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Fair Division of Scarce Resources: A Brief Survey of Claims Problems

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Abstract

This paper studies the fair division of a scarce resource among competing claimants, a problem commonly referred to as the claims problem in the literature. This model has numerous practical applications, ranging from estate division and rationing to bankruptcy and even fundamental taxation issues. An axiomatic approach is utilized to examine the properties of division rules, which are represented as mathematical axioms. The paper presents a comprehensive examination of prominent division rules, including the proportional, egalitarian, and parametric rules, and provides their respective axiomatic characterizations. Ultimately, the paper presents a comprehensive framework for achieving a fair and equitable allocation of scarce resources among competing claimants.

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1. Introduction

What is the best way to divide a resource when competing claims exceed the available amount? This is known as a "claims problem", and a division rule is used to determine how the resource should be allocated among the claimants.

The model we are studying has many applications in real-life situations. One of the primary applications is estate division, where a man dies and his estate is insufficient to cover his debts, and we need to decide how to divide it among his creditors. The problem of inheritance is the earliest recorded example of the rationing problem, as evidenced by sources such as O'Neill (1982) and Rabinovitch (1973) who cite examples from the Babylonian Talmud. Another related application is the bankruptcy problem, where the liquidation value of a bankrupt firm needs to be allocated among its creditors (see Aumann and Maschler (1985)).

Another example of the application of this model is rationing, where a firm has to allocate a limited amount of a commodity among its consumers who have placed orders for it. This rationing problem can also occur at the level of nations, where scarce resources such as food, clean water, medical supplies, or shares of the global carbon budget must be distributed among different states or provinces. Similarly, when international agencies distribute aid to impoverished countries, they often have to contend with limited resources that are insufficient to meet all the needs.

Our model can also serve as a formalization of basic taxation problems. In this scenario, the agents are taxpayers whose incomes exceed the cost of a given project. The question arises as to how much each taxpayer should contribute towards the overall cost in a fair and equitable manner (see Young (1988, 1990)).

More generally, this model can be used when a resource needs to be allocated among a group of agents when the amount is insufficient to satisfy their claims, needs, or demands.

In this paper, we will follow the axiomatic approach. We begin with examining the properties of rules, which are expressed as mathematical axioms. These axioms represent our intuition about how a rule should operate in different situations. An axiomatic study typically concentrates on a small set of properties, examines their logical connections, and explores the consequences of imposing these properties in different combinations. The results obtained through the axiomatic approach can be categorized into two types. In some cases, certain properties are found to be incompatible, resulting in an impossibility theorem. On the other hand, a list

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of properties may be found to be compatible, and the family of rules satisfying them can be described which is called an axiomatic characterization of the rule. The main goal of the axiomatic program is to delineate the boundary between compatible and incompatible lists of properties and to obtain comprehensive and explicit descriptions of which rules or families of rules satisfy all of the compatible properties. For readers interested in exploring the axiomatic characterization of division rules in more depth, we recommend the comprehensive surveys by Moulin (2002) and Thomson (2019).

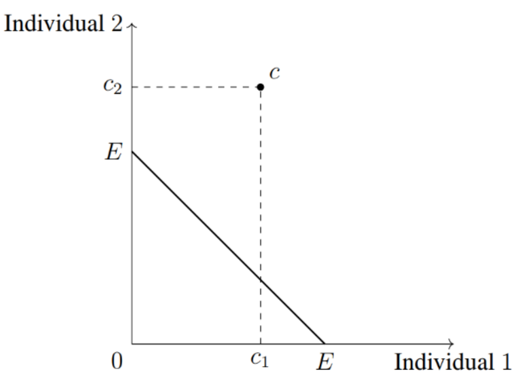
The paper is structured as follows. In Section 2, we present preliminaries for the claims problem. In Section 3, we discuss various well-known division rules from the literature. Next, in Section 4, we introduce standard axioms that capture important properties such as efficiency, fairness, monotonicity, and consistency and we provide axiomatic characterizations for the division rules discussed in Section 3. Finally, in Section 5, we offer concluding remarks.

2. Preliminaries

An infinitely divisible resource $E \in \mathbb{R}_+$ is to be divided among a group of agents N where individual $i \in N$ has a claim $c_i \in \mathbb{R}_+$ on the resource. The vector $c = (c_i)_{i\in N}$ represents the claims of all individuals. We assume that N is finite and subset of the set of natural numbers indexed by $\{1, 2, \dots, n\}$.

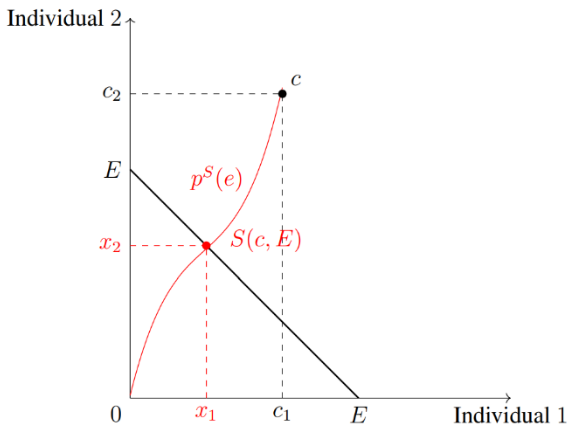
To define a "claims problem", we require that the total claims of the individuals exceed the amount of the resource available, that is, $\sum_{i\in N} c_i \geq E$. The pair $(c, E) \in \mathbb{R}_+^N \times \mathbb{R}_+$ is referred to as "claims problem". Let the set C^N represent the entire collection of problems under consideration. We represent the claims problem for two individuals with $c_1 < c_2$ in Figure 1 below.

Figure 1. Claims Problem for $|N| = 2$ and $c_1 < c_2$.



Our objective is to find a solution that assigns a list of "awards" to each claimant, such that their sum of awards equals the resource. We further assume that each claimant receives an award that is non-negative and no greater than their claim. Formally, for each problem (c, E) a vector $x \in R_+^N$ is awarded such that $0 \leq x \leq c$ and the division is balanced, i.e., $\sum_{i\in N} x_i = E$ The set of awards vectors for a problem (c, E) is denoted by $X(c, E)$. A division rule is represented as a function denoted by S, which assigns an awards vector in $X(c, E)$ to each problem (c, E). For a two-person problem, the division rule can be represented by *path of awards*, denoted by $p^S(e)$ for the division rule $S(c, E)$, where $e \in [0, \sum_{i\in N} c_i]$, as shown in Figure 2 below.

Figure 2. $p^S(e)$, the path of awards for the division rule $S(c, e)$ for $|N| = 2$ and $c_1 < c_2$ where $e \in [0, c_1 + c_2]$.



3. Division Rules

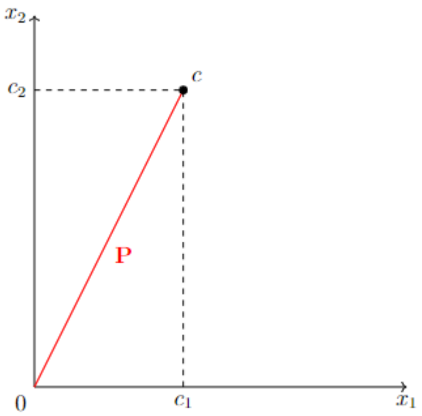
In this section, we will present the most prominent division rules in the literature. Their axiomatic characterization will be provided in Section 4.

Proportionality has been the primary method for handling simple division problems for a long time, as documented in history. Aristotle, who is frequently quoted in this context, considered proportionality as equity as his maxim states: "A just act necessarily involves at least four terms: two persons for whom it is in fact just, and two shares in which its justice is exhibited. And there will be the same equality between the shares as between the persons, because the shares will be in the same ratio to one another as the persons ... What is just in this sense is what is proportional and what is unjust is what violates the proportion."

Accordingly, the proportional rule in claims problems ensures that each claimant should receive a share of the available resources that is proportional to their claim. The proportional rule embodies Aristotle's idea that equals should be treated equally and unequals unequally, by distributing resources in proportion to the size of each claim. In this way, the proportional rule ensures that each person receives a share that is proportional to their contribution to the situation, and thus achieves a sense of justice and equity.

The Proportional (P) Rule: For all $(c, E) \in C^N$, $P(c, E) = \lambda E$ where $\lambda = \frac{c}{\sum_{i \in N} c_i}$

Figure 3. The path of awards for the proportional rule where $c_1 < c_2$.

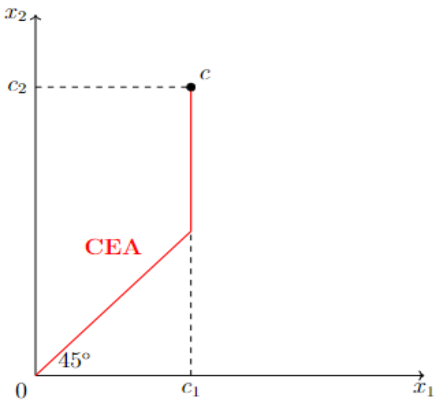


We can view proportional division as a type of equal division. Specifically, in the context of resolving competing claims, proportional division treats all "units of claims" equally, regardless of who holds them, and allocates these units equally among claimants. Each claimant is then awarded partial payments corresponding to the units allocated to them.

We are now shifting our attention from equal division based on equality per unit of claim to equal division in absolute terms. The next rule retains the idea of equal division provided that no individual receives more than their claim. Numerous medieval scholar such as Maimonides has advocated this rule.

The Constrained Equal Awards (CEA) Rule: For all $(c, E) \in C^N$, $CEA(c, E) = (min\{c_i, \lambda\})_{i \in N}$, where $\lambda \in \mathbb{R}_+$ is such that $\sum_{i \in N} min\{c_i, \lambda\} = E$.

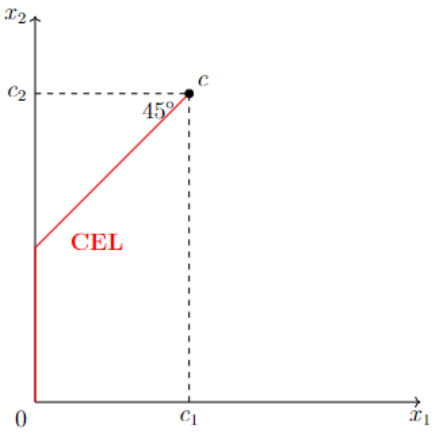
Figure 4. The path of awards for the CEA rule where $c_1 < c_2$.



The next rule shares similarities with the constrained equal awards rule in that it promotes the concept of equality. However, it takes a different perspective by focusing on the losses experienced by claimants instead of what they receive. The rule attempts to make losses as equal as possible while ensuring that no one receives a negative amount. This rule has been mentioned in Maimonides' writings as well.

The Constrained Equal Losses (CEL) Rule: For all $(c, E) \in C^N$, $CEL(c, E) = (max\{c_i - \lambda, 0\})_{i \in N}$ where $\lambda \in \mathbb{R}_+$ is such that $\sum_{i \in N} max\{c_i - \lambda, 0\} = E$.

Figure 5. The path of awards for the CEL rule where $c_1 < c_2$.



Both the CEA and the CEL rules aim to equalize, respectively, the awards x_i and the losses $(c_i - x_i)$ across individuals while respecting the feasibility constraints of the division rule. It is easy to see that $CEA(c, E)$ is the unique solution that maximizes the leximin ordering over awards, which lexicographically maximizes the smallest coordinate x_i, then the next-smallest coordinate, and so on.[1] Similarly, $CEL(c, E)$ is the unique maximizer of "leximin" ordering applied to the vector of losses $(c_i - x_i)$.

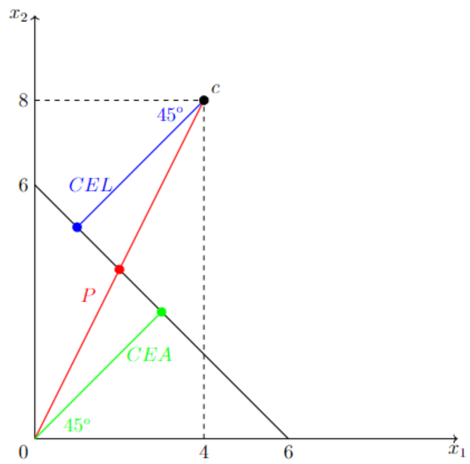
Example. Let us provide an example to illustrate the three main rules defined above. Consider a bankruptcy problem with an estate value of $E = 6$. Individual 1 has a claim of $c_1 = 4$, and individual 2 has a claim of $c_2 = 8$. Under the proportional rule, the estate is divided in proportion to the claims, meaning each individual would receive half of their claims. In this case, individual 1 would receive $x_1 = 2$, and individual 2 would receive $x_2 = 4$. Alternatively, under the CEA rule, the awards are equalized, resulting in $x_1 = x_2 = 3$. Both individuals would receive an equal share of 3. On the other hand, the CEL rule equalizes the losses, which means the difference between the claim and the award is the same for both individuals. In this scenario, $c_1 - x_1 = c_2 - x_2 = 3$. As

[1] To formally define the lexicographic ordering, let $x, y \in \mathbb{R}^n$ and let $x^*, y^* \in \mathbb{R}^n$ be obtained by rearranging the coordinates of x, y in increasing order. We say x and y are indifferent with respect to leximin ordering if $x^* = y^*$. We say that x is preferred to y with respect to leximin ordering if there exists an integer $m = 0,1, \ldots, n-1$ such that $x_i^* = y_i^*$ for $i = 1,2, \ldots, m$ and $x_{m+1}^* > y_{m+1}^*$.

a result, individual 1 would receive $x_1 = 1$, while individual 2 would receive $x_2 = 5$.

In Figure 6, we present the path of awards for P, CEA, and CEL rules based on the example mentioned above.

Figure 6. The path of awards for P, CEA, and CEL rules where $E = 6,\ c_1 = 4,\ \ c_2 = 8$.



The issue commonly referred to as the *contested garment problem* concerns a dispute between two individuals over the rightful ownership of a garment, which leads to conflicting claims regarding its value. The question then arises as to how the value of the garment should be fairly distributed between the two parties. A solution to this problem is presented in a passage from the Talmud, which states that in the case where "Two hold a garment... If one of them says, "It is all mine", and the other says "Half of it is mine", ... the former receives three quarters and the latter receives one quarter."[2]

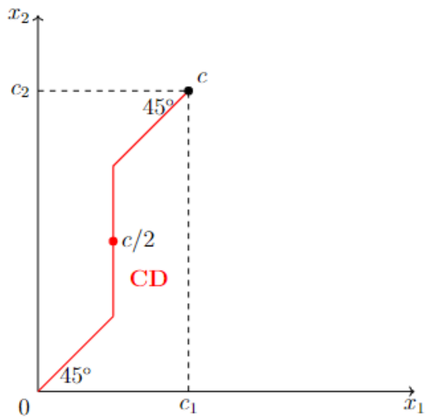
To formally define the rule, let us consider a scenario involving two claimants, represented by i and j. In this scenario, each claimant specifies a claim value, denoted by c_i and c_j, respectively. The fundamental principle behind the rule is that if one claimant specifies a claim value, they are effectively conceding the remaining value to the other claimant. More specifically, if $E - c_1$ is non-negative, then claimant i concedes the amount $E - c_i$ to claimant j, and if $E - c_i$ is negative, then claimant i concedes nothing to claimant j. Similarly, if $E - c_j$ is non-negative, then claimant j concedes the amount $E - c_j$ to claimant i, and if $E - c_j$ is negative, then claimant j concedes nothing to claimant i.

In the first step of the rule, each claimant is assigned the amount that the other has conceded to them. In the second step, the remaining value, which is the part that is truly contested, is divided equally between the claimants.

The Concede-and-Divide (CD) Rule: Let $|N| = 2$. For all $(c, E) \in C^N$,

$$CD_i(c, E) = max(E - c_j, 0) + \frac{E - max(E - c_j, 0) - max(E - c_i, 0)}{2}$$ for all distinct $i, j \in N$.

Figure 7. Path of awards for the CD rule where $c_1 < c_2$.



We will now introduce a division rule that can be viewed as a generalization of the CD rule for more than two claimants. Aumann and Maschler (1985) defined the *Talmud rule* to extend the rules in the Talmud to situations where there are more than three people. Although the Talmud doesn't provide an example for such cases, they found the underlying reasoning compelling, and the rule coincides with the recommendations for two people (contested garment problem) and three people (marriage contract problem)[3]. To define the Talmud rule formally, it involves dividing the claims in half and applying the CEA rule to each half until it is fulfilled. Then, the CEL rule is applied to the remaining half claims.

The Talmud (T) Rule: For all $(c, E) \in C^N$, and for all $i \in N$

$$T_i(c, E) = \begin{cases} min\left\{\frac{c_i}{2}, \lambda\right\} & \text{if } E \leq \frac{\sum_{i \in N} c_i}{2} \\ c_i - min\left\{\frac{c_i}{2}, \lambda\right\}, & \text{otherwise} \end{cases}$$

where $\lambda \in \mathbb{R}_+$ is such that $\sum_{i \in N} T_i(c, E) = E$.

[2] See Baba Metzia, Babylonian Talmud I. All references to the relevant passages of the Talmud and of the secondary literature are taken from O'Neill (1982), Aumann ve Maschler (1985), and Dagan (1996).

[3] Kethubot 93a by Rabbi Nathan: If a man who was married to three wives died and the kethubah of one was one maneh (100 zuz), of the other two hundred zuz, and of the third three hundred zuz, and the estate (was worth) only one maneh (one hundred zuz), the (the sum) is divided equally. If the estate (was worth) two hundred zuz (the claimant) of the maneh (one hundred zuz) receives fifty zuz (and the claimants respectively) of the two hundred and three hundred zuz (receive each) three gold denarii (seventy-five zuz). If the estate (was worth) three hundred zuz (the claimant) of the maneh receives fifty zuz and (the claimant) of the two hundred zuz (receives) a maneh (one hundred zuz) while (the claimant) of the three hundred zuz (receives) six gold denarii (one hundred and fifty zuz). Similarly, if three persons contributed to a joint fund and they had made a loss or profit they share in the same manner.

Now, we focus our attention on the family of rules introduced by Young (1987), known as parametric rules. To formally define this family, we represent each function f in Φ as follows: $f: \mathbb{R}_+ \times (\underline{\lambda}, \bar{\lambda}) \rightarrow \mathbb{R}_+$, where $-\infty \leq \underline{\lambda} < \bar{\lambda} \leq \infty$. These functions are continuous and non-decreasing with respect to their second argument. Additionally, for every $c_0 \in \mathbb{R}_+$, they satisfy the conditions $f(c_0, \underline{\lambda}) = 0$ and $f(c_0, \bar{\lambda}) = c_0$.

To apply this definition to a problem $(c, E) \in C^N$, we determine λ such that the sum of the values taken by the functions $\{f(c_i, .)\}_{i \in N}$ when their second argument is λ equals E. We then choose the vector $\{f(c_i, \lambda)\}_{i \in N}$ as the awards vector. It is formally defined as follows:

The Parametric (PAR) Rules: For all $(c, E) \in C^N$, $f(c, E) = (f(c_i, \lambda))_{i \in N}$ where $\lambda \in \mathbb{R}_+$ is such that $\sum_{i \in N} f(c_i, \lambda) = E$.

We can interpret $f(c_0, \lambda)$ as the reward needed for an individual with a claim of c_0 to attain a welfare evaluation at λ. This evaluation remains independent of the individual's identity, as well as the identities of other claimants with their respective claims. Consequently, parametric rules exhibit the consistency principle, which will be further discussed in the next section. It is worth noting that the parametric rules family encompasses a substantial number of division rules, among which are the aforementioned rules, namely the proportional, CEA, CEL, and Talmud rules.

4. Axiomatic Characterizations

This section presents axiomatic characterizations of the rules introduced in the previous section. Our first set of axioms consists of nonnegativity, balance, and claim boundedness, which are assumed as a definition of the rule. Here we are presenting them as separate axioms.

Our first axiom is that a rule should allocate the entire endowment for each problem.

Balance: For all $(c, E) \in C^N$, $\sum_{i \in N} S_i(c, E) = E$.

The next axiom ensures that no individual receives a negative amount.

Non-negativity: For all $(c, E) \in C^N$, $S_i(c, E) \geq 0$ for all $i \in N$.

The following axiom establishes an upper bound on awards, stating that each individual should receive at most his claim.

Claim boundedness: For all $(c, E) \in C^N$, $S_i(c, E) \leq c_i$ for all $i \in N$.

All three axioms presented above are fundamental to the definition of the rule and are highly relevant to the context of claims problems such as bankruptcy and inheritance discussed in the introduction. These axioms embody crucial aspects of fair division, and it is difficult to conceive of a rule that does not adhere to them.

The axioms discussed in this section revolve around a fundamental notion in the theory of fair allocation, which asserts that agents with similar characteristics should be treated equally. However, when agents differ in dimensions that are not accounted for in the model, unequal treatment may be necessary. Our subsequent axiom encapsulates this fundamental concept of symmetry. Note that all the rules defined in Section 3 satisfy this axiom.

Equal treatment of equals: For all $(c, E) \in C^N$, and for all $\{i, j\} \subseteq N$, if $c_i = c_j$ then $S_i(c, E) = S_j(c, E)$.

Our next axiom establishes a lower bound for a claimant's award in a problem. This lower bound is based on the difference between the endowment and the sum of the claims of the other agents, if this difference is non-negative, and 0 otherwise. Formally, $m_i(c, E) = max\{E - \sum_{j \neq i} c_j, 0\}$ represents the minimal right of individual i. We require that each claimant receive at least his minimal right.[4]

Minimal rights lower bounds on awards: For all $(c, E) \in C^N$, $S_i(c, E) \geq m_i(c, E)$ for all $i \in N$.

Our next requirement on a rule is that, for each problem, it is possible to calculate the awards vector either directly or in two steps, as follows: in the first step, the rule assigns to each claimant their minimal right. In the second step, the rule revises the claims down by the minimal rights and distributes the residual endowment. The revised claims are ensured to be non-negative, and their sum is at least as large as the remainder, making the problem of the second step well-defined.

Minimal rights first: For all $(c, E) \in C^N$, $S(c, E) = m(c, E) + S(c - m(c, E), E - \sum_{i \in N} m_i(c, E))$.

Our next axiom is an invariance property which stipulates that for each problem, truncating claims at the endowment should not affect the awards vector it chooses. Given $(c, E) \in C^N$, let the truncated claim of individual i be denoted by $t_i(c, E) = min(c_i, E)$.[5]

Claims truncation invariance: For all $(c, E) \in C^N$, we have $S(c, E) = S(t(c, E), E)$.

We will now introduce two more invariance axioms concerning changes in the endowment. The first axiom, known as “composition down”, applies to situations where the endowment decreases. In this scenario, there are two possible approaches: one can either cancel the initial division and reapply the rule to obtain new awards based on the revised value, or use the awards calculated based on the initial value as claims and reapply the rule to divide the revised value. The

[4] The minimal rights lower bounds axiom is satisfied by all rules satisfying balance, non-negativity, and claim boundedness.

[5] The concept of truncation was initially presented by Aumann and Maschler (1985). Claims truncation invariance was later introduced by Curiel et al. (1987), and subsequently proposed as a formal axiom by Dagan (1993).

"composition down" axiom ensures that both approaches result in the same awards vector.[6]

Composition down: For all $(c, E) \in C^N$, and $E' < E$, we have $S(c, E') = S(S(c, E), E')$.

We now turn our attention to the opposite scenario of what we considered above. The "composition up" axiom applies to situations where the endowment decreases. In this new situation, we have two options similar to those we discussed earlier. The first option is to cancel the initial division and apply the rule again to divide the revised value. The second option is to let the claimants keep their initial awards, adjust their claims based on these awards, and reapply the rule to divide the incremental value. This would result in each claimant receiving their assignment in two installments. As before, the "composition up" axiom requires that both options result in the same awards vector.[7]

Composition up: For all $(c, E) \in C^N$, and $E' > E$ such that $\sum_{i \in N} c_i \geq E'$, we have $(c, E') = S(S(c, E), E')$.

Before we start with our axiomatic characterizations, it is crucial to highlight the significance of "duality" in claims problems, as it profoundly influences our analysis. A "claims problem" can be approached from two distinct perspectives. The first perspective centers on the resources that are available, while the second perspective focuses on the deficit, which represents the difference between the total claims and the available resources. This notion of "symmetry" between the perspectives becomes evident in definitions like the constrained equal awards and constrained equal losses rules. This symmetry is also fundamental to the lower and upper bounds, as well as the two composition properties. We define two problems as dual if they share identical claims vectors, and the endowment in one problem is equivalent to the deficit in the other.

Dual of rule $S = S^d$: For all $(c, E) \in C^N$, $S^d(c, E) = c - S(c, \sum_{i \in N} c_i - E)$

It is important to note that the constrained equal awards (CEA) and the constrained equal losses (CEL) rules are dual rules to each other. A rule is considered self-dual when its dual is equal to itself, i.e., $S^d = S$. From a geometric perspective, self-duality can be interpreted as the path of awards demonstrating symmetry in relation to half of the claims. It is worth mentioning that both the proportional rule and the Talmud rule are self-dual.

The concept of duality can be extended to properties of rules as follows: Two properties are considered dual if whenever a rule satisfies one of them, its dual satisfies the other. Notably, the properties of "non-negativity" and "claim boundedness" are dual to each other. Similarly, the properties of "truncation invariance" and "minimal rights first" are dual to each other. Likewise, the properties of "composition down" and "composition up" exhibit duality. A property is self-dual if it is identical to its dual. For instance, the property of "equal treatment of equals" is a self-dual property.

Now we will start with the characterization of the constrained equal awards (CEA) rule which is due to Dagan (1996).

Theorem 1. The constrained equal awards (CEA) rule is the unique rule satisfying equal treatment of equals, claims truncation invariance, and composition up.

The next characterization theorem is the dual of the theorem mentioned above, wherein we replace each rule and property with its dual counterpart.

Theorem 2. The constrained equal losses (CEL) rule is the unique rule satisfying equal treatment of equals, minimal rights first, and composition down.

Next, we give a characterization of the concede-and-divide (CD) rule, which is self-dual and can be regarded as a hybrid of the constrained equal awards (CEA) and constrained equal losses (CEL) rules. This characterization is provided in Dagan (1996).

Theorem 3. For $|N| = 2$, the concede-and-divide (CD) rule is the unique rule satisfying equal treatment of equals, claims truncation invariance, and minimal rights first.

The concede-and-divide (CD) rule can be also characterized by minimal rights first and self-duality. Additionally, by the use of duality, its characterization can be achieved by combining claims truncation invariance and self-duality.

Now, we will provide a characterization of the proportional rule which is due to Young (1988).

Theorem 4. The proportional rule is the unique rule satisfying composition up and self-duality.

Furthermore, through the application of duality, another characterization of the proportional rule can be achieved by combining composition down and self-duality.

Finally, we proceed to present the axiomatic characterization of the parametric rules. To achieve this, we introduce the consistency axiom, which basically states that if we remove one individual (or multiple individuals) from society N, along with the resources allocated to these individual(s) within N, the allocation of shares within the reduced society remains unchanged.

To formally define the concept of consistency, let N represent the initial population of claimants, $(c, E) \in C^N$ the problem they encounter, and $x = S(c, E)$ be the awards vector chosen by the rule S. Now, consider a scenario where some claimants leave with their respective awards, and let $M \subset N$ denote the population of remaining claimants. Consider the

[6] Moulin (1987) introduced this axiom as "path independence" for surplus sharing problems, while Moulin (2000) referred to it as "upper composition".

[7] Young (1988) introduced the "composition" axiom in the context of taxation, while Moulin (2000) refers to it as the "lower composition" property.

reduced problem of (c_M, E_M) where c_M is the claim vector of the remaining individuals with the residual endowment of $E_M = \sum_{i \in M} x_i$. In this reduced problem, we demand that the rule assigns to each $i \in M$ the same amount as initially, namely x_i.

Consistency: For all $M \subset N$, and all $(c, E) \in C^N$, if $S(c, E) = x$, then $S(c_M, \sum_{i \in M} x_i) = x_M$.

To achieve the desired characterization, we should introduce the continuity axiom, which stipulates that small changes in the data of the problem being solved should not result in large changes in the awards vector.

Continuity: For all sequences $\{(c^t, E^t)\}$ and for all $(c, E) \in C^N$, if $(c^t, E^t) \to (c, E)$ then $S(c^t, E^t) \to S(c, E)$.

Finally, we are ready to present the characterization of the parametric rules, which is due to Young (1987).

Theorem 5. The parametric rules are the only rules satisfying equal treatment of equals, continuity, and consistency.

5. Conclusion

The claims problem is a fundamental problem that arises in many real-life situations, such as estate division, bankruptcy, rationing, and taxation. The allocation of a resource among competing claimants requires the use of a division rule, which is determined based on a set of axioms that capture important properties such as efficiency, fairness, monotonicity, and consistency.

In this paper, we have presented a survey of various well-known division rules from the literature and provided axiomatic characterizations for them. The axiomatic approach has allowed us to examine the logical connections between different properties and to obtain comprehensive and explicit descriptions of which rules or families of rules satisfy all the compatible properties.

In summary, the claims problem remains a vibrant area of research that continues to attract the attention of economists, mathematicians, and computer scientists. The axiomatic approach has been a valuable tool for studying this problem, and we hope that this brief survey has provided a useful introduction to this approach and to the different division rules that have been proposed.

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