A COMPUTATIONAL EFFICIENT DATA DETECTION METHOD FOR DOWNLINK OF AN MC-CDMA SYSTEM

Nihat KABAOĞLU[1], Rana ORTAÇ KABAOĞLU [2]

[1]Istanbul Medeniyet University, Department of Electrical and Electronics Engineering,
Faculty of Engineering and Natural Science, Uskudar, İstanbul, Turkey
nihat.kabaoglu@medeniyet.edu.tr
[2]Istanbul University, Department of Electrical and Electronics Engineering,
Faculty of Engineering, Avcilar, Istanbul, Turkey
rana@istanbul.edu.tr

Abstract: *In this paper, an Expectation-Maximization (EM) based iterative data detection method for downlink of a multicarrier system is proposed. The proposed method has low computational cost when compared with the other iterative ones since it does not require any matrix inversion if the requirements on users' data are met. The performance of the resulting algorithm is compared with Minimum Mean Squared Error (MMSE) estimator in terms of Symbol Error Rate (SER) for downlink of a Multi Carrier-Code Division Multiple Access (MC-CDMA) system in the presence of frequency selective channels using computer simulations. It is illustrated that the proposed algorithm outperforms the MMSE.*

Keywords: *Data detection, Multicarrier system, EM algorithm.*

1. Introduction

CDMA (Code Division Multiple Access) technique, which allows spreading of the users over the same physical channel with different codes assigned to each ones by using spread spectrum technology, is used as channel access method in various radio communication technologies. In addition to the resilience to CDMA technique, ICI (Interchannel Interference) and jamming problems, TDMA (Time Division Multiple Access) and FDMA (Frequency Division Multiple Access: Frequency (Multiple-Access) techniques, it is preferred both in today's systems and indispensable for next generation systems because it can support multimedia applications which require high data rate [1].

On the other hand, Orthogonal Frequency-Division Multiplexing (OFDM), which is a multi-carrier modulation technique with better performance over single carrier systems in multi-channel damping environments encountered in high-speed mobile communication systems, has become an area of interest in radio communication in itself. A broadband signal (high-speed data) is also an efficient way of reducing the amount of OFDM ISI (Inter-Symbol Interference), which is based on the realization of communication by splitting it into many narrowband signals (lower data rates). With the development of digital integrated technology, OFDM is used for many radio standards such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB-T), IEEE 802.11a, and IEEE 802.16a [2].

Both the advantages of multi-carrier modulation and the flexibility provided by the spread spectrum technique have encouraged many researchers to propose and develop a new technique called MC-SS (Multi-carrier Spread Spectrum), which is a combination of these two techniques. This combination can be used by many researchers independently of each other, such as Multi-tone Code Division Multiple Access (MT-CDMA), Multi-Carrier Code Division Multiple Access (MC-CDMA) and Multicarrier Direct Sequence Code Division Multiple Access (MC-DS-CDMA) [3] - [7]. In the last two decades, the MC-CDMA technique has been a powerful alternative to conventional modulation techniques for the downlink of a system due to its high spectral efficiency and its low receiver complexity. Today, it is still preferred as a popular multiple access technique for next generation systems [8]-[11].

If the channel delay exceeds a symbol duration, serious multiple-access interference (MAI) and inter-symbol-interference (ISI) are encountered in multi-carrier systems due to multipath propagation. As a result, the capacity of a multi-carrier system is limited. The effect of MAI can be partially removed by using orthogonal codes, but it cannot be possible to completely remove it because of the corruption on the orthogonality of users' codes and time

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delays. These effects on the MAI can be further reduced by using channel and data estimation of all active users. With this motivation, researchers began to present iterative PIC-based multi-user data detection studies for CDMA-based systems to enhance their performance which is sensitive to initial values [12], [13]. It means that they need high quality data and channel estimation in the initial stage. That is why ML type algorithms having high performance such as EM and space-alternating generalized expectation-maximization (SAGE) is preferred. This is also the main reason for us to propose an iterative EM algorithm. Although this type of algorithm gives an opportunity to jointly estimate data and channel coefficients, it is a computationally complex and slower algorithm if the interest is in both of them [13]. Hence, ML type joint estimation methods for CDMA based system are generally used in the uplink communication [14], [15]. For this reason, in this paper, we focus on only data estimation.

2. System Model

An MC-CDMA based cellular communication system in which there are K active users controlled by same the base station is considered. In this system, each active user's data is first multiplied with their own spreading codes, and then each chip of spreaded data symbol is mapped on to different subcarrier. After mapping and parallel to serial conversion, a cyclic prefix is inserted into the resulting signal. The obtained signal is transmitted through a frequency selective multipath fading channel whose impulse response is expressed as

$$g_k(t) = \sum_{s=1}^{S} h_{k,s}\delta\left(t-\tau_{k,s}\right). \tag{1}$$

Here, S is the number of paths; $h_{k,s}$ and $\tau_{k,s}$ are complex fading coefficients and delay of the sth path, respectively.

In this work, a synchronous downlink MC-CDMA system is assumed. The obtained signal at the output of the kth user's matched filter is

$$\begin{aligned}\mathbf{y} &= \sum_{n=1}^{K}\mathbf{Q}_{nk}\mathbf{b}_n + \mathbf{w}\\ &= \mathbf{Q}_{kk}\mathbf{b}_k + \underbrace{\underbrace{\sum_{n=1,n\neq k}^{K}\mathbf{Q}_{nk}\mathbf{b}_n}_{MAI} + \underbrace{\mathbf{n}}_{NOISE}}_{\mathbf{w}},\end{aligned} \tag{2}$$

where $\mathbf{y} = \left[y(1), y(2), \ldots, y(M)\right]^T$; $\mathbf{Q}_{nk} = \rho_{nk}\mathbf{f}\mathbf{h}_k\mathbf{I}_M$; ρ_{nk} represents the cross correlation between nth user's spreading and kth user's spreading code; $\mathbf{f} \in \mathbb{C}^{1\times S}$ represents the Fourier transform vector; $\mathbf{h_k} \in \mathbb{C}^{S\times 1} \sim CN\left(0, \mathbf{C}_{h_k}\right)$; $\mathbf{b}_n \in \mathbb{C}^{M\times 1} \sim CN\left(0, \mathbf{C}_{b_n}\right)$ for $n = 1, 2, \ldots, k, \ldots, K$ are independent and identically distributed; $\mathbf{n} \in \mathbb{C}^{M\times 1} \sim CN\left(0, \frac{\sigma^2}{P}\mathbf{I}_M\right)$ which represents the output of the matched filter. (2) can be written in a more compact form as follows:

$$\mathbf{y} = \mathbf{Q}_{kk}\mathbf{b}_k + \mathbf{w} \tag{3}$$

MAI can be modelled using Gaussian distribution when the number of active users increases. Assuming that all users' data are independent from each other and they have a common covariance matrix $\mathbf{C}_{b_k}$, $\sum_{n=1,n\neq k}^{K}\rho_{nk}^2 = (K-1)\rho^2$ and $h = \mathbf{f}\mathbf{h}_k$, statics of $\mathbf{w}$ can be written as follows:

$$\begin{aligned}&\boldsymbol{\mu}_w = \mathbf{0},\\ &\mathbf{C}_w = (K-1)\rho^2|h|^2\mathbf{C}_{b_n} + \frac{\sigma^2}{P}\mathbf{I}_M .\end{aligned} \tag{4}$$

When the length of the spreading sequence is increased, covariance matrix $\mathbf{C}_w$ can be approximetely expressed as $\mathbf{C}_w \approx (K-1)\rho^2|h|^2\mathbf{C}_{b_n}$.

3. Data Detection

In case of having the channel coefficients, Maximum Likelihood (ML) estimation of kth user's data vector can be given as

$$\hat{\mathbf{b}}_k = \underset{\mathbf{b}_k}{\arg\max}\log l\left(\mathbf{b}_k;\mathbf{y},\mathbf{h}_k\right) \tag{5}$$

where $l\left(\mathbf{b}_k;\mathbf{y},\mathbf{h}_k\right)$ is likelihood function of $\mathbf{b}_k$. Direct maximization of (5) needs more computational efforts for large values of K and M, but it can be easily solved iteratively. The aim of the work is to design a data detector that iterates between received raw data and given channel state information using iterative algorithms such as Expectation-Maximization (EM) and Space Alternating Generalized EM Algorithm (SAGE). In this work, we prefer EM algorithm to find a solution to (5) since it is one of them having high performance. EM algorithm needs complete and incomplete data sets. Complete and incomplete data sets for signal model in (3) can be chosen as $\{\mathbf{y},\mathbf{h}_k\}$ and $\mathbf{y}$ respectively. The vector to be estimated is $\mathbf{b}_k$ that maximizes the expected value of the logarithmic likelihood (log-likelihood) function with respect to $\{\mathbf{y},\mathbf{h}_k\}$ given $\mathbf{y}$ under the current estimate of the parameter $\mathbf{b}_k$ by using the following two steps.

3.1. Expectation Step (E-Step)

The E-Step to execute the EM algorithm is to find expected average of log-likelihood function that can be expressed as

$$Q\left(\mathbf{b}_k|\mathbf{b}_k^{(i)}\right) = E\left[\ln p\left(\mathbf{y},\mathbf{h}_k,\mathbf{b}_k\right)|\mathbf{y},\mathbf{b}_k^{(i)}\right], \tag{6}$$

where $\mathbf{b}_k^{(i)}$ is the estimation of $\mathbf{b}_k$ at ith iteration step and

$$\ln p(\mathbf{y},\mathbf{h}_k,\mathbf{b}_k) = \ln p(\mathbf{y}|\mathbf{h}_k,\mathbf{b}_k) + \ln p(\mathbf{h}_k|\mathbf{b}_k) + \ln p(\mathbf{h}_k) \quad (7)$$

By neglecting the term independent of $\mathbf{h}_k$, (2.7) can be re-written as

$$\ln p(\mathbf{y},\mathbf{h}_k,\mathbf{b}_k) \approx -(\mathbf{y}-\mathbf{Q}_{kk}\mathbf{b}_k)\mathbf{C}_w^{-1}(\mathbf{y}-\mathbf{Q}_{kk}\mathbf{b}_k)^\dagger - \mathbf{b}_k^\dagger\mathbf{C}_{b_k}^{-1}\mathbf{b}_k \quad (8)$$

where $(.)^\dagger$ denotes the conjugate transpose. Inserting (8) in (6), the conditional expected value in (6) can be expressed as

$$\begin{aligned} Q(\mathbf{b}_k|\mathbf{b}_k^{(i)}) &= -E\left[(\mathbf{y}-\mathbf{Q}_{kk}\mathbf{b}_k)\mathbf{C}_w^{-1}(\mathbf{y}-\mathbf{Q}_{kk}\mathbf{b}_k)^\dagger \| \mathbf{y},\mathbf{b}_k^{(i)}\right] \\ &\quad - E\left[\mathbf{b}_k^\dagger\mathbf{C}_{b_k}^{-1}\mathbf{b}_k|\mathbf{y},\mathbf{b}_k^{(i)}\right] \end{aligned} \quad (9)$$

When (9) is arranged, we have

$$\begin{aligned} Q(\mathbf{b}_k|\mathbf{b}_k^{(i)}) &= -\mathbf{y}^\dagger\mathbf{C}_w^{-1}\mathbf{y} - \mathbf{b}_k^\dagger\mathbf{C}_{b_k}^{-1}\mathbf{b}_k + \mathbf{f}E\left[\mathbf{h}_k|\mathbf{y},\mathbf{b}_k^{(i)}\right]\mathbf{y}^\dagger\mathbf{C}_w^{-1}\mathbf{b}_k \\ &\quad + E\left[\mathbf{h}_k^\dagger|\mathbf{y},\mathbf{b}_k^{(i)}\right]\mathbf{f}^\dagger\mathbf{b}_k^\dagger\mathbf{C}_w^{-1}\mathbf{y} \\ &\quad - E\left[\mathbf{h}_k^\dagger\mathbf{f}^\dagger\mathbf{f}\mathbf{h}_k|\mathbf{y},\mathbf{b}_k^{(i)}\right]\mathbf{b}_k^\dagger\mathbf{C}_w^{-1}\mathbf{b}_k \end{aligned} \quad (10)$$

To compeletly solve (10), we need the conditional statistics of $\mathbf{h}_k$. They can be obtained from its posteriori probability density function which is given as follows

$$p(\mathbf{h}_k|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{h}_k)p(\mathbf{h}_k) \quad (11)$$

where

$$\begin{aligned} \boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)} &= \boldsymbol{\Sigma}_{\mathbf{h}_k|\mathbf{y}}^{-1}\mathbf{f}^\dagger\mathbf{b}_k^{(i)\dagger}\mathbf{C}_w^{-1}\mathbf{y} \\ \boldsymbol{\Sigma}_{\mathbf{h}_k|\mathbf{y}}^{(i)} &= \left(\mathbf{C}_{h_k}^{-1} + \mathbf{f}^\dagger\mathbf{b}_k^{(i)\dagger}\mathbf{b}_k^{(i)}\mathbf{f}\right)^{-1} \end{aligned} \quad (12)$$

After computing the expectation on the right hand side of (10), it can be re-written as follows

$$\begin{aligned} Q(\mathbf{b}_k|\mathbf{b}_k^{(i)}) &= -\mathbf{y}^\dagger\mathbf{C}_w^{-1}\mathbf{y} + \mathbf{f}\boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)}\mathbf{y}^\dagger\mathbf{C}_w^{-1}\mathbf{b}_k + \boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)\dagger}\mathbf{f}^\dagger\mathbf{b}_k^\dagger\mathbf{C}_w^{-1}\mathbf{y} \\ &\quad - \left[tr\left(\mathbf{f}^\dagger\mathbf{f}\boldsymbol{\Sigma}_{\mathbf{h}_k|\mathbf{y}}^{(i)}\right) + \boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)\dagger}\mathbf{f}^\dagger\mathbf{f}\boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)}\right]\mathbf{b}_k^\dagger\left(\mathbf{C}_w^{-1} + \mathbf{C}_{b_k}^{-1}\right)\mathbf{b}_k \end{aligned} \quad (13)$$

3.2. Maximization Step (M-Step)

In the M-Step of the proposed algorithm is to update the estimate of data sequence at the next itertaion step according to

$$\mathbf{b}_k^{(i+1)} = \underset{\mathbf{b}_k}{\arg\max} \log Q\left(\mathbf{b}_k|\mathbf{b}_k^{(i)}\right) \quad (14)$$

In order to obtain $\mathbf{b}_k^{(i+1)}$, it is necessary to take the derivative of $Q(\mathbf{b}_k|\mathbf{b}_k^{(i)})$ with respect to $\mathbf{b}_k$, and equate the resulting equation to zero. Hence, when no coding is employed, we can obtain $\mathbf{b}_k^{(i+1)}$ as follows

$$\mathbf{b}_k^{(i+1)} = \beta^{(i)}\left(\mathbf{I}_M + \mathbf{C}_w C_{b_k}^{-1}\right)^{-1}\mathbf{y} \quad (15)$$

where

$$\beta^{(i)} = \frac{\boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)\dagger}\mathbf{f}^\dagger}{tr\left(\mathbf{f}^\dagger\mathbf{f}\boldsymbol{\Sigma}_{\mathbf{h}_k|\mathbf{y}}^{(i)}\right) + \boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)\dagger}\mathbf{f}^\dagger\mathbf{f}\boldsymbol{\mu}_{\mathbf{h}_k|\mathbf{y}}^{(i)}} \quad (16)$$

Inserting (16) in (15), we finally have

$$\mathbf{b}_k^{(i+1)} = \frac{\beta^{(i)}}{\left[1 + (K-1)\rho^2|h|^2\right]}\mathbf{y}\ . \quad (17)$$

It should be noted that (17) does not require any matrix inversion under the assumption that all users' data is independent from each other and they have a common covariance matrix.

4. Computer Simulations

In this section, we present computer simulation results to reveal the performance of the proposed EM algorithm based on a power controlled cellular MC-CDMA system. The number of active users in the system is K=8, the length of data frame for each user is M=32, the number of cahnnel taps is S=4. The orthogonal Gold sequence code is selected as spreading code and its processing gain P equals to the number of subcarriers N_c=128. It is assumed that the system operates with Quadrature Pahase Shift Keying (QPSK) signaling, and its channel characteristics is frequency selctive multipath fading channel. We analyse the performance of the proposed algorithm in the presence of both the perfect Channel State Information (CSI) and imperfect CSI by comparing with MMSE estimation method.

In Figure 1, SER performance of the proposed algorithm is presented as a function of signal to noise ratio (SNR) by means of solid and dashed curves when receiver has both perfect and imperfect CSI respectively. It can be concluded that the proposed EM algorithm performs better than MMSE method for both CSI cases. The proposed algorithm could be expected to perform further, but its performance is lower because of the approximation made for the covariance matrix of the data sequence. Another reason for relatively poor performance may be that the number of active users is insufficient to model MAI as a Gaussian distribution.

In Figure 2, assuming that receiver has perfect CSI, the SER is plotted as a function of the number of iterations for several values of SNR to examine the convergence rate of the EM algorithm. It can be seen that three or four iterations are sufficient in order for the proposed EM algorithm to converge, depending on SNR as well on initial values.

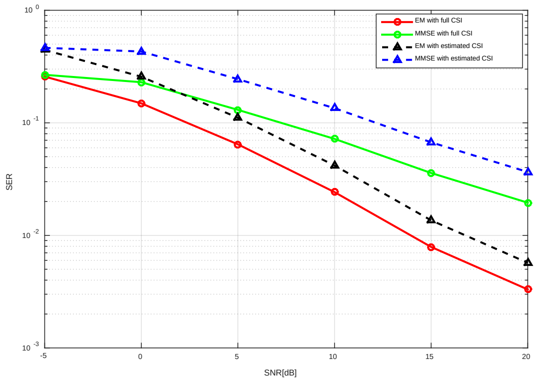

Figure 1. SER versus SNR simulation results

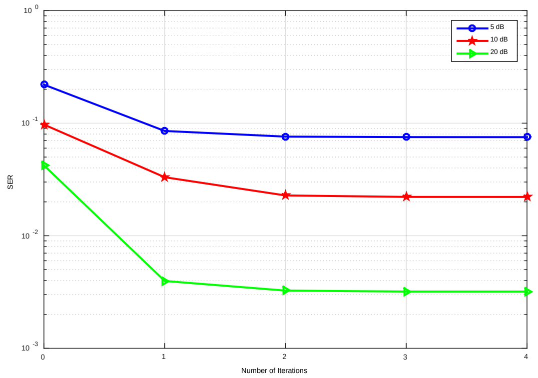

Figure 2. SER versus number of iterations for different SNR

4. Conclusion

In this work, we have presented a computationally efficient EM based data detection method for downlink of an MC-CDMA system. It has a simple implementation since it does not require any matrix inversion when the requirements on users' data are met. Simulation studies have indicated that SER performance of the proposed EM algorithm significantly outperforms conventional MMSE algorithm. Moreover, it is seen that the proposed EM algorithm has the same SER values as MMSE with an average gain of 4 dB.

5. References

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Nihat Kabaoğlu received the B.S. and M.S. degrees in electronics engineering from Istanbul University, and the Ph.D. degree in electronics and telecommunication engineering from Istanbul Technical University, Istanbul, Turkey, in 1997, 2000, 2005 respectively. After working in different positions at Istanbul University, The Scientific and Technological Research Council of Turkey, Kadir Has University and Maltepe University, he joined to Istanbul Medeniyet University as an associate professor in 2015. His general research interests cover estimation theory, statistical signal processing, communication theory, wireless sensor networks and array processing. His current research areas are focused on wireless communication concepts with specific attention to channel estimation for spread-spectrum and multicarrier systems, on beamforming design for next generation systems and on optimal power allocation in wireless sensor networks.

Rana Ortaç Kabaoğlu received the B.S. and M.S. degrees in electrical and electronics engineering from Istanbul University, and the Ph.D. degree in control engineering from Istanbul Technical University, Istanbul, Turkey, in 1997, 2001, 2010 respectively. At 1998, she joined to Istanbul University, electrical and electronics engineering department as a reseach assistant. She has been working as an assosiated professor at the same department since 2015. Her research areas are control systems, modelling, intelligent systems, fault dedection and control.