvements in mathematical performance, real-world relevance, and language proficiency upon the conclusion of their investigations. In line with the "Every child can learn" principle of the Ministry of National Education (2005), it can be deduced that every child, albeit at their own pace and through different means, can learn mathematics under suitable conditions. Therefore, it is crucial to tailor the mathematics instruction to the unique combination of learning styles, prior knowledge, and hearing-impaired status of each student. Those who are hard of hearing require greater economic support, social and cultural inclusion, and educational opportunities. In keeping with the principle of equal opportunity in education, studies must be conducted to develop learning environments that align with the individual characteristics of hearing-impaired individuals to meet their educational needs. Therefore, it is essential for teachers to utilize a range of teaching strategies and resources to enhance the mathematical learning process for students with HI (Adeniyi & Kuku, 2020; Burley et al., 1994; Jadhav & Gathoo, 2018; Makinde, 2012; Mtuli, 2015; Ovadiya, 2021). Adopting a multidimensional approach is essential in meeting the specific learning requirements of students with HI (Pradhan, 2020; Tongwa & Atemnkeng, 2019). It is critical to note that the significance of family support, instructional approaches, teaching materials, and notably academic studies cannot be denied. However, there is a lack of research on the mathematics education of hearing-impaired individuals, which hinders our understanding of their learning experiences. As a result, the educational environments of individuals with HI are often overlooked. To ensure the academic success of individuals with HI, their cognitive and emotional strengths and weaknesses in the classroom must be identified. Examining the research conducted about how hearing-impaired individuals learn mathematics can help researchers, teachers, and parents comprehend their situation in mathematics education, the outcome of studies, and recommendations to improve educational environments (Wilkens & Hehir, 2008). This can also provide an opportunity to create accessible mathematics curricula for hearing-impaired individuals. It is crucial to understand the trajectory of changes in the educational process, its status, and the results it produces in order to effectively track how mathematics is taught to students with HI (Opfer et al., 2020). The objective of this research is to analyze the graduate theses on the education of individuals with HI in mathematics teaching from the Databases of National Thesis Center of the Council of Higher Education in Türkiye through the lens of the following research questions:

1. What is the aim of the theses?
2. How do the theses handle study design?
3. How are participant characteristics determined in the theses?
4. What are the main results of the theses?
5. What teaching practices are recommended in the theses?

Importance of the research

The significance of this research is profound. This research will contribute to the existing body of knowledge by offering a unique perspective on the mathematical education of individuals with HI, focusing on the current situation in Türkiye. The choice of Türkiye as the research context is pivotal; Türkiye has a unique educational landscape that is influenced by various social, cultural, and political factors. There is a paucity of research focusing specifically on the mathematical education of individuals with HI in Türkiye, which creates a critical gap in the literature (Atış & Doğaner, 2022; Gürefe, 2018; Solak Berigel & Karal, 2021; Şimşek & Çağlıyan, 2020). Furthermore, the extant body of literature encompasses research conducted in Türkiye pertaining to the distance education of individuals with disabilities (Karabey et al., 2020), pedagogical approaches in the education of individuals with disabilities (Kızılcık, 2022), and the education of individuals with visual impairments (Coşkunçay & Horzum, 2022). These studies provide a restricted amount of material that specifically pertains to the mathematics education of individuals with HI. This is expected to contribute to the subject's corpus of knowledge and aid in the formulation of educational policies in similar contexts. Within the above framework, the objective of this study is to gather and analyze graduate theses pertaining to the education of students with hearing impairment in Turkey. The rationale behind selecting graduate theses stems from the assertion made by Karadağ (2009) that the examination of theses pertaining to a certain scientific discipline holds significance in elucidating the overall perspective of said discipline. By analyzing the teaching practices proposed in these theses, the research will provide valuable insights that could improve the educational experience of students with HI. The outcomes of this investigation could be instrumental in shaping educational policies. Moreover, this study has particular importance for Türkiye as it may influence educational policymaking in the country. By identifying the obstacles and proposing solutions in the mathematical education of individuals with HI, this research could guide Türkiye in creating more inclusive and effective educational policies. Educational policies could be made more inclusive and effective by identifying the obstacles encountered in the mathematical education of individuals with HI and proposing solutions to these obstacles. Moreover, this study has the potential to increase awareness regarding the education of individuals with HI. This can help teachers, parents, and the broader community better comprehend the educational needs of individuals with HI. In the Türkiye context, where disability education might not always receive the attention it deserves, this study can act as a catalyst for change by informing and mobilizing the educational community. This research may promote more inclusive, effective, and equal educational opportunities for individuals with disabilities by enhancing general knowledge and comprehension regarding the education of people with disabilities.

METHOD

In this research, a systematic review approach was employed. By utilizing definite criteria for inclusion and exclusion, a systematic review determines which studies should be incorporated in a compilation and integrates the outcomes of those studies in the context of research questions (Petticrew & Roberts, 2006). According to Bellibaş and Gümüş (2018), systematic review studies can be conducted in three ways: meta-analysis, meta-synthesis, and descriptive content analysis. In this research, descriptive content analysis, which is a methodological research design that scrutinizes the patterns found in related subjects, was performed. The present study systematically examined the theses that had previously been subjected to a jury review. This scrutiny entails evaluating the aims, study design, characterization of hearing-impaired students, main results, and recommendations for improving the mathematics instruction of individuals with HI.

Inclusion and Exclusion Criteria

Determining the criteria for the inclusion and exclusion of theses in systematic literature review studies is a crucial step. The reliability of the findings is significantly influenced by these criteria. In this research, included studies were selected based on several criteria. Firstly, the theses should have been published by 30th September 2022. Secondly, only students with HI should have been included in the participant group. Thirdly, the theses should have been related to education and instruction regarding the teaching of mathematics. Fourthly, the theses should have focused on mathematical education. Fifthly, the study should have completed the jury process and deemed appropriate as a thesis study in its field. Sixthly, the theses should have been based on original data. Finally, the theses should have been fully available in the text.

Several criteria were also used to exclude certain theses from this research. Firstly, the theses should not have been conducted with students who have comorbidity or multiple disabilities. Secondly, the theses should not have relied on previous research. Finally, theses that were not freely available in databases were also excluded.

Research Identification Process

The theses were acquired through exploration of the database of the National Thesis Center of the Higher Education Council of Türkiye. In addition to the Turkish language, English keywords were utilized for the search process, specifically, all possible keywords such as “hearing impairment”, “hearing impaired”, “hearing loss”, “işitme engeli”, “işitme engelli”, and “işitme kaybı” for the subject of “education and instruction”. The search code is (("hearing impairment" OR "hearing impaired" OR "hearing loss" OR "işitme engeli" OR "işitme engelli" OR "işitme kaybı") AND ("education" OR "instruction")). During the initial stage of the search process, a total of 471 theses were identified, with the study including theses published until September 30, 2022. In the second stage (identification phase), the titles, abstracts, and keywords of the identified research were reviewed, and the relevance of these theses to mathematical education was later determined, whereby the theses that were unrelated to mathematics education were discarded. A total of 28 theses were compiled because of the scanning process. These theses were then analyzed for consistency with the research criteria based on the basic research criteria, and the full-text accessibility of each thesis was verified, with those that were inaccessible being excluded. This method resulted in obtaining twenty-three theses, while five were eliminated. Thirteen duplicate theses were disqualified, and the final step was to ensure that students with disabilities are not further disadvantaged. In this way, one thesis was eliminated. The study incorporated nine theses, as illustrated in Figure 1, and Table 1 displays the characteristics of the theses included in the research based on the information obtained from this source.

Table 1. *The characteristics of the theses included in the research*

Code, Author, Year, Type, University	**Code, Author, Year, Type, University**
T1, Şen, 1990, M, Anadolu University	T6, Cağlıyan, 2018, M, Gazi University
T2, Güldür, 2005, M, Anadolu University	T7, Doğan Fırat, 2018, PhD, İnönü University
T3, Yıldırım, 2009, M, Eskişehir Osmangazi University	T8, Ada, 2021, M, Bursa Uludağ University
T4, Tanrıdiler, 2012, PhD, Anadolu University	T9, Ertaş, 2021, M, Ege University
T5, Gürefe, 2015, PhD, Gazi University	

(T: Thesis, M: Master’s Thesis, PhD: Philosophy of Doctorate Thesis)

Three out of the nine theses were composed as doctoral theses, whereas the remaining six were crafted as master's theses, as delineated in Table 1. The prime motive behind these theses was to facilitate the instruction of mathematics to students with HI. The preponderance of theses in Table 1 were inscribed at Anadolu and Gazi University.

Analysis Process

Employing content analysis, a standardized methodology was adopted, and a comprehensive reporting template was crafted, through which data from each thesis illustrated in Figure 1 were examined. Key variables were carefully considered during this process: publication year, university, thesis type, author's full name, research aim, study design, and participant characteristics. The latter included grade level, age, gender, participant number, status of HI, communication method, and the definition of HI. Additionally, the main results of each thesis and recommendations for improving pedagogical practices were thoroughly reviewed.

In the research examination form, each thesis was scrutinized based on the following components: research aim, study design, participant characteristics, main results, and recommended teaching practices.

This study utilized the research problems, purpose statements, objectives, and findings of the theses to carry out an analysis of the aims of the theses. The study design of the theses was examined in three categories: research models, types of data collection tools, and the data collection process. In this context, attention was paid to the research models, as well as how the data collection tools were obtained following a particular process. Characteristics of the participants, including which age group, at what grade level, gender, number of participants included in the study, their preferred communication methods, and their hearing status were assessed. Also, the definition of hearing impairment preferred in the theses was taken into consideration. While analyzing the main results obtained in the theses and the given recommendations, results and recommendations that concern the teaching of mathematics to students with hearing impairments were considered. Other results and recommendations have been noted within the scope of this study.

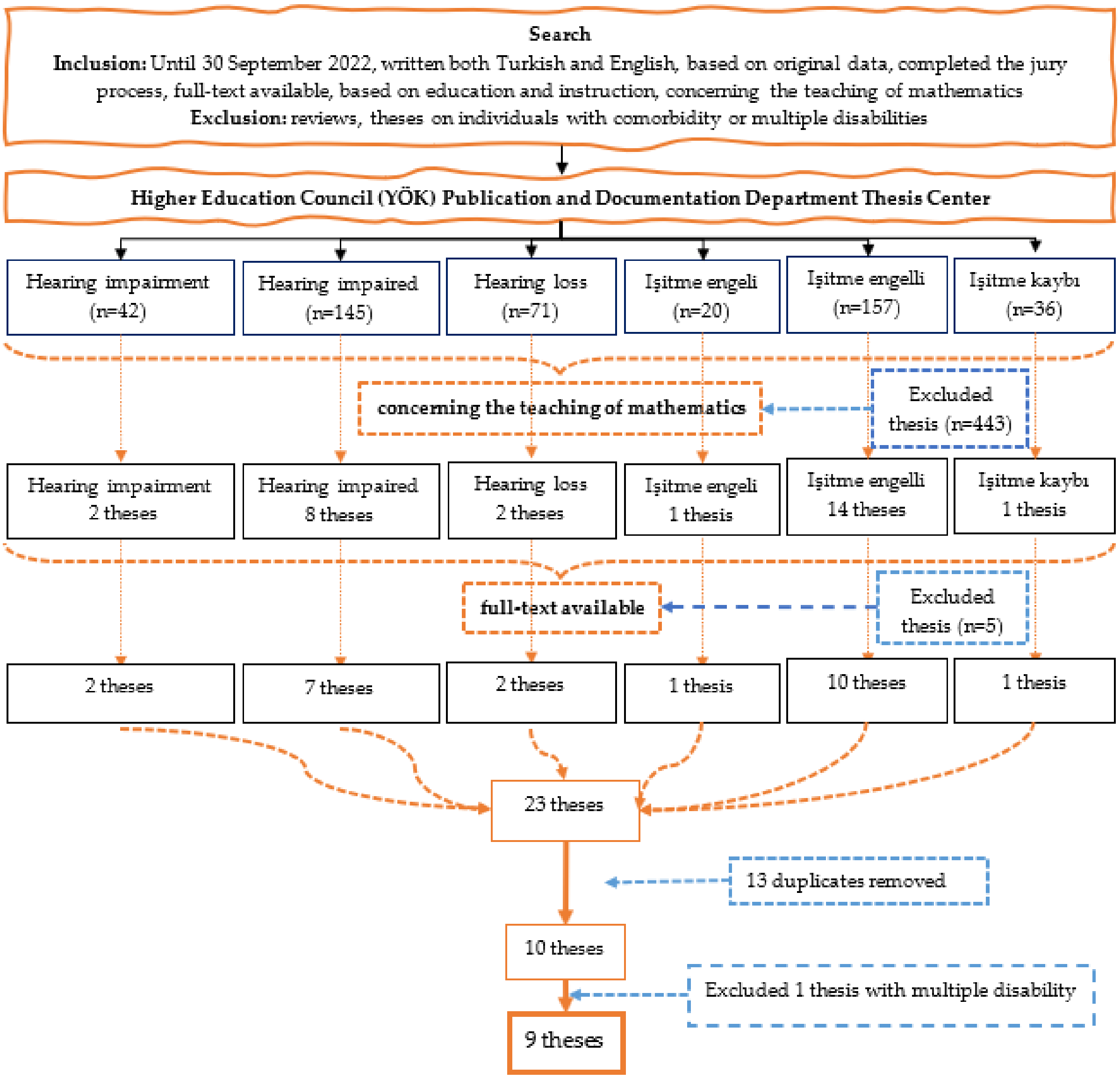

Figure 1. *The review search strategy* (Adapted from: Moher etc., 2009).

Inter-Coder Reliability

Using the "consensus / [consensus + disagreement] X 100" formula prescribed by Gast (2010), the inter-coder reliability of the research was determined by comparing the results of the initial analysis conducted by the researcher to those of a subsequent analysis conducted by the same researcher two months later. The obtained level of coding consistency was a notable 95%. For the 5% of the group that could not be aligned, a final decision was made with the assistance of an additional researcher with a Ph.D. in mathematics education.

Ethical Declaration

This study is an investigation of the theses published openly to public on YÖKTEZ. Therefore, there is no ethical issue in the conduct of the research and no ethical board approval is required. Additionally, it is important to note that this study rigorously complies with the ethical rules and regulations set forth by the Council of Higher Education in Turkey (YÖK). This commitment guarantees that the research upholds academic integrity and demonstrates due regard for intellectual property rights.

RESULTS

In this section, the aims of the theses, the study design used, the characteristics of students with HI, the main results, and the recommended teaching practices were presented.

The aim of the theses

To ascertain the aim of the theses, it is of utmost importance to accurately discern the purpose statements. Table 2 provides an exposition of the purpose statement and the corresponding themes contained in the body of each thesis. Subsequently, five primary categories were discerned, namely: determining an impact of something, performing due diligence, identifying relationships, examining perspectives, and proposing a teaching model.

Table 2. *The aim of the theses*

	Theme	Statement of The Purpose
T1	-Determining an impact of something (method)	-To determine whether the programmed teaching method will aid in the instruction of mathematics to hearing-impaired students.
T2	-Performing due diligence -Identifying relationships	-To evaluate the mathematical problem-solving behavior of students with HI. -To ascertain the problem-based scores of hearing-impaired students because of solving four-operation mathematical problems. -To ascertain the total scores of hearing-impaired students based on their ability to solve four-operation mathematical problems. -To identify student characteristics that may influence hearing-impaired students' behavior when solving four-operation mathematical problems.
T3	-Determining an impact of something (activity)	-To assess the impact of geometry activities created using the dynamic geometry program Euclidean Reality on students' Van Hiele geometry levels, attitudes, and accomplishments based on their hearing status.
T4	-Determining an impact of something (instructional setting, method) -Identifying relationships -Proposing teaching model	-To analyze in a systematic, regular, reflective, and cyclical manner the interactions of various variables that affect the Balanced Mathematics Teaching in the real systematic environment of hearing-impaired students. -To investigate the changes that occur in students and teachers during the teaching-learning process of Balanced Mathematics. -To propose a model for teaching mathematics to hearing-impaired students.
T5	-Performing due diligence	-To investigate the semiotic resources used by hearing impaired students to express certain geometric concepts
T6	-Performing due diligence -Identifying relationships	-To determine the Van Hiele geometric thinking levels and geometry self-efficacy of hearing-impaired middle school students -To ascertain whether certain variables affect the predetermined situations -To investigate the connection between Van Hiele geometric thinking levels and geometry self-efficacy

T7	-Determining an impact of something (material)	-To investigate the impact of abacus education on students' operational fluency and its continuity on operational fluency when teaching addition and subtraction operations to hearing-impaired fourth graders.
T8	-Determining an impact of something (method) -Examining perspectives	-To investigate the effect of peer teaching on academic success of hearing-impaired eighth graders in fractions, their math attitudes, and their perspectives on peer teaching
T9	-Examining perspectives	-To determine the views of the students with HI regarding the digital teaching material designed for subtraction that necessitates decimal breaking

Table 2 reveals that some graduate theses (T2, T4, T6, T8) concerning the instruction of mathematics to hearing-impaired students were established with multiple aims. Predominantly, these theses analyze the influence of specific teaching environments, methods, activities, or materials. The effectiveness of programmed, balanced, and peer teaching methods as chosen instructional strategies were scrutinized. Their contributions to mathematics education, impact on students and teachers, and effects on student achievement were evaluated. Investigations also encompassed the impact of software-assisted dynamic geometry activities on the geometrical thinking levels, geometry attitudes, and success rates of hearing-impaired students. Furthermore, the influence of teaching addition and subtraction with an abacus, either as a low-level technology or tangible material, on operational fluency was assessed.

An equal number of theses (n=3) studied the situations of hearing-impaired students relative to other factors, and emphasis on the interrelationships between some factors that shape the instructional settings. Theses identified student circumstances by considering elements like problem-solving behaviors and test scores of those with HI, as well as the semiotic resources used to elucidate geometric concepts. One thesis aimed to identify and analyze the geometry self-efficacy and Van Hiele's geometric thinking levels of hearing-impaired students, evaluating these in the context of various variables. Theses that centered on student characteristics and teaching methodologies probed the relationships between diverse factors influencing learning environments. Therefore, one thesis investigated the impact of student characteristics on four-operational problem-solving processes, while other explored various elements affecting balanced mathematics teaching. Another thesis assessed the relationship between Van Hiele geometric thinking levels and geometry self-efficacy of hearing-impaired students. Theses studying perspectives examined student views on peer teaching and digital teaching material utilization in classrooms. One singular thesis aimed to propose a teaching model for mathematics instruction to hearing-impaired students.

The study design of the theses

To comprehend the research design, it is essential to comprehend the context in which the research data were gathered. To ascertain the theses' research designs, research models, data collection tools, and data collection processes were analyzed. In this three-fold context, Table 3 displays the research design of the theses.

Table 3. *The study design used in theses*

Study design		Theses
Research models	Experimental design	T1, T3
	Single-subject design	T7
	Descriptive model	T2
	Survey design	T6
	Case study	T5, T9
	Phenomenology	T5
	Action research	T4
	Mixed methods	T8
Data collection tools	Tests and scales	T1, T2, T3, T6, T8
	Video and audio recordings	T2, T4, T5, T7, T9
	Documents (reflective diaries, lesson plans,	T4, T9

	etc.)	
	Observations and interviews	T5, T8, T9
Data collection process	Pre-test and post-test applications	T1, T3, T8
	Group and individual applications	T2, T4
	Lon-term education and intervention processes	T3, T4, T7, T8
	Semi-structured and unstructured observations	T5, T9

As illustrated by Table 3, it is apparent that the study design of the theses is approached by employing a variety of research models, multiple data collection tools, and distinct data collection processes, resulting in each thesis adopting a unique approach. Notably, the research models employed among the theses are diverse. The research models include experimental design (T1, T3), case study (T5, T9), single-subject design (T7), descriptive model (T2), survey model (T6), phenomenology (T5), action research (T4), and mixed methods (T8). The T1 and T3 coded theses utilized a pre-experimental design of a single group pre-test-post-test model, while the variable canon model was employed in the T7 coded thesis. Similarly, the theses employed an extensive range of techniques for data collection, encompassing tests and scales (T1, T2, T3, T6, T8), audio and video recordings (T2, T4, T5, T7, T9), documents such as reflective diaries and lesson plans (T4, T9), interviews and observations (T5, T8, and T9). Regarding the process of data collection, the theses included a multitude of long-term education and intervention processes (T3, T4, T7, T8), pre-test and post-test applications (T1, T3, T8), group applications as well as individual applications (T2, T4), and semi-structured in addition to unstructured observations (T5, T9).

The participants' characteristics of the theses

The participants in the theses were examined in terms of grade level, age, gender, number of participants, status of HI, communication methods, and the definition of HI. Information about the participants in the theses is presented in Table 4.

Table 4. *Characteristics of participants*

	Grade level/Age	Gender	Participants' number/Status of HI/Communication method	Definition of HI
T1	7^{th} grade HI students	Not specified	-25 - Students studying at deaf schools and diagnosed with HI	Definition associated with difficulties in development, adaptation, and communication
T2	6^{th} –8^{th} grade students with HI students/ aged between 11 years 9 months-17 years 7 months	Not specified	-In total 19 -Students with severe (n=4) and very severe (n=15) hearing loss -Hearing aid usage periods are between 1 year 6 months and 14 years.	Definitions pertaining to the results of hearing tests and the deviation from normal hearing thresholds
T3	-6^{th} grade typical students/not specified -8^{th} grade HI students/not specified	-15F-12M -14F-11M In total 29 F-23M	-52 -Students diagnosed with HI in schools for the deaf and inclusion schools	Definition related to the degrees and grouping of HI
T4	-7^{th} grade students with HI/aged between 13-15	-5F – 3M	-8 -8 students with bilateral sensorineural hearing loss studying at the school where	General description of the presence of hearing loss

	-Average age is 13,5		hearing impaired students are educated. -1 moderate, 4 severe, 2 very severe hearing loss students, -2 cochlear implant users, 6 behind-the-ear hearing aid users in both ears	
T5	-9th – 11th grade students with HI/ aged between 17-18	-2F – 1M	-3 -Students studying at a high school for the deaf, using hearing aids and having a good command of sign language -Advanced (n=2) and very severe (n=1) Hearing impaired -Congenital (n=1) and HI as a result of febrile illness (n=2)	Definition of difficulties in using verbal language and communication in daily life
T6	-5th – 8th grade students with HI / not specified	-64F – 62M	-126 -Students studying at a middle school for the students with HI and knowing sign language at different levels -Cochlear implant use (n=20), hearing aid use (n=69), fm system use (n=16), no device use (n=21)	Definition related to special education and support education service need
T7	-4th grade / aged between 10-11	-3F	-3 -Students studying at primary school for the hearing impaired who use the verbal-auditory method as a communication method and use cochlear implants.	Definition pertaining to the results of hearing tests and the deviation from normal hearing thresholds
T8	-8th grade / not specified	-4F – 8M	-12 -Students who know sign language studying at middle school for the hearing impaired	Definition of an individual's condition, which is characterized by diminished hearing sensitivity
T9	-Primary school level -Middle school level -Teachers	-2M -3F – 1M -1F – 1M -In total 4F – 4M	-8 -Students who study in middle and primary schools for the deaf, do not use hearing aids, and use Turkish sign language -4 secondary school students (3 girls, 1 boy), 2 primary school students (2 boys) - 2 deaf mathematics teachers (1 secondary school, 1 primary school level)	Definition of HI determined through hearing tests

HI: hearing impairment, F: female, M: male

According to Table 4, the research findings suggest that there is a presence of students with HI across various grade levels in the theses. The grade levels of the students range from primary school to high school. Furthermore, there have been comparative studies conducted on students with normal growing. The age range of the participants falls between 10 to 18 years, and although it is not explicitly stated in some theses, it is generally understood that primary, middle, and high school students are typically the subjects of theses. The focus of the theses was to evaluate students who received education in schools specifically designed for individuals with HI, as well as those who attended inclusive schools. It is worth noting that all participants in the theses were diagnosed with HI. The gender distribution of the participants varies; however, the analysis indicates that there is no significant disparity between the number of male and female students in the theses.

Regarding HI status, it was observed that there exist investigations regarding the instruction of mathematics to pupils with diverse HI within theses. Varying necessities and challenges of students were deliberated, considering factors such as the extent of hearing loss, school category, and communication methods. The hearing loss of the students who were selected as participants in the theses ranged from a minor to an exceedingly severe degree. Furthermore, the duration of the students' utilization of hearing aids, the types of hearing deficits, and the causes of hearing loss were also found to be variable.

The research findings indicated that eight distinct interpretations of the term "hearing impairment" were favored. Specifically, in T2 and T7, HI is defined as the variance between hearing tests results and the standard hearing thresholds. In T1, HI is defined as difficulties in developing, adapting, and communicating, whereas in T3, it is defined based on the degrees and categorization of HI. HI in T4 is described in terms of the presence of hearing loss and difficulties in utilizing verbal language and communication in daily life in T5. T6, on the other hand, defines HI as necessitating special education and support services, while T8 characterizes it as an inability caused by a decrease in hearing sensitivity. Finally, T9 defines HI as hearing loss identified through hearing tests.

The main results of the theses

Table 5 presents the main results on the instruction of mathematics to students with HI as outlined in theses. Through a comparative analysis of the two viewpoints, the main results were scrutinized, encompassing the impact and importance of diverse techniques and tools, as well as the process of acquiring mathematical knowledge.

Table 5. *The main results of the theses*

	Themes	Main results
T1	-The impact and importance of diverse techniques and tools	1.Students with HI can be instructed using a programmed teaching method. 2.Students with HI from all institutions benefited equally from the programmed teaching method in mathematics class.
T2	-The process of acquiring mathematical knowledge	1.Hearing impaired pupils demonstrated low to moderate problem-solving success. 2.As the problem's level of intricacy increases, the students' performance declines. 3.The mean hearing loss is not an independent variable that influences problem-solving abilities. 4.It has been determined that hearing aids are not being utilized properly. 5.It was determined that students did not acquire sufficient and necessary problem-solving experience.
T3	-The impact and importance of diverse techniques and tools -The process of acquiring mathematical	1.Computer-assisted education has led to improvements in the academic achievement and attitudes of both typically hearing and hearing-impaired students. 2.While there was no significant difference in terms of Van Hiele geometry levels among students with HI, there was a significant difference among students with normal hearing.

	knowledge	
T4	-The impact and importance of diverse techniques and tools	1.The students' math performance, their capacity to relate to the real world, and their language skills all improved. 2.It has been determined that the teacher has improved in terms of writing lesson plans, developing applications, and employing direct and indirect learning-teaching strategies.
T5	-The impact and importance of diverse techniques and tools -The process of acquiring mathematical knowledge	1.Sign language should be added to the definition of semiotic source because it has been determined that students with HI use sign language to describe polygons. 2.It was observed that, when defining concepts, pupils employed gestures more than sign language. 3.It has been determined that the features of polygons in the students' minds are manifested by gestures derived from their semiotic resources.
T6	-The impact and importance of diverse techniques and tools --The process of acquiring mathematical knowledge	1.The students' geometry self-efficacy varied according to their sign language proficiency and their grades in mathematics from the first semester. 2.The correlation between the students' Van Hiele geometric thinking levels and their geometry self-efficacy was found to be weakly positive. 3.There was no significant difference in the geometry self-efficacy of students with HI in the fifth, sixth, seventh, and eighth grades, regardless of whether they received support education or not or whether they had other HI in their family.
T7	-The impact and importance of diverse techniques and tools -The process of acquiring mathematical knowledge	1.It has been determined that abacus education is effective for increasing accuracy rates while making simple addition and subtraction operations straightforward and quick for hearing impaired students, as well as for increasing the average number of high-level numbers processed. 2.It has been determined that abacus education is effective in teaching addition and subtraction fluency in operations to the students with HI.
T8	-The impact and importance of diverse techniques and tools -The process of acquiring mathematical knowledge	1.The academic achievement of eighth-grade hearing-impaired pupils in fractions increased statistically significantly because of peer teaching. 2.There was no statistically significant shift in the attitudes of students toward mathematics. 3.The students stated that they found the peer-teaching method beneficial and enjoyable, that they retained their prior knowledge and gained a deeper understanding of the subject matter. 4.The students reported having difficulty being understood and being understood during the peer teaching process, as well as finding mathematics problematic.
T9	-The impact and importance of diverse techniques and tools	1.The devised digital instructional materials enhanced student learning and reduced distractions. 2.Students demonstrated interest in and proficiency with the material; they also requested further development in various subjects.

Based on the findings presented in Table 5, the main results of the theses referred to the impact and importance of diverse techniques and tools employed in the realm of mathematics education for students with HI. Several theses (T1, T3, T4, T5, T6, T7, T8, T9) offer solutions to increase the mathematical performance of students with HI. Various instructional practices have proven to be effective in enhancing the mathematical performance of students with HI. Notable examples include the implementation of programmed teaching methods, computer-assisted education, abacus education, peer teaching methods, and the utilization of digital teaching materials. According to the T4, there exists a close relationship between students' mathematical performance and their skill to connect mathematical concepts to real-life situations and language skills. T9 demonstrates that the integration of technology in mathematics instruction can facilitate and enhance students' learning process. Moreover, it has been claimed that the presence of hearing loss in isolation is not a significant factor in determining how students with HI learn mathematics. Additional factors, such as the experiences of students and the methods employed in teaching, also exert a significant influence. In T5 and T6, the significance of sign language is underscored in the context of mathematics education for students with HI, with T5 further emphasizing the importance of gestures.

The T2, T3, and T5, on the other hand, highlight particular differences in how hearing-impaired students acquire mathematics. T2 points out the weaknesses in the math performance of hearing-impaired students, whereas T3 claims that they perform similarly in math to students with normal hearing. For instance, in T2, it was discovered that hearing-impaired students had difficulties verifying the accuracy of their operation, drawing a figure or diagram to represent the problem, estimating the problem's result, performing the operations necessary to solve the problem and recording the result, comparing the result they found with their prediction before the solution, and explaining the reason. In other words, T2 asserts that students with HI perform poorly in problem-solving skills, that their success decreases as the difficulty of the problem increases, that hearing loss has no impact on their problem-solving skills, and that the effective usage of hearing aids is inadequate. T3 demonstrates that the academic achievement of students with HI in geometry may not be enhanced through computer-assisted instruction. The T5, on the other hand, presents several viewpoints on the significance of sign language in the mathematics learning process of students with HI as well as the significance of using gestures as a semiotic resource. The T6, T7, and T8 also highlight additional differences in how students with HI learn mathematics. According to T6, there are differences between students' self-efficacy in geometry, sign language proficiency, and mathematics grades. T7 shows how abacus education improves students with HI students' math performance. T8 asserts that while the peer teaching method improves the academic achievement of students with HI in fractions, there is no change in their attitudes about mathematics. Namely, even if the peer teaching method is helpful for hearing impaired students, math is still challenging for them.

The recommended teaching practices of the theses

Figure 2 depicts the recommended teaching practices for theses. Five themes are used to examine recommendations for teaching mathematics to hearing-impaired students. These include recommendations for utilization within the classroom, recommendations for structuring the learning environment, recommendations regarding student characteristics, recommendations for supporting students, and recommendations for evaluating students.

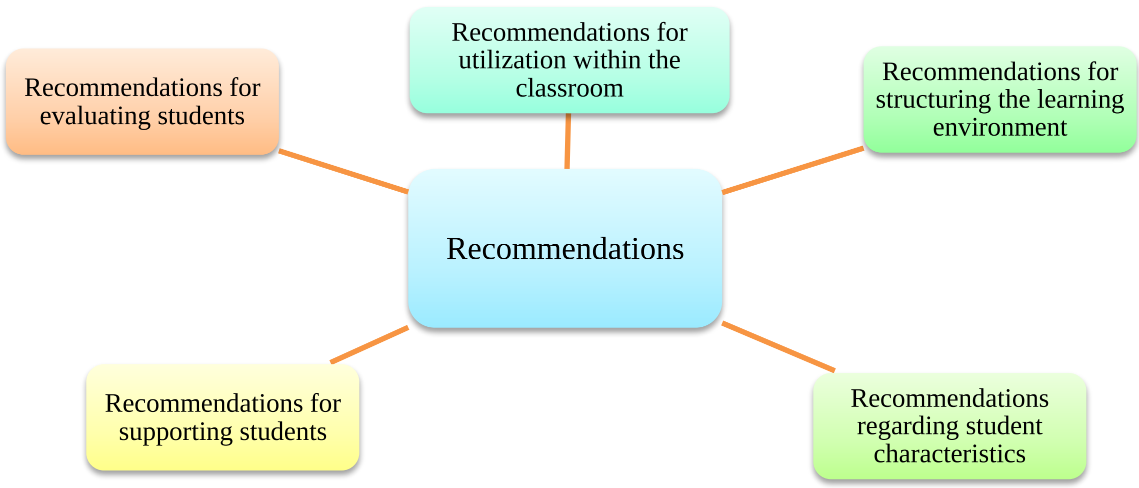

Figure 2. *Recommendations presented in the theses for teaching mathematics to hearing-impaired students*

Figure 2 shows a range of recommendations provided in the theses to support hearing-impaired students in mathematics instruction. T2 specifies that problem-solving activities ought to be long-term, geared towards individual needs, and incorporate meaningful language inputs. The implementation of sign language is proposed in T5 and T6 to encourage effective communication and support between students and teachers. Abacus training is recommended in T7 to enhance the operational fluency and attention of hearing-impaired students, while peer teaching in T8 is put forth as a means of supporting mathematics learning and improving communication skills. The second theme of recommendations highlighted in the theses focuses on methods proposed for use in the learning environment. Accordingly, in T1, the programmed instruction method is suggested to facilitate mathematics learning for hearing-impaired students, and in T2, problem-solving activities are recommended to foster development of mathematical skills. The third theme of recommendations focuses on learning environment. T3 proposes dynamic geometry software to enrich geometry learning for hearing-impaired students, while T5 advocates for mathematics books to contain definitions and properties of concepts as well as signs and concrete models involving these concepts. The fourth theme of recommendations pertains to student characteristics. T2 suggests that problem-solving skills of hearing-impaired students should be assessed based on their characteristics, while T3 recommends the use of Euclidean Reality software suitable for all levels, given that hearing-impaired students may have varying degrees of hearing loss. Another theme of recommendations focuses on the evaluation of hearing-impaired students. T4 recommends preparing booklets of enrichment activities related to the mathematical topics and developing evaluation tools to assess mathematical knowledge of hearing-impaired students. T9 suggests that digital materials on various topics could be developed to evaluate mathematics learning of hearing-impaired students.

DISCUSSION AND CONCLUSION

This study investigates the intricate landscape of mathematics education for students with HI, focusing on an analysis of Turkish graduate theses. The aim is to uncover the existing educational circumstances and suggested teaching practices in this field. The investigation is based on data obtained from the YÖK National Thesis Center in Türkiye. This data provides valuable information regarding the aims, research methodology, participant profiles, and primary findings of the theses, and recommended teaching practices. This study enhances our comprehension of the complicated relationship between instructional practices and educational results within this unique situation at hand. While the research does not primarily focus on generalization, its objective is to provide a comprehensive overview of the educational landscape depicted in Turkish graduate theses. Specifically, the research examines the teaching of mathematics to this specific population.

Addressing the multifaceted dimensions of mathematics education for hearing-impaired students, the analyzed theses spotlight key issues ranging from pedagogical approaches to the invaluable insights of the students themselves. The theses addressed several subjects, including impacts, due diligence, relationship identification, perspectives, and a proposed teaching method, in alignment with their respective aims. The primary emphasis of these theses was to examine the impacts of teaching mathematics to students with HIs. researchers investigated the impact of different approaches, tools, activities, and resources on learning outcomes. The investigation revealed the success of programmed, balanced, and peer teaching methods, underscoring the need for diverse, individualized teaching strategies for this particular student population. Another consistent theme was the examination of various factors affecting learning environments and student outcomes. The interconnected aspects of student characteristics, problem-solving behaviors, test scores, and semiotic resources were all brought into sharp light by the relationship-focused theses. Teachers' struggles in selecting the appropriate teaching methods were highlighted, pointing the gaps in understanding the factors influencing learning outcomes of hearing-impaired students in mathematics instruction (Sarıkaya & Börekçi, 2016). This emphasizes the necessity for further research into the intricate interactions between diverse variables such as student characteristics and problem-solving behaviors. The importance of understanding students' perspectives was also evident, revealing the value of their feedback in creating an inclusive educational setting (Nurhasanah & Suryaman, 2022). Lastly, innovation in educational approaches was signified by a thesis proposing a tailored teaching model for math education. This resonates with the need for a multifaceted teaching strategy that addresses the unique educational requirements of hearing-impaired students, as substantiated by various teaching methods such as programmed, balanced, and peer teaching and tools such as software-assisted dynamic geometry activities and abacuses (Pradhan, 2020; Tongwa & Atemnkeng, 2019).

Unveiling the intricate tapestry of methodological diversity, the examined theses collectively signal the imperative for an integrative research paradigm in the realm of mathematics education for hearing-impaired students. The study designs employed in the theses were notably varied, encompassing a spectrum of research models, data collection tools, and processes. This methodological diversity facilitates a broader assessment of educational practices targeting students with HI. Given this context, future research in the mathematics education of hearing-impaired students would benefit from embracing a similarly diverse set of methodologies. The research designs employed ranged from experimental to phenomenological, encapsulating case studies, action research, single-subject designs, surveys, descriptive models, and mixed methods, among others. This varied landscape of research models reflects the myriad questions that have been addressed, underscoring the multi-layered inquiry needed to cater to the unique educational demands of hearing-impaired students. In light of this, no single research design could adequately capture the intricate nuances of this specialized pedagogical environment. Innovative data collection methods, such as longitudinal studies, might serve to augment this multi-methodological approach. Indeed, a variety of methodologies is indispensable for an exhaustive evaluation of complex educational practices (Miller-Young & Poth, 2022; Ocean & Hicks, 2021). The application of varied data collection tools provided researchers with more in-depth and comprehensive insights into the mathematics education of hearing-impaired students. While some theses utilized standardized tests and scales to measure students' mathematical skills and attitudes, others employed methods such as video and audio recordings, reflective diaries, lesson plans, and document analyses to gain more detailed knowledge. Moreover, observations and interviews further enriched researchers' grasp of both student and teacher experiences. Employing such diverse data collection tools can enable teachers and researchers to acquire a more detailed understanding of the mathematical skills and attitudes of students with HI, as well as to gain better insights into their experiences and perspectives (Rishaelly, 2017; Sarı & Pürsün, 2018). Similarly, the data collection process demonstrated a range of approaches, from long-term educational and intervention processes to pre-test and post-test applications, individual and group applications, and structured to unstructured observations. These divergent methods offered avenues for teachers and researchers to explore ways of enhancing the mathematical skills of students with HI. For instance, some theses employed pre-tests and post-tests as a significant measure, particularly in research using single-group pre-test post-test designs and mixed methods. Others adopted more descriptive and survey models to study the status and abilities of these students. Such long-term educational interventions enable a sustainable evaluation of the impact of mathematical practices on hearing-impaired students (Elliot et al., 2020; Tanrıdiler, Uzuner & Girgin, 2015). Studies conducted through methodologies such as action research and single-subject experimental designs allow for closely tracking of educational processes and their effects. This is paramount for understanding the evolving trajectory of educational process and its outcomes in the context of teaching mathematics to students with HI (Opfer et al., 2020).

Probing the intricacies of participant characteristics, the analyzed theses spotlight the heterogeneity inherent in the educational settings for students with HI, thereby illuminating the need for tailored, context-sensitive approaches. Detailed examinations were conducted on the characteristics of the participants, such as grade level, age, gender, the number of participants, school types, status of hearing impairment, communication methods, and definition of HI. Age information was either unspecified or ranged from 10 to 18 years. When gender distribution was articulated, it was either balanced or left unmentioned. Such variations in gender data underscore the role of gender considerations in educational research, as learning environments, attitudes, and outcomes can differ between male and female students (Meyer et al., 1994; She, 2000). This highlights the need for educators to be attuned to these potential divergences when formulating teaching plans. The number of participants in the theses varied dramatically, ranging from as few as three to as many as 126. This range underscores the diverse scales at which these investigations were conducted, each with its own implications for the scope and detail of data collection. Theses involving larger participants pools could lean towards more generalizable conclusions, whereas small-scale theses might offer more detailed insights. Likewise, the students' grade levels spanned from fourth to eleventh grades, and in one instance, teachers were also included as participants. The inclusion of teachers not only adds another layer of complexity but also elevates the discourse around the importance of teacher training and pedagogical strategies in the context of educating students with HI (Li & Miloň, 2022).

Navigating the complex terrain of student diversity in education for the hearing-impaired, the analyzed theses illustrate the intricate variables at play, such as school type, HI status, and communication methods, thereby punctuating the need for an equally nuanced pedagogical lens. Factors like the type of school attended, HI status, and preferred communication methods were meticulously analyzed to adress the diverse challenges and needs of the students. The students were part of various educational settings, including schools specifically for the hearing-impaired and inclusive schools. This diversity in educational settings indicates the profound impact that the type of school can have on educational experiences and outcomes (Li & Miloň, 2022). The status of HI among the participants also varied widely. Some theses focused on students with severe or extremely severe HI, while others included students with varying degrees of impairment. The varying degrees of HI necessitate the development of tailored teaching strategies that cater to the unique requirements of each student (Shields & Lennox, 2017). Additionally, comparative studies were conducted that included students with normal development patterns. While hearing-impaired students are generally considered to have a slower learning process (Drigas et al., 2005; Epstein et al., 1994; Reuterskiold et al., 2010), these comparative studies enrich our understanding of the specific educational impacts of HI. As for communication methods, they varied significantly across the student population. Methods ranged from sign language and the verbal-auditory method to the use of various hearing aids, such as FM systems, behind-the-ear hearing aids, cochlear implants, and no-device use. Individuals with HI have been shown to naturally adopt sign language in conducive environments (Goldin-Meadow, 1993) and use it as a visual-spatial language for communication (Sandler & Lillo-Martin, 2006; Valli & Lucas, 2000). The heterogeneity in communication methods amplifies the need for adaptable, inclusive teaching strategies in the mathematics education of hearing-impaired students. While the emphasis on the verbal-auditory method was not pronounced in the theses, it has been suggested that verbal communication skills for students with HI should be improved (Li & Miloň, 2022; Rudge et al., 2022). Lastly, the term "hearing impairment" was interpreted in eight different ways across the analyzed theses. This terminological diversity underscores the multifaceted nature of HI, affecting students experiences and outcomes in a myriad of ways (Woods, 2022). This comprehensive understanding can further support the development of diverse, tailored educational strategies, acknowledging the unique challenges that students with HI may encounter.

Charting the landscape of instructional innovation, the analyzed theses shed light on a repertoire of effective teaching methods and technological tools, thereby providing a roadmap for enhancing the mathematical learning experiences of students with hearing impairment. These theses provide in-depth insights into an array of techniques and tools that are effective in teaching mathematics to students with HI. Various teaching strategies-such as programmed teaching, computer-assisted education, abacus-based methods, peer teaching, and the employment of digital materials- were underscored for their positive influence on learning outcomes (Adeniyi & Kuku, 2020; Makinde, 2012). A strong correlation emerged between students' mathematical proficiency and their ability to employ these concepts in real-world contexts. This finding accentuates the significance of fostering practical application skills in students, a focus supported by existing literature (Kemp & Vidakovic, 2021; Loi et al., 2020). Moreover, the incorporation of technology into the educational process was shown to markedly enhance both the learning experience and academic outcomes. Such technological integration is not only beneficial but also crucial for optimizing the educational trajectory of students with HI (Liu et al., 2006; Pagliaro, 1998; Suarsana et al., 2019). Interestingly enough, the theses revealed that HI is not the sole variable affecting learning outcomes in mathematics; other factors like students' backgrounds and the specific pedagogical approaches employed also have a considerable impact. This suggests that a multifaced approach to understanding learning outcomes is necessary, one that considers variables beyond just HI (Tongwa & Atemnkeng, 2019).

Unpacking the synergies of sign language and personalized teaching strategies, the analyzed theses illuminate the complexities and opportunities in fostering mathematical proficiency among students with HI. The theses underscore the pivotal role of sign language as an essential instrument in teaching mathematics to students with HI, particularly when augmented by gestures (Krause, 2019). The research not only identified challenges-such as the difficulty in portraying mathematical problems and verifying calculations, and rationalizing solutions - but also revealed that these students can achieve academic success comparable to their hearing peers when equipped with suitable instructional approaches like computer-assisted teaching (Reuterskiold et al., 2010). A notable correlation emerged between self-efficacy in geometry, proficiency in sign language, and mathematics grades. The utilization of specific pedagogical techniques like abacus-based education and peer teaching appeared to significantly bolster academic performance, especially when accounting for the interconnected dynamics of self-efficacy, sign language proficiency, and academic grades in mathematics (Mtuli, 2015). However, it was observed that even these effective strategies might not substantially alter students' perceptions of fundamental mathematical challenges. This observation accentuates the complex nature of educating students with HI in mathematics, emphasizing multitude of contributing factors and the efficacy of customized teaching approaches.

Serving as a navigational guide through the complex realm of mathematics education for hearing-impaired students, this study categorizes its findings into five pivotal areas: classroom utilization, learning environment structure, consideration of student characteristics, student support, and evaluation methods. The overarching point of these recommendations emphasizes a learner-centered approach, adapting teaching methodologies and learning environments to each student's unique attributes and needs. In the realm of classroom utilization, recommendations showcase long-term individualized problem-solving activities (Ovadiya, 2021; Kelly et al., 2003), the role of sign language (Heslinga & Nevenglosky, 2012), abacus training (Jadhav & Gathoo, 2018), and peer teaching (Burley et al., 1994) as key elements. This focus on classroom utilization aligns with the study’s theme of a learner-centered approach. When it comes to structuring learning environments, the recommendations point toward the incorporation of programmed instruction, dynamic geometry software, and comprehensive mathematics textbooks. Such structural elements take on added significance given that hearing-impaired individuals have access to fewer learning environments compared to their typically developing peers (Wilkens & Hehir, 2008). In the context of student characteristics, the study highlights the need to evaluate problem-solving skills tailored to each student's unique traits along with the use of adaptable software for varying degrees of HI. For student support, the development of enrichment activity booklets and diverse digital materials were observed to be particularly beneficial, a domain where contributions could be invaluable. In evaluation, the study accentuates the necessity for innovative tools to assess the mathematical knowledge of hearing-impaired students (Lahdenperä et al., 2023). The frequent references to sign language, problem-solving activities, and technological aids like dynamic geometry software and digital materials underline their importance in this educational context. Finally, the consistent integration of these elements into teaching practices is crucial (Heslinga & Nevenglosky, 2012; Kelly et al., 2003).

RECOMMENDATIONS

The results of this research present actionable recommendations that are beneficial for educators, policymakers, and researchers in the realm of mathematics education for students with HI. Emphasizing long-term, individualized problem-solving activities can significantly enhance the comprehension of mathematical concepts among these students. Nevertheless, it is crucial to acknowledge that the research is predominantly grounded in graduate theses originating from Türkiye, hence constraining its applicability to broader educational settings. Furthermore, the integration of sign language as a teaching tool for students with HI is of paramount importance. Given the compelling evidence surrounding the advantages of abacus training and peer teaching, educators are encouraged to consider these methods as part of their instructional strategies. Although the aforementioned teaching methods have exhibited potential, the study highlights a deficiency in the accessibility of specialized training for educators in these particular approaches.

Adjusting the learning environment to include programmed instruction and the utilization of dynamic geometry software can notably elevate the educational experiences of students with HI. However, the present study emphasizes the constraints associated with the lack of comprehensive technological infrastructure in educational environments, which hinders the effective use of these technologies. Alongside these modifications, there's a clear necessity for more detailed mathematics textbooks tailored specifically to the unique learning needs of these students. It is imperative to acknowledge that the availability of these specific materials is presently constrained, underscoring the necessity for additional advancement. When assessing these students, evaluations of their problem-solving skills should be individualized, reflecting the variety and nuances within the HI category. To achieve this, the adoption of adaptable software solutions specifically designed for this demographic is recommended. Nevertheless, the research highlights a constraint in the accessibility and flexibility of these particular software solutions.

In terms of resources, offering enrichment activity booklets alongside a diverse array of digital materials is vital to meeting the educational requirements of students with HI. These resources play a significant role in enhancing their learning trajectories. The primary constraint in this context pertains to the limited availability of resources that educational institutions may encounter while attempting to provide materials that are more comprehensive and diverse in nature. Moreover, there is an urgent call to design bespoke assessment methods that can accurately gauge the mathematical capabilities of this particular student group, ensuring that educators and policymakers have a deeper, more insightful perspective on their academic growth. However, the approaches examined in this study demonstrate a deficiency in the availability of customized assessment tools.

From a policy and broader system perspective, it is essential for policymakers to prioritize integrating these teaching strategies into tailored educational blueprints for students with HI. There's also a pronounced demand for continuous teacher training programs that emphasize best practices specifically tailored for this student group. Initiatives in this direction promise not only to uplift the quality of education but also to foster an inclusive and accommodating learning atmosphere. Nevertheless, the findings of the study suggest that the implementation of systemic changes tends to be a gradual process, so highlighting a constraint in the immediate practicality of these suggestions.

Lastly, the current research landscape indicates discernible gaps in our understanding of education for students with HI. These voids in knowledge present prime opportunities for future scholarly exploration. By leveraging diverse research methodologies and data collection tools, the academic community can gain a more enriched and comprehensive understanding of effective educational practices for students with HI.

Conflict Interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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