Research Article GU J Sci, Part A, 10(4): 402-416 (2023) 10.54287/gujsa.1333067

Gazi University
Journal of Science
PART A: ENGINEERING AND INNOVATION
http://dergipark.org.tr/gujsa

An Improvement in Estimating the Population Mean Based on Family of Estimators with Different Application Areas

Ceren UNAL[1*] Cem KADILAR[1]

[1] Department of Statistics, Faculty of Science, Hacettepe University, Ankara, Türkiye

Keywords
Population Mean
Family of Estimators
Auxiliary Information
Non-Response
Efficiency

Abstract
In a sampling study, the complete information for the necessary variables may not always be available in practice. Therefore, the non-response situation has been considered for estimating the unknown population parameters with different types of estimators. The families of estimators are proposed for the population mean in the case of non-response under two different cases with the approach of an exponential function. Their properties are derived in detail. We compare these estimators with the main estimators in the literature to present the efficiencies, theoretically. Moreover, the three different empirical studies are illustrated and, in that way, we found that the theoretical conclusion is supported by the obtained results numerically for each data set.

Cite

Unal, C., & Kadilar, C. (2023). An Improvement in Estimating the Population Mean Based on Family of Estimators with Different Application Areas. *GU J Sci, Part A*, *10*(4), 402-416. doi:10.54287/gujsa.1333067

Author ID (ORCID Number)

0000-0002-9357-1771 Ceren UNAL
0000-0003-4950-9660 Cem KADILAR

Article Process

Submission Date	26.07.2023
Revision Date	13.09.2023
Accepted Date	28.11.2023
Published Date	11.12.2023

1. INTRODUCTION

In sampling theory, the sample describes as a sub-group of the population and is utilized to avoid the difficulty of money, time, labor, etc. which originated from the population. Under these circumstances, the choice of sample and sampling method becomes evident. The process of the estimation for any unknown population parameters begins after determining the sample. The estimator, which is a mathematical equation, utilize for estimating these parameters. In general, the most efficient estimator is preferred compared to others. Here, one of the most appropriate methods is the utilize of information of auxiliary variable (x) for increasing efficiency. Many researchers propose different types of estimators to estimate the mean of the population utilizing auxiliary variable information. At this point, Yadav and Zaman (2021) proposed ratio type estimators using non-conventional and conventional parameters. Tailor and Lone (2014); Mehta and Tailor (2020); Singh and Nigam (2020) and Yadav et al. (2021) suggested various ratio type estimators for estimation of population mean using different sampling methods. Oncel Cekim & Kadilar (2018); Javed et al. (2019); Shabbir and Onyango (2022) and Oncel Cekim (2022) introduced unbiased estimators under various sampling methods.

When complete information is obtained on both the variable of study (y) and the variable of auxiliary (x), some of the important estimators for estimating the population mean ($\bar{Y}$) in literature are as follows:

Cochran (1940, 1977) is introduced both classical ratio and classical regression estimators for $\bar{Y}$, respectively

$$t_R = \frac{\bar{y}}{\bar{x}}\bar{X}, \tag{1}$$

$$t_{reg} = \bar{y} + b(\bar{X} - \bar{x}), \tag{2}$$

*Corresponding Author, e-mail: cerenunal@hacettepe.edu.tr

where $\bar{x}$ and $\bar{y}$ are the sample mean due to x and y, respectively. $\bar{X}$ is the population mean for x. b represents the regression coefficient of Y on X.

The MSE equations of (1) and (2) are given by

$$\mathrm{MSE}(t_R) = \lambda \bar{Y}^2 (C_y^2 + C_x^2 - 2C_{yx}), \tag{3}$$

$$\mathrm{MSE}(t_{reg}) = \lambda \bar{Y}^2 C_y^2 (1 - \rho_{xy}^2), \tag{4}$$

respectively, where $f = \frac{n}{N}, \lambda = \frac{1-f}{n}, C_x^2 = \frac{S_x^2}{\bar{X}^2}, C_y^2 = \frac{S_y^2}{\bar{Y}^2}, C_{xy} = \rho_{xy} C_x C_y$. Here, coefficient of f means sampling rate. The coefficient of population correlation is denoted as ρ_{xy} .

A family of estimators has been defined by Khoshnevisan et al. (2007). This family of estimators and their minimum MSE are given as follows:

$$t_K = \bar{y} \left(\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)a\bar{X}+b} \right)^g, \tag{5}$$

and

$$\mathrm{MSE}_{min}(t_K) = \lambda \bar{Y}^2 C_y^2 (1 - \rho_{xy}^2). \tag{6}$$

Later, studies have concentrated on creating modified modified estimators. Numerous researchers introduce various kinds of estimators. Among these type of estimators, Bahl and Tuteja (1991) were the first to provide estimators using exponential function strategy as

$$t_{BT} = \bar{y} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}} \right), \tag{7}$$

and MSE of Eq. (7) is given as

$$\mathrm{MSE}(t_{BT}) = \lambda \bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} - C_{yx} \right). \tag{8}$$

After this estimator in Eq. (7), Shabbir et al. (2014); Özel Kadilar (2016); Zaman and Kadilar (2019, 2021a, 2021b); Ahmad et al. (2021) provided exponential type of estimators under various sampling methods.

All of the mentioned estimators are defined in a case that the variables have only response units. This situation can be considered in theory but may not be obtained in practice. Therefore, the estimation in the presence of non-response units has become prominent in recent years. In literature, Hansen and Hurwitz (1946) presented a new method to deal with this situation.

In the Hansen and Hurwitz (1946) procedure, a sample size of n units is drawn from the population of N units with SRSWOR, which is denoted as $= \{S_1, S_2, \ldots, S_N\}$. For y and x respectively, the individual elements for the i^{th} unit in the population are represented as (y_i, x_i). This method divides the size of the population N into two parts: the number of respondent units (N_1) and the number of non-respondent units (N_2). Similar to this situation, the sample size, n, is also split into two parts, which are referred to as n_1 and n_2. In addition, a sub-sample size of r (where $r = {}^{n_2}/_{p}$) is drawn from the n_2 units by means of extra effort. Some studies refer to this aspect of the method as the subsampling technique. It is important to note that p (where $p > 1$) represents the inverse of the sampling rate in the sample of size n in the second phase. This means that this technique can be used to estimate using the total number of units $(n_1 + r)$, which replaces n. Using this procedure, the unbiased estimator with total $(n_1 + r)$ units for the nonresponse population was introduced by Hansen and Hurwitz (1946) as

$$t_{HH} = w_1\bar{y}_1 + w_2\bar{y}_{2(r)}, \tag{9}$$

where $w_2 = {n_2}/{n}$ and $w_1 = {n_1}/{n}$. For clarity, w_1 is the proportion of respondent units while w_2 is the proportion of non-respondent units for the sample. For the study variable, the $\bar{y}_{2(r)}$ and $\bar{y}_1$ refer the sample means due to the *r* and n_1 units, respectively.

The variance of $t_{HH}$is given by

$$V(t_{HH}) = \bar{Y}^2\left(\lambda C_y^2 + \frac{W_2(p-1)}{n}C_{y(2)}^2\right). \tag{10}$$

Here, $C_{y(2)}^2 = {S_{y(2)}^2}/{\bar{Y}^2}$.

There are two main forms of these non-response problems. Firstly, Case I is defined as units that do not respond on the *y* only. Secondly, Case II is defined by units that do not respond at both the *y* and *x*. The $(\bar{X})$ is known for both of these cases. In theoretical terms, $\bar{x}^*$ and $\bar{y}^*$ refer to the mean of the sample for x and y in the presence of nonresponse. Following the pioneering work of Hansen and Hurwitz (1946), the researchers propose the estimators for the $(\bar{Y})$ by taking into account the two different non-response cases.

For the Case I, Rao (1986) defines the following classical ratio and classical regression estimators, respectively, as follows:

$$t_R^* = \frac{\bar{y}^*}{\bar{x}}\bar{X}, \tag{11}$$

$$t_{reg}^* = \bar{y}^* + b(\bar{X} - \bar{x}), \tag{12}$$

where $b^* = {S_{xy}^*}/{S_x^{*2}}$. To obtain the MSE of the estimators in Eq. (11) – Eq. (12), we have $\bar{y}^* = \bar{Y}(1 + e_0^*)$ and $\bar{x} = \bar{X}(1 + e_1)$.

Then, $E(e_0^*) = E(e_1) = 0, E(e_1^2) = \lambda C_x^2, E\left({e_0^*}^2\right) = \lambda C_y^2 + \frac{W_2(p-1)}{n}C_{y(2)}^2$, and , $E(e_0^* e_1) = \lambda\rho_{xy}C_yC_x$.

By utilizing the provided these definitions, the MSE of the t_R^* and t_{reg}^* are given as

$$MSE(t_R^*) = \bar{Y}^2\left(\lambda\left(C_y^2 + C_x^2 - 2C_{yx}\right) + \frac{W_2(p-1)}{n}C_{y(2)}^2\right), \tag{13}$$

$$MSE\left(t_{reg}^*\right) = \bar{Y}^2\left(\lambda C_y^2\left(1 - \rho_{xy}^2\right) + \frac{W_2(p-1)}{n}C_{y(2)}^2\right). \tag{14}$$

Singh et al. (2010) presented the first exponential estimators, utilizing Eq. (7) for Case I, in accordance with Bahl and Tuteja (1991) as

$$t_{BT}^* = \bar{y}^*\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right), \tag{15}$$

whose MSE is given by

$$MSE(t_{BT}^*) = \bar{Y}^2\left(\lambda\left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right) + \frac{W_2(p-1)}{n}C_{y(2)}^2\right). \tag{16}$$

For the Case 2, Cochran (1977) suggested the following classical ratio and classical regression estimators as

$$t_R^{**} = \frac{\bar{y}^*}{\bar{x}^*}\bar{X}, \tag{17}$$

$$t_{reg}^{**} = \bar{y}^* + b(\bar{X} - \bar{x}^*), \tag{18}$$

respectively.

Singh et al. (2010) also proposed the estimator using the exp. function for the Case II as

$$t_{BT}^{**} = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right). \tag{19}$$

To obtain the MSE of the estimators in Eq. (17) – Eq. (19), we have $\bar{x}^* = \bar{X}(1 + e_1^*)$.

Then, $E(e_1^*) = 0$, $E\left({e_1^*}^2\right) = \lambda C_x^2 + \frac{W_2(p-1)}{n} C_{x(2)}^2$, $E(e_0^* e_1^*) = \lambda \rho_{xy} C_y C_x + \frac{W_2(p-1)}{n} \rho_{xy(2)} C_{y(2)} C_{x(2)}$, and using these equations, the MSE of the mentioned estimators are, respectively, obtained by

$$\mathrm{MSE}(t_R^{**}) = \bar{Y}^2 \left(\lambda\left(C_y^2 - 2C_{yx} + C_x^2\right) + \frac{W_2(p-1)}{n}\left(C_{y(2)}^2 + C_{x(2)}^2 - 2C_{yx(2)}\right)\right), \tag{20}$$

$$\mathrm{MSE}(t_{reg}^{**}) = \bar{Y}^2 \left(\lambda C_y^2\left(1 - \rho_{xy}^2\right) + \frac{W_2(p-1)}{n}\left(C_{y(2)}^2 + \rho_{xy}^2 \frac{C_y^2}{C_x^2} C_{x(2)}^2 - 2\rho_{xy} \frac{C_y}{C_x} C_{yx(2)}\right)\right), \tag{21}$$

$$\mathrm{MSE}(t_{BT}^{**}) = \bar{Y}^2 \left(\lambda\left(C_y^2 - C_{yx} + \frac{C_x^2}{4}\right) + \frac{W_2(p-1)}{n}\left(C_{y(2)}^2 + \frac{C_{x(2)}^2}{4} - C_{yx(2)}\right)\right), \tag{22}$$

and $\rho_{xy(2)} = \frac{C_{yx(2)}}{C_{y(2)} C_{x(2)}}$ is the coefficient of population correlation for the non-response group.

On the line of these estimators, Sanaullah et al. (2019), Anieting et al. (2020); Ahmad et al. (2022); Ahmadini et al. (2022); Fatima et al. (2022); Rehman and Shabbir (2022) and Sharma et al. (2022) recently proposed various type of estimators for the estimation of $\bar{Y}$ under both cases in the literature. Especially, there are also many proposed estimators using exponential function strategy in the literature under non-response scheme. Under the non-response condition, Khan et al. (2023) suggested a new exp-ratio type estimator using double sampling for estimating the $\bar{Y}$. Zahid et al. (2022) proposed a generalized dual to exp-ratio type estimator. Kumar and Bhougal (2011) modified ratio-product type exp. estimator following Singh et al. (2008) study. Kumar (2013); Yunusa and Kumar (2014) and Unal and Kadilar (2021, 2022a, 2022b) proposed estimators using exp. function for the estimating $\bar{Y}$. Kumar and Kumar (2017) and Pal and Singh (2017, 2018) proposed various estimators taking the advantage of the exp. function. Dansawad (2019) introduced a class of exp. type estimators. Singh and Usman (2019a, 2019b) proposed a general family of exp. type and the ratio-product type difference-cum-exp. type estimators, respectively in their studies.

The estimator that is proposed by Khoshnevisan et al. (2007) and given in Eq. (5) is important in the literature and has formed the basis for many studies. In this present study, this estimator was specifically used and proposed again by adding an exponential function in the case of non-response in Section 2. Results of the efficiency comparisons are made theoretically and numerically, as well, which are obtained in Sections 3 and 4, respectively. In final part, Section 5 introduces the results of the study.

2. THE ADAPTED ESTIMATORS

Following Khoshnevisan et al. (2007), we suggest the new estimator with adapt the family of estimators in Eq. (5) under two different cases as first case and second case.

2.1. The adapted estimators for the first case:

The first proposed family of estimators is given by

$$t_{C1} = \bar{y}^* \exp\left(\frac{a\bar{X} + b}{\alpha_1(a\bar{x} + b) + (1 - \alpha_1)(a\bar{X} + b)} - 1\right), \tag{23}$$

where α_1 is a chosen constant which using for the MSE minimum. In Eq. (23), the values of α and b can be correlation coefficient, coefficient of variation, skewness, kurtosis etc.

In terms of e_0^* and e_1, we have

$$t_{C1} = \bar{Y}\left(1 + e_0^* - \alpha_1\theta e_1 + \frac{3\alpha_1^2\theta^2 e_1^2}{2} - \alpha_1\theta e_0^* e_1\right) \quad (24)$$

where $\theta = \frac{a\bar{X}}{a\bar{X}+b}$.

If $\bar{Y}$ is subtracted and get the expected value from both sides in Eq. (24):

$$E(t_{C1} - \bar{Y}) = B(t_{C1}) = \bar{Y}\lambda C_x^2\alpha_1\theta\left(\frac{3\alpha_1\theta}{2} - \rho_{yx}\frac{C_y}{C_x}\right). \quad (25)$$

When taking square of Eq. (25), we get MSE of the t_{C1} estimator as

$$MSE(t_{C1}) = \bar{Y}^2\left(\lambda\left(C_y^2 + \alpha_1^2\theta^2 C_x^2 - 2\alpha_1\theta C_{yx}\right) + \frac{W_2(p-1)}{n}C_{y(2)}^2\right). \quad (26)$$

By the minimization of (26), the MSE of the t_{C1} is min. for the optimal value of

$$\alpha_1^* = \left(\frac{1}{\theta}\rho_{xy}\frac{C_y}{C_x}\right). \quad (27)$$

We get the min MSE of the t_{C1}, using the value of α_1^* in Eq. (26), as follows:

$$MSE_{min}(t_{C1}) = \bar{Y}^2\left(\lambda C_y^2\left(1 - \rho_{xy}^2\right) + \frac{W_2(p-1)}{n}C_{y(2)}^2\right). \quad (28)$$

It is important to note that the $MSE_{min}(t_{C1})$ equals the $MSE\left(t_{reg}^*\right)$ in Eq. (14) under the first case.

We see that there are 10 different θ as $(\theta_1, \theta_2, \ldots, \theta_{10})$ in Table 1.

Table 1. *θ Values*

$\theta_i, i = 1,2,\ldots,10$	α	b
1	1	1
2	1	$\beta_2(x)$
3	1	C_x
4	1	ρ
5	$\beta_2(x)$	C_x
6	C_x	$\beta_2(x)$
7	C_x	ρ
8	ρ	C_x
9	$\beta_2(x)$	ρ
10	ρ	$\beta_2(x)$

2.2. The adapted estimators for the second case:

The second proposed family of estimators is given by

$$t_{C2} = \bar{y}^*\exp\left(\frac{a\bar{X}+b}{\alpha_2(a\bar{x}^*+b)+(1-\alpha_2)(a\bar{X}+b)} - 1\right), \quad (29)$$

where α_2 is a chosen constant that determines the MSE of the proposed estimator minimum. We can also generate some members for the t_{C2} estimator under the second case as in Table 1 by replacing $\bar{x}$ with $\bar{x}^*$.

In terms of e_i^* $(i = y, x)$, we can write

$$t_{C2} = \bar{Y}\left(1 + e_0^* - \alpha_2\theta e_1^* + \frac{3\alpha_2^2\theta^2 e_1^{*2}}{2} - \alpha_2\theta e_0^* e_1^*\right). \quad (30)$$

Using the t_{C1} estimator's similar procedure, we arrive at the bias and MSE of the t_{C2}, respectively, as follows:

$$B(t_{C2}) = \bar{Y}\left(\lambda\left(\frac{3\alpha_2^2\theta^2}{2}C_x^2 - \alpha_2\theta C_{yx}\right) + \frac{W_2(p-1)}{n}\left(\frac{3\alpha_2^2\theta^2}{2}C_{x(2)}^2 - \alpha_2\theta C_{yx(2)}\right)\right), \quad (31)$$

$$MSE(t_{C2}) = \bar{Y}\left(\lambda\left(C_y^2 + \alpha_2^2\theta^2 C_x^2 - 2\alpha_2\theta C_{yx}\right) + \frac{W_2(p-1)}{n}\left(C_{y(2)}^2 + \alpha_2^2\theta^2 C_{x(2)}^2 - 2\alpha_2\theta C_{yx(2)}\right)\right), \quad (32)$$

We obtain the optimal value of α_2 by the minimization of the MSE equation in Eq. (32) as

$$\alpha_2^* = \frac{\lambda C_{yx} + \frac{W_2(p-1)}{n}C_{yx(2)}}{\theta\left(\lambda C_x^2 + \frac{W_2(p-1)}{n}C_{x(2)}^2\right)}. \quad (33)$$

Using the value of α_2^*, the $MSE_{min}(t_{C2})$ equation is determined as follows:

$$MSE_{min}(t_{C2}) = \bar{Y}^2\left[\lambda C_y^2 + \frac{W_2(p-1)}{n}C_{y(2)}^2 - \frac{\left(\lambda C_{yx} + \frac{W_2(p-1)}{n}C_{yx(2)}\right)^2}{\lambda C_x^2 + \frac{W_2(p-1)}{n}C_{x(2)}^2}\right]. \quad (34)$$

3. EFFICIENCY COMPARISONS

To prove the efficiency, comparison of the t_{C1} and t_{C2} estimators has been made with the mentioned classical estimators under both cases, respectively. The efficiency conditions have also been stated. Firstly, we utilize Eq. (10), (13), (16), and Eq. (29) to compare the efficiencies of the t_{C1} with the t_{HH}, t_R^*, and t_{BT}^* for the Case I. Here, comparison between the t_{C1} estimator and the regression estimator t_{reg}^* is not included because the minimum MSEs of the estimators are equal to each other. We obtain the following efficiency conditions of the t_{C1} estimator.

$$[MSE(t_{HH}) - MSE_{min}(t_{C1})] = \lambda\rho_{xy}^2 C_y^2 > 0, \quad (35)$$

$$[MSE(t_R^*) - MSE_{min}(t_{C1})] = \left(C_x - \rho_{xy}C_y\right)^2 > 0, \quad (36)$$

$$[MSE(t_{BT}^*) - MSE_{min}(t_{C1})] = \left(\frac{C_x}{2} - \rho_{xy}C_y\right)^2 > 0. \quad (37)$$

The t_{C1} estimator perform better at the optimal value of α_1 than t_{HH}, t_R^*, and t_{BT}^* estimators, according to the conditions between Eq. (35) – Eq. (37), as these conditions are always satisfied.

Secondly, we compare the MSEs of the t_{C2} with the $t_{HH}, t_R^{**}, t_{reg}^{**}$ and t_{BT}^{**} for the Case II. Using Eq. (10), (20), (21), (22), and Eq. (34), we respectively have

$$[MSE(t_{HH}) - MSE_{min}(t_{C2})] = \left(\lambda C_{yx} + \frac{W_2(p-1)}{n}C_{yx(2)}\right)^2 > 0, \quad (38)$$

$$[MSE(t_R^{**}) - MSE_{min}(t_{C2})] = \left(\left(\lambda C_x^2 + \frac{W_2(p-1)}{n}C_{x(2)}^2\right) - \left(\lambda C_{yx} + \frac{W_2(p-1)}{n}C_{yx(2)}\right)\right)^2 > 0, \quad (39)$$

$$[MSE(t_{BT}^{**}) - MSE_{min}(t_{C2})] = \left(\left(\lambda C_{yx} + \frac{W_2(p-1)}{n}C_{yx(2)}\right) - \frac{1}{2}\left(\lambda C_x^2 + \frac{W_2(p-1)}{n}C_{x(2)}^2\right)\right)^2 > 0, \quad (40)$$

$$iv)\ \left[\mathrm{MSE}\left(t_{reg}^{**}\right)-\mathrm{MSE}_{min}(t_{C2})\right]=\left(\left(\frac{W_2(p-1)}{n}C_{x(2)}^2\rho_{yx}\frac{C_y}{C_x}\right)-\left(\frac{W_2(p-1)}{n}C_{yx(2)}\right)\right)^2>0. \quad (41)$$

The conditions Eq. (38) – Eq. (41) are always satisfied, thus we conclude that the t_{C2} estimator perform better at the optimal value of α_2 than the compared estimators.

4. EMPIRICAL STUDY

As we show that the proposed t_{C1} and t_{C2} estimators, using the optimal values of α_1 andα_2, respectively, are always the most efficient estimators among compared estimators for the first and second cases, respectively, in Section 3, we obtain the ranges of α_1 and α_2 values that make the proposed families of estimators, respectively, more efficient than other estimators, based on the different values of *p*, in this section. We also compute the MSE values and using these values obtain the percent relative efficiencies (PRE) for each proposed and compared estimators by using Eq. (43) as below:

$$PRE(t_*)=\frac{Var(t_{HH})}{MSE(t_*)}x100\ . \quad (42)$$

In this equation, t_* symbolizet_R^*, t_{BT}^*, t_{C1}, t_R^{**}, t_{BT}^{**}, t_{reg}^{**}, and t_{C2} estimators, respectively. For the comparison, the reference estimator is t_{HH} estimator. We have utilized the popular data sets of three populations in Unal and Kadilar (2019), referred by many studies in literature, as well. In this way, we try to prove the performance of the t_{C1} and t_{C2} estimators for the first and second cases, respectively, in practice. In this section, we have utilized three distinct datasets from various sources.

The first dataset (Population 1) consists of seventy observations indicating the population of the village and cultivated area (Khare & Srivastava, 1993). This Population 1 represents the cultivated area as the variable of study "y" and the village population as the variable of auxiliary "x". The second dataset (Population 2) originates from Khare and Sinha (2009) and involves the variable of study being the number of agriculture labors, while the variable of auxiliary is the area of the village. Lastly, the third dataset (Population 3) is obtained from Satici and Kadilar (2011). In Population 3, the variable of study is the number of successful students, and the variable of auxiliary is the number of teachers. The underlying population parameters are briefly summarized for Populations 1-3 as follows:

Population 1. *(Khare & Srivastava, 1993)*

N=70, n=35	$\bar{X}$ =1755.53	$\rho_{yx(2)}$ =0.45	C_{yx} =0.39	$C_{yx(2)}$ =0.10
λ=0.014	$\bar{Y}$ =981.29	ρ_{yx} =0.78	C_x =0.80	$C_{x(2)}$ =0.57
f=0.50	W_2 =0.2	$\beta_2(x)$ =0.34	C_y =0.63	$C_{y(2)}$ =0.41

Population 2. *(Khare & Sinha, 2009)*

N=96, n=40	$\bar{X}$ =144.87	$\rho_{yx(2)}$ =0.72	C_{yx} =0.82	$C_{yx(2)}$ =1.41
λ=0.01458	$\bar{Y}$ =137.92	ρ_{yx} =0.77	C_x =0.81	$C_{x(2)}$ =0.94
f=0.42	W_2 =0.25	$\beta_2(x)$ =1.19	C_y =1.32	$C_{y(2)}$ =2.08

Population 3. *(Satici & Kadilar, 2011)*

N=261, n=90	$\bar{X}$ =306.44	$\rho_{yx(2)}$ =0.97	C_{yx} =3.19	$C_{yx(2)}$ =1.46
λ=0.01	$\bar{Y}$ =222.58	ρ_{yx} =0.97	C_x =1.76	$C_{x(2)}$ =1.23
f=0.35	W_2 =0.25	$\beta_2(x)$ =21.36	C_y =1.87	$C_{y(2)}$ =1.22

In Table 2, we observe the values of θ_i that are utilized to find the MSE values of the t_{C1} and t_{C2} by Eq. (26) and Eq. (32), respectively, considering the data of Populations 1-3.

Table 2. *The values of θ_i for Populations 1-3*

	Populations		
$\theta_i, i = 1,2,\ldots,10$	*I*	*II*	*III*
1	0.9994307	0.9931446	0.9967473
2	0.9998066	0.9917850	0.9348373
3	0.9995440	0.9944399	0.9942909
4	0.9995570	0.9947130	0.9968429
5	0.9986581	0.9953621	0.9997313
6	0.9997586	0.9898775	0.9618934
7	0.9994470	0.9934809	0.9982033
8	0.9994139	0.9927910	0.9941184
9	0.9986965	0.9955902	0.9998518
10	0.9997515	0.9893572	0.9329893

As discussed in Section 2, the min. MSE equation in Eq. (28) for the t_{C1} estimator is equivalent to the MSE equation of the t^*_{reg} estimator in Eq. (14) for Case I. Therefore, when determining the ranges of α_1 values that make the t_{C1} estimator performs better than other estimators, the regression estimator is not taken into consideration. For the first case, we obtain the ranges of α_1 values that make the proposed t_{C1}estimator more efficient than other compared estimators, based on the different values of *p*, in Tables 3-5. In other words, the ranges of α_1 values, as presented in Tables 3-5 for Populations 1-3, respectively, demonstrate that the t_{C1} estimator exhibits the min.

Table 3. *The α_1 values range for the family of t_{C1} estimators for Population 1*

	p				
$\theta_i, i = 1,2,\ldots,10$	*2*	*3*	*4*	*5*	*6*
1	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)
2	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)
3	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)
4	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)
5	(0,501; 0,716)	(0,501; 0,716)	(0,501; 0,716)	(0,501; 0,716)	(0,501; 0,716)
6	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)
7	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)
8	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)
9	(0,501; 0,716)	(0,501; 0,716)	(0,501; 0,716)	(0,501; 0,716)	(0,501; 0,716)
10	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)	(0,501; 0,715)

Table 4. *The α_1 values range for the family of t_{C1} estimators for Population 2*

	p				
$\theta_i, i = 1,2,\dots,10$	*2*	*3*	*4*	*5*	*6*
1	(1,007; 1,520)	(1,007; 1,520)	(1,007; 1,520)	(1,007; 1,520)	(1,007; 1,520)
2	(1,009; 1,522)	(1,009; 1,522)	(1,009; 1,522)	(1,009; 1,522)	(1,009; 1,522)
3	(1,006; 1,518)	(1,006; 1,518)	(1,006; 1,518)	(1,006; 1,518)	(1,006; 1,518)
4	(1,006; 1,517)	(1,006; 1,517)	(1,006; 1,517)	(1,006; 1,517)	(1,006; 1,517)
5	(1,005; 1,516)	(1,005; 1,516)	(1,005; 1,516)	(1,005; 1,516)	(1,005; 1,516)
6	(1,011; 1,525)	(1,011; 1,525)	(1,011; 1,525)	(1,011; 1,525)	(1,011; 1,525)
7	(1,007; 1,519)	(1,007; 1,519)	(1,007; 1,519)	(1,007; 1,519)	(1,007; 1,519)
8	(1,008; 1,520)	(1,008; 1,520)	(1,008; 1,520)	(1,008; 1,520)	(1,008; 1,520)
9	(1,010; 1,516)	(1,010; 1,516)	(1,010; 1,516)	(1,010; 1,516)	(1,010; 1,516)
10	(1,011; 1,525)	(1,011; 1,525)	(1,011; 1,525)	(1,011; 1,525)	(1,011; 1,525)

Table 5. *The α_1 values range for the family of t_{C1} estimators for Population 3*

	p				
$\theta_i, i = 1,2,\dots,10$	*2*	*3*	*4*	*5*	*6*
1	(1,004; 1,061)	(1,004; 1,061)	(1,004; 1,061)	(1,004; 1,061)	(1,004; 1,061)
2	(1,07; 1,131)	(1,07; 1,131)	(1,07; 1,131)	(1,07; 1,131)	(1,07; 1,131)
3	(1,006; 1,063)	(1,006; 1,063)	(1,006; 1,063)	(1,006; 1,063)	(1,006; 1,063)
4	(1,004; 1,061)	(1,004; 1,061)	(1,004; 1,061)	(1,004; 1,061)	(1,004; 1,061)
5	(1,001; 1,058)	(1,001; 1,058)	(1,001; 1,058)	(1,001; 1,058)	(1,001; 1,058)
6	(1,040; 1,099)	(1,040; 1,099)	(1,040; 1,099)	(1,040; 1,099)	(1,040; 1,099)
7	(1,002; 1,059)	(1,002; 1,059)	(1,002; 1,059)	(1,002; 1,059)	(1,002; 1,059)
8	(1,006; 1,064)	(1,006; 1,064)	(1,006; 1,064)	(1,006; 1,064)	(1,006; 1,064)
9	(1,001; 1,057)	(1,001; 1,057)	(1,001; 1,057)	(1,001; 1,057)	(1,001; 1,057)
10	(1,072; 1,133)	(1,072; 1,133)	(1,072; 1,133)	(1,072; 1,133)	(1,072; 1,133)

When we examine Tables 3 and 4, we see that the ranges of α_1 values are nearly the same for all θ_i, because all θ_i values are nearly 1 for the Populations 1 and 2, as given in Table 2. However, in Table 5, we see that the ranges of α_1 values are different with each other, according to the parameter θ_i, because θ_i values differ with each other for the Population 3, as given in Table 2. In addition, it is surprising that the values of *p* do not affect the ranges of α_1 values for all the populations in the Case 1.

For the second case, we obtain the ranges of α_2 values that make the proposed t_{C2}estimator more efficient than other compared estimators, based on the different values of *p*, in Tables 6-8. For this case, the ranges of α_2 values for the efficiency of the second proposed t_{C2} estimator, relative to others, are provided in Tables 6-8 for Populations 1-3, respectively. These ranges are based on different values of p and obtained for allθ_i $(i = 1,2,\dots,10)$.

Table 6. *The α_2 values range for the family of t_{C2} estimators for Population 1*

	p				
$\theta_i, i = 1,2,\dots,10$	*2*	*3*	*4*	*5*	*6*
1	(0,509; 0,607)	(0,501; 0,546)	(0,494; 0,500)	(0,454; 0,500)	(0,421; 0,500)
2	(0,509; 0,607)	(0,501; 0,545)	(0,494; 0,500)	(0,453; 0,500)	(0,421; 0,500)
3	(0,509; 0,607)	(0,501; 0,546)	(0,494; 0,500)	(0,454; 0,500)	(0,421; 0,500)
4	(0,509; 0,607)	(0,501; 0,546)	(0,494; 0,500)	(0,454; 0,500)	(0,421; 0,500)
5	(0,510; 0,608)	(0,501; 0,546)	(0,495; 0,500)	(0,454; 0,500)	(0,422; 0,500)
6	(0,509; 0,607)	(0,501; 0,545)	(0,494; 0,500)	(0,454; 0,500)	(0,421; 0,500)
7	(0,509; 0,607)	(0,501; 0,546)	(0,494; 0,500)	(0,454; 0,500)	(0,421; 0,500)
8	(0,509; 0,607)	(0,501; 0,546)	(0,494; 0,500)	(0,454; 0,500)	(0,421; 0,500)
9	(0,510; 0,608)	(0,501; 0,546)	(0,495; 0,500)	(0,454; 0,500)	(0,422; 0,500)
10	(0,509; 0,607)	(0,501; 0,545)	(0,494; 0,500)	(0,454; 0,500)	(0,421; 0,500)

Table 7. *The α_2 values range for the family of t_{C2} estimators for Population 2*

	p				
$\theta_i, i = 1,2,\dots,10$	*2*	*3*	*4*	*5*	*6*
1	(1,264; 1,512)	(1,264; 1,628)	(1,264; 1,695)	(1,264; 1,738)	(1,264; 1,769)
2	(1,266; 1,514)	(1,266; 1,630)	(1,266; 1,697)	(1,266; 1,741)	(1,266; 1,771)
3	(1,262; 1,510)	(1,262; 1,626)	(1,262; 1,693)	(1,262; 1,736)	(1,262; 1,767)
4	(1,262; 1,510)	(1,262; 1,626)	(1,262; 1,692)	(1,262; 1,736)	(1,262; 1,766)
5	(1,261; 1,509)	(1,261; 1,624)	(1,261; 1,691)	(1,261; 1,735)	(1,261; 1,765)
6	(1,268; 1,517)	(1,268; 1,633)	(1,268; 1,701)	(1,268; 1,744)	(1,268; 1,775)
7	(1,266; 1,512)	(1,264; 1,628)	(1,264; 1,694)	(1,264; 1,738)	(1,264; 1,768)
8	(1,264; 1,513)	(1,264; 1,629)	(1,264; 1,696)	(1,264; 1,739)	(1,264; 1,770)
9	(1,261; 1,509)	(1,261; 1,624)	(1,261; 1,691)	(1,261; 1,734)	(1,261; 1,765)
10	(1,269; 1,518)	(1,269; 1,634)	(1,269; 1,701)	(1,269; 1,745)	(1,269; 1,776)

When we examine the ranges in Tables 6 and 7 in detail for the Case II, again we can simply say that there is no important difference for the range values of α_2 in Populations 1 and 2; on the other hand, when we examine the ranges in Table 8, there is a clear difference for the range values according to θ_i $(i = 1,2,\dots,10)$ for Population 3, because of the same reason as in Case I. It is also surprising that there is no effect of the values of p on the ranges of α_2 values for all of the populations in Case II, as well.

The PRE results of t_{C1} and t_{C2} estimators with respect to the competing estimators are presented in Tables 9–10 for the Population 1, 2, and 3, respectively, under both cases.

Table 8. *The α_2 values range for the family of t_{C2} estimators for Population 3*

	p				
$\theta_i, i = 1,2,\dots,10$	2	3	4	5	6
1	(1,013; 1,032)	(1,004; 1,027)	(1,004; 1,016)	(1,004; 1,007)	(1,001; 1,003)
2	(1,080; 1,100)	(1,070; 1,095)	(1,070; 1,083)	(1,070; 1,074)	(1,067; 1,069)
3	(1,015; 1,034)	(1,006; 1,029)	(1,006; 1,018)	(1,006; 1,010)	(1,004; 1,005)
4	(1,013; 1,032)	(1,004; 1,027)	(1,004; 1,016)	(1,004; 1,007)	(1,001; 1,003)
5	(1,010; 1,029)	(1,001; 1,024)	(1,001; 1,013)	(1,001; 1,004)	(0,998; 1,000)
6	(1,050; 1,069)	(1,040; 1,064)	(1,040; 1,053)	(1,040; 1,044)	(1,037; 1,039)
7	(1,011; 1,030)	(1,002; 1,025)	(1,002; 1,014)	(1,002; 1,006)	(1,000; 1,001)
8	(1,016; 1,035)	(1,006; 1,029)	(1,006; 1,018)	(1,006; 1,010)	(1,004; 1,005)
9	(1,010; 1,029)	(1,001; 1,024)	(1,001; 1,013)	(1,001; 1,004)	(0,998; 1,000)
10	(1,082; 1,102)	(1,072; 1,097)	(1,072; 1,085)	(1,072; 1,076)	(1,070; 1,071)

Table 9. *The PRE results for all data sets under the Case I with respect to t_{HH}*

	Pop. 1					*Pop. 2*					*Pop. 3*				
	p					*p*					*p*				
	2	3	4	5	6	2	3	4	5	6	2	3	4	5	6
t_{HH}	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
t_R^*	143.1	135.7	130.4	126.5	123.5	138.0	122.2	115.7	112.1	109.9	524.0	344.4	271.7	232.3	207.6
t_{BT}^*	200.3	177.6	163.3	153.4	146.2	122.4	113.8	109.9	107.8	106.4	247.4	209.4	187.0	172.2	161.7
t_{C1}	***207.0***	***182.2***	***166.7***	***156.1***	***148.5***	***140.3***	***123.4***	***116.5***	***112.7***	***110.4***	***525.7***	***345.1***	***272.1***	***232.6***	***207.8***

Table 10. *The PRE results for all data sets under the Case II with respect to t_{HH}*

	Pop. 1					*Pop. 2*					*Pop. 3*				
	p					*p*					*p*				
	2	3	4	5	6	2	3	4	5	6	2	3	4	5	6
t_{HH}	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
t_R^{**}	124.4	108.6	98.9	92.3	87.5	202.3	194.4	190.7	188.6	187.2	1719.3	1735.3	1748.0	1758.3	1766.9
t_{BT}^{**}	208.3	188.9	176.2	167.2	160.5	148.1	144.4	142.7	141.7	141.0	330.8	334.8	337.9	340.5	342.6
t_{reg}^{**}	209.0	184.9	169.7	159.3	151.7	218.5	211.1	207.6	205.6	204.3	1726.5	1731.2	1734.9	1737.9	1740.4
t_{C2}	***210.8***	***189.2***	***176.2***	***167.5***	***161.2***	***220.7***	***215.0***	***212.6***	***211.3***	***210.6***	***1729.2***	***1739.3***	***1749.2***	***1758.4***	***1766.9***

Boldfaced values indicate the "best" performances.

From Tables 9–10, it is shown that the t_{C1} and t_{C2} estimators perform better than all other compared estimators for all data sets under both cases. Accordingly, it can be inferred that among competing estimators, t_{C1} and t_{C2} are the most effective ones in general. For the t_{C1} estimator, it is observed that the PRE value decreased as the value of p increased in all populations. For the t_{C2} estimator, a similar situation is observed only in Populations 1 and 2. In Population 3, it is concluded that PRE values increase as p increases. At this point, Figures 1 and 2 represents the PRE results of the proposed t_{C1} and t_{C2} estimators for the Population 1, 2, and 3, respectively. In both cases, the highest PRE values obtained in Population 3 are noteworthy.

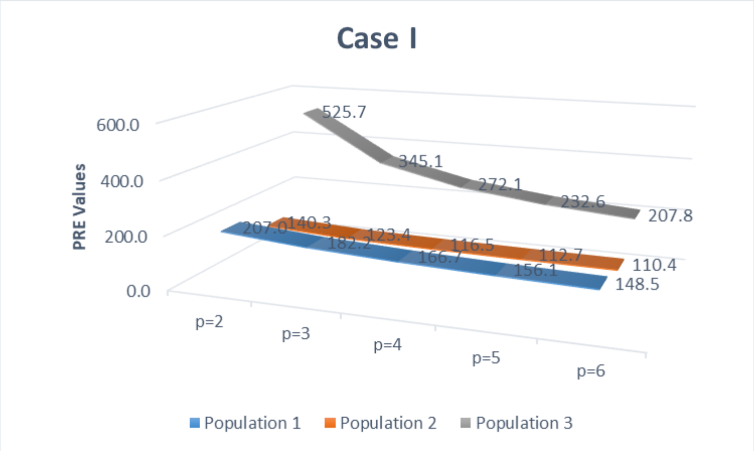

Figure 1. *The PRE results of t_{C1} estimator for all populations*

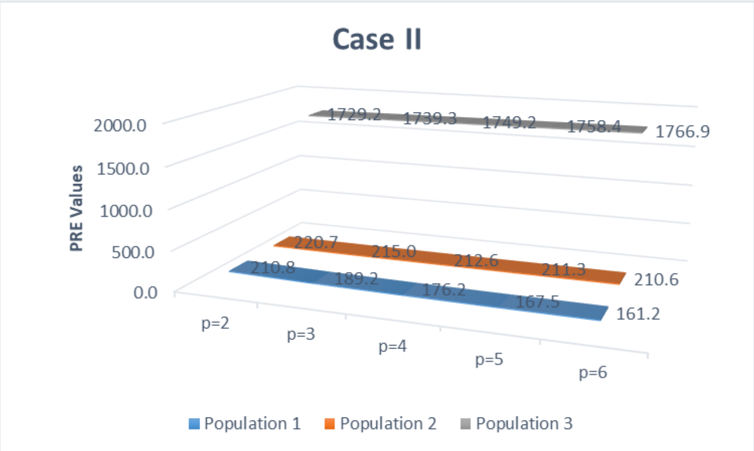

Figure 2. *The PRE results of t_{C2} estimator for all populations*

5. CONCLUSION

We consider the estimation of the $\bar{Y}$ when non-response occurs in two different cases and propose two families of estimators, t_{C1} and t_{C2}, using the exponential function under these cases. The bias and minimum MSE of the t_{C1} and t_{C2}estimators are obtained. We compare the proposed estimators with the mentioned estimators in theory and in application using three different data sets. We demonstrate that the t_{C1} and t_{C2} estimators are always recommended based on theory in Section 3 and obtain the efficiency intervals of α_1 and α_2 for the first and the second proposed families of estimators in practice using three different population data in Section 4. Additionally, PRE values are included in the application. When we look at the compared and proposed estimators, it is seen that the values of the suggested estimators are the highest for both cases and these values increase even more, especially for Population 3.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

REFERENCES

Ahmad, S., Arslan, M., Khan, A., & Shabbir, J. (2021). A generalized exponential-type estimator for population mean using auxiliary attributes. *Plos One*, *16*(5), e0246947. https://www.doi.org/10.1371/journal.pone.0246947

Ahmad, S., Hussain, S., Aamir, M., Khan, F., Alshahrani, M. N., & Alqawba, M. (2022). Estimation of finite population mean using dual auxiliary variable for non-response using simple random sampling. *Aims Mathematics*, *793*, 4592-4613. https://www.doi.org/10.3934/math.2022256

Ahmadini, A. A. H., Yadav, T., Yadav, S. K., & Al Luhayb, A. S. M. (2022). Restructured searls family of estimators of population mean in the presence of nonresponse. *Frontiers in Applied Mathematics and Statistics*, *8*, 969068. https://www.doi.org/10.3389/fams.2022.969068

Anieting, A. E., Enang, E. I., & Onwukwe, C. E. (2020). Efficient estimator for Population mean in stratified double sampling in the presence of nonresponse using one auxiliary variable. *Statistics*, *4*(2), 40-50. https://www.doi.org/10.52589/AJMSS-YF4V11QV

Bahl, S., & Tuteja, R. K. (1991). Ratio and product type exponential estimators. *Journal of Information and Optimization Sciences*, *12*(1), 159-164. https://www.doi.org/10.1080/02522667.1991.10699058

Cochran, W. G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. *The Journal of Agricultural Science*, *30*(2), 262-275. https://www.doi.org/10.1017/S0021859600048012

Cochran, W. G. (1977) *Sampling Techniques*. John Wiley and Sons, New-York

Dansawad, N. (2019). A class of exponential estimator to estimate the population mean in the presence of non-response. *Naresuan University Journal: Science and Technology*, *27*(4), 20-26. https://www.doi.org/10.14456/nujst.2019.33

Fatima, M., Shahbaz, S. H., Hanif, M., & Shahbaz, M. Q. (2022). A modified regression-cum-ratio estimator for finite population mean in presence of nonresponse using ranked set sampling. *AIMS Mathematics*, *7*(4), 6478-6488. https://www.doi.org/10.3934/math.2022361

Hansen, M. H., & Hurwitz, W. N. (1946). The problem of non-response in sample surveys. *Journal of the American Statistical Association*, *41*(236), 517-529. https://www.doi.org/10.1080/01621459.1946.10501894

Javed, M., Irfan, M., & Pang, T. (2019). Hartley-Ross type unbiased estimators of population mean using two auxiliary variables. *Scientia Iranica*, *26*(6), 3835-3845. https://www.doi.org/10.24200/sci.2018.5648.1397

Khare, B. B., & Sinha, R. R. (2009). On class of estimators for population mean using multi-auxiliary characters in the presence of non-response. *Statistics in Transition*, *10*(1), 3-14.

Khare, B. B., & Srivastava, S. (1993). Estimation of population mean using auxiliary character in presence of non-response. *National Academy Science Letters*, *16*, 111-114.

Khan, A., Ali, A., Ijaz, M., Azeem, M., & El-Morshedy, M. (2023). An exponential ratio type estimator of the population mean in the presence of non-response using double sampling. *Journal of Statistics Applications and Probability*, *12*(1), 191-205. https://www.doi.org/10.18576/jsap/120118

Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N., & Smarandache, F. (2007). A general family of estimators for estimating population mean using known value of some population parameter(s). *Far East Journal of Theoretical Statistics*, *22*, 181-191.

Kumar, S. (2013). Improved exponential estimator for estimating the population mean in the presence of non-response. *Communications for Statistical Applications and Methods*, *20*(5), 357-366. https://www.doi.org/10.5351/CSAM.2013.20.5.357

Kumar, S, & Bhougal, S. (2011). Estimation of the population mean in presence of non-response. *Communications for Statistical Applications and Methods*, *18*(4), 537-548. https://www.doi.org/10.5351/CKSS.2011.18.4.537

Kumar, K., & Kumar, M. (2017). Improved exponential ratio and product type estimators for population mean in the presence of nonresponse. *Bulletin of Mathematics and Statistics Research*, *5*(2), 68-76.

Mehta, P., & Tailor, R. (2020). Chain ratio type estimators using known parameters of auxiliary variates in double sampling. *Journal of Reliability and Statistical Studies*, *13*(2-4), 243-252. https://www.doi.org/10.13052/jrss0974-8024.13242

Oncel Cekim, H., & Kadilar, C. (2018). New families of unbiased estimators in stratified random sampling. *Journal of Statistics and Management Systems*, *21*(8), 1481-1499. https://www.doi.org/10.1080/09720510.2018.1530176

Oncel Cekim, H. (2022). Modified unbiased estimators for population variance: An application for COVID-19 deaths in Russia. *Concurrency and Computation: Practice and Experience*, *34*(22), e7169. https://www.doi.org/10.1002/cpe.7169

Özel Kadilar, G. (2016). A new exponential type estimator for the population mean in simple random sampling. *Journal of Modern Applied Statistical Methods*, *15*(2), 207-214. http://www.doi.org/10.22237/jmasm/1478002380

Pal, S. K., & Singh, H. P. (2017). A class of ratio-cum-ratio-type exponential estimators for population mean with sub sampling the nonrespondents. *Jordan Journal of Mathematics and Statistics*, *10*(1), 73-94.

Pal, S. K., & Singh, H. P. (2018). Estimation of finite population mean using auxiliary information in presence of non-response. *Communications in Statistics-Simulation and Computation*, *47*(1), 143-165. https://www.doi.org/10.1080/03610918.2017.1280161

Rao, P. S. R. S. (1986). Ratio estimation with sub sampling the non-respondents. *Survey Methodology*, *12*(2), 217-230.

Rehman, S. A., & Shabbir, J. (2022). An efficient class of estimators for finite population mean in the presence of non-response under ranked set sampling (RSS). *Plos One*, *17*(12), e0277232. https://www.doi.org/10.1371/journal.pone.0277232

Sanaullah, A., Ehsan, I., & Noor-Ul-Amin, M. (2019). Estimation of mean for a finite population using sub-sampling of non-respondents. *Journal of Statistics and Management Systems*, *22*(6), 1015-1035. https://www.doi.org/10.1080/09720510.2019.1572981

Satici, E., & Kadilar, C. (2011). Ratio estimator for the population mean at the current occasion in the presence of non-response in successive sampling. *Hacettepe Journal of Mathematics and Statistics*, *40*(1), 115-124.

Shabbir, J., Haq, A., & Gupta, S. (2014). A new difference-cum-exponential type estimator of finite population mean in simple random sampling. *Revista Colombiana de Estadística*, *37*(1), 199-211. https://www.doi.org/10.15446/rce.v37n1.44366

Shabbir, J., & Onyango, R. (2022). Use of an efficient unbiased estimator for finite population mean. *Plos One*, *17*(7), e0270277. https://www.doi.org/10.1371/journal.pone.0270277

Sharma, P., Pal, S. K., & Singh, H. P. (2022). Improved estimators of population mean under nonresponse in successive sampling. *Mathematical Problems in Engineering*, *2022*(1), 1-8. https://www.doi.org/10.1155/2022/1349689

Singh, R., Chauhan, P., & Sawan, N. (2008). On linear combination of ratio and product type exponential estimator for estimating the finite population mean. *Statistics in Transition - New Series*, *9*, 105-115.

Singh, R., Kumar, M., Chaudhary, M. K., & Smarandache, F. (2010). Estimation of mean in presence of non-response using exponential estimator. *Multispace & Multistructure Neutrosophic Transdisciplinarity (100 Collected Papers of Sciences)*, (vol IV, pp. 758-768). https://doi.org/10.48550/arXiv.0906.2462

Singh, H. P., & Nigam, P. (2020). Ratio-Ratio-Type exponential estimator of finite population mean in double sampling for stratification. *International Journal of Agricultural and Statistical Science*, *16*(1), 251-257.

Singh, G. N., & Usman, M. (2019a). Ratio-to-product exponential-type estimators under non-response. *Jordan Journal of Mathematics and Statistics*, 12(4), 593-616.

Singh, G. N., & Usman, M. (2019b). Efficient combination of various estimators in the presence of non-response. *Communications in Statistics Simulation and Computation*, *50*(8), 2432-2466. https://www.doi.org/10.1080/03610918.2019.1614618

Tailor, R., & Lone, H. A. (2014). Separate ratio-type estimators of population mean in stratified random sampling. *Journal of Modern Applied Statistical Methods*, *13*(1), 223-233. https://www.doi.org/10.22237/jmasm/1398917580

Unal, C., & Kadilar, C. (2019). Exponential type estimator for the population mean in the presence of non-response. *Journal of Statistics and Management Systems*, *23*(3), 603-615. https://www.doi.org/10.1080/09720510.2019.1668158

Unal, C., & Kadilar, C. (2021). Improved family of estimators using exponential function for the population mean in the presence of nonresponse. *Communications in Statistics - Theory and Methods*, *50*(1), 237-248. https://www.doi.org/10.1080/03610926.2019.1634818

Unal, C., & Kadilar, C. (2022a). A new population mean estimator under non-response cases. *Journal of Taibah University for Science*, *16*(1), 111-119. https://www.doi.org/10.1080/16583655.2022.2034343

Unal, C., & Kadilar, C. (2022b). Exponential type estimators using sub-sampling method with applications in agriculture. *Journal of Agricultural Sciences (Tarim Bilimleri Dergisi)*, *28*(3), 457-472. https://www.doi.org/10.15832/ankutbd.915999

Yadav, S. K., Sharma, D. K., & Mishra, S. S. (2021). New modified ratio type estimator of the population mean using the known median of the study variable. *International Journal of Operational Research*, *41*(2), 151-167. https://www.doi.org/10.1504/IJOR.2021.115625

Yadav, S. K., & Zaman, T. (2021). Use of some conventional and non-conventional parameters for improving the efficiency of ratio-type estimators. *Journal of Statistics and Management Systems*, *24*(5), 1077-1100. https://www.doi.org/10.1080/09720510.2020.1864939

Yunusa, O., & Kumar, S. (2014). Ratio-cum-product estimator using exponential estimator in the presence of non-response. *Journal of Advanced Computing*, *3*(1), 1-11. https://www.doi.org/10.7726/jac.2014.1001

Zahid, S., Kamal, A., & Makhdum, M. (2022). Generalized Dual to Exponential Ratio Type Estimator for the Finite Population Mean in the Presence of Nonresponse. In: O. O. Awe, K. Love, & E. A. Vance (Eds.), *Promoting Statistical Practice and Collaboration in Developing Countries* (2nd ed., pp. 249-263). Chapman and Hall/CRC. https://www.doi.org/10.1201/9781003261148

Zaman, T., & Kadilar, C. (2019). Novel family of exponential estimators using information of auxiliary attribute. *Journal of Statistics and Management Systems*, *22*(8), 1499-1509. https://www.doi.org/10.1080/09720510.2019.1621488

Zaman, T., & Kadilar, C. (2021a). Exponential ratio and product type estimators of the mean in stratified two-phase sampling. *AIMS Mathematics*, *6*(5), 4265-4279. https://www.doi.org/10.3934/math.2021252

Zaman, T., & Kadilar, C. (2021b). New class of exponential estimators for finite population mean in two-phase sampling. *Communications in Statistics-Theory and Methods*, *50*(4), 874-889. https://www.doi.org/10.1080/03610926.2019.1643480