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Intergenerational Equity with Heterogeneous Individual Time Preferences in a Model of Optimal and Sustainable Growth*

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ABSTRACT

One of the ethical objections made to inter-generational equity is the violation of consumer sovereignty. To address this concern, this paper presents a continuous-time Overlapping Generations Model (OLG) suitable for the treatment of sustainability issues, which distinguishes the intra- and inter-generational discount factor (Calvo and Obstfeld, 1988) with taking into account heterogeneities of time preferences among individuals. We find that consumption for older patient individuals is always higher than older impatient agents while the social planner decides to allocate equally the consumption between patient and impatient young individuals. Along age, the consumption of patient old agents always increases relatively, whose speed depends on intertemporal elasticity of substitution. Our results show that the consumption of patient and impatient young individuals are interdependent. We also show that the Hotelling rule for renewable resources is not affected regardless of the share of patient and impatient agents in society. Finally, we find that the effect of static increase of the individual discount rate of a patient or impatient agent on sustainable income depends on the level of aggregate consumption.

Keywords : *Environment ; equity ; overlapping generations model ; sustainability*

1. INTRODUCTION

The debate around discounting, notably concerning the sustainability of growth has intensified with the prospect of climate change (Stern, 2006; Weitzman, 2007; Heal, 2009). In the mainstream literature, social welfare functions are assumed to take into account consumer sovereignty, including also time preferences. One of the main question that we ask ourselves in this paper is “how is it possible to reconcile social preferences that are convenient with inter-generational neutrality and still use a representative agent models with homogenous time preferences?”. One way to cope with this question is to distinguish the intra-generational discount rate from the social planner’s discount rate, which is equivalent to inter-generational discount factor.

In this paper, we use a continuous-time overlapping generations model (Yaari, 1965; Blanchard and Fisher, 1985; Calvo and Obstfeld, 1988) with physical capital and renewable natural resources with two-stage optimization. As such, our framework is different from other articles using OLG framework. The framework that we use belongs to Calvo and Obstfeld (1988), which makes the distinction between intra-generational and inter-generational utility discounting by solving the analytical model with two-stage dynamic optimization problem. While some other studies have shown that optimal sustainable economic development with inter-generational equity is possible with the presence of backstop technology (Endress et al., 2005; Heal, 2000; Ayong Le Kama, 2001), in these articles, we see that individual impatience is neglected, as it is the case in most articles in economic literature. Our paper aims mainly to fill this gap and to take into account not only the aspect of inter-generational equity but also intra-generational equity, without also neglecting the heterogeneities of time preferences in a society.

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Our framework is not only getting inspired by Calvo and Obstfeld (1988) but also an adaptation of Endress et al. (2014) which tries to answer the question that we have asked above and treats the individual utility with homogenous time preferences. This framework does not consider the existing heterogeneities concerning the time preferences among different agents. In our framework, our main contribution consists of making a distinction between patient and impatient agents who have different individual discounting factors. This important extension allows us to analyze the interdependencies concerning the consumption between different types of agents and also to analyze how the consumption profile of different types of agents changes along the age of the given individual.

We also show that social planner allocates equally the consumption of young patient and impatient agents. This equality concerning the consumption is not the case for old agents. For a given age, patient old agents consume always more than impatient old agents at the optimum. Additionally, one of the important result to mention in this paper is that even though there exists heterogeneities in individual time preferences, the model collapses to a standard infinitely lived agent (ILA) model. Calvo and Obstfeld (1988), Marini and Scaramozzino (1995) and Schneider et al. (2012) have also shown that the continuous-time OLG model collapses to standard ILA model but these papers have shown this result for homogeneous agents. We hereby extend this result to heterogeneous case. Moreover, we observe that individual discount rates have an impact on forever-sustainable income, which depends on the level of aggregate consumption.

The issue of sustainability and inter-generational fairness started to be treated after the report of the World Commission on Environment and Development (1987). Howarth and Norgaard (1992) examine sustainability and inter-generational fairness with a discrete-time OLG model in the context of climate change, but this article does not treat the consumer sovereignty problem nor does it make a distinction between individual and inter-generational time preferences in order to address concerns about individual sovereignty.

Inter-generational equity and sustainability is one of the very actual debated subjects in economics and in many other disciplines like philosophy and sociology. Many of environmental economists worry that current generations will not leave enough bequests of natural resources for future generations. Some important philosophers like Kant and Rawls are concerned mainly by the dictatorship of future on present. Kant (1784) argues that supporting today the burdens of nature for the sake of later generations is disconcerting. Rawls (1999) is concerned that utilitarianism may lead us to demand big sacrifices from poorer generations for the benefits of future generations which will be better off.

On the other hand, there is also another reverse-sided dictatorship, which is that of present on future. This kind of dictatorship is present when there exists a positive discounting factor. It is evident that positive discounting is representing an asymmetry between present and future, especially for very distant future, which is also valid for management of both renewable and non-renewable resources. A majority of neoclassical theories of sustainable growth uses positive discounting. This is interesting because sustainability issue is deeply linked to inter-generational equity. In economic literature, inter-generational equity is perceived as zero discount factor use. For this purpose, the seminal work Ramsey (1928), which defends the idea that discounting is “ethically indefensible”, is often cited by researchers. Samuelson and Solow (1956) generalize Ramsey (1928) by allowing zero discount factor for any number of capital goods. Another important approach, Solow (1974) in which author pretends to be “more Rawlsian than Rawls”, defends the idea that we must act as if the pure rate of time preference is equal to zero. There exist also other studies, which insist on the importance of inter-generational equity. Heal (2009) considers the positive discounting as a “discrimination rate across generations”. The optimal growth trajectory of an economy is unsustainable with a positive discount factor, which implies a declining consumption, which approaches to zero for very future generations. Therefore, it is clear that the positive discounting factor is inconsistent with the idea of inter-generational equity.

Nonetheless, there are also objections concerning zero utility discounting for inter-generational equity. Dasgupta (2011) and Heal (1983) argue that in a simple cake-eating problem, if the initial level of consumption is low, this level will remain the same, which means the economy will be channelled to a low-level of consumption. Another interesting idea comes from Dasgupta (2011) where setting the inter-generational discount factor to zero may provide an unfair advantage for future generations who will benefit from a higher stock of knowledge capital. According to Dasgupta (2011), this problem can be tempered by a huge elasticity of marginal utility, which captures the social tolerance for inter-generational equity.

The remainder of the paper is as follows, in section 2, we introduce the model framework and our main results. Section 3 characterizes the inter-generational neutrality case with zero interest rate. In this section, we also focus on the impact of individual discount rates on forever-sustainable income. Section 4 presents a discussion on the limits of the present paper and proposes some extensions for further research. Section 5 concludes.

2. MODEL AND RESULTS

We consider an economy made up of overlapping generations of heterogeneous individuals in terms of discount rate, which shows patience level. In a simple model configuration, each generation contains one patient individual and another impatient individual who live to age N. At each time t, society is made of these two types of individuals who range in age from 0 to N, and no two individuals with same patience level have the same age[1]. The patient and impatient individual of age τ have consumption good in amount $c_\beta(t,\tau)$ and $c_\alpha(t,\tau)$ and enjoy the utility $u(c_\beta(t,\tau))$ and $u(c_\alpha(t,\tau))$. The two types of individuals born at time T measure remaining lifetime utility, U_{T_β} and U_{T_α}, according to the formula where β is the discount factor for patient and α for the impatient agents ($\alpha > \beta$).

Firstly, as we will give the utility functions at individual level, these discount factors can also be interpreted as the intra-generational discount rate. Following equations 2.1 and 2.2 show the individual utility of patient and impatient agents. An important assumption is that we will use the same utility function for both types of agents for the sake of simplicity, as it has been also done in Zhong-Li and Löfgren (2000).

$$U_{T_\beta} = \int_{\tau=0}^{N} u(c_\beta(T+\tau,\tau))\, e^{-\beta\tau} d\tau \qquad 2.1$$

$$U_{T_\alpha} = \int_{\tau=0}^{N} u(c_\alpha(T+\tau,\tau))\, e^{-\alpha\tau} d\tau \qquad 2.2$$

Consequently, the social welfare can be calculated as a weighted average of the utilities of these two types of agents with δ, the constant share of patient agents in the society. In the aggregated social welfare, we will introduce a unique inter-generational discount factor, which concerns the whole society. The social planner's aim is to choose the optimal consumption level for each type of agent $c_\beta(t,\tau)$ and $c_\alpha(t,\tau)$ $\forall t,\tau$ (time path for each type of individual consumption) subject to the following differential equations of the two stocks. The optimization problem of the social planner is given by:

[1] For example, there exists only one impatient agent who is X years old and another X year old individual who is patient. To sum up, there exists only 2 individuals who are X years old, and one of them is patient and another is impatient.

$$\max_{c(t,\tau)} \widetilde{W} = \delta \int_{T=-N}^{\infty} U_{T_\beta}\, e^{-r\tau} dT + (1-\delta) \int_{T=-N}^{\infty} U_{T_\alpha}\, e^{-r\tau} dT \qquad 2.3$$

$$s.t.\ \dot{K} = F(K,R) - \gamma K - \Theta(X)R - \delta \int_0^N c_\beta(t,\tau)\, d\tau - (1 - \delta) \int_0^N c_\alpha(t,\tau)\, d\tau \qquad 2.4$$

$$\dot{X} = G(X) - R \qquad 2.5$$

$K(0)$ and $X(0)$ are given

Production is given by $Y = F(K,R)$ where K represents the capital stock which depreciates with a constant rate γ. R is the extraction from a renewable resource which grows at rate $G(X)$. There exist also extraction costs, which is stock-dependant unit cost $\Theta(X)$. There are not any special assumptions about these functions.

It can be possible to solve this problem as an optimal control problem. We can easily see that the two equations of motion show the rate of variation in terms of pure time t. The application of the Maximum Principle for this problem can be simplified by reformulating the objective function in terms of time t, instead of generational index, T. For this one, we pass from the separability of social welfare by individual to separability by time period through a transformation of the two time variable systems. To sum up, we switch from (T,τ) to (t,τ) by puting $T = t - \tau$ and we maintain $\tau = \tau$. This kind of reformulation also exists in Burton (1993) in which the author makes a welfare analysis involving both individual and generational discount factors. Let V_i represent the aggregate utility of each type of agent. We, therefore, can define the aggregate utility for each agent before reformulating our optimization program:

$$V_\beta(C) = \int_0^N u(c_\beta(t,\tau))\, e^{-(\beta-r)\tau} d\tau \qquad 2.6$$

$$V_\alpha(C) = \int_0^N u(c_\alpha(t,\tau))\, e^{-(\alpha-r)\tau} d\tau \qquad 2.7$$

So, the new optimization problem becomes:

$$\max_{c(t,\tau)} W = \delta \int_0^{\infty} V_\beta(C)\, e^{-r\tau} d\tau + (1-\delta) \int_0^{\infty} V_\alpha(C)\, e^{-r\tau} d\tau \qquad 2.8$$

$$s.t.\ \dot{K} = F(K,R) - \gamma K - \Theta(X)R - C \qquad 2.9$$

$$\dot{X} = G(X) - R \qquad 2.10$$

$K(0)$ and $X(0)$ are given

where $V_\beta(C)$ and $V_\alpha(C)$ show the aggregate utility of patient and impatient agents. We show how to derive 2.8 from 2.3 in Appendix A and how to derive $V_\beta(C)$ and $V_\alpha(C)$ in Appendix B.

$$\max_{c(t,\tau)} W = \int_0^\infty (\delta V_\beta(C) + (1-\delta)V_\alpha(C))e^{-r\tau}d\tau = \int_0^\infty V(C)\, e^{-r\tau}d\tau \qquad 2.11$$

In the following section, we explain how to solve the maximization problem in two stages.

2.1. How to solve the optimization problem in two stages?

In ***the first stage of the maximization problem***, the social planner will weight V with δ in order to maximize the utility V_i of each type of agent. We write as it follows;

$$\max_{c(t,\tau)} W = \delta \int_0^N u(c_\beta(t,\tau))\, e^{-(\beta-r)\tau}d\tau + (1-\delta)\int_0^N u(c_\alpha(t,\tau))\, e^{-(\alpha-r)\tau}d\tau \qquad 2.12$$

$$s.t.\ \delta \int_0^N c_\beta(t,\tau)\, d\tau + (1-\delta)\int_0^N c_\alpha(t,\tau)\, d\tau \le C \qquad 2.13$$

C represents the aggregate level of output for the consumption of the society. This problem can be solved as a two-stage maximization problem. In the first stage, we establish a relationship between $c^*(t,0)$ and the optimal path of individual consumption $c^*(t,\tau)$ for both type of agents. The second stage consists on the solution of the problem given by equation 2.8. First, we put the first order conditions from our maximization problem that maximizes V. To solve it, we write the Lagrangian as follows:

$$\mathcal{L} = \delta \int_0^N u(c_\beta(t,\tau))\, e^{-(\beta-r)\tau}d\tau + (1-\delta)\int_0^N u\big(c_\alpha(t,\tau)\big)\, e^{-(\alpha-r)\tau}d\tau + \lambda\left[C - \delta\int_0^N c_\beta(t,\tau)\, d\tau - (1-\delta)\int_0^N c_\alpha(t,\tau)\, d\tau\right] \qquad 2.14$$

The solution of the first order conditions

$$\frac{\partial \mathcal{L}}{\partial c_\beta} = u'\left(c_\beta(t,\tau)\right) e^{-(\beta-r)\tau} - \lambda = 0 \qquad 2.15$$

$$\frac{\partial \mathcal{L}}{\partial c_\alpha} = u'\big(c_\alpha(t,\tau)\big)e^{-(\alpha-r)\tau} - \lambda = 0 \qquad 2.16$$

will give the following relationship:

$$\frac{u'(c_\beta(t,\tau))}{u'\big(c_\alpha(t,\tau)\big)} = e^{-(\alpha-\beta)\tau} \qquad 2.17$$

Proposition 1. *With a given age τ, older patient individuals consume always more than older impatient agents regardless of the share of impatient agents in society.*

Before passing to the explanation of our proposition, we find it useful to give a definition of old and young agent. We suppose that agents at age $\tau = 0$ are considered to be young, and other agents with age τ different from zero are old agents. The relationship given by equation 2.15 shows us that the consumption of old patient individuals is relatively higher than its of impatient individuals when the difference between the individual discount factor of old patient (β) and impatient agents (α) gets bigger. The fact that old patient agents consume more can be explained by other factors that are exogenous to our model. Patient old agents, as they are patient, can have savings from past. So, in this case, it can be plausible for those agents to consume more at a given age τ.

Proposition 2. *Consumption of patient old agents is higher than the consumption of impatient old agents with age τ.*

Another interesting point to study is the consumption profile of different types of individual along age τ. Given individual discount rates, we can see easily that individual consumption of old patient agents increases according to its level for old impatient agents. We can explain this situation by the fact that patient agents conserve more of their budget for ulterior consumption.

Proposition 3. *Consumption of patient old agents increases less rapidly along age τ when intertemporal elasticity of substitution is lower.*

This is a quite intuitive result. The consumption of patient agents is less sensitive to discount factor when the intertemporal elasticity of substitution is lower. That's why, in the case where the intertemporal elasticity of substitution is lower, the consumption of old patient agents will increase less rapidly along age τ.

In order to analyze the consumption decisions concerning young individuals, we give $\tau = 0$ as it has been used in Endress et al. (2014),

$$u'\left(c_\beta(t,0)\right) - \lambda = 0 \qquad 2.18$$

$$u'\left(c_\alpha(t,0)\right) - \lambda = 0 \qquad 2.19$$

and solving the first order conditions, we obtain the following result:

$$u'(c_\beta(t,0)) = u'\left(c_\alpha(t,0)\right) \qquad 2.20$$

Proposition 4. *At the social optimum, there exists a constant relationship between the marginal utility of patient young agents and the marginal utility of impatient young agents, which implies also that the social planner allocates the consumption equally between patient young and impatient young individuals. A constant relationship would not necessarily imply equal shares. Equal shares is a special case with $\alpha = \beta$.*

In this case, we see clearly that the marginal utility of young patient agents is equivalent to the marginal utility of young impatient agents, which implies that the social planner allocates equally the consumption between patient and impatient agents at the age $\tau = 0$, which is the date agents were born[2]. We can interpret this result as to be convenient with equity between young individuals. Logically, at the date of born 0, social planner is not capable of distinguishing if one agent is patient or impatient. So, it can be plausible that he allocates the consumption equally between young agents at the age $\tau = 0$.

[2] Formally, we have $c_\beta(t,0) = c_\alpha(t,0)$.

2.2. How to solve the optimization problem in two stages?

In order to see how the allocation of the consumption between old and young agents changes according to age and individual discount rates, we are combining separately 2.15 - 2.18 and 2.16 - 2.19, we have;

$$u'\left(c_\beta(t,0)\right)e^{-(\beta-r)\tau} = u'\left(c_\beta(t,\tau)\right) \qquad 2.21$$

$$u'\left(c_\alpha(t,0)\right)e^{-(\alpha-r)\tau} = u'\left(c_\alpha(t,\tau)\right) \qquad 2.22$$

In the following part, we will treat three cases concerning intra-generational and inter-generational discount factors. This analysis will permit us to see how a young generation can consume a bigger part of society's aggregate consumption and vice versa. We will treat the case for patient agents. The same analysis is also valid for impatient agents.

Proposition 5. *The consumption profile of different generations along age τ differs according to Case 1 and Case 3.*

Case 1: $\beta > r$

In this case, we will have $u'\left(c_\beta(t,0)\right) < u'\left(c_\beta(t,\tau)\right)$ which implies $c_\beta(t,\tau) < c_\beta(t,0)$. Each generation's consumption is affected by the difference between individual discount rates of each type of agents and inter-generational discount rate. As the marginal utility of old patient agents is higher, young patient agents consume a larger part of the aggregate consumption.

Case 2: $\beta = r$

One can easily see that marginal utility of different generations is equal, which means that all generations will enjoy the same marginal utility of consumption. As we have $u'\left(c_\beta(t,0)\right) = u'\left(c_\beta(t,\tau)\right)$, the consumption of old patient agents is equal to the consumption of young patient agents.

Case 3: $\beta < r$

In this case, we will have $u'\left(c_\beta(t,0)\right) > u'\left(c_\beta(t,\tau)\right)$. It is evident that young patient agents will have the chance to consume a smaller part of the aggregate consumption.

It is also important to stress out that consumption profile of different generations will have different profiles along age τ according to Case 1 and Case 3 that we have analysed. For example, in the first case, as the individual discount rate is higher than inter-generational discount factor, the marginal utility of the old generation is higher, which implies that young generation consume a larger share of the aggregate consumption along τ. We will have the symmetric result for the Case 3 in which the marginal utility of the old generation is lower, which shows that the old generation's part of consumption in aggregate consumption is higher.

As we have different types of agents. It could be so interesting to focus on a case in which we have $\beta < r < \alpha$, which implies that patient agents would have a lower discount rate than the social planner's discount rate and impatient agents would have higher discount rate. So, in this case, the individual consumption profile of old patients and young patients in the society would be symmetric along τ. Recall that we don't treat this kind of case, which would be in contradiction with our section about inter-generational equity in which we treat the inter-generational discount factor to be equal to zero.

Following the very standard approach in the neoclassical sustainable growth theory, we assume that the utility function takes the constant elasticity of marginal utility form. This assumption is necessary in order to have a balanced growth path. We use the following utility function;

$$u\big(c_i(t,\tau)\big) = -(c_i(t,\tau))^{-(\theta-1)} \text{ with } \theta > 1 \qquad 2.23$$

This one implies at the following optimum relationship:

$$c_\beta^*(t,\tau) = c_\beta^*(t,0)e^{\frac{-(\beta-r)\tau}{\theta}} \text{ and } c_\alpha^*(t,\tau) = c_\alpha^*(t,0)e^{\frac{-(\alpha-r)\tau}{\theta}} \qquad 2.24$$

We write the aggregate utility function of the society $V(C)$ as a function of C as it follows:

$$V(C) = -\left[\frac{C}{M}\right]^{-(\theta-1)} D \qquad 2.25$$

where M and D represent the aggregation coefficient and a constant parameter respectively which we present in Appendix B. The aggregation factor has also some important insights. When the difference between the social planner and individuals' discount rate is higher, then the allocation of the consumption between generations over time differ. This means that the static allocation of the consumption by the social planner between different generations at a given time t is important.

In ***the second stage of the maximization problem***, we write the current-value Hamiltonian for maximizing $V(C)$ as follows:

$$\mathcal{H} = V(C) + \lambda[F(K,R) - \gamma K - \Theta(X)R - C] + \psi[G(X) - R] \qquad 2.26$$

We generate Keynes-Ramsey condition and Hotelling rule for renewable resources:

$$\frac{V''(C)}{V'(C)}\dot{C} = r - [F_K - \gamma] \qquad 2.27$$

$$F_R - \theta(X) = \frac{1}{F_K - \gamma - r}\big[\dot{F_R} + (F_R - \Theta(X))G'(X) - \Theta'(X)G(X)\big] \qquad 2.28$$

As an important result, we can observe that the aggregation coefficient M and constant parameter D cancel out when we derive Keynes-Ramsey condition. This means that the aggregate quantities are governed by generational discount rate r and not by individual discount rates α and β.

Proposition 6. *Hotelling rule is not affected by the share of patient and impatient agents in the society, which changes the share of consumption at the aggregate level between these two type of agents in terms of patience level.*

Calvo and Obstfeld (1988), Marini and Scaramozzino (1995) show that the continuous time OLG modelling collapses to infinitely lived representative agent model for the case of homogeneous agents. In this paper, we have additionally showed that even in the case with heterogeneity in time-preferences, the continuous-time OLG model reduces to standard representative agent model.

By computing the derivatives for equation 2.27 Keynes-Ramsey condition can be written as follows:

$$F_K - \gamma = \theta \frac{\dot{C}}{C} + r \qquad 2.29$$

The conditions that we have found for Keynes-Ramsey and Hotelling show that in an OLG model, the optimum trajectory of the aggregate consumption and the resource extraction path is governed by inter-generational discount rate r and not by intra-generational discount factors but this result does not mean that intra-generational discount can not factor into consumption decisions.

At the steady state, we have;

$$\dot{C} = 0 \Rightarrow F_K = \gamma + r \qquad 2.30$$

Thus, before reaching the steady state, we would have $F_K > \gamma + r$ which implies $\dot{C} > 0$.

The fact that two conditions that we have given above are not governed by intra-generational discount does not mean that this one fails to factor into consumption decisions. Returning to the first stage of the optimization problem, we can make comparative static analysis in order to see how the intra-generational discount affects the consumption decisions. See Appendix C for the details.

Proposition 7. *When the intra-generational discount factor of impatient agents increases according to inter-generational discount rate, not only the consumption of young impatient agents increases, but also the consumption of young patient agents increases. This one is also valid for the intra-generational discount factor of patient agents.*

This result gives us interesting insights concerning the allocation of consumption across different types of young individuals. Young agents with different discounting factor are all in interaction. This corresponds also to economic reality. The same cohorts in terms of age influence each other's consumption. This result is also in the same line with Cowan et al. (2004) who argue that consumption of an agent is affected by peer group of similar consumers.

3. INTERGENERATIONAL NEUTRALITY

Discounting favors present generations at the expense of the future generations (Ramsey, 1928). It is therefore possible to manage this ethical question in a way that we give zero as value for the discount factor. The technical problem with this one is that welfare function is immediately infinite for any consumption path. So, the consumption path will not converge to zero. This problem can be overcome by using the specification proposed by Ramsey (1928). Note also that, contrary to the first section, we assume in this section that there does not exist extraction costs for renewable resources. This assumption permits us not to deal with tedious computations concerning our analysis of the impacts of individual discount rate on forever-sustainable income. We define the following maximization problem as follows:

$$\max_{C_t} W = \int_{t=0}^{\infty} (V(C_t) - V(\hat{C}))dt \qquad 3.1$$

where $V(\hat{C})$ is the bliss point or golden rule of consumption. The new Hamiltonian of our maximization problem is the following:

$$\mathcal{H} = \left[V(C_t) - V(\hat{C})\right] + \lambda[F(K,R) - \gamma K - C] + \psi[G(X) - R] \qquad 3.2$$

The Hamiltonian is time-independent, which we can name an autonomous control problem. So, we will have $\frac{\partial \mathcal{H}}{\partial t} = 0$. In the optimal path, $\frac{\partial \mathcal{H}}{\partial t} = \frac{\partial \mathcal{H}}{\partial \tau} = 0$, which yields $\mathcal{H} = 0$. We reformulate the problem as follows;

$$\begin{aligned} V(\hat{C}) &= V(C_t) + \lambda[F(K,R) - \gamma K - C] + \psi[G(X) - R] \\ &= V(C_t) + \lambda\dot{K} + \psi\dot{X} \end{aligned} \qquad 3.3$$

$$V(\hat{C}) = -\left[\frac{C}{M}\right]^{-(\theta-1)} D + \lambda\dot{K} + \psi\dot{X} \qquad 3.4$$

In this way, we have found the forever-sustainable income which is the maximum value of social utility at any time t, which is equal to the value of consumption, net investment and natural resources. The latter gives also a definition of Gross Net National Income, (GNNP) which is defined by Weitzman (1976). In this case of inter-generational neutrality, not only the income is sustained forever but also the income does not have a declining profile as it was the case in the neoclassical growth models with positive discounting rate.

Proposition 8. *The overall effect of static increase of individual discount rate on forever-sustainable income depends on the level of aggregate consumption of society.*

Case 1: **Negative effect on forever-sustainable income**

$$\frac{\partial \lambda}{\partial x} < 0 \text{ and } \frac{\partial V(C)}{\partial x} < 0 \text{ if } \mathrm{C(t)} < \frac{\delta M}{(\theta-1)D}$$

Case 2: **Ambiguous effect on forever sustainable income**

$$\frac{\partial \lambda}{\partial x} < 0 \text{ and } \frac{\partial V(C)}{\partial x} > 0 \text{ if } \mathrm{C(t)} > \frac{\delta M}{(\theta-1)D}$$

where $x = \beta - r$. We recall that this analysis is also valid for $x = \alpha - r$, which is for impatient agents. See Appendix D for the proof.

It is worth mentioning that the individual discount rate has an impact on sustainable income. We find that there exist two different channels concerning this impact.

First, when the aggregate consumption is under a threshold[3], the increase in individual discount rate of patient or impatient agents, which implies a higher share of aggregate consumption for young generations if $\alpha > \beta > r$[4], decreases the aggregate utility of consumption $V(C)$. In this first case, there exists a second channel, which concerns λ. When we look at equations 2.15 and 2.16, it is so easy to see that a static

[3] We find analytically this threshold.
[4] It is possible to say that generally agents are more impatient than the social planner. So, this assumption can be plausible in many cases.

increase of individual discount rates decrease the shadow price of capital λ. This causes the overall effect of a static increase of individual discount factor to decrease the forever-sustainable income of the economy.

Second, when the aggregate consumption is above a threshold, the static increase of individual discount rates if $\alpha > \beta > r$ increases the aggregate utility of consumption $V(C)$. The second channel confirms always that a static increase of individual discount rates decreases the shadow price of capital λ. As a nutshell, the overall effect of the static increase of individual discount rates is ambiguous.

Concerning the first case, we see that when the aggregate consumption level is under the threshold, the fact that young individuals consume more than old individuals decreases the forever-sustainable income. This result is quite interesting and gives theoretical support for some surveys about environmental concern of different age groups. Denniss (2005) defends the idea that old people care more about environment than young people in Australia with a data containing 56344 respondents.

4. FURTHER EXTENSIONS AND DISCUSSION

It is sure that this paper has some limits. One of the important limits is that we are using additive utility functions in the sense that utility of individuals at any date t depends neither on past nor on future. That's why, as it is possible to see at the first stage of our optimization problem, for any date, the social planner takes the same decision. In order to visualize this one, we draw the following Figure 4.1.

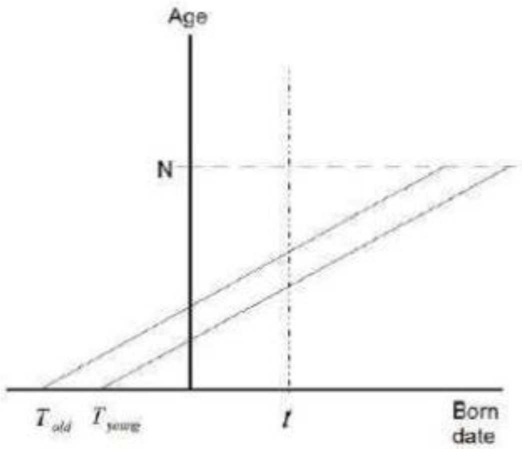

Figure 1. Generations over time

It is straightforward to say that social planner maximizes the utility of old and young generations at date t and does not take into account what has happened in the past concerning the consumption of different generations. So, even if there were inequalities for consumption among generations in the past, the social planner does not do anything in order to compensate for this inequality of consumption between generations. Further research must aim to focus on this limit. For this extension, one can benefit from Obstfeld (1990), using recursive utility functions, which we can express in a discrete-time model with a Bellman equation. For the moment, we limit our representation to a homogeneous individual.

$$U(C_t) = u(c_t) + U(C_{t+1})e^{-\theta(c_t)} \qquad 4.1$$

where $\theta(c_t)$ is the discount factor depending on consumption. To give a concise idea, the time-additive setup that we have used in this paper implies $\theta(c) = \theta$, which is a constant. Note that $\theta(c)$ and $u(c)$ are twice differentiable and strictly increasing and concave functions. An increasing $\theta(c)$ may be justified as follows: the more individuals consume, the more they become patient. This may be called as a wealth effect on the patience level of individuals. This functional form is a member of recursive utility functions, which obeys:

$$U(C_t) = W\big(c_t, U(C_{t+1})\big) \tag{4.2}$$

It is also possible to convert the discrete-time framework to continuous-time framework according to Obstfeld (1990):

$$U(C_0) = \int_0^{\infty} U(c_t)\, e^{\left(-\int_0^t \theta(c_s)ds\right)} dt \tag{4.3}$$

C_0 shows the consumption path which originates from $t = 0$. Notice also that 4.3 is the continuous-time analogue of the recursivity condition that we have defined in 4.1.

From this extension with non-additive time-preferences, we anticipate that marginal utility from consumption of different types of individuals on a given date will also depend on the consumption on other dates. For example, it is plausible to anticipate that consumption increases during a high interest rate. The main difference in this approach will be that the long-run target level of wealth is attended to be more well-defined. This one is also intuitive because when social planner optimizes the utility of different individuals at a date t, with non-additive time preferences, he can take into account the utility of ulterior periods like $t + 1$.

5. CONCLUSION

The analysis of different types of individual discount rate has given different results than those given by Endress et al. (2014). Our results have shown how these heterogeneities on individual discount factors can have various impacts on consumption profile of generations. An interesting finding is that the consumption profile between impatient and patient agents of same generation and different generations does not only depend on the level of individual discount rate but also depends on the level of individual discount rate relative to social planner's discount rate and on age τ.

From the first-stage of our optimization problem, it is also easy to see that at the optimum, the allocation of consumption between different types of agents within the young generation is equal while for the older generations it is optimal that patient agents consume more than impatient agents. We have also found that the Hotelling rule for renewable resources is not affected regardless of the share of patient and impatient agents across the society. Another interesting result is that the static increase of the discount rate of patient agents, which implies an increase in consumption of patient agents leads also to an increase of consumption of young impatient agents. This kind of interaction between similar agents in terms of age is also supported by Cowan (2004).

This paper has also investigated sustainability issues by a continuous-time OLG model. We show analytically that forever sustainable income is also affected by individual discount factors but the effect of the individual discount rates is not certain and they depend on the level of aggregate consumption of society. As we have discussed on the previous section, our setup with time-additive preferences has some limits that social planner does not consider the utility of other time periods when he maximizes the utility at a given date. Extending our model to non-additive time preferences à la Epstein (see Epstein 1986 and Obstfeld 1990) is highly desirable and planned in our future research program.

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APPENDIX:

A. We replace 2.1 and 2.2 in 2.3, so we have;

$$\widetilde{W} = \delta \int_0^\infty \int_\tau^N u_\beta\,(c(t,\tau))e^{-\beta\tau}e^{-r(t-\tau)}d\tau dt \qquad \text{A.1}$$

$$+(1-\delta)\int_0^\infty \int_\tau^N u_\alpha\,(c(t,\tau))e^{-\alpha\tau}e^{-r(t-\tau)}d\tau dt$$

We can distinguish quickly $V(C_\beta)$ and $V(C_\alpha)$ in this expression with $T = t - \tau$.

$$\mathrm{u}\big(c_i(t,\tau)\big) = -(c_i(t,\tau))^{-(\theta-1)}, \theta > 1 \qquad \text{A.2}$$

$$c_\beta^*(t,\tau) = c_\beta^*(t,0)e^{\frac{-(\beta-r)\tau}{\theta}} \text{ and } c_\alpha^*(t,\tau) = c_\alpha^*(t,0)e^{\frac{-(\alpha-r)\tau}{\theta}} \qquad \text{A.3}$$

B. We can substitute the each equation of A.3 in the consumption constraint of 2.14 to solve for the aggregate consumption of patient and impatient agents.

$$C_\beta(t) = \delta C = \int_0^N c_\beta^*(t,\tau)\,d\tau = \int_0^N c_\beta^*(t,0)e^{\frac{-(\beta-r)\tau}{\theta}}d\tau \qquad \text{B.1}$$

$$= \left(\frac{\theta\left(1-e^{\frac{-(\beta-r)N}{\theta}}\right)}{\beta-r}\right)c_\beta^*(t,0) = M_1 c_\beta^*(t,0)$$

$$C_\alpha(t) = (1-\delta)C = \int_0^N c_\alpha^*(t,\tau)\,d\tau = \int_0^N c_\alpha^*(t,0)e^{\frac{-(\alpha-r)\tau}{\theta}}d\tau \qquad \text{B.2}$$

$$= \left(\frac{\theta\left(1-e^{\frac{-(\alpha-r)N}{\theta}}\right)}{\alpha-r}\right)c_\alpha^*(t,0) = M_2 c_\alpha^*(t,0)$$

with $M_1 = \left(\frac{\theta\left(1-e^{\frac{-(\beta-r)N}{\theta}}\right)}{\beta-r}\right)$, $M_2 = \left(\frac{\theta\left(1-e^{\frac{-(\alpha-r)N}{\theta}}\right)}{\alpha-r}\right)$. Then $C = M_1 c_\beta^*(t,0) + M_2 c_\alpha^*(t,0)$. From 2.20;

$$c_\alpha^*(t,0) = c_\beta^*(t,0) \qquad \text{B.3}$$

Now, as

$$V(C) = Max\left[\delta \int_0^N u\left(c_\beta(t,\tau)\right) e^{-(\beta-r)\tau} d\tau + (1-\delta)\int_0^N u\left(c_\alpha(t,\tau)\right) e^{-(\alpha-r)\tau}\, d\tau\right] \qquad \text{B.4}$$

$$= \delta \int_0^N u(c_\beta^*(t,\tau)) e^{-(\beta-r)\tau} d\tau + (1-\delta)\int_0^N u(c_\alpha^*(t,\tau)) e^{-(\alpha-r)\tau} d\tau$$

and we have

$$\mathrm{u}\left(c_\beta^*(t,\tau)\right) = -(c_\beta^*(t,\tau))^{-(\theta-1)} = -(c_\beta^*(t,0)e^{\frac{-(\beta-r)\tau}{N}})^{-(\theta-1)} \qquad \text{B.5}$$

$$\mathrm{u}\left(c_\alpha^*(t,\tau)\right) = -(c_\alpha^*(t,\tau))^{-(\theta-1)} = -(c_\alpha^*(t,0)e^{\frac{-(\beta-r)\tau}{N}})^{-(\theta-1)} \qquad \text{B.6}$$

when we plug B.5 and B.6 in B.4 we obtain $V(C)$ as

$$V(C) = -\begin{bmatrix} \delta\left(c_\beta^*(t,0)\right)^{-(\theta-1)} \int_0^N e^{\frac{-(\beta-r)(1-\theta)\tau}{\theta}} e^{-(\beta-r)\tau} d\tau \\ +(1-\delta)(c_\alpha^*(t,0))^{-(\theta-1)} \int_0^N e^{\frac{-(\alpha-r)(1-\theta)\tau}{\theta}} e^{-(\alpha-r)\tau} d\tau \end{bmatrix} \qquad \text{B.7}$$

where $\int_0^N e^{\frac{(\beta-r)(1-\theta)\tau}{\theta}} e^{-(\beta-r)\tau} d\tau$ can be rewritten as $\int_0^N e^{\frac{-(\beta-r)\tau}{\theta}} d\tau$. By B.3, we have $c_\alpha^*(t,0) = c_\beta^*(t,0)$, then $V(C)$ becomes

$$V(C) = -\left(c_\beta^*(t,0)\right)^{-(\theta-1)}\left[\delta\left(\frac{\theta\left(1-e^{\frac{-(\beta-r)N}{\theta}}\right)}{\beta-r}\right) + (1-\delta)\left(\frac{\theta\left(1-e^{\frac{-(\alpha-r)N}{\theta}}\right)}{\alpha-r}\right)\right] \qquad \text{B.8}$$

If $D = \delta\left(\frac{\theta\left(1-e^{\frac{-(\beta-r)N}{\theta}}\right)}{\beta-r}\right) + (1-\delta)\left(\frac{\theta\left(1-e^{\frac{-(\alpha-r)N}{\theta}}\right)}{\alpha-r}\right)$, then

$$V(C) = -\left(c_\beta^*(t,0)\right)^{-(\theta-1)} D \qquad \text{B.9}$$

By using again B.3, we can write $C = c_\beta^*(t,0)(M_1 + M_2) = c_\beta^*(t,0)M$ where $M = M_1 + M_2$. Then, we will have

$$V(C) = -\left[\frac{C}{M}\right]^{-(\theta-1)} D \quad \text{B.10}$$

C. In the previous section, we have found $C = c_\beta^*(t,0)(M_1 + M_2) = c_\beta^*(t,0)M$. We prove that $\frac{\partial \widetilde{M}}{\partial \beta} > 0$ where $\widetilde{M} = \frac{1}{M} = \frac{1}{M_1+M_2}$. In order to not face to tedious derivative computations, we write;

$$M_1 = \frac{\theta(1 - e^{-bx})}{x} \quad \text{C.1}$$

where $x = \beta - r$ and $b = \frac{N}{\theta}$ and

$$M_2 = \frac{\theta(1 - e^{-by})}{y} \quad \text{C.2}$$

where $y = \alpha - r$.

$$\frac{\partial M_1}{\partial x} = \frac{\theta(1 + bx)e^{-bx} - 1}{\theta(1 - e^{-bx})^2} \quad \text{C.3}$$

It is sufficient to look at the sign of the numerator. The term $\theta(1 + bx)e^{-bx}$ attains 1 only if $x = 0$. Therefor for $x \neq 0$, the numerator is negative. This yields;

$$\frac{\partial M_1}{\partial x} = \frac{\theta(1 + bx)e^{-bx} - 1}{\theta(1 - e^{-bx})^2} < 0 \quad \text{C.4}$$

The same reasoning is valid also for M_2. So, in this case, $\frac{\partial M}{\partial x} < 0$. As we know that $\frac{\partial M}{\partial x} < 0$, we can deduce that $\frac{\partial \widetilde{M}}{\partial \beta} > 0$.

D. We take the derivative of $V(C)$ with respect to $x = \beta - r$;

$$\frac{\partial V(C)}{\partial x} = (\theta - 1)\left[\frac{M}{C}\right]^{-\theta} D\,\frac{\theta(1 + bx)e^{-bx} - 1}{\theta(1 - e^{-bx})^2} - \left[\frac{M}{C}\right]^{-(\theta-1)} \delta\,\frac{\theta(1 + bx)e^{-bx} - 1}{\theta(1 - e^{-bx})^2} \quad \text{D.1}$$

It is possible to see that $\frac{\partial V(C)}{\partial x}$>0 if;

$$\mathrm{C} > \frac{\delta M}{(\theta - 1)D} \quad \text{D.2}$$