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MODEL OF STUDYING ELECTROMAGNETIC FIELD AND WAVES THEORY VIA COMPUTER SIMULATION

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ABSTRACT: Model of studying the electromagnetic theory, mathematics, computing and data visualization in purpose to comprehend the main ideas of theories and practical applications are proposed. To learn physical processes and system's properties by computer simulation and to learn simulation by solution of physical tasks is the concept of proposed model. Integrated course of electromagnetic (EM) field and wave theories, mathematical solution of equations, computing techniques based on solutions of well known tasks as well as current problems of applied electrodynamics are considered. Application examples of electromagnetic in scientific research, modern technologies represent the abstract theories in realistic existence, help to understand deeply theoretical course, appreciate significance of (EM) theories in many fields of daily life. Knowledge of main theories of electromagnetic combined with mathematics and computing is necessary for solving the electrodynamics problems such as (EM) waves scattering and diffraction, interaction of (EM) field and objects of different electric and geometric properties, basis of linear and nonlinear optical techniques, etc. Selected tasks of electrodynamics are constructed of several modules: formulation of physical problem, theories and methods of solution – physical and mathematical, specifics of problem, approximation and application cases, computer simulation, analysis. Each completed module expands outlook, develops skills, intuition, self-confidence, encourages participants be more motivated, active in learning and improvement of knowledge in multi disciplines. Proposed model is presented by considering one task - EM waves scattering on a single cylindrical body, applicable in radio physics, transmitting and detecting systems, aerosol studies for particles of different origin. Estimation of EM field components, scattering characteristics, theoretical predictions based on analytical solutions and numerical simulations are considered.

Keywords: Electrodynamics, wave scattering, simulation

INTRODUCTION

The way to comprehend the physical phenomenon and main properties of systems pass through the deliberate study of well-known tasks and model systems. At any level of education, solution of physical tasks is based on multi-disciplinary knowledge of physics, mathematics, computing as well as skills and ability of analytical thinking. Modeling, simulation and visualization are the tools simplifying the study of systems and processes however simulation itself is the subject of deep learning and analysis. To learn physical processes and system's properties by computer simulation and to learn simulation by solution of physical tasks is the concept of proposed model of learning [1].

Study of model tasks and applications in scientific research and advanced technologies represent the abstract theories in realistic existence, helps to understand deeply and appreciate significance of electromagnetic (EM)

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field theories in many directions of radio physics and optics, spectroscopic techniques, detecting and transmitting systems, aerosol and medical studies, etc. Specific knowledge and experience needed for solving of modern physical problems should be gained and expanded step by step alongside with topics rising from process of investigation and research. This approach is known and well-proven in practice of scientific and educational activities.

Proposed model is demonstrated for studying EM field and wave theories, related mathematics, computing and data visualization, namely for one task of electrodynamics - EM waves scattering on a single cylindrical body. Solution of task gives possibility to analyze the physical picture of phenomena, understand the process of interaction of EM field and body, estimate characteristics and scattering properties of body of different electric and geometric parameters, find out the appropriate solutions of mathematical equations, learn the developing of computing program and specifics of computing and visualization.

Selected task is constructed of several modules. Subtasks chosen on the basis of skills and experience of participants, which makes the work in team more efficient and motivative. Each completed module expands outlook, improves knowledge in multi discipline, develops skills, intuition, self-confidence.

Scattering of EM waves on single objects, as cylinders and spheres is of great interest because of its wide application in antennas, detecting systems, aerosol studies [2-5].

Single body solutions are been used as the models for testing a complex tasks, single body properties are taking into account while combining and studying the multi-element system, etc. There are many naturally occurring particles, such as some viruses, ice needles, fibers, which are best represented as cylinders long compared with their diameter [6]. In our research studies single particles of cylindrical and spherical shapes are considered as approximated physical models of virions (of viruses, bacteriophages) of icosahedral, prolate and rod-shaped morphology, and used for studying virus-like particles (VLPs) of biological or artificial origin [7].

FORMULATION OF PHYSICAL PROBLEM

The treatment of EM scattering by a bodies is the problem of EM theory. Based on macroscopic approach to the problem, Maxwell equations for homogeneous, isotropic, free of charge medium are considered using time-harmonic dependence by time-factor $\exp(-i\omega t)$:

$$\nabla \times \vec{E} = i\omega\mu_q\mu_0\vec{H} \quad , \tag{1}$$

$$\nabla \times \vec{H} = -i\omega\varepsilon_q\varepsilon_0\vec{E} \quad , \tag{2}$$

where $\vec{E}$ and $\vec{H}$ are electric and magnetic field vectors; ω is the angular friquency; ε_q - permittivity and μ_q - permeability of q medium; $\varepsilon_o = 8{,}85 \cdot 10^{-12}$ F/m; $\mu_o = 1{,}26 \cdot 10^{-6}\,\mathrm{H/m}$.

Some mathematical transformations of eq.s (1), (2) leads to Helmholtz's (wave) equation:

$$\Delta\vec{E} + k_q^2\vec{E} = 0 \quad , \tag{3}$$

where $\Delta = \nabla^2$ is Laplace scalar operator. Wave vector $k_q = k\sqrt{\varepsilon_q\mu_q}$ is prescribed to (q) medium and $k = \omega\sqrt{\varepsilon_o\mu_o}$ to the free space, using subscripts q=1 for core, q=2 for coat, q=3 for surrounded areas.

Study of EM scattering on single coated cylinder of circular (in XOY plane) cross section is considered. Cylinder long (L/d>>5) compared with its diameter (d) may be approximated by cylinder of infinite (L) length [6]. Incident plane monochromatic EM wave from positive direction of axis (x), makes angle θ with the direction of x . If incident wave is indipendent on coordinate z ($\partial/\partial z \equiv 0$) and $H_z \equiv 0$, $E_r \equiv 0$, $E_\varphi \equiv 0$, the scattered wave will be of the same stucture as incident wave, so electric component of incident EM wave could be written as follows (E_o =const):

$$E_z^{(\mathrm{o})} = E_o e^{-ik_3(x\cos\theta + y\sin\theta) - i\omega t} \quad , \tag{4}$$

coordinates and wave vector are given in Cartesian (x, y, z) and Cylindrical (r, φ, z) coordinate systems

$$x = r\cos\varphi\,, \qquad y = r\sin\varphi\,, \qquad z = z \tag{5}$$

$$k_x = -k\cos\theta\,, \qquad k_y = -k\sin\theta \quad . \tag{6}$$

Magnetic component of EM wave is derived from eq. (1) $\quad H_\varphi = -\frac{1}{i\omega\mu_q\mu_0}\frac{\partial}{\partial r}E_z \quad .$ (7)

Definition of scattered field in outside area ($b \le r \prec \infty$), inside areas of coat ($a \le r \le b$) and core ($0 \le r \le a$) of cylinder is goal of problem in purpose of study physical charecteristics of system, as well as the effects caused by EM field – body interection.

METHOD OF SOLUTION

Problem in view should be studied as physical and mathematical tasks. In these respects, let us consider:

1. Physical Subtasks:

1.1. Determination of scattered fields, which should satisfy an eq. (3), radiation condition outside the cylinder ($kr >> 1$) and finiteness inside the areas of cylinder under the condition [4]: $(E_z \cdot grad\,\varepsilon) = 0$. (8)

1.2. Definition of boundary conditions for electric and magnetic field components at the boundary surfaces separating the surrounding medium-coat-core. It requires the continuity of tangential components of EM field and leads to relations:

$E_z^{(o)} + E_z^{(sc)} = E_z^{(co)}$ and $H_\varphi^{(o)} + H_\varphi^{(sc)} = H_\varphi^{(co)}$ at $r = b$ $0 \le \varphi \le 2\pi$ (9)

$E_z^{(co)} = E_z^{(in)}$ and $H_\varphi^{(co)} = H_\varphi^{(in)}$ at $r = a$ $0 \le \varphi \le 2\pi$ (10)

1.3. Estimation of scattering characteristics of system for:

a) Near field as the lines of equal amplitudes (E_z) and equal phases ($\varphi_E = arctg \frac{\operatorname{Im} E_z}{\operatorname{Re} E_z}$),

b) Far-field ($kr \to \infty$) as extinction (σ_{ext}) cross section and an angular dependence $F(\varphi)$ of scattered EM field ($E_z^{(sc)}$) [6,8]. The presence of the particles in EM field has resulted in extinction of the incident wave, therefore the extinction cross section is defined by the sum of scattering and absorbing cross sections $\sigma_{ext} = \sigma_{sc} + \sigma_{abs}$. If the medium in which the particle is embedded is nonabsorbing, extinction cross section $\sigma_{ext} = \sigma_{sc}$. Using asymptotic expression of Hankel functions [2, 9] $H^{(1)}{}_s(\eta) \square \sqrt{\frac{2}{\pi\eta}} e^{i(\eta - \frac{2s+1}{4}\pi)}$, in a case of cylindrical bodies the formula for σ_{sc} and scattering pattern $F(\varphi)$ take the forms:

$$\sigma_{sc} = \frac{4}{k} I \qquad (11)$$

$$F(\varphi) = e^{-i\pi/4} \sqrt{2/\pi k} \cdot f(\varphi) \qquad (12)$$

Expressions for I and $F(\varphi)$ are defined by solution of physical problem, by means of the values of coefficients A_s as well as geometric and electric parameters of system:

$$I = \frac{1}{2\pi} \int_0^{2\pi} f(\varphi) f^*(\varphi) d\varphi = \sum_{m=-\infty}^{m=\infty} |A_m|^2 \qquad (13)$$

$$f(\varphi) = \sum_{m=-\infty}^{m=\infty} i^{-m} A_m e^{im\varphi} \qquad (14)$$

"*" denotes the complex conjugate of function.

2. Mathematical Subtasks:

2.1. Determination the solutions of an eq. (3) in cylindrical coordinate system.

Using expression [2,10] for $\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$ (15)

and seperation of variables method applicable if boundaries of body coincide with coordinate surfaces of coordinate system in which the wave equation is separable. E_z component of EM field could be presented as

$$E_z(r, \varphi, z) = \Re(r)\Omega(\varphi)Z(z) \qquad (16)$$

It leads to well-known ordinary differential equations

$$\frac{d^2\Re}{dr^2} + \frac{1}{r}\frac{d\Re}{dr} + (p^2 - \frac{m^2}{r^2})\Re = 0 \quad (17) \qquad \frac{\partial^2\Omega}{\partial\varphi^2} + m^2\Omega = 0 \quad (18) \qquad \frac{d^2 Z}{\partial z^2} + \chi^2 Z = 0 \quad (19)$$

$k_q{}^2 = p^2 + \chi^2$, $m = 0, 1, 2, ...,$. In a case, considered above, we have taken into account that $\partial/\partial z \equiv 0$, therefore we could write down the separable solutoion to (17), (18): $E_z = \Re(k_q r) e^{im\varphi}$, (20) function $\Re(k_q r)$ is the solution of Bessel equation (17).

2.2. Determination of EM fields in each area of cylinder.
Fields in the areas of cylinder are expressed by the sum of multi-pole waves, using Bessel functions of m order and first (Bessel), second (Neumann) or third kind (Hankel) based on conditions under consideration.

$$E_z^{(sc)} = \sum_{m=-\infty}^{\infty} A_m \mathrm{H}_m^{(1)}(k_3 r) e^{im\varphi - i\omega t} \quad , \quad b \le r \prec \infty \quad , \quad 0 \le \varphi \le 2\pi \tag{21}$$

$$E_z^{(\text{in})} = \sum_{m=-\infty}^{\infty} B_m J_m(k_1 r) e^{im\varphi - i\omega t} \quad , \quad 0 \le r \le a \quad , \quad 0 \le \varphi \le 2\pi \tag{22}$$

$$E_z^{(\text{co})} = \sum_{m=-\infty}^{\infty} [C_m J_m(k_2 r) + D_m N_m(k_2 r)] e^{im\varphi - i\omega t} \quad , \quad a \le r \le b \quad , \quad 0 \le \varphi \le 2\pi \tag{23}$$

$\{A_m\}, \{B_m\}, \{C_m\}$ and $\{D_m\}$ are unknown multipole coefficients.

2.3. Presentation of incident $E_z^{(\text{o})}$ field by Bessel functions.

Using (4)-(6), and Fourier representation for Bessel function [2]: $e^{-i\eta\cos\psi} = \sum_{s=-\infty}^{\infty} i^{-s} J_s(\eta) e^{is\psi}$, (24)

incident wave may be expressed as $E_z^{(\text{o})} = E_o e^{-i\omega t} \sum_{m=-\infty}^{\infty} i^{-m} J_m(k_3 r) e^{im\varphi} e^{-im\theta}$ (25)

2.4. Functional transformations.
Using boundary conditions (9)-(10), field expressions (21)-(23), (25) and relation (7) between electric and magnetic components, we get the functional relations:

$$E_o \sum_{m=-\infty}^{\infty} i^{-m} J_m(k_3 b) e^{im\varphi} e^{-im\theta} + \sum_{m=-\infty}^{\infty} A_m \mathrm{H}_m^{(1)}(k_3 b) e^{im\varphi} = \sum_{m=-\infty}^{\infty} [C_m J_m(k_2 b) + D_m N_m(k_2 b)] e^{im\varphi} \tag{26}$$

$$E_o \sum_{m=-\infty}^{\infty} i^{-m} J_m^{/}(k_3 b) e^{im\varphi} e^{-im\theta} + \sum_{m=-\infty}^{\infty} A_m \mathrm{H}_m^{(1)/}(k_3 b) e^{im\varphi} = W_{23} \sum_{m=-\infty}^{\infty} [C_m J_m^{/}(k_2 b) + D_m N_m^{/}(k_2 b)] e^{im\varphi} \tag{27}$$

$$\sum_{m=-\infty}^{\infty} [C_m J_m(k_2 a) + D_m N_m(k_2 a)] e^{im\varphi} = \sum_{m=-\infty}^{\infty} B_m J_m(k_1 a) e^{im\varphi} \tag{28}$$

$$\sum_{m=-\infty}^{\infty} [C_m J_m^{/}(k_2 a) + D_m N_m^{/}(k_2 a)] e^{im\varphi} = W_{12} \sum_{m=-\infty}^{\infty} B_m J_m^{/}(k_1 a) e^{im\varphi} \tag{29}$$

where "/" denotes the derivative with respect to an argument, $W_{pq} = \frac{W_p}{W_q} = \frac{k_p \mu_q}{k_q \mu_p}$, $W_q = = \sqrt{\frac{\varepsilon_q}{\mu_q}}$. (30)

Integrate (26)-(29) over the interval (0, 2π), using Wronskian determinant [9] of cylindrical functions

$\overline{W}\{J_s(\eta), N_s(\eta)\} = \frac{2}{\pi\eta}$ (31) and the Kronecker delta [9] $\delta_{m,s} = \frac{1}{2\pi} \int_0^{2\pi} e^{i(m-s)\varphi} d\varphi$, (32)

($\delta_{m,s}$ is 0 if $m \ne s$ and 1 otherwise), we get the set of algebraic equations with respect to multipole coefficients $\{A_s\}, \{B_s\}, \{C_s\}$ and $\{D_s\}$.

2.5. Determination the multipole coefficients of scattered fields.
Solving the system of algebraic equations, $\{A_s\}, \{B_s\}, \{C_s\}$ and $\{D_s\}$ coefficients are determined by formulas:

$$B_s = \frac{E_o i^{-s} e^{-is\theta} \frac{2i}{\pi k_3 b}}{\frac{\pi k_2 a}{2} \{U_s \Gamma_s - G_s \Lambda_s\}} \quad , \tag{33}$$

$$A_s = -E_o i^{-s} e^{-is\theta} \frac{\{U_s \Im_s - G_s L_s\}}{\{U_s \Gamma_s - G_s \Lambda_s\}} \quad , \tag{34}$$

$$D_s = -\frac{\pi k_2 a}{2} B_s G_s^m (k_2 a, k_1 a) \quad , \tag{35}$$

$$C_s = B_s U_s \frac{\pi k_2 a}{2} \quad , \tag{36}$$

where the notations Γ_s, Λ_s, $\Im_s$, L_s, G_s, U_s define the functions as follows:

$$\Gamma_s = \mathrm{H}_s^{(1)/}(k_3 b) J_s(k_2 b) - W_{23} \mathrm{H}_s^{(1)}(k_3 b) J_s^{/}(k_2 b) \quad , \tag{37}$$

$$\Lambda_s = \mathrm{H}_s^{(1)/}(k_3 b) N_s(k_2 b) - W_{23} \mathrm{H}_s^{(1)}(k_3 b) N_s^{/}(k_2 b) \quad , \tag{38}$$

$$\Im_s = J_s^{/}(k_3 b) J_s(k_2 b) - W_{23} J_s(k_3 b) J_s^{/}(k_2 b) \quad , \tag{39}$$

$$L_s = J_s^{/}(k_3 b) N_s(k_2 b) - W_{23} J_s(k_3 b) N_s^{/}(k_2 b) \quad , \tag{40}$$

$$G_s = J_s(k_1 a) J_s^{/}(k_2 a) - W_{12} J_s^{/}(k_1 a) J_s(k_2 a) \quad , \tag{41}$$

$$U_s = J_s(k_1 a)N_s^{/}(k_2 a) - W_{12} J_s^{/}(k_1 a) N_s(k_2 a) \quad . \tag{42}$$

3. Special Cases:

By formulas (33), (34) we could estimate the multi-pole coefficients for special cases, namely:

3.1. Single dielectric cylinder without coat. If consider nonmagnetic ($\mu_q \approx 1$) medium and assume that $k_3 \equiv k$, $k_2 \equiv k_1$, $b \equiv a$, we will define $w_{23} = \sqrt{\varepsilon_1}$ and $W_{12} = \frac{W_1}{W_2} = 1$. The expressions for B_s and A_s are of exactly the same as that for B_s and A_s obtained in [1,6]. The expression for A_s is of the form:

$$A_s = -E_o i^{-s} e^{-is\theta} \frac{[J_s^{/}(ka)J_s(k_1 a) - \sqrt{\varepsilon_1} J_s(ka) J_s^{/}(k_1 a)}{[\mathrm{H}_s^{(1)/}(ka)J_s(k_1 a) - \sqrt{\varepsilon_1}\mathrm{H}_s^{(1)}(ka)J_s^{/}(k_1 a)]} \quad . \tag{43}$$

3.2. Single metallic cylinder of infinite length. Assuming $\varepsilon_1 \to \infty$, in (43), multipole coefficient A_s takes the form $$A_s = A_s = -E_o i^{-s} e^{-is\theta} \frac{J_s(ka)}{\mathrm{H}_s^{(1)}(ka)}, \tag{44}$$

3.3. Relativly low ($ka < 0.5$) and high $ka > 3$ frequencies.
Based on asymptotic expressions of Bessel and Hankel functions formulas derived above may be used for analysis of fileds qualitatively different for low ($ka < 0.5$) and high $ka > 3$ frequencies [4] as well as particles of small ($ka << 1$) and large ($ka >> 1$) sizes in comparision with the wave lenght (λ) of incident wave.

SPECIFICS OF PROBLEM

For numerical estimation and visualization the programs based on Matlab v7.0.4. are created. Bessel function J = besselj(m, ξ) computes the Bessel function of the first kind, H = besselh(m, ξ) computes the Hankel function, for each element of the complex array ξ. The order m need not be an integer, but must be real. Computing (EM) fields and characteristics by formulas (21)-(23), (11),(12) we determine the number m of terms of series along with estimation of convergence of series and multi-pole coefficients with algorithm within the given accuracy 10^{-6}. The number of terms of series may be determined by the empirical formula: $m \geq 2\left[\left(k_1 a\right)+1\right]$.

VIZUALIZATION

For constructing the complete picture of system we have to calculate and analyze the main characteristics of system. In this paper we demonstrate some of them in a case of normal incident of wave $(\vartheta = 0°)$. In Fig. 1, is presented the near field characteristic by the lines of equal amplitudes in the interval ($-\frac{\lambda}{2}, \frac{\lambda}{2}$) along the axes x and y, for homogeneous cylinder ($ka = 2$) of permitivity $\varepsilon_1 \equiv \varepsilon = 1.7$, and far field characteristic by the scattering pattern of cylinder of $\varepsilon_1 \equiv \varepsilon = 1.7$ and 55 in polar coordinate system. The scattering pattern describing the angular dependence of scattered field is given in Cartesian coordinate system for cylinder of different parameters $ka = 1$; π , $k_1 a = \pi$, $\varepsilon = 55$ (Fig. 2 (left)) and coated cylinder of parameter $ka = 1$; permittivities of core and coat of cylinder correspondingly are equal $\varepsilon_1 = 55$, $\varepsilon_2 = 2$, coat parameters are $kb = 1.1$ and 1.4 (Fig.2 (right)). The value of permittivity $(\varepsilon = 55)$ for cylinder has been used while modelling the virus as a long thin rod of a homogeneous bulk material [11].

CONCLUSION

Model of studying the electromagnetic (EM) field and wave theories, related mathematical equations, computing and visualization techniques based on solutions of well known tasks of applied electrodynamics are considered. EM wave scattering on a single particle, namely coated cylinder as an example system for model demonstration is proposed. Theoretical and numerical solution of tasks applied for investigation of physical characteristics of virus-like particles (VLPs) is presented. Analysis shows that deliberate consideration and study of subtasks makes possible to achieve the intended goals - gaining the higher level knowledge and getting the complete physical picture of system under consideration. The proposed concept simplifies the process of understanding the difficult themes and theories targets the answers on questions "why" and "how", its findings are applicable to other disciplines as well.

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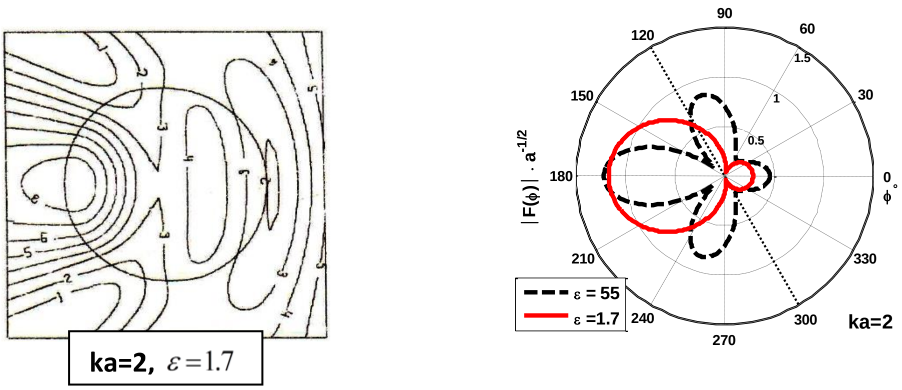

Figure 1. Lines of Equal Amplitudes (Left), Scattering Pattern in

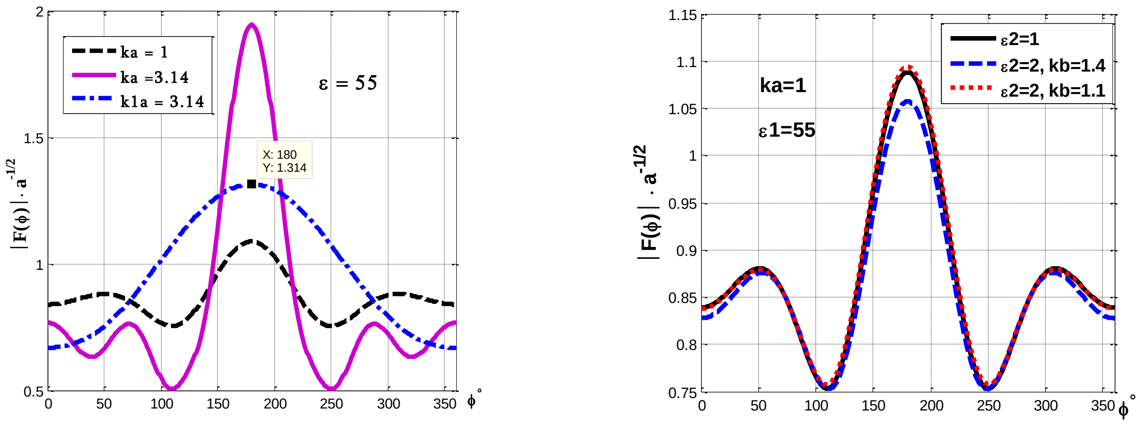

Figure 2. Scattering Pattern of Cylinder in Cartesian Coordinate System, for $\vartheta = 0^\circ$

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