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A Novel Hyperchaotic Financial System with Hyperbolic Sinusoidal Nonlinearity: From Theoretical Analysis to Adaptive Neural Fuzzy Controller Design

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ABSTRACT Chaotic systems are known to be extremely sensitive to initial conditions, meaning small changes can have a significant impact on the outcomes. By analyzing the average profit margin in relation to chaotic dynamics, companies can conduct sensitivity analysis to assess the potential impact of various factors on their profitability. This analysis can help identify critical variables or scenarios that may significantly affect profit margins. In this article, we have proposed a hyperchaotic financial system with hyperbolic sinusoidal non-linear variables applied to the average profit margin. Furthermore, we have investigated the stability of the hyperchaotic financial dynamics model to provide information to companies to assess the consistency and reliability of their profitability. In addition, fundamental dynamic behavior like Lyapunov exponents, bifurcation analysis, coexisting attractors have been reported. Finally, a nonlinear feedback control approach is developed to train an adaptive neural fuzzy controller. The application of Lyapunov theory confirms that this nonlinear feedback controller can effectively minimize the synchronization error within a finite duration. The results from simulations establish the effectiveness of the proposed neural fuzzy controller architecture in controlling the synchronization of two hyperchaotic financial models. Additionally, the simulation includes a comparison between the performance of the nonlinear controller and the adaptive neural fuzzy controller.

KEYWORDS
Chaotic system
Financial system
Dynamical analysis
Complexity analysis
Adaptive neural fuzzy controller

INTRODUCTION

Chaotic behavior in financial systems often leads to increased volatility and unpredictability (Guegan 2009; Vogl 2022; Inglada-Perez 2020). By studying the average profit margin in relation to chaotic dynamics, companies can gain a better understanding of the potential risks they face (Lux 1998; Shi *et al.* 2022; Xin *et al.* 2013). This understanding can help in developing risk management strategies and contingency plans to mitigate the adverse effects of financial instability on profitability. Moreover, chaotic behavior in financial markets can create opportunities for profit. The market fluctuations and price volatility can allow skilled investors and traders to capitalize on short-term price movements and generate profits through strategic buying and selling (Ma and Li 2020; Musaev *et al.* 2022). The ability to identify patterns or trends within chaotic behavior can provide a competitive advantage in capturing these profit opportunities.

Financial chaotic systems arise in business modelling and they have many applications in science and engineering. Many studies

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have discussed the financial model using hyperchaotic systems approach, such as Yu *et al.* (2012) proposed hyperchaos financial system with addition variable average profit margin using quadratic nonlinear term and designed speed feedback controllers and linear feedback controllers for stabilizing hyperchaos to unstable equilibrium points. Xin (2009) constructed a new 4D continuous autonomous financial hyperchaotic system using a nonlinear state feedback controller. In their study, Vargas *et al.* (2015) introduced an adaptive controller aimed at achieving synchronization in hyperchaotic finance systems. The proposed controller addresses uncertainties that may arise from factors such as unknown system parameters and disturbances that vary over time or are dependent on the system's state. Szumiński (2018) conducted an analysis of integrability for complex dynamical systems within the context of the financial hyperchaotic model, utilizing differential Galois theory.

The primary focus of this study was to demonstrate the non-integrability of the examined system across a broad range of functions. Additionally, the research aimed to identify specific parameter values that may indicate integrability within the system. Jahanshahi *et al.* (2019) apply the four-dimensional financial hyperchaotic system and propose a unique control method for suppressing chaos and achieving synchronization in this nonlinear system. This study approach combines fuzzy logic with a fast disturbance observer and ITSMC. Chen *et al.* (2021) developed suitable control strategies for achieving synchronization between two financial systems that have different initial conditions. They also provided mathematical evidence demonstrating the effectiveness of the control law employed in their study. Kai *et al.* (2017) proposed a 4D hyperchaotic financial system derived from an existing three-dimensional nonlinear financial system. They expand upon this system by incorporating a controller term to account for the impact of control on the overall dynamics.

On the other hand, numerous findings regarding the analysis and control of financial hyperchaotic systems have been documented in the existing literature. Bekiros *et al.* (2021) introduce an optimal mixed H2/$H\infty$ control approach based on type-2 fuzzy logic for a hyperchaotic financial system. Their investigation centers around the dynamical properties of the system in the presence of coexisting attractors. Cao (2018) designed a four-dimensional hyperchaotic finance system, which can generate double-wing chaotic and hyperchaotic attractors with three equilibrium points. Hajipour *et al.* (2018) developed a sliding mode control strategy to regulate a hyperchaotic financial model and to ensure stability of the proposed system in the face of unwanted dynamics and disturbances. To achieve this, they applied an adaptive sliding mode control scheme that aims to drive the system's states towards desired set points.

Xu *et al.* (2021) investigate the $H\infty$ control problem for a hyperchaotic finance system with an energy-bounded disturbance, employing a delayed feedback controller. Through the utilization of quadratic system theory, an augmented Lyapunov functional, integral inequalities, and rigorous mathematical derivations, they establish a sufficient condition based on linear matrix inequalities. This condition ensures that the closed-loop system attains desirable performance characteristics, including boundedness, $H\infty$ performance, and asymptotic stability. Li *et al.* (2022) introduce a novel approach to establish adequate conditions that ensure the presence and stabilization of positive solutions in a specific hyper-chaotic financial model. Their study also investigates a nonlinear chaotic financial system with diffusion by incorporating the concepts of Laplacian semigroup and impulsive control. Rao and Zhu (2021) demonstrate the theoretical importance of guiding the actual financial market. Their study highlights that implementing positive and accurate macroeconomic control measures at specific frequencies can promote market stability and result in higher positive interest rates.

Specifically, the main contributions and novelty of this study can be summarized into the following points:

1. The system consists of a total eleven terms including with sinusoidal hyperbolic non-linear variables applied to the average profit margin
2. The system exhibits multistability and coexistence attractors
3. A nonlinear feedback control approach is developed to train an adaptive neural fuzzy controller

The structure of this research article is presented as follows. First, we provide a concise description of the mathematical model for the hyperchaotic financial system. Next, we carry out a dynamic analysis of the new hyperchaotic financial system. We also discuss the multistability and coexisting hyperchaotic attractors for the new hyperchaotic financial system. As a control application, we present an adaptive neural fuzzy controller for the synchronization of the new hyperchaotic financial systems and describe the simulation results. Finally, we conclude this research article with a summary of the main results.

MODELING OF THE NOVEL FINANCIAL RISK SYSTEM

A chaotic-based financial system due to Gao and Ma Gao and Ma (2009) can be expressed as follows

$$\begin{cases} \dot{z}_1 = z_3 + (z_2 - a)z_1 \\ \dot{z}_2 = 1 - bz_2 - z_1^2 \\ \dot{z}_3 = -z_1 - cz_3 \end{cases} \quad (1)$$

where a represents the savings, b denotes the per investment cost, and c signifies the elasticity of commercial demands, all of which are positive constants. Considering the respective values as follows: $a = 0.9$, $b = 0.2$, $c = 1.2$, and the initial state of the system is $(z_1(0), z_2(0), z_3(0)) = (1, 2, 0.5)$, the system (1) exhibits chaotic behavior. The Lyapunov exponents of the Gao-Ma system (1) are calculated as $LE_1 = 0.0833$, $LE_2 = 0$ and $LE_3 = -0.6987$. Using these values, the Kaplan-Yorke dimension of the Gao-Ma system (1) is evaluated as follows:

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.1192 \quad (2)$$

Furthermore, Yu *et al.* Yu *et al.* (2012) proposed 4-D hyperchaotic finance system with added average profit margin. The 4D hyperchaotic finance system Yu *et al.* (2012) has the following dynamics:

$$\begin{cases} \dot{z}_1 = z_3 + (z_2 - a)z_1 + z_4 \\ \dot{z}_2 = 1 - bz_2 - z_1^2 \\ \dot{z}_3 = -z_1 - cz_3 \\ \dot{z}_4 = -dz_1z_2 - pz_4 \end{cases} \quad (3)$$

where parameters a, b, c, d, p are held positive. Yu *et al.* Yu *et al.* (2012) showed that the system (3) exhibits hyperchaotic behavior for the following values: $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$ and $p = 0.17$. For MATLAB simulations, the initial state of the system (3) is chosen as $Z(0) = (1, 2, 0.5, 0.5)$. The Lyapunov exponents of the Hyperchaotic Yu finance system (3) are calculated as $LE_1 = 0.0344$, $LE_2 = 0.0180$, $LE_3 = 0$ and $LE_4 = -1.1499$.

Using these values, the Kaplan-Yorke dimension of the hyperchaotic Yu system (3) is evaluated as follows:

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.0456 \tag{4}$$

This study proposes a new finance model by combining the Gao-Ma finance model (1) and the Yu finance model (3). Specifically, we consider the effect of hyperbolic sinusoidal nonlinearity in the z_4 dynamics of the system. The new hyperchaotic finance system has the following form:

$$\begin{cases} \dot{z}_1 &= z_3 + (z_2 - a)z_1 + qz_4 \\ \dot{z}_2 &= 1 - bz_2 - z_1^2 \\ \dot{z}_3 &= -z_1 - cz_3 \\ \dot{z}_4 &= -dz_1z_2 - p\sinh(z_1) \end{cases} \tag{5}$$

Eq. (5) which represents a new financial 4-D system consists of the parameters a, b, c, d, p, q that are held as positive constants taking the following values: $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.3$, $p = 0.15$ and $q = 0.1$. Suppose also that we consider the initial state as $Z(0) = (1, 2, 0.5, 0.5)$.

The calculation of Lyapunov exponents for the 4-D system (5) yields the following values: $LE_1 = 0.0382$, $LE_2 = 0.0298$, $LE_3 = 0$ and $LE_4 = -1.0865$ which establish the hyperchaotic behavior of the system. Using these values, the Kaplan-Yorke dimension of the new hyperchaotic financial system (5) is evaluated as follows:

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.0625 \tag{6}$$

Table 1 gives a comparison of the Lyapunov exponents, maximal Lyapunov exponent (MLE) and the Kaplan-Yorke dimension (D_{KY}) of the three financial systems expressed by (1), (3) and (5). Table 1 demonstrates that the maximum Lyapunov exponent (MLE) value of the new system (5) is greater than that of the financial systems (1) and (3). Moreover, it is also shown that the Kaplan-Yorke dimension of the new system (5) is greater than that of the financial systems (1) and (3). These are the advantages of the proposed financial chaotic system (5). A disadvantage of the new hyperchaotic system is that it includes a hyperbolic sinusoidal nonlinearity in its dynamics and for this reason, designing an electronic circuit or field programmable gate array (FPGA) design of the new financial system (5) is complicated.

Figure 1 exhibits the plot for the strange attractors and phase portraits of system (5).

Next, we solve the following system of equations for finding the equilibrium point for system (5).

$$\begin{cases} 0 &= z_3 + (z_2 - a)z_1 + qz_4 \\ 0 &= 1 - bz_2 - z_1^2 \\ 0 &= -z_1 - cz_3 \\ 0 &= -dz_1z_2 - p\sinh(z_1) \end{cases} \tag{7}$$

Table 1 Lyapunov Exponents, MLE and Kaplan-Yorke Dimension of Three Financial Systems

Financial System	Lyapunov Exponents (LEs)	MLE Value	D_{KY}
Gao-Ma Financial System (1)	$LE_1 = 0.0833$ $LE_2 = 0$ $LE_3 = -0.6987$	0.0833	2.1192
Hyperchaotic Yu System (3)	$LE_1 = 0.0344$ $LE_2 = 0.0180$ $LE_3 = 0$ $LE_4 = -1.1499$	0.0344	3.0456
New Financial System (5)	$LE_1 = 0.0381$ $LE_2 = 0.0298$ $LE_3 = 0$ $LE_4 = -1.0865$	0.0381	3.0625

By performing a straightforward calculation, it can be determined that the new financial hyperchaotic system (5) possesses a unique equilibrium point given by $E_0 = (0, 1/b, 0, 0)$. For the hyperchaotic case, $b = 0.2$. In this special case, the new financial hyperchaotic system (5) has the unique equilibrium point $E_0 = (0, 5, 0, 0)$. We can establish the Jacobian matrix at $E_0 = (0, 5, 0, 0)$ to be as follows:

$$J = \begin{bmatrix} 4.10 & 0 & 1 & 0.1 \\ 0 & -0.2 & 0 & 0 \\ -1 & 0 & -1.5 & 0 \\ -1.65 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

We find that the matrix J has the eigenvalues: $\lambda_1 = 0.0484$, $\lambda_2 = 3.8712$, $\lambda_3 = -0.2$ and $\lambda_4 = -1.3196$. This shows that $E_0 = (0, 5, 0, 0)$ is a saddle point and unstable equilibrium for system (5).

DYNAMICAL ANALYSIS

In this section, we conduct an analysis of the dynamical properties of the new hyperchaotic financial model (5) as a function of its parameters. To achieve this, we utilize various tools such as spectrum of Lyapunov exponents, bifurcation diagrams, and phase plots.

This section of the paper aims to explore how the parameters affect the behavior of the recently developed hyperchaotic finance system. To achieve this, we will utilize Lyapunov exponents spectrum, bifurcation diagrams, and phase plots. Specifically, we will focus on the case where $Z_0 = (0.4, 0.2, 0.4, 0.4)$.

Bifurcation Diagram and Lyapunov Exponent

Dynamics when** a **varies Figure 2 depicts the bifurcation diagram and Lyapunov exponents spectrum of the new hyperchaotic finance system (5), where the parameter a is varied within the range $[0, 4]$, while the other parameters remain constant: $b = 0.2$, $c = 1.5$, $d = 0.3$, $p = 0.15$, and $q = 0.1$. Notably, the Lyapunov spectrum

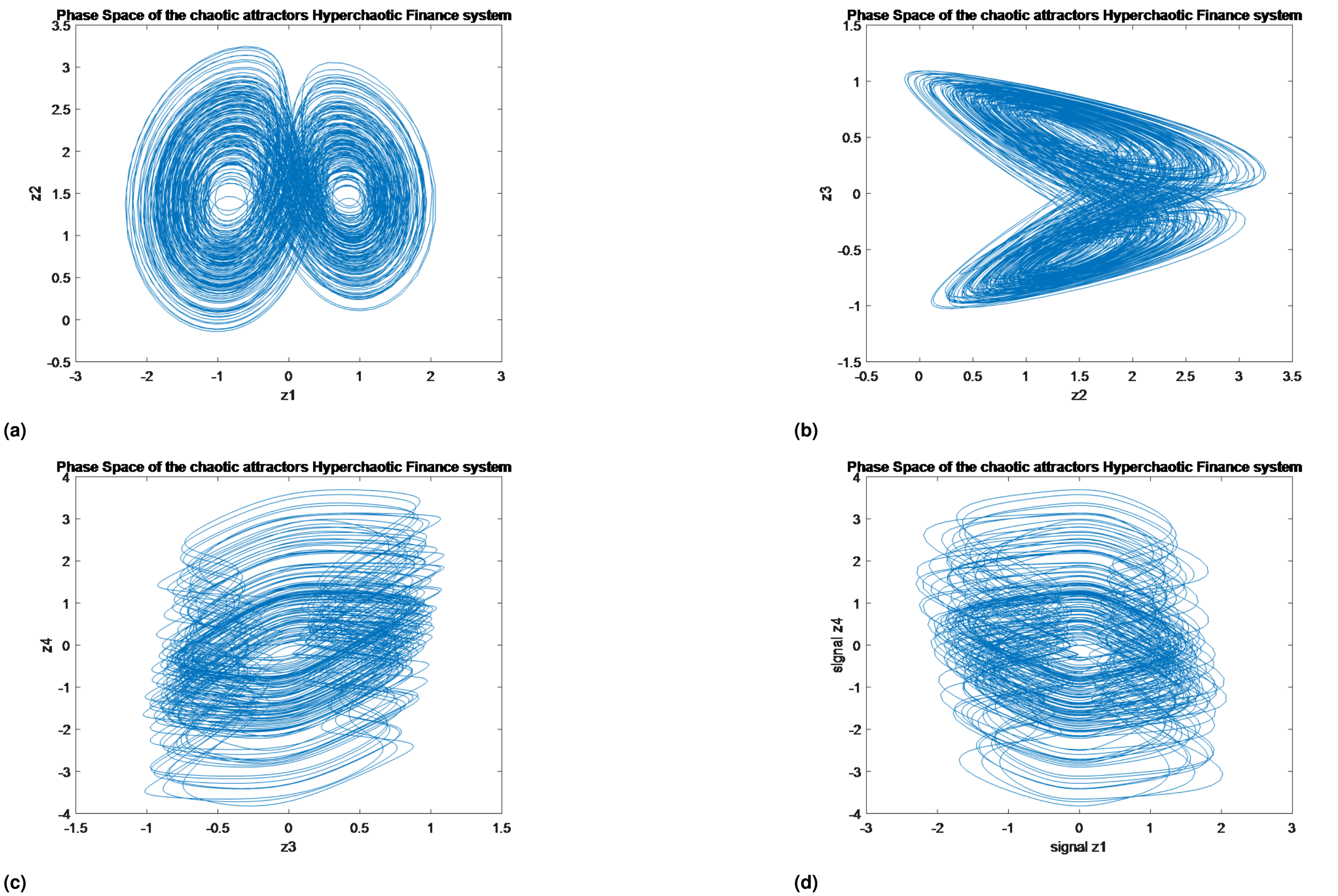


Figure 1 Phase portraits of the new hyperchaotic financial system (5) using MATLAB in (a) $z_1 - z_2$ plane (b) $z_2 - z_3$ plane, (c) $z_3 - z_4$ plane and (d) $z_1 - z_4$ plane

results presented in Figure 2b align with the findings obtained from the bifurcation diagram shown in Figure 2a.

When the value of a lies within the interval $[0, 2.85]$, the system (5) demonstrates the presence of two positive Lyapunov exponents, indicating an extreme hyperchaotic nature. The Kaplan-Yorke dimension for this behavior is measured to be $DKY = 3.055$. For plots, we specifically selected a to be 0.5. As a result, Figure 3a illustrates the $z_1 - z_2$ attractor, visually representing the hyperchaotic behavior exhibited by the system (5) with the respective Lyapunov exponents having the following values:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.053 \\ 0.013 \\ 0 \\ \text{-}1.042 \end{pmatrix} \tag{9}$$

When the parameter a fall within the range $[2.86, 3.10]$, the system (5) demonstrates a positive maximal Lyapunov exponent, indicating the presence of chaotic behavior. The Kaplan-Yorke dimension, measured to be $DKY = 3.011$, further confirms the complex nature of hyperchaotic finance system (5). Specifically selecting a to be 2.95 for our plots, we observe the $z_1 - z_2$ attractor depicted in Figure 3b, which illustrates the chaotic behavior exhibited by the system (5). The respective values for Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.0260 \\ 0 \\ \text{-}0.012 \\ \text{-}1.032 \end{pmatrix} \tag{10}$$

When the parameter a fall within the range $[3.11, 4]$, the system (5) exhibits periodic behavior without complexity. For plots, we specifically selected a to be 3.5. As a result, Figure 3c, presents the $z_1 - z_2$ attractor, visually representing the periodic behavior displayed by the new 4D hyperchaotic finance system (5). Specific to this case, the respective Lyapunov exponents are recorded as the followings:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \text{-}0.042 \\ \text{-}0.045 \\ \text{-}1.011 \end{pmatrix} \tag{11}$$

Dynamics when b varies In Figure 4, the bifurcation diagram and Lyapunov exponents spectrum are depicted for the finance system (5) as the parameter b changes within the range of $[0, 0.5]$.

Figure 2 Characteristics of the system (5) in terms of a) Bifurcation diagram and b) LEs spectrum when $a \in [0, 4]$.

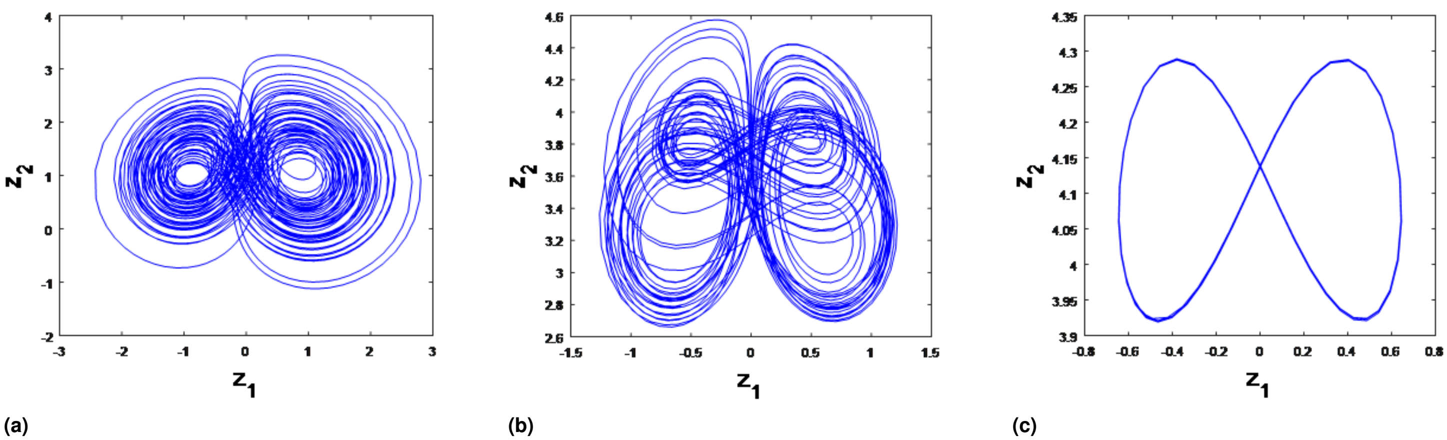


Figure 3 Output Matlab simulation (a) $z_1 - z_2$ hyperchaotic attractor of the system (5) when $a = 0.5$. (b) $z_1 - z_2$ chaotic attractor of system (3) when $a =$ 2.95 and (c) $z_1 - z_2$ periodic orbit of the system (5) when $a = 3.5$.

When the parameter b falls within the range $[0, 0.29]$, the system (5) displays the presence of two positive Lyapunov exponents, signifying an extreme hyperchaotic nature. The Kaplan-Yorke dimension for this behavior is measured to be $D_{KY} = 3.063$. For plots, we specifically chose b to have a value of 0.05. Consequently, Figure 5a displays the $z_1 - z_3$ attractor, providing a visual representation of the hyperchaotic behavior exhibited by the system (5). The associated Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} . = \begin{pmatrix} 0.053 \\ 0.013 \\ 0 \\ \text{-}1.042 \end{pmatrix} \tag{12}$$

When the parameter b is within the range $[0.30, 0.35]$, the system (5) showcases a positive maximal Lyapunov exponent, indicating the presence of chaotic behavior. The measurement of the Kaplan-Yorke dimension as $D_{KY} = 3.010$ further validates the complex nature of the system (5). For plots, we specifically selected b to have a value of 0.31. As a result, the $z_1 - z_3$ attractor presented in Figure 5b vividly portrays the chaotic behavior exhibited by the system (5). The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.017 \\ 0 \\ \text{-}0.006 \\ \text{-}1.101 \end{pmatrix} \tag{13}$$

When the parameter b is in the range of $[0.036, 0.5]$, the system (5) exhibits periodic behavior without complexity. For plots, we specifically selected b to be 0.5. As a result, Figure 5c presents the $z_1 - z_3$ attractor, visually representing the periodic behavior displayed by the system (5). The corresponding Lyapunov exponents are as follows:

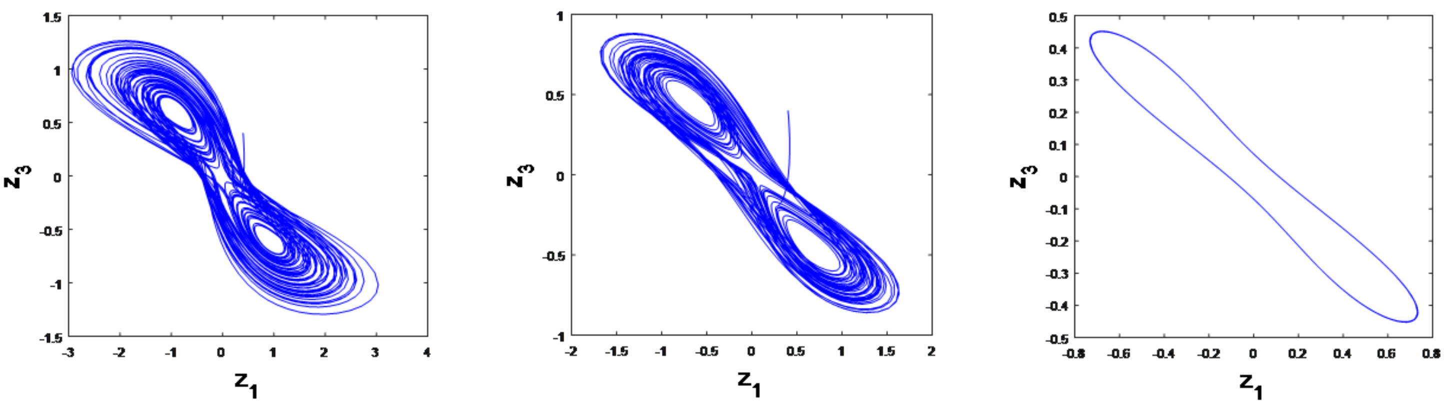
$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \text{-}0.013 \\ \text{-}0.222 \\ \text{-}1.051 \end{pmatrix} \tag{14}$$

Dynamics when c varies For system (3), the Bifurcation diagram and Lyapunov exponents spectrum are depicted as in Figure 6 as the value of c ranges from 0 to 1.5.

When c belongs to the intervals $([0, 0.84], [0.98, 1.01])$, the system (5) has one zero and three negative Lyapunov exponents,

(a) (b)

Figure 4 Characteristics of the system (5) in terms of a) Bifurcation diagram and b) LEs spectrum when $b \in [0, 0.5]$.



(a) (b) (c)

Figure 5 Output Matlab simulation (a) $z_1 - z_3$ hyperchaotic attractor of system (3) when $b = 0.5$. (b) $z_1 - z_3$ chaotic attractor of the system (5) when $b = 0.31$ and (c) $z_1 - z_3$ periodic orbit of the system (5) when $b = 0.5$.

indicating the emergence of periodic behavior. For plots, we have chosen the parameter $c = 0.5$. Subsequently, Figure 7a illustrates the $z_2 - z_3$ attractor, which clearly demonstrates the periodic behavior of the system (5). The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \text{-0.025} \\ \text{-0.133} \\ \text{-0.134} \end{pmatrix} \tag{15}$$

When c belongs to the intervals $([0.84, 0.87], [0.91, 0.98], [1.01, 1.15])$, the system (5) demonstrates a positive maximal Lyapunov exponent, and the Kaplan-Yorke dimension yields a fractional value of $D_{KY} = 3.072$. These characteristics indicate that the system (5) exhibits complex chaotic behavior. For plots, we have selected the parameter $c = 1.02$. Subsequently, Figure 7b illustrates the $z_2 - z_3$ attractor, clearly showcasing the chaotic behavior of the system (5). The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.061 \\ 0 \\ \text{-0.025} \\ \text{-0.502} \end{pmatrix} \tag{16}$$

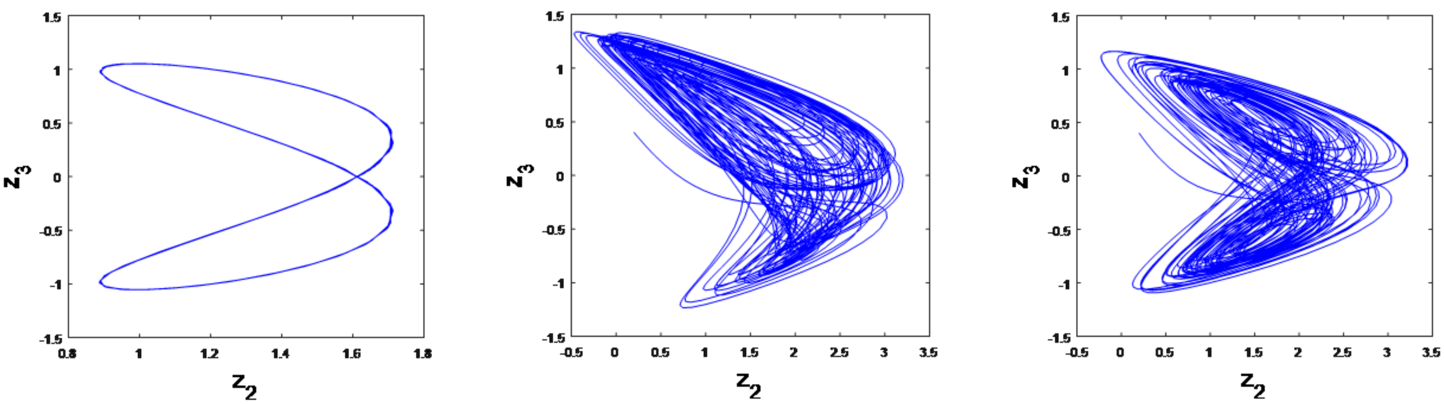
In the range where c belongs to $([0.87, 0.91], [1.15, 1.5])$, the system (5) showcases two positive Lyapunov exponents, indicating the presence of extreme hyperchaotic behavior. The system also possesses a fractional value of the Kaplan-Yorke dimension, with $D_{KY} = 3.090$. For plots, we have chosen the parameter $c = 1.35$. Subsequently, Figure 7c illustrates the $z_2 - z_3$ attractor, effectively demonstrating the hyperchaotic behavior of the system (5). The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.057 \\ 0.027 \\ 0 \\ \text{-0.929} \end{pmatrix} \tag{17}$$

Dynamics when d varies For the system (5), the Bifurcation diagram and Lyapunov exponents spectrum are plotted as in Figure 8 when d varies in the region $[-0.3, 0.3]$.

(a) (b)

Figure 6 Characteristics of system (3) in terms of a) Bifurcation diagram and b) LEs spectrum when c ε [0, 1.5].



(a) (b) (c)

Figure 7 Output Matlab simulation (a) $z_2 - z_3$ hyperchaotic attractor of the system (5) when $c = 0.5$. (b) $z_2 - z_3$ chaotic attractor of the system (5) when $c = 1.02$ and (c) $z_2 - z_3$ periodic orbit of the system (5) when $c = 1.35$.

When d belongs to the interval $[-0.3, -0.14]$, the system (5) exhibits four negative Lyapunov exponents, indicating its convergence to a stable state. Figure 9a illustrates the $z_1 - z_4$ attractor for $d = -0.3$. The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} -0.029 \\ -0.405 \\ -0.409 \\ -1.126 \end{pmatrix} \tag{18}$$

When the value of d falls within the range $[-0.13, 0.03]$, the system (5) demonstrates the presence of one positive Lyapunov exponent, resulting in a fractional value of the Kaplan-Yorke dimension $D_{KY} = 3.034$. This indicates that the system (5) exhibits complex chaotic behavior. Figure 9b showcases the $z_1 - z_4$ attractor for $d = 0$, illustrating the chaotic behavior of the system (5). The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} -0.055 \\ 0 \\ -0.020 \\ -1.040 \end{pmatrix} \tag{19}$$

When the value of d lies within the interval $[0.04, 0.3]$, the system (5) showcases two positive Lyapunov exponents, indicating its extreme hyperchaotic behavior. Moreover, the Kaplan-Yorke dimension takes on a fractional value of $D_{KY} = 3.051$. Figure 9c represents the plotted $z_1 - z_4$ attractor for $d = 0.2$, effectively demonstrating the hyperchaotic behavior of the system (5). The corresponding Lyapunov exponents are as follows:

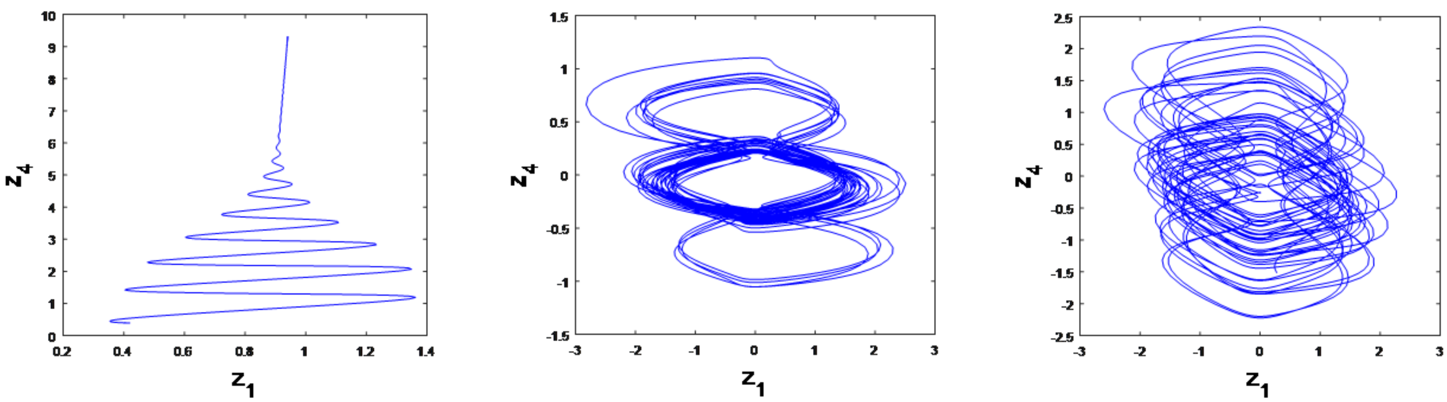
$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.032 \\ 0.023 \\ 0 \\ -1.077 \end{pmatrix} \tag{20}$$

Dynamics when p varies Figure 10 illustrates the Bifurcation diagram and Lyapunov exponents spectrum of the system (5) while the value of p varies between 0 and 3.

When the parameter p falls within the range $[0, 0.7]$, the system (5) exhibits extreme hyperchaotic behavior. The Kaplan-Yorke dimension for this behavior is measured to be $DKY = 3.062$. Figure 11a displays the $z_2 - z_4$ hyperchaotic attractor for $p = 0.1$. The associated Lyapunov exponents are as follows:

(a) (b)

Figure 8 Characteristics of the system (5) in terms of a) Bifurcation diagram and b) LEs spectrum when $d \in [-0.3, 0.3]$.



(a) (b) (c)

Figure 9 Output Matlab simulation (a) $z_1 - z_4$ hyperchaotic attractor of the system (5) when $d = 0.5$. (b) $z_1 - z_4$ chaotic attractor of the system (5) when $d = 0$ and (c) $z_1 - z_4$ periodic orbit of the system (5) when $d = 0.2$.

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.041 \\ 0.026 \\ 0 \\ -1.086 \end{pmatrix} \tag{21}$$

When the parameter p is within the range $[0.8, 1.7]$, the system (5) showcases a positive maximal Lyapunov exponent, indicating the presence of chaotic behavior with $D_{KY} = 3.033$. Figure 11b portrait the $z_2 - z_4$ chaotic attractor exhibited by the system (5) for $p = 1.2$. The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.076 \\ 0 \\ -0.039 \\ -1.119 \end{pmatrix} \tag{22}$$

When the parameter p is in the range of $[1.8, 3]$, the system (5) exhibits periodic behavior without complexity. Figure 11c presents the $z_2 - z_4$ attractor, visually representing the periodic behavior displayed by the system (5) for $p = 3$. The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.020 \\ -0.023 \\ -1.146 \end{pmatrix} \tag{23}$$

Dynamics when q varies For system (5), the Bifurcation diagram and Lyapunov exponents spectrum are illustrated as in Figure 12 as the value of q ranges from 0 to 0.5.

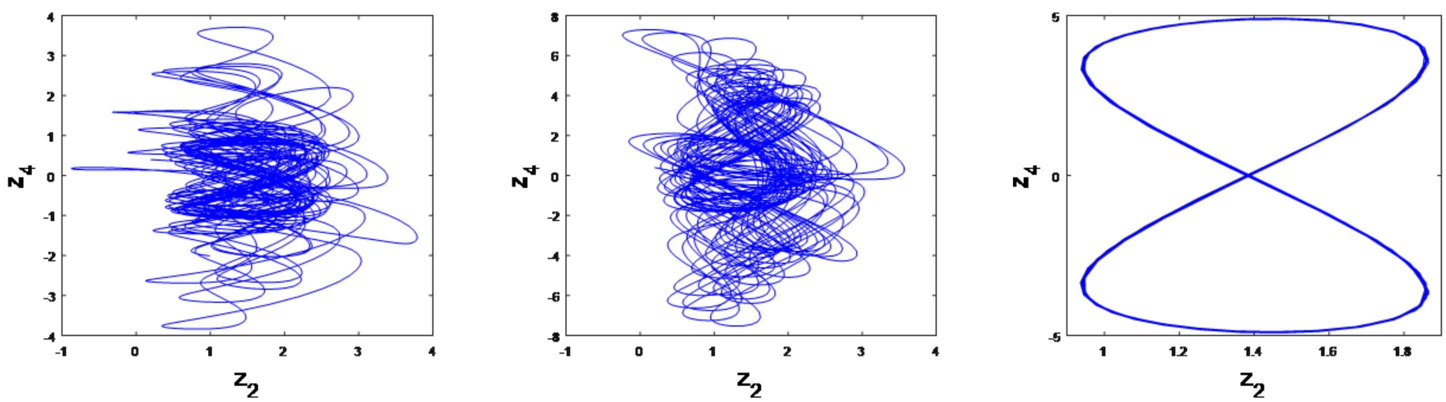
When $q = 0$, the system (5) exhibits periodic behavior. Figure 13a illustrates the $z_3 - z_4$ attractor, which clearly demonstrates the periodic behavior of the system (5). The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -0.010 \\ -0.947 \end{pmatrix} \tag{24}$$

When q belongs to the interval $[0.01, 0.28]$, system (3) exhibits complex hyperchaotic behavior with $D_{KY} = 3.055$. For plots, we have selected the parameter $q = 0.2$. Subsequently, Figure 13b illustrates the $z_3 - z_4$ attractor. The corresponding Lyapunov exponents are as follows:

(a) (b)

Figure 10 Characteristics of the system (5) in terms of a) Bifurcation diagram and b) LEs spectrum when p ε [0, 3].



(a) (b) (c)

Figure 11 Output Matlab simulation (a) $z_2 - z_4$ hyperchaotic attractor of the system (5) when $p = 0.1$. (b) $z_2 - z_4$ chaotic attractor of the system (5) when $p = 1.2$ and (c) $z_2 - z_4$ periodic orbit of the system (5) when $p = 3$.

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.048 \\ 0.013 \\ 0 \\ -1.106 \end{pmatrix} \tag{25}$$

In the range where q belongs to $[0.29, 0.5]$, the system (5) generates chaotic behavior. The system also possesses a fractional value of the Kaplan-Yorke dimension, with $DKY = 3.017$. Figure 13c illustrates the $z_3 - z_4$ attractor for $q = 0.5$. The corresponding Lyapunov exponents are as follows:

$$\begin{pmatrix} LE_1 \\ LE_2 \\ LE_3 \\ LE_4 \end{pmatrix} = \begin{pmatrix} 0.093 \\ 0 \\ -0.074 \\ -1.122 \end{pmatrix} \tag{26}$$

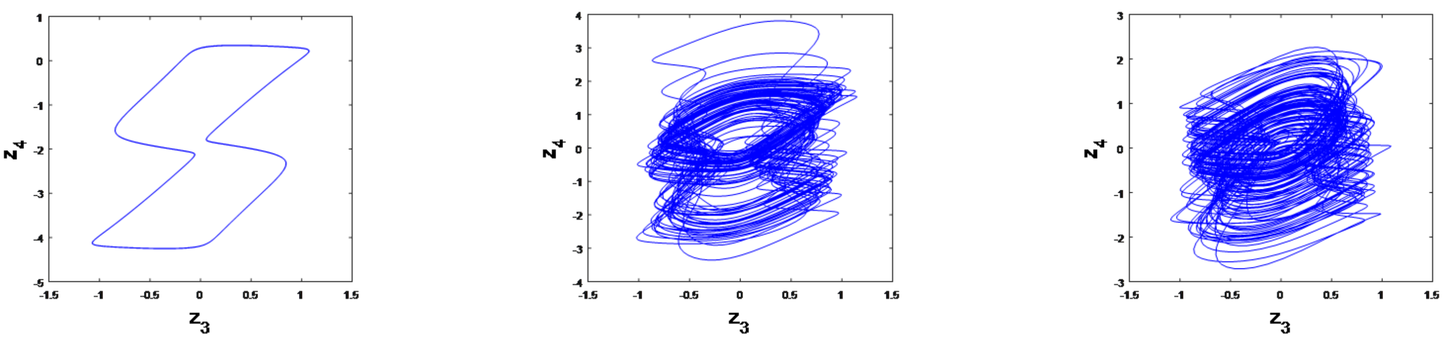
Multistability and Coexisting Attractors The new 4D hyperchaotic finance system (5) possesses the ability to exhibit multiple coexisting attractors. To explore this phenomenon further, we examine the system's behavior using two distinctive starting points: Z_{01} = (0.4, 0.2, -0.4, 0.4) and Z_{02} = (0.4, 0.2, 0.4, 0.4).

When the parameters $a = 0.9$, $b = 0.2$, $c = 1$, $d = 0.3$, $p = 0.15$, and $q = 0.1$ are held constant, the finance system (5) exhibits two distinct periodic behaviors based on its initial conditions, as illustrated in Figure 14. The figure showcases three separate representations denoted as (a), (b), and (c).

In the presence of parameters, $a = 0.9$, $b = 0.2$, $c = 1.05$, $d = 0.3$, $p = 0.15$, and $q = 0.1$, the finance system (5) reveals the presence of two distinct chaotic attractors, determined by the selection of initial conditions (Z_{01} or Z_{02}), as depicted in Figure 15. The figure encompasses three different representations, labeled as (a), (b), and (c).

(a) (b)

Figure 12 Characteristics of the system (5) in terms of a) Bifurcation diagram and b) LEs spectrum when $q \in [0, 0.5]$.



(a) (b) (c)

Figure 13 Output Matlab simulation (a) $z_3 - z_4$ hyperchaotic attractor of the system (5) when $q = 0.1$. (b) $z_3 - z_4$ chaotic attractor of the system (5) when $q = 0.2$ and (c) $z_3 - z_4$ periodic orbit of the system (5) when $q = 0.5$.

SYNCHRONIZATION OF HYPERCHAOTIC FINANCE SYSTEM WITH GENERAL ADAPTIVE NEURAL FUZZY CONTROLLER METHOD

Nonlinear Control Design

To synchronize two hyperchaotic financial systems modelled by the dynamics (5), first consider the following master-slave equations where index m is for master and index s is for slave.

$$\begin{cases} \dot{z}_{m1} &= z_{m3} + (z_{m2} - a)z_{m1} + qz_{m4} \\ \dot{z}_{m2} &= 1 - bz_{m2} - z_{m1}^2 \\ \dot{z}_{m3} &= -z_{m1} - cz_{m3} \\ \dot{z}_{m4} &= -dz_{m1}z_{m2} - p\sinh(z_{m1}) \end{cases} \tag{27}$$

$$\begin{cases} \dot{z}_{s1} &= z_{s3} + (z_{s2} - a)z_{s1} + qz_{s4} + u_1 \\ \dot{z}_{s2} &= 1 - bz_{s2} - z_{s1}^2 + u_2 \\ \dot{z}_{s3} &= -z_{s1} - cz_{s3} + u_3 \\ \dot{z}_{s4} &= -dz_{s1}z_{s2} - p\sinh(z_{s1}) + u_4 \end{cases} \tag{28}$$

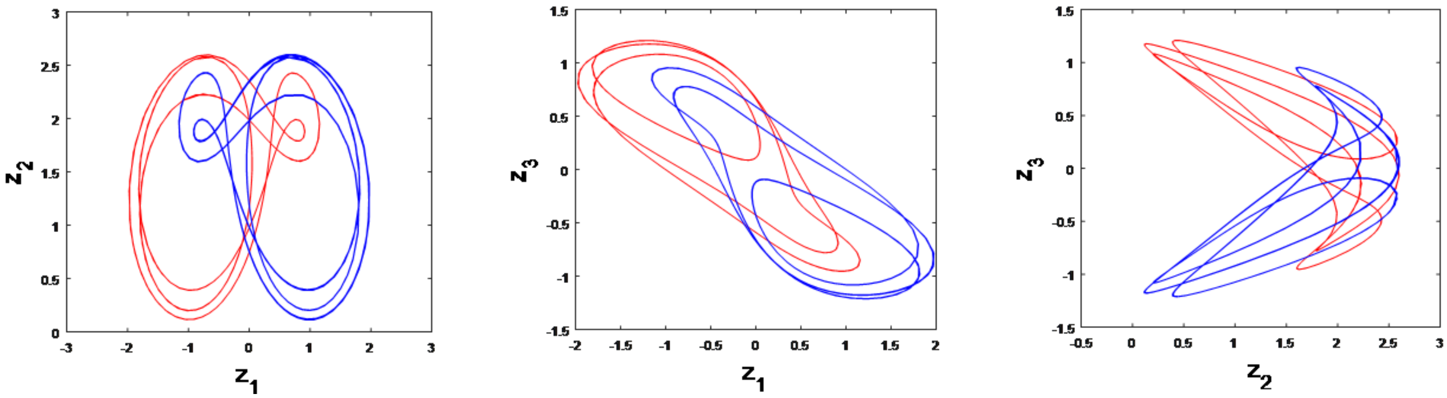
where u_i is the controller (decision variable) that should direct the equations of the slave system to the equations of the master system. Figure 16 shows that by having different values of initial conditions for the two hyperchaotic financial systems, the behavior of the system will be different.

The first goal in designing a nonlinear controller is to determine the system error equation. So:

$$\begin{cases} e_1 &= z_{s1} - z_{m1} \\ e_2 &= z_{s2} - z_{m2} \\ e_3 &= z_{s3} - z_{m3} \\ e_4 &= z_{s4} - z_{m4} \end{cases} \tag{29}$$

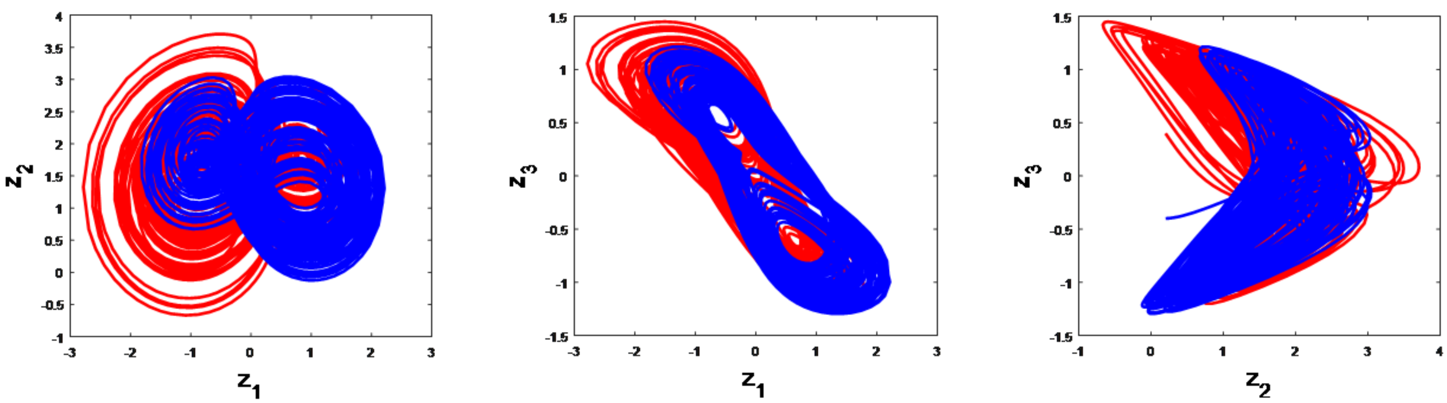
The main goal in controller design is to reach

$$\lim_{t \to \infty} \|e_i(t)\| = 0 \quad (i = 1, 2, 3, 4) \tag{30}$$



(a) (b) (c)

Figure 14 Matlab plots of the coexisting periodic/periodic attractors generated by the new 4D finance system (5), where the attractors generated from Z_{01} are in blue color, while attractors generated from Z_{02} are in red color. (a) $z_1 - z_2$ plane, (b) $z_1 - z_3$ plane and (c) $z_2 - z_3$ plane.



(a) (b) (c)

Figure 15 MATLAB plots of the coexisting chaotic/chaotic attractors generated by the new finance system (5), where the attractors generated from Z_{01} are in blue color, while the attractors generated from Z_{02} are in red color. (a) $z_1 - z_2$ plane, (b) $z_1 - z_3$ plane and (c) $z_2 - z_3$ plane.

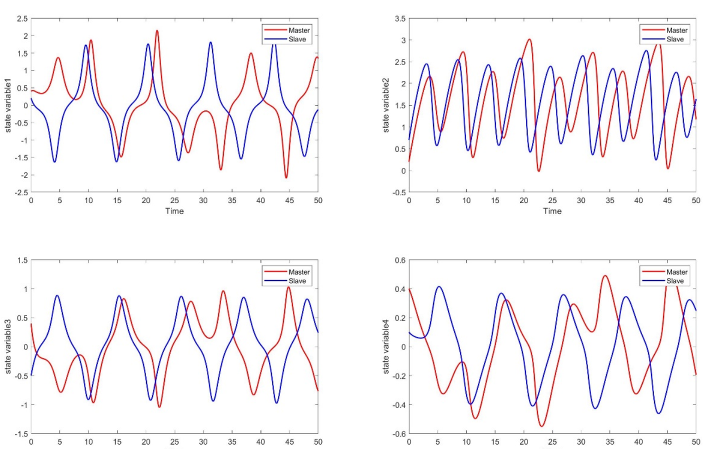


Figure 16 Behavior of two 4D hyperchaotic systems for initial conditions $z_{m1}(0) = 0.4, z_{m2}(0) = 0.2, z_{m3}(0) = 0.4, z_{m4}(0) = 0.4$ and $z_{s1}(0) = 0.2, z_{s2}(0) = 0.7, z_{s3}(0) = -0.5, z_{s4}(0) = 0.1$

From equation (29), we derive:

$$\begin{cases} \dot{e}_1 &= e_3 + qe_4 + (z_{s2} - a)z_{s1} - (z_{m2} - a)z_{m1} + u1 \\ \dot{e}_2 &= -be_2 - z_{s1}^2 + z_{m1}^2 + u_2 \\ \dot{e}_3 &= -e_1 - ce_3 + u_3 \\ \dot{e}_4 &= -d(z_{s1}z_{s2} - z_{m1}z_{m2}) - p(\sinh(z_{s1}) - \sinh(z_{m1})) + u_4 \end{cases} \quad (31)$$

Theorem 1. *The slave hyperchaotic finance system (28) will synchronize with the hyperchaotic finance system (27) if the controller is chosen as follows.*

$$\begin{cases} u_1 &= -e_3 - qe_4 - (z_{s2} - a)z_{s1} + (z_{m2} - a)z_{m1} + \lambda_1 e_1 \\ u_2 &= be_2 + z_{s1}^2 - z_{m1}^2 + \lambda_2 e_2 \\ u_3 &= e_1 + ce_3 + \lambda_3 e_3 \\ u_4 &= d(z_{s1}z_{s2} - z_{m1}z_{m2}) + p(\sinh(z_{s1}) - \sinh(z_{m1})) + \lambda_4 e_4 \end{cases} \quad (32)$$

Proof. To prove the stability of the candidate Lyapunov function, it is considered as follows

$$V_i(e) = \sum_{i=1}^{4} \frac{1}{2} e_i^2 = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \quad (33)$$

□

If we derive from the equation (31), we have

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 \quad (34)$$

By substituting equation (29) and (31) in the equation (34) and finally by substituting the proposed nonlinear controller, we have:

$$\dot{V} = \lambda_1 e_1^2 + \lambda_2 e_2^2 + \lambda_3 e_3^2 + \lambda_4 e_4^2 < 0 \quad (35)$$

where $\lambda_i, (i = 1, 2, 3, 4)$ are the controlling gains chosen to be negative. This completes the proof. ■

Nonlinear control design simulation results

Considering the same initial conditions as before, also the controller gain is equal to $\lambda_i(i = 1, 2, 3, 4) = -2$, Figure 17 shows the synchronization of two hyper-chaotic finance systems given by (27) and (28). The controller is applied to the model since $t = 20$. Figure 17 shows the synchronization of two hyperchaotic systems by the proposed nonlinear method.

Figure 18 shows the behavior of the non-linear controller (decision variable) that has been applied to the hyper-chaotic slave system (28) from $T = 20$. Figure 19 shows that the synchronization error in the proposed method tends to zero after passing a short period of time.

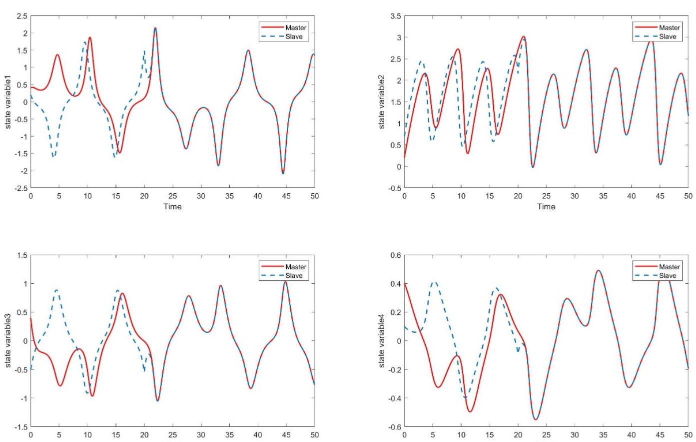


Figure 17 Synchronization of two hyperchaotic finance systems (27) and (28) in a non-linear way

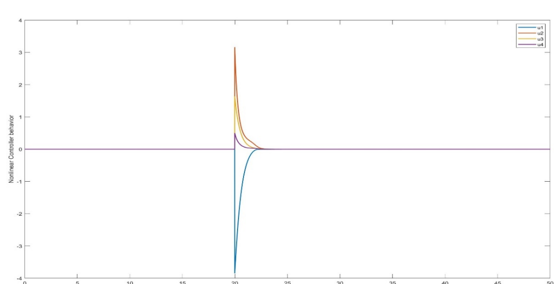


Figure 18 Non-linear controller behavior (decision variable)

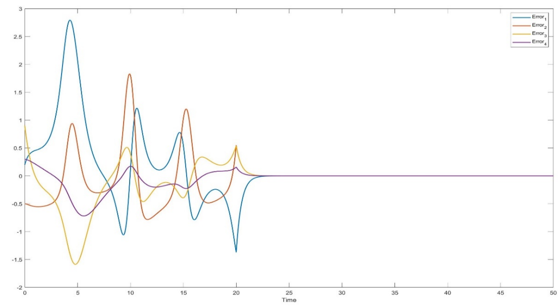


Figure 19 Synchronization error of two hyperchaotic finance systems (27) and (28) by nonlinear control method

By increasing the gain of the nonlinear controller, the convergence speed of the error can be adjusted to zero. But this issue will face problems in the real world. To prove, gains are equal to $\lambda_i(i = 1, 2, 3, 4) = -3$ will be considered (See Figure 20). Also, to ensure the performance of the proposed method, the controller application time has also been changed. The initial conditions are unchanged. The synchronization error of two hyperchaotic systems with gains can be seen in Figure 21.

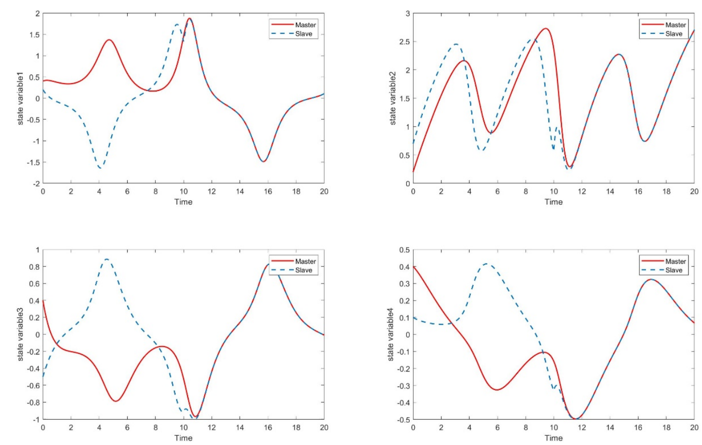


Figure 20 Synchronization of two hyperchaotic finance systems (27) and (28) with gains $\lambda_i(i = 1, 2, 3, 4) = -3$

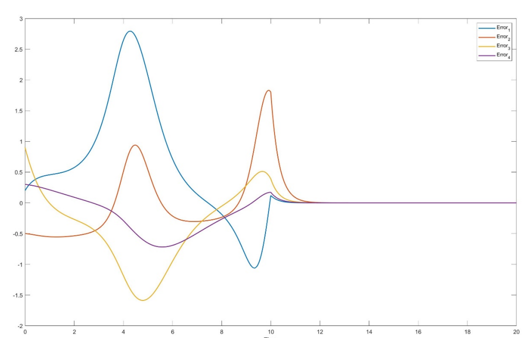


Figure 21 Synchronization error of two hyperchaotic finance systems (27) and (28) with gains

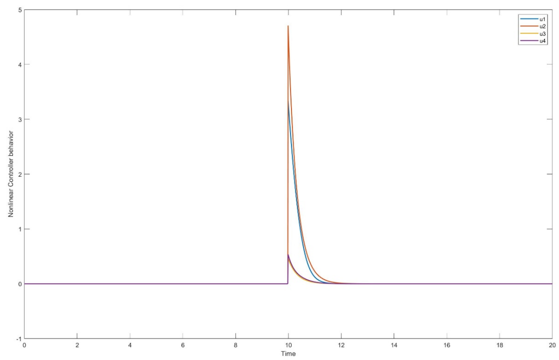


Figure 22 Behavior of nonlinear controller with gains $\lambda_i(i = 1,2,3,4) = -3$

Training and Test of General Adaptive Neural Fuzzy Controller

For ANFIS training, the training data must be determined first. The block diagram in Figure 23 shows how the training data is extracted from the nonlinear controller. As can be seen in Figure 23, training data is entered in ANFIS training block and training is completed by setting P_1 to P_4 parameters. The parameters P_1 to P_4 are given in Table 2. An important principle in fuzzy neural network training is what part of controller behavior and error to use for training.

In Figure 22, the controller's action time is $t = 10$, which has reached zero at the approximate time of $t = 12.5$. Also, in Figure 21, in the interval $10 < t < 12.5$, the system error has reached zero. Therefore, training data should be selected from time t equal to $10 < t < 12.5$. This is precisely the learning of the nature of hyperchaotic systems. Note that the time step is equal to 0.01.

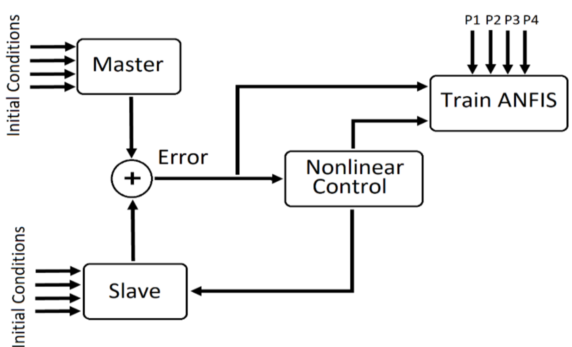


Figure 23 The concept of learning the adaptive neural fuzzy network with the nonlinear control method

Considering that the condition for stopping training is to reach zero error or the number of epochs, in this article, the number of epochs of training has led to stopping training. Figure 24 shows that the fuzzy neural network has trained the nonlinear controller well. Figure 25 shows the error of the proposed method for training.

Numerical Simulation and Comparison between the performance of two methods of controlling nonlinear and fuzzy adaptive neural feedback

Now the exquisite controller design is finished. ANFIS controller is replaced by non-linear controller. In the numerical simulation, the initial conditions of the slave financial finance system are set equal to $z_{s1}(0) = 0.1, z_{s2}(0) = -0.2, z_{s3}(0) = 0.1, z_{s4}(0) = 0.5$, and

Table 2 Proposed fuzzy neural network architecture

Total number of trainings	250
Number of training pairs	83
Number of check pairs	83
Number of test pairs	84
Number of input / output membership function (P1)	5
Input / Output Membership Function Type (P2)	'gaussmf'
epoch (P3)	80
Training error (P4)	0

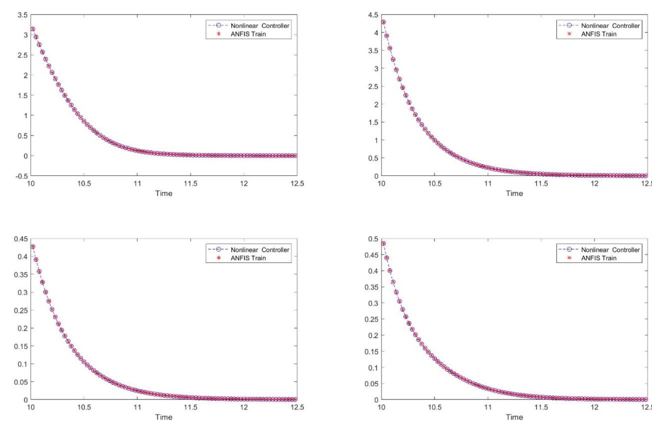


Figure 24 Comparison between the behavior of the nonlinear controller and the trained behavior of the ANFIS controller

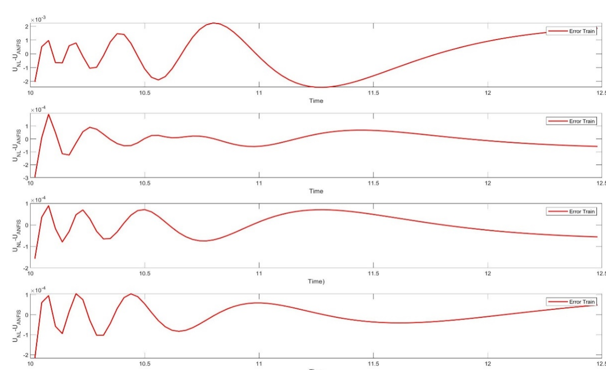


Figure 25 ANFIS controller training error

the initial conditions for the master financial hyperchaotic system are taken as $z_{m1}(0) = 0.4, z_{m2}(0) = 0.2, z_{m3}(0) = 0.4, z_{m4}(0) = 0.4$. To ensure the performance of the controller, the time of its application to the system will also be changed. Figure 26 shows the synchronization of two hyperchaotic finance systems (27) and (28) using ANFIS control method. The application time of ANFIS controller is $t = 13$.

In the last part of the numerical simulation, the performance of two non-linear and infinite control methods is compared. For this purpose we regulate the initial conditions of the two hyperchaotic

Table 3 Comparison of mean least square error

Type Control	MSE (E1)	MSE (E2)	MSE (E3)	MSE (E4)
Nonlinear	0.0983	0.1379	0.0270	0.0098
ANFIS	0.0980	0.1382	0.0273	0.0098

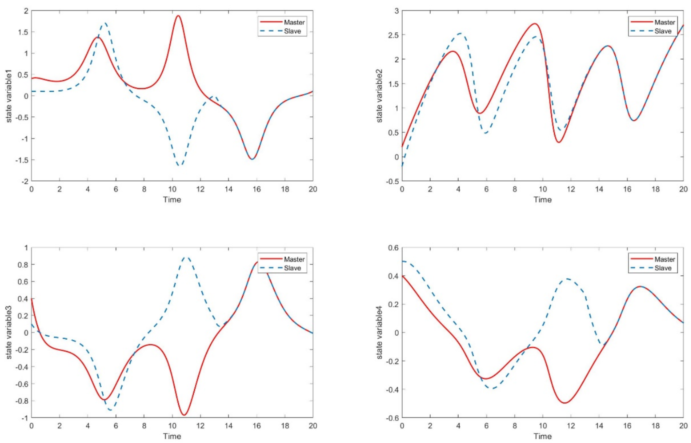


Figure 26 Synchronization of two hyperchaotic finance systems (27) and (28) by ANFIS control method

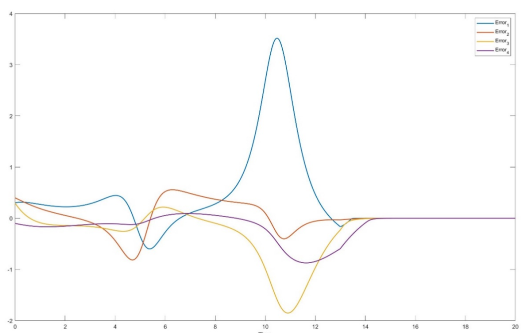


Figure 27 Synchronization error of two hyperchaotic finance systems (27) and (28) using ANFIS control method

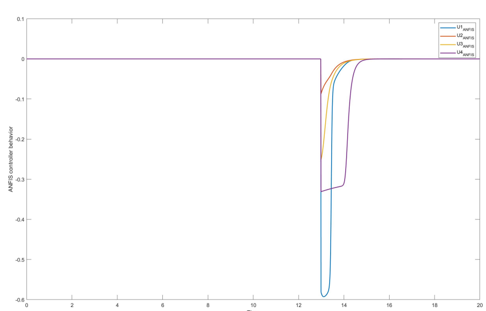


Figure 28 ANFIS controller behavior for synchronization of two hyperchaotic finance systems (27) and (28)

finance systems (27) and (28) as unchanged, while that of the slave hyperchaotic system equals to $z_{s1}(0) = 0.9, z_{s2}(0) = 0.4, z_{s3}(0) = 1, z_{s4}(0) = 0.25$.

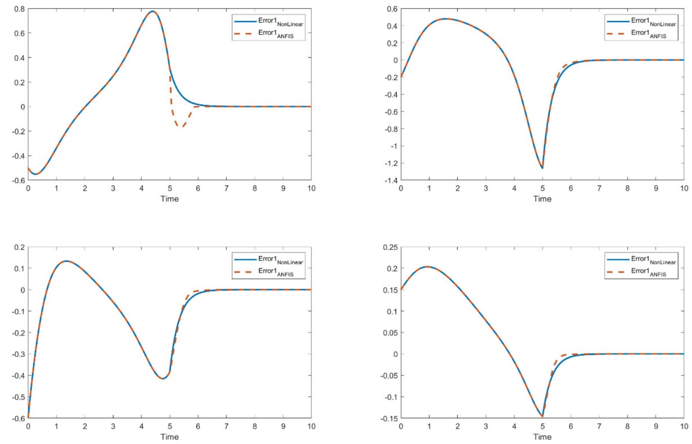


Figure 29 comparing the error of two controllers with the same simulation conditions

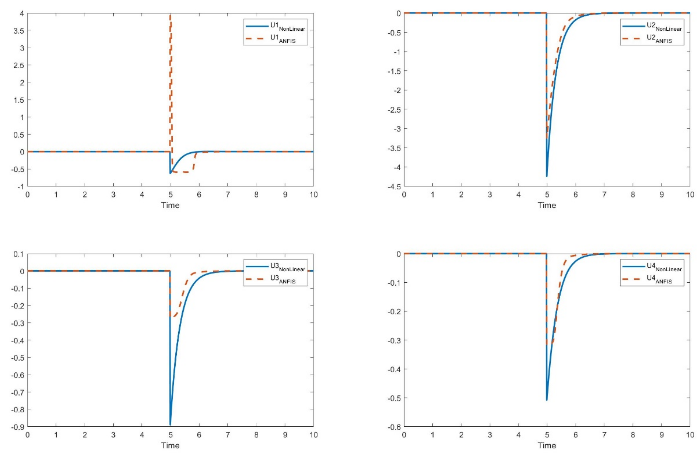


Figure 30 The comparison of the behavior of two controllers with the same simulation conditions

The gains of the nonlinear controller are equal to $\lambda_i(i = 1, 2, 3, 4) = -3$. The controller is applied at time t=5. Figure 29 shows the error of two non-linear and ANFIS controllers for each variable of the financial hyper chaotic system. As can be seen from Figure 29, the error behavior in all variables of the hyperchaotic system was almost the same for both methods. Table 3 shows the mean of the least squares error for each variable.

Figure 30 shows the behavior of two nonlinear and ANFIS controllers while the simulation conditions of both were the same. Controller behavior in the real world represents implementation costs. Therefore, this behavior must be analyzed after designing the controller. In the analysis of the behavior of the controller for the first variable, the range of the ANFIS controller is much higher than the nonlinear controller, but in the fourth variable, the range of the nonlinear controller is more than the ANFIS. In the other two variables (second and third), the range of the nonlinear controller is higher.

CONCLUSION

In this article, we investigated the new 4D hyperchaotic financial system with sinusoidal hyperbolic non-linear variables applied to the average profit margin. The bifurcation diagrams, Lyapunov exponents, and multistability, have all been used to explain the complexity behavior of new 4D hyperchaotic financial system. Finally, a nonlinear control and adaptive neural fuzzy controller are designed for demonstrate the performances of the proposed approach. The main finding is the simulation results show that the proposed neural fuzzy controller architecture well controls the synchronization of the new 4D hyperchaotic financial systems taken as master and slave systems.

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Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

LITERATURE CITED

Bekiros, S., H. Jahanshahi, F. Bezzina, and A. A. Aly, 2021 A novel fuzzy mixed h2/h ∞ optimal controller for hyperchaotic financial systems. Chaos, Solitons & Fractals **146**: 110878.

Cao, L., 2018 A four-dimensional hyperchaotic finance system and its control problems. Journal of Control Science and Engineering **2018**: 1–12.

Chen, H., L. Yu, Y. Wang, and M. Guo, 2021 Synchronization of a hyperchaotic finance system. Complexity **2021**: 1–7.

Gao, Q. and J. Ma, 2009 Chaos and hopf bifurcation of a finance system. Nonlinear Dynamics **58**: 209–216.

Guegan, D., 2009 Chaos in economics and finance. Annual Reviews in Control **33**: 89–93.

Hajipour, A., M. Hajipour, and D. Baleanu, 2018 On the adaptive sliding mode controller for a hyperchaotic fractional-order financial system. Physica A: Statistical Mechanics and its Applications **497**: 139–153.

Inglada-Perez, L., 2020 A comprehensive framework for uncovering non-linearity and chaos in financial markets: Empirical evidence for four major stock market indices. Entropy **22**: 1435.

Jahanshahi, H., A. Yousefpour, Z. Wei, R. Alcaraz, and S. Bekiros, 2019 A financial hyperchaotic system with coexisting attractors: Dynamic investigation, entropy analysis, control and synchronization. Chaos, Solitons & Fractals **126**: 66–77.

Kai, G., W. Zhang, Z. Wei, J. Wang, A. Akgul, *et al.*, 2017 Hopf bifurcation, positively invariant set, and physical realization of a new four-dimensional hyperchaotic financial system. Mathematical Problems in Engineering **2017**.

Li, X., R. Rao, and X. Yang, 2022 Impulsive stabilization on hyperchaotic financial system under neumann boundary. Mathematics **10**: 1866.

Lux, T., 1998 The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions. Journal of Economic Behavior & Organization **33**: 143–165.

Ma, Y. and W. Li, 2020 Application and research of fractional differential equations in dynamic analysis of supply chain financial chaotic system. Chaos, Solitons & Fractals **130**: 109417.

Musaev, A., A. Makshanov, and D. Grigoriev, 2022 Statistical analysis of current financial instrument quotes in the conditions of market chaos. Mathematics **10**: 587.

Rao, R. and Q. Zhu, 2021 Exponential synchronization and stabilization of delayed feedback hyperchaotic financial system. Advances in Difference Equations **2021**: 1–13.

Shi, J., K. He, and H. Fang, 2022 Chaos, hopf bifurcation and control of a fractional-order delay financial system. Mathematics and Computers in Simulation **194**: 348–364.

Szumiński, W., 2018 Integrability analysis of chaotic and hyperchaotic finance systems. Nonlinear Dynamics **94**: 443–459.

Vargas, J. A., E. Grzeidak, and E. M. Hemerly, 2015 Robust adaptive synchronization of a hyperchaotic finance system. Nonlinear Dynamics **80**: 239–248.

Vogl, M., 2022 Controversy in financial chaos research and nonlinear dynamics: a short literature review. Chaos, Solitons & Fractals **162**: 112444.

Xin, B., Y. Li, *et al.*, 2013 0-1 test for chaos in a fractional order financial system with investment incentive. In *Abstract and Applied Analysis*, volume 2013, Hindawi.

Xin, L., 2009 Modified projective synchronization of a new hyperchaotic system via nonlinear control. Communications in Theoretical Physics **52**: 274.

Xu, E., K. Ma, and Y. Chen, 2021 H ∞ control for a hyperchaotic finance system with external disturbance based on the quadratic system theory. Systems Science & Control Engineering **9**: 41–49.

Yu, H., G. Cai, and Y. Li, 2012 Dynamic analysis and control of a new hyperchaotic finance system. Nonlinear Dynamics **67**: 2171–2182.

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