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REVIEW ARTICLE

Analytical Solution of Three-Dimensional Stable Conduction Heat Transfer on a Composite Plate with Internal Energy Generation

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ABSTRACT

In this study, an analytical solution is presented for the conductive heat transfer of a plate made of composite materials at stable conditions with energy generation and radiation flux within the composite plate. The proposed exact solution is useful for studying heat transfer in tanks and composite panels. In addition, the coefficient of thermal conductivity tensor for composite materials is introduced and a method for determining the coefficient based on the properties of the matrix is introduced. The heat transfer equation in the Cartesian coordinate system for composite materials is obtained when the fibers in each layer are wrapped around the plate. Then, using the method of separating the variables, an exact solution for this equation is presented under certain boundary conditions.

Contents

1. Introduction

Today, the use of composite materials for the construction of equipment, machinery and structures has developed significantly. The use of these materials results in lightening of equipment and structures (while maintaining mechanical strength) and lowering costs. The use of these materials in some industries (chemical industry, gas turbine blades, military industry, aerospace and so on) is unrivaled with isotropic materials. Scientific studies have focused more on the behavior of these materials than on their mechanical and thermal properties and less on other phenomena such as heat transfer in these materials. So far, some activities have been carried out in the field of heat transfer analysis in non-isotropic materials. Initial analyzes focused mainly on one-dimensional heat transfer in non-isotropic crystals. So far, activities have been carried out in the field of heat transfer analysis in non-isotropic materials. Preliminary analyzes mainly focused on one-dimensional heat transfer in non-isotropic crystals [1, 2].

Gradually, with the development of knowledge of composite materials, heat transfer in these materials has also been considered. Mulholland's paper on the phenomenon of intermittent diffusion in orthotropic cylinders is one of the first activities in this field [3]. Today, continuous activities are carried out to present new formulations and to study heat transfer in composite materials.

Today, Golovchan [4] and Shi-qiang [5] have conducted research in pursue of introducing new formulations and to study heat transfer in composite materials. Greengard examined the theory of heat transfer and the determination of conductive properties in composites in his article [6].

The examination of heat transfer in the production process of composite materials is also of great importance. For

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example, the application of heat transfer in non-isotropic materials made by Newnham [7] has been done by the finite element method. As an example of a one-dimensional analytical heat transfer solution, a study by San [8] offers a non-permanent solution in a composite block.

Oseloka [9] obtained a response for heat transfer at composite interfaces using Green's functions and integral formulation of the heat transfer equation . In another study, Lu et al. [10] and Halpin et al. [11] developed exact solutions for the discontinuous heat transfer in a composite structure in radius and axis directions and under different boundary and initial conditions.

In this study, three-dimensional stable heat transfer in a compound cube was investigated. In addition, an analytical solution for the temperature distribution of the cube is obtained under the simultaneous influence of solar radiation and a constant temperature from inside and outside, and natural heat transfer is investigated. In this study, the method of separation of variables was used to better solve heat transfer equations.

2. Modeling, Equations and Boundary Conditions

In this study, stable conduction heat transfer in the composite plate was investigated. Figure 1 shows a plate at x, y and z positions.

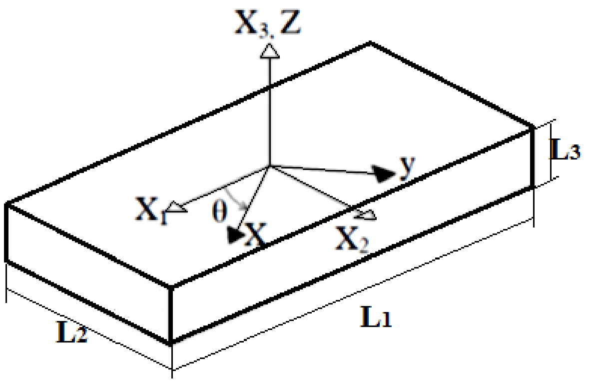

Figure 1 3D coordinate systems

It is also assumed that the internal energy production is a function of r and is defined as follows:

$$\dot{g}=\frac{\gamma}{r^2}\dot{q} \tag{1}$$

Here (γ) is a constant coefficient. A cubic element similar to Figure 1 should be considered to determine the heat transfer equation. In the Cartesian coordinate system, the Fourier equation hip in the orthotropic material is as follows:

$$\begin{Bmatrix} q_x \\ q_y \\ q_z \end{Bmatrix} = -\begin{bmatrix} \bar{k}_{11}\bar{k}_{12}\bar{k}_{13} \\ \bar{k}_{21}\bar{k}_{22}\bar{k}_{23} \\ \bar{k}_{31}\bar{k}_{22}\bar{k}_{33} \end{bmatrix} \begin{Bmatrix} \frac{\partial T}{\partial x} \\ \frac{1}{r}\frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{Bmatrix} \tag{2}$$

The following equations are written for a better understanding of heat transfer in orthotropic materials:

$$\begin{aligned} \bar{k}_{11}\frac{1}{r}\frac{\partial}{\partial x}\left(r\frac{\partial T}{\partial x}\right)+\bar{k}_{22}\frac{1}{x^2}\frac{\partial^2 T}{\partial y^2}+\bar{k}_{33}\frac{\partial^2 T}{\partial z^2} & \\ +\left(\bar{k}_{12}+\bar{k}_{21}\right)\frac{1}{r}\frac{\partial^2 T}{\partial y\partial x} & \\ +\left(\bar{k}_{13}+\bar{k}_{31}\right)\frac{\partial^2 T}{\partial x\partial z}+\frac{\bar{k}_{13}}{r}\frac{\partial T}{\partial x} & \\ +\left(\bar{k}_{23}+\bar{k}_{32}\right)\frac{1}{r}\frac{\partial^2 T}{\partial y\partial z}+\frac{\gamma}{r^2}\dot{q} & \\ =\rho c\frac{\partial T}{\partial t} & \end{aligned} \tag{3}$$

In the above equations:

$$\begin{aligned} \bar{k}_{11} &= k_{22} \\ \bar{k}_{22} &= m_l^2 k_{11}+n_l^2 k_{22} \\ \bar{k}_{33} &= n_l^2 k_{11}+m_l^2 k_{22} \\ \bar{k}_{12} &= \bar{k}_{21}=0 \\ \bar{k}_{13} &= \bar{k}_{31}=0 \\ \bar{k}_{23} &= \bar{k}_{32}=2m_l n_1\left(k_{11}-k_{22}\right) \end{aligned} \tag{4}$$

Plugging Equation 3 in Equation 4, we get:

$$\begin{aligned} k_{22}\frac{1}{r}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\left(m_l^2 k_{11}+n_l^2 k_{22}\right)\frac{1}{r^2}\frac{\partial^2 T}{\partial y^2} & \\ +\left(n_l^2 k_{11}+m_l^2 k_{22}\right)\frac{\partial^2 T}{\partial z^2} & \\ +2m_l n_1\left(k_{11}-k_{22}\right)\frac{1}{r}\frac{\partial^2 T}{\partial y\partial z} & \\ +\frac{\gamma}{r^2}\dot{q}=\rho c\frac{\partial T}{\partial t} & \end{aligned} \tag{5}$$

If the boundary conditions are not a function of z and the temperature gradient of z is zero and the steady-state temperature gradient term is zero, then:

$$k_{22}\frac{1}{r}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\left(m_l^2 k_{11}+n_l^2 k_{22}\right)\frac{1}{r^2}\frac{\partial^2 T}{\partial y^2}+\frac{\gamma}{r^2}\dot{q}=0 \tag{6}$$

Outside the plate, there is a boundary condition for both natural convection and solar radiation, therefore:

$$-k_{22}\frac{\partial T}{\partial z}=-Q+h\left(T-T_{\infty}\right) \tag{7}$$

where T_{∞} is the ambient temperature, h is the heat transfer coefficient on the outer surface of the plate at ambient temperature, and Q is the radiative flux from the sun, and the following equation hip is obtained:

$$Q=\varepsilon\sigma A{T_s}^4 \tag{8}$$

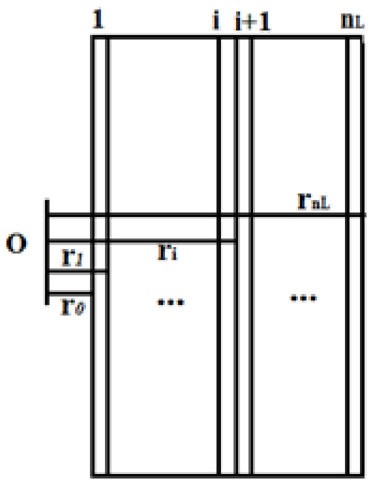

Figure 2 Placing of the plate in the composite material

Figure 2 shows a composite plate made up of several layers, where the direction of the fibers in each layer may be different from the adjacent layer. Therefore, Equation 4 will be different for each of the layers and there will be continuity of temperature and continuity of temperature flux between layers with new boundary conditions. If i and i+1 are the boundaries between the two layers r=r_i, then:

$$T^{(i)} = T^{(i+1)} \& - k_{22}\frac{\partial T^{(i)}}{\partial r} = -k_{22}\frac{\partial T^{(i+1)}}{\partial r} \tag{9}$$

2.1. Analytical Conductivity Solution in Composite Plate Material

In this section, the analytical solution of the conduction heat transfer equation of Equation (4) in the conditions described in the previous section presents the stable temperature distribution with energy generation in the cylinder and the boundary case of simultaneous displacement and radiation inside the cylinder. By writing Equation (4) again, Equation (8) will be obtained [12]:

$$\alpha^2\frac{\partial^2 T}{\partial x^2} + \alpha^2\frac{1}{r}\frac{\partial T}{\partial x} + \frac{1}{\beta^2}\frac{1}{x^2}\frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{r^2}\dot{q} = 0 \tag{10}$$

where

$$\alpha^2 = k_{22} \quad ve\frac{1}{\beta^2} = n_l^2 k_{11} + m_l^2 k_{22} \tag{11}$$

Here, the decomposition method is used to solve Equation (8).

$$T(z, y) = R(z)\,G(y) \tag{12}$$

By applying Equation (10) to Equation (8) the heat transfer equation is split by the following two equation s:

$$\begin{cases} \dfrac{\partial^2 G}{\partial y^2} + \beta^2\mu^2 G = 0 \\ \alpha^2 r^2\dfrac{\partial^2 R}{\partial x^2} + \alpha^2 r\dfrac{\partial R}{\partial x} + (\gamma\dot{q} - \mu^2)R = 0 \end{cases} \tag{13}$$

In this equation, the parameter (μ) represents the eigenvalues of the heat transfer equation and its value is calculated from the application of boundary conditions. Equation 11 is also called the Cauchy-Euler differential equation.

$$R(r) = \begin{cases} c_1 r^{(\lambda_n)} + c_2 r^{(-\lambda_n)} n > 0 \\ c_3 r^{(\vartheta_n)} + c_4 r^{(-\vartheta_n)} n = 0 \end{cases} \tag{14}$$

Here:

$$\mu_n = \frac{n}{\beta}, \vartheta = \sqrt{\frac{\gamma\dot{q}}{\alpha^2}}, \lambda_n = \sqrt{\frac{\gamma\dot{q} - \mu^2}{\alpha^2}} \tag{15}$$

In this study, the response of the Euler part of the Fourier series with respect to the internal elongation (r_o) is measured to equate all the Fourier series coefficients (with the temperature dimension) of the temperature distribution. To homogenize the boundary condition inside the cube by applying the variable (τ =T-T_{in}) to Equation (9), the following equation for the temperature distribution is obtained:

$$\tau^{(i)}(r, y) = a_0^{(i)}(\frac{r}{r_0})^{\vartheta} + b_0^{(i)}(\frac{r}{r_0})^{-\vartheta} + \sum_{n=1}^{\infty}\left(a_n^{(i)}(\frac{r}{r_0})^{\lambda_i} + b_n^{(i)}(\frac{r}{r_0})^{-\lambda_i}\right) + \left(c_n^{(i)}(\frac{r}{r_0})^{\lambda_i} + d_n^{(i)}(\frac{r}{r_0})^{\lambda_i}\right) \tag{16}$$

In the above equation hip, the coefficients (a_n, b_n, c_n, and d_n) are constant coefficients (in terms of temperature) of the Fourier series. Also, in Equation (14), the headings and indices represent layer (i) of the composite sheet. Boundary conditions are necessary to determine the Fourier coefficients. Since the temperature in the cylinder is assumed to be a constant value (T_{in}) in this study, this situation appears as ($\tau(r = r_0, \varphi) = 0$) in Equation (14), and by applying it, the following equation s are obtained for the Fourier coefficients of the first layer:

$$a_0^{(1)} + b_0^{(1)} = 0 \tag{17}$$

$$a_n^{(1)} + b_n^{(1)} = 0 \tag{18}$$

$$c_n^{(1)} + d_n^{(1)} = 0 \tag{19}$$

Boundary conditions of temperature continuity and heat flux continuity Equation 14 are established between the layers. By plugging Equations 15, 16 and 17 in Equation 14, the following results are obtained at the boundary of the layers at the composite interface (radius r=r_i) [12]:

$$a_0^{(i)} + b_0^{(i)} = 0 \tag{20}$$

$$a_0^{(i+1)} + b_0^{(i+1)} = 0 \tag{21}$$

$$a_n^{(i)}(\frac{r_i}{r_0})^{\lambda_i} + b_n^{(i)}(\frac{r_i}{r_0})^{-\lambda_i} - a_n^{(i+1)}(\frac{r_i}{r_0})^{\lambda_{i+1}} + b_n^{(i+1)}(\frac{r_i}{r_0})^{-\lambda_{i+1}} = 0 \tag{22}$$

$$c_n^{(i)}(\frac{r_i}{r_0})^{\lambda_i} + d_n^{(i)}(\frac{r_i}{r_0})^{-\lambda_i} - c_n^{(i+1)}(\frac{r_i}{r_0})^{\lambda_{i+1}} + d_n^{(i+1)}(\frac{r_i}{r_0})^{-\lambda_{i+1}} = 0 \quad (23)$$

$$a_n^{(i)}(\frac{r_i}{r_0})^{\lambda_i-1} + b_n^{(i)}(\frac{r_i}{r_0})^{-\lambda_i-1} - a_n^{(i+1)}\left(\frac{\lambda_i}{\lambda_{i+1}}\right)(\frac{r_i}{r_0})^{\lambda_{i+1}-1} + b_n^{(i+1)}\left(\frac{\lambda_i}{\lambda_{i+1}}\right)(\frac{r_i}{r_0})^{-\lambda_{i+1}-1} = 0 \quad (24)$$

$$c_n^{(i)}(\frac{r_i}{r_0})^{\lambda_i-1} + d_n^{(i)}(\frac{r_i}{r_0})^{-\lambda_i-1} - c_n^{(i+1)}\left(\frac{\lambda_i}{\lambda_{i+1}}\right)(\frac{r_i}{r_0})^{\lambda_{i+1}-1} + d_n^{(i+1)}\left(\frac{\lambda_i}{\lambda_{i+1}}\right)(\frac{r_i}{r_0})^{-\lambda_{i+1}-1} = 0 \quad (25)$$

(18,19,20,21,22 and 23) It should be noted that their equation hip is valid only at the boundary between layers and has no validity outside the plate. On the outer surface of the plate, the boundary condition of natural displacement and radiation is established simultaneously and must be placed in relation to Equations (14), (5) for it to be applied. In this case, the following equations were obtained:

$$k_{22}(a_0^{(nL)}(\frac{\vartheta}{r_0})(\frac{r_{nL}}{r_0})^{\vartheta-1} + b_0^{(nL)}(\frac{\vartheta}{r_0})(\frac{r_{nL}}{r_0})^{-\vartheta-1}) + h(T_\infty - T_{in}) = \frac{Q}{\pi} \quad (26)$$

$$a_n^{(nL)}\left[k_{22}((\frac{\lambda_n}{r_0})(\frac{r_{nL}}{r_0})^{\lambda_n-1} + h(\frac{r_{nL}}{r_0})^{\lambda_n}\right] + b_n^{(nL)}\left[h(\frac{r_{nL}}{r_0})^{-\lambda_n} - k_{22}((\frac{\lambda_n}{r_0})(\frac{r_{nL}}{r_0})^{-\lambda_n-1}\right] = \begin{cases} 0 \longrightarrow n = odd \\ \dfrac{2Q}{\pi(1-n^2)} \longrightarrow n = even \end{cases} \quad (27)$$

$$c_n^{(nL)}\left[k_{22}((\frac{\lambda_n}{r_0})(\frac{r_{nL}}{r_0})^{\lambda_n-1} + h(\frac{r_{nL}}{r_0})^{\lambda_n}\right] + d_n^{(nL)}\left[h(\frac{r_{nL}}{r_0})^{-\lambda_n} - k_{22}((\frac{\lambda_n}{r_0})(\frac{r_{nL}}{r_0})^{-\lambda_n-1}\right] = \begin{cases} 0 \longrightarrow n > 1 \\ \dfrac{q''}{2} \longrightarrow n = 1 \end{cases} \quad (28)$$

According to Equations (15) - (24) it is concluded that the sum (a_o and b_o) is equal in all layers and its values can be calculated from Equations (25 and 26). The coefficients in this study are determined using the Gaussian elimination method and the cross-coefficient matrix, and the same method can be used to determine the coefficients (a_n, b_n, c_n and d_n).

3. Conclusion

In this study, the equation hip of tensor and heat transfer in composite materials is introduced and the method of determining the conductivity coefficients of these materials is discussed, and then an analytical solution for the heat transfer of the composite plate in two-dimensional Cartesian coordinates is presented. The proposed definitive solution can be used directly on vehicles or machines for clothing with composite materials. The equation hips obtained for stable conductive heat transfer in a composite plate (25 and 26) are the general form of heat transfer for this plate and can be used for all thermal boundary conditions. Boundary conditions are only effective in the amount of Fourier coefficients of the above equation hip, and these coefficients can be obtained using any desired boundary condition inside and outside the plate. It should also be noted that the interlayer thermal conditions related to temperature and heat flux continuity are independent of the boundary conditions inside and outside the plate. In this study, a boundary condition where the temperature inside the plate is constant and the outer surface is constant under the simultaneous effect of free transfer heat transfer and solar radiation is investigated.

Conflict of Interest

The authors declared no conflict of interest.

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