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FRACTIONAL ORDER ANALYSIS OF THE 4-DIMENSIONAL HYPERCHAOTIC PANG SYSTEM AND ITS ADAPTIVE SYNCHRONIZATION

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Abstract: Fractional calculus is an effective method used to analyze the dynamics of nonlinear systems and provide more precise results. In this study, firstly, the 4-dimensional Pang system is introduced and its dynamic analyses demonstrating the hyperchaotic structure are given. Then, fractional-order calculations of the system are presented and the dynamics of the system for different fraction orders are investigated. At this point, according to the results obtained from Lyapunov exponents and phase-space representation, the Pang system exhibits periodic, chaotic, and hyperchaotic behaviors in different fractional orders. The results obtained at the end of this study present that the system is hyperchaotic for the fractional order of 3.52 and it is also confirmed that more accurate results are obtained than the integer-order analysis. In the next part of the study, adaptive synchronization of the fractional-order system is performed. Three different cases are examined and it is demonstrated that synchronization is achieved in all cases.

Anahtar Kelimeler: Fractional order systems, Synchronization, Hyperchaos

4-Boyutlu Hiperkaotik Pang Sisteminin Kesir Dereceli Analizi ve Adaptif Senkronizasyonu

Öz: Kesir dereceli hesaplamalar doğrusal olmayan sistemlerin dinamiklerini analiz etmekte kullanılan ve daha kesin sonuçlar elde edilmesini sağlayan etkili bir yöntemdir. Bu çalışmada, öncelikle 4 boyutlu Pang sistemi tanıtılmış ve hiperkaotik yapısını gösteren dinamik analizleri verilmiştir. Daha sonra sistemin kesir dereceli hesaplamaları yapılarak farklı kesir dereceleri için sahip olduğu dinamikler incelenmiştir. Bu kapsamda, Lyapunov üstelleri ve faz-uzayı gösteriminden elde edilen sonuçlara göre, Pang sistemi farklı kesir derecelerinde periyodik, kaotik ve hiperkaotik davranışlar sergilemektedir. Çalışmanın sonunda elde edilen sonuçlar, sistemin 3,52 kesir derecesi için hiperkaotik yapıda olduğunu göstermiştir. Elde edilen bu sonuç, tamsayı dereceli modele göre kesir dereceli yapı ile daha kesin sonuçlara ulaşıldığını doğrulamıştır. Çalışmanın ilerleyen kısmında, elde edilen kesir dereceli sistemin adaptif senkronizasyonu gerçekleştirilmiştir. Üç farklı durum incelenerek her durumda senkronizasyonun sağlandığı gösterilmiştir.

Keywords: Kesir dereceli sistemler, Senkronizasyon, Hiperkaos

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1. INTRODUCTION

Chaotic systems were first introduced in 1963 by Edward Lorenz's work on weather forecasting (Lorenz, 1963). The fact that a nonlinear autonomous system has at least one positive Lyapunov exponent and the system has at least 3 dimensions indicates that the type of the system is chaotic. In addition, chaotic systems have characteristic features such as sensitivity to initial conditions, unpredictable dynamics, and complex system structure. Hyperchaotic systems, on the other hand, are similar to chaotic systems and exhibit more complex dynamics. Since the 1960s, many chaotic and hyperchaotic systems with different structures have been proposed and these systems have been applied in fields such as communication, encryption, control, and electronics (Abd El-Maksoud et al., 2019; Huang et al., 2021; Liao et al., 2022; Nwachioma & Pérez-Cruz, 2021; F. Wang et al., 2019; J. Wang et al., 2019).

Integer order systems do not accurately and precisely reflect the dynamics of chaotic structures that are sensitive to initial values and parameters. For this reason, the dynamics of a model can be obtained more clearly by using fractional order calculations and it is possible to achieve more accurate results in applications such as control. The best-known first work on fractional order systems was done by Liouville and Riemann (Oldham & Spanier, 1974). However, other proposed methods such as the Caputo method (Caputo, 1967) and the Grünwald-Letnikov method (Scherer et al., 2011) are frequently used in the generation of fractional order systems.

The degree of a system is expressed as the sum of all individual degrees, that is, the size of the system (Lu, 2006). In autonomous chaotic structures, the system size should be at least 3 as an integer and 4 in hyperchaotic models. However, it has been shown that this degree may be decreased with fractional order analyses. For instance, the well-known Chua circuit exhibits chaotic behavior with a system degree of 2.7 (Qammer, 1995). On the other hand, Wu et al. calculated the smallest system degree required to obtain the hyperchaotic dynamics as 2.88 (Wu et al., 2009).

The basic principle of synchronization lies in the convergence of trajectories of two systems, commonly named master (drive) and slave (response) systems (Gularte et al., 2021). It can be also explained as adjusting the signals coming from the slave part to act as the master system (Yılmaz et al., 2022). Chaos synchronization has been studied to control nonlinear dynamical systems after the pioneering research of Pecora and Carroll (Pecora et al., 1997). Various approaches have been proposed since then to achieve synchronization. Adaptive synchronization (Sajjadi et al., 2020), projective synchronization (Al-Obeidi & AL-Azzawi, 2019), complete synchronization (P. Wang et al., 2019), hybrid synchronization (Singh et al., 2021) and sliding mode control (Liao et al., 2022; Vaidyanathan et al., 2021) are examples of leading methods used to synchronize both chaotic and hyperchaotic systems.

Fractional order analysis has been implemented in chaotic and hyperchaotic systems due to its more realistic nature and giving more accurate results to physical phenomena (Meng et al., 2021). Furthermore, synchronization of fractional order dynamical systems has also been intensively addressed in the literature. Bouridah and Wang have proposed an image encryption algorithm based on the synchronization of fractional order hyperchaotic systems (Bouridah et al., 2021; S. Wang et al., 2020). In addition, chaos-based fractional order synchronization has also been investigated in different studies (Lin et al., 2021, 2022; Meng et al., 2021).

The main contribution of this paper is to provide adaptive synchronization between an integer order and a fractional order hyperchaotic Pang system besides obtaining integer order synchronization and fractional order synchronization separately. The main features of the study can be described as follows:

- Introducing four-dimensional hyperchaotic Pang system with phase portraits and Lyapunov exponents analysis.

- Applying fractional order analysis to four-dimensional Pang system and demonstrating hyperchaotic dynamics when the system degree is 3.52.
- Implementing the adaptive synchronization method to a fractional order system and determining the suitable control function.
- Synchronizing a fractional order hyperchaotic slave system to an integer order hyperchaotic master system besides illustrating the integer order and fractional order synchronization individually.

In the next section of the study, the Pang system is introduced, and the dynamic analysis of the system is given. In Section 3, the Caputo method is explained, and fractional analyses of the system are performed. Here, the dynamics of the system for different fractional degrees are examined and the existence of periodic, chaotic, and hyperchaotic structures is shown with Lyapunov exponents and phase portraits. Then adaptive synchronization of the fractional order hyperchaotic Pang system is provided in Section 4 and simulation results for different cases are given in the following section. Finally, in Section 6, the results obtained from this study are evaluated.

2. 4-DIMENSIONAL HYPERCHAOTIC PANG SYSTEM

The 4-dimensional hyperchaotic system proposed by Pang (Pang & Liu, 2011) in 2010 is shown in Eq. (1):

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= cy - xz + w \\ \dot{z} &= -bz + xy \\ \dot{w} &= -k_1 x - k_2 y \end{aligned} \tag{1}$$

Here, *x, y, z,* and *w* represent the state variables of the system, while *a, b, c,* k_1, and k_2 are system parameters and fixed values. When (a, b, c, k_1, k_2) are selected as *(36, 3, 20, 2, 2)*, hyperchaotic behavior is observed in the system. The phase-space representation of the *x-z* and *y-z* planes of the system for the given constant values is presented in Figure 1.

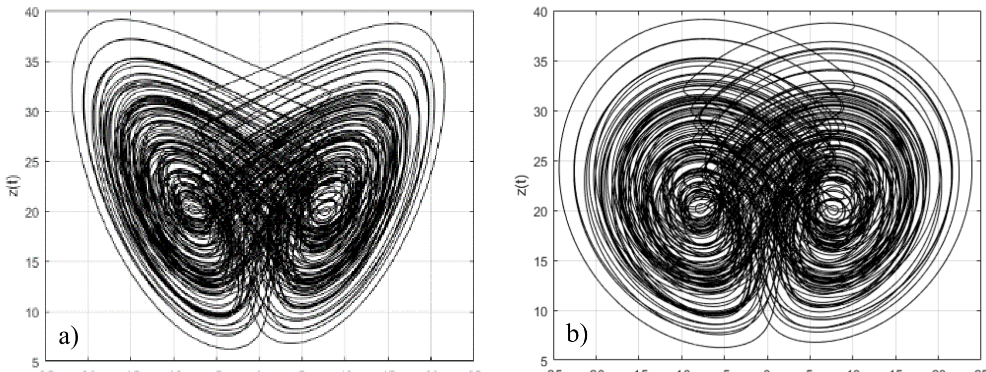

Figure 1:
Phase-space representation of the Pang system: a) for the x-z plane, b) for the y-z plane

Lyapunov exponents of the Pang system given in Eq. (1) are calculated for the values $(a, b, c, k_1, k_2) = (36, 3, 20, 2, 2)$ and obtained as follows:

$$\lambda_{L1} = 1.3963 \quad \lambda_{L2} = 0.1518 \quad \lambda_{L3} = 0.0029 \quad \lambda_{L4} = -20.4796$$

The positive results of λ_{L1} and λ_{L2} from the obtained Lyapunov exponents confirmed the hyperchaotic nature of Eq. (1). Furthermore, Lyapunov exponents are calculated for the gradually increasing k_2 parameter in the system and the graph created is shown in Figure 2. When the k_2 parameter is in the range of *[0-27]*, the first and second Lyapunov exponents are positive and the system exhibits hyperchaotic behavior in this range.

3. FRACTIONAL PANG SYSTEM AND DYNAMIC ANALYSIS

There are several methods used to obtain fractional order systems in the literature. The Riemann-Liouville method and the Caputo method are the most well-known methods and have been applied to different systems. In this study, the Caputo definition given in Eq. (2) is used to obtain the fractional order hyperchaotic Pang system:

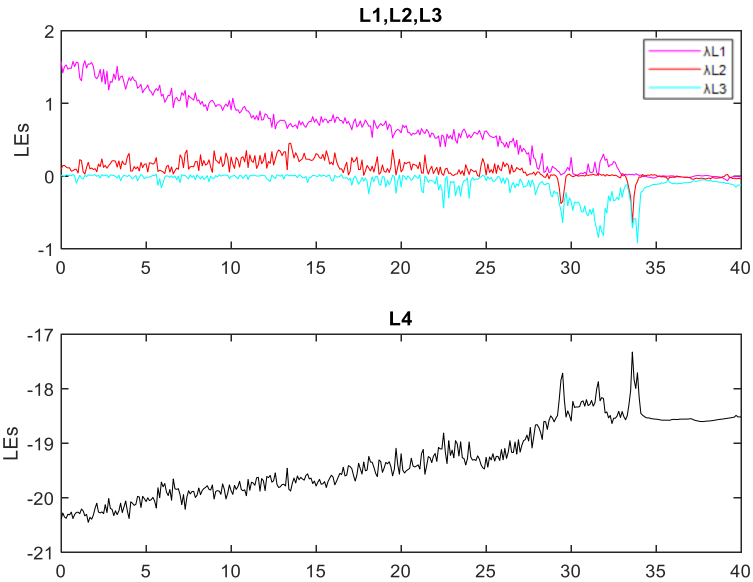

Figure 2:
Lyapunov exponents calculated for varying k_2 parameter of the Pang system.

$$D_t{}^q f(t) = \begin{cases} \frac{1}{\Gamma(n-q)} \int_0^t \frac{f(\tau)}{(t-\tau)^{q+1-n}} d\tau, & n-1 < q < n \\ \frac{d^n}{dt^n} f(t), & q = n \end{cases} \tag{2}$$

The expression D_t^q given in Eq. (2) represents the Caputo derivative operator, the value $\Gamma(.)$ represents the Euler Gamma function, and the value n is an integer not less than the value q. If $q = 1$ in the given equation, the classical integer order system structure is obtained.

The fractional equivalent of the Pang system formed by Caputo's definition, is included in Eq. (3):

$$\begin{aligned} D_t{}^{q_1} x &= a(y-x) \\ D_t{}^{q_2} y &= cy - xz + w \\ D_t{}^{q_3} z &= -bz + xy \\ D_t{}^{q_4} w &= -k_1 x - k_2 y \end{aligned} \tag{3}$$

where q_1, q_2, q_3, and q_4 represent fractional degrees and can take values in the range $0 < q_i < 1$. In order for the obtained fractional order system to have nonlinear dynamics, the system structure must be unstable. For this purpose, the characteristic equation of the system for the equilibrium point $E\ (0,\ 0,\ 0,\ 0)$ and constant values $(a,\ b,\ c,\ k_1,\ k_2) = (36,\ 3,\ 20,\ 2,\ 2)$ is calculated as follows:

$$\Delta(\lambda) = (\lambda + 3)(\lambda^3 + 16\lambda^2 - 718\lambda)$$

Accordingly, the eigenvalues of the system are:

$$\lambda_1 = -3 \qquad \lambda_2 = 19.964 \qquad \lambda_3 = -35.964 \qquad \lambda_4 = 0$$

The fact that the eigenvalues consist of negative and positive numbers indicates that the system is unstable.

Dynamic analysis of the fractional order system given in Eq. (3) is carried out for different fractional degrees using $q_1 = q_2 = q_3 = q_4 = q$ values. Accordingly, phase portraits of the *x-z* plane obtained for fractional degrees $q = 0.7$, $q = 0.88$, and $q = 0.89$, respectively, are given in Figure 3. Phase space representation illustrates that the system is periodic when $q = 0.7$ and exhibits chaotic behavior when $q = 0.88$. On the other side, when the q fractional degree is changed to 0.89, the system switches to the hyperchaotic structure. With the time series graph given in Figure 4, the behavior of the System (3) for the periodic and hyperchaotic dynamics is shown.

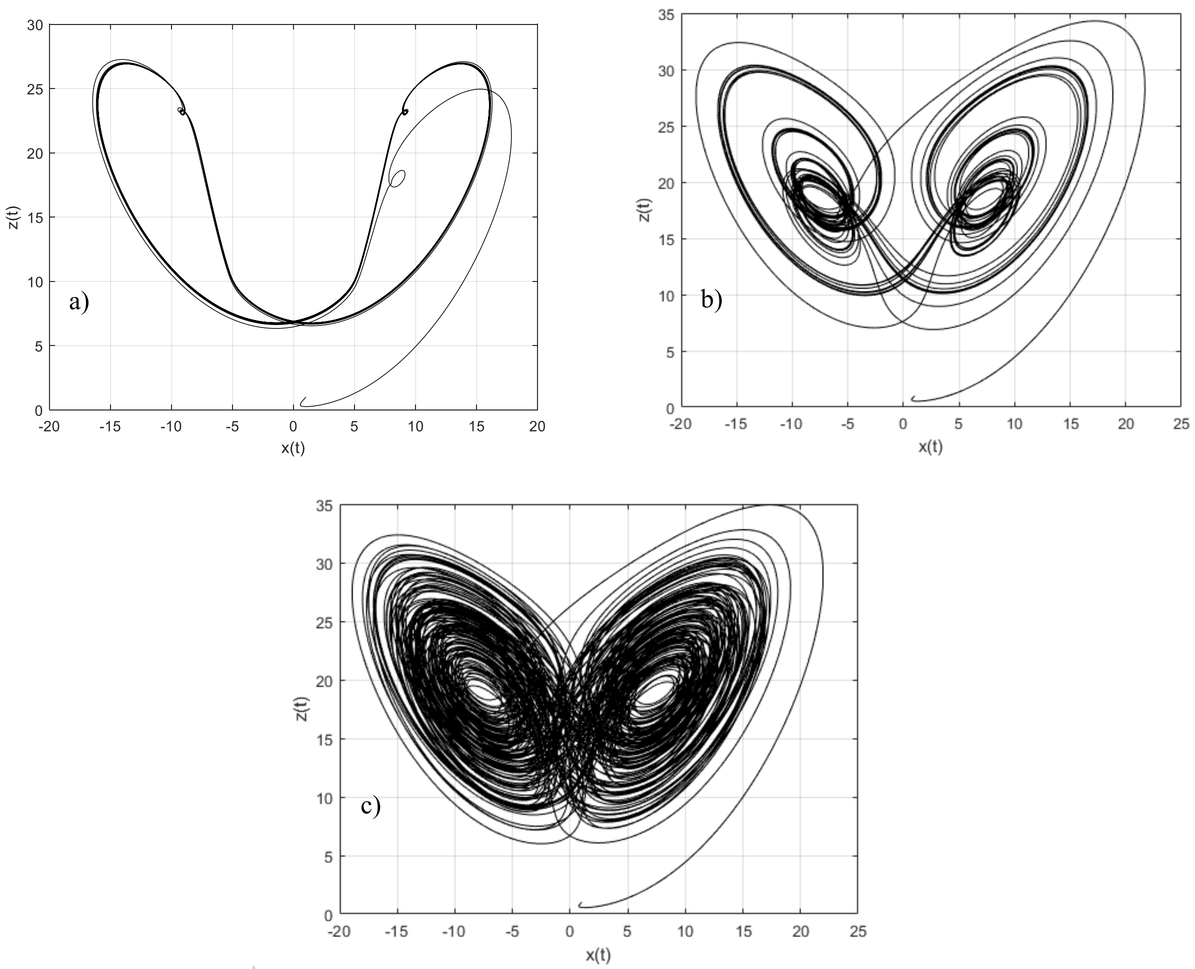

Figure 3:
Phase portraits of the Pang system for different fraction degrees. a) when q = 0.7, periodic behavior; b) when q = 0.88, chaotic behavior; c) when q = 0.89, hyperchaotic behavior

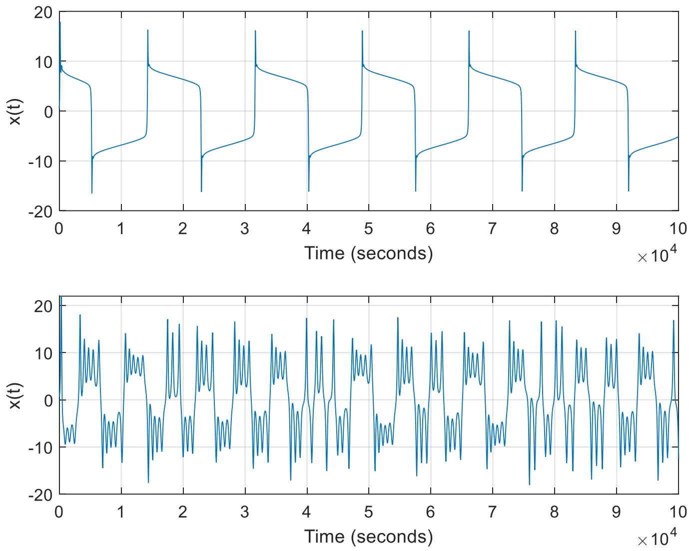

Figure 4:
Time series graph of fractional order Pang System: For q = 0.7 (above); for q = 0.89 (below)

Lyapunov exponents of the System (3), which are analyzed for different fraction degrees, are also calculated and the result obtained for the fraction degree interval *[0.7 1]* is given in Figure 5. When the given graph is examined, it is seen that the Pang system has two positive Lyapunov exponents starting from the fraction degree $q = 0.89$. For this reason, for the Pang system to exhibit hyperchaotic behavior, the smallest fraction degree must be *0.89* and the system size must be at least *3.56*. Lyapunov exponent values obtained for different fraction degrees of the system are presented in Table 1. Values given in this table demonstrate that by increasing the fractional degree, the system exhibits periodic, chaotic, and hyperchaotic dynamics, respectively.

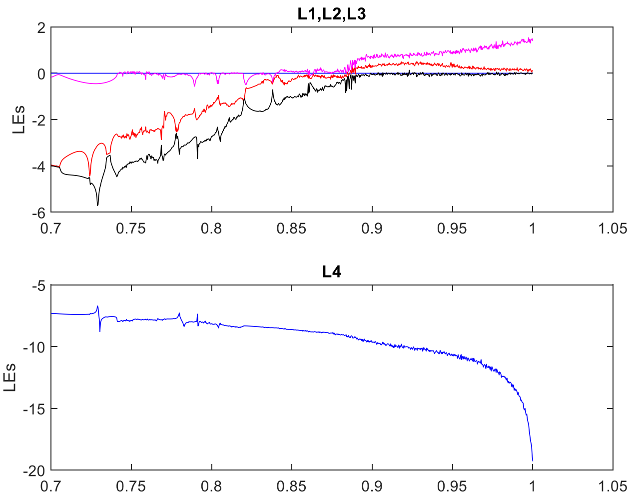

Figure 5:
Lyapunov exponents for the interval of q = [0.7 1] of the Pang system.

Table 1. Values of Lyapunov Exponents Calculated for Different Fraction Degrees of the Pang System and System Structure

Fractional Order	λ_{L1}	λ_{L2}	λ_{L3}	λ_{L4}	System Structure
$q = 0.70$	-0.01	-3.96	-3.99	-7.31	Periodic
$q = 0.75$	0.03	-2.56	-3.83	-7.89	Chaotic
$q = 0.80$	0.03	-1.43	-2.72	-8.15	Chaotic
$q = 0.88$	0.12	-0.19	-0.44	-9.06	Chaotic
$q = 0.89$	0.42	0.23	-0.07	-9.38	Hyperchaotic
$q = 0.95$	0.92	0.39	0.00	-10.74	Hyperchaotic
$q = 1$	1.39	0.15	0.00	*-20.48*	Hyperchaotic

3.1. The Effect of k_2 Change on the System When $q = 0.89$

After obtaining the hyperchaotic structure for the value of $q = 0.89$ in the fractional order system, the dynamic structure is examined according to the change of the system parameters. Lyapunov exponents are calculated again by keeping the $(a, b, c, k_1) = (36, 3, 20, 2)$ values constant and changing the k_2 parameter within the range of $[0\ 40]$. In the graph given in Figure 6, λ_{L1} and λ_{L2} are positive for $k_2 = [1\ 11]\ U\ [12.5\ 18]$ values, and the system has hyperchaotic dynamics in this range. On the other hand, for the values of $18 < k_2 < 36$, it is observed that the system is in a chaotic structure.

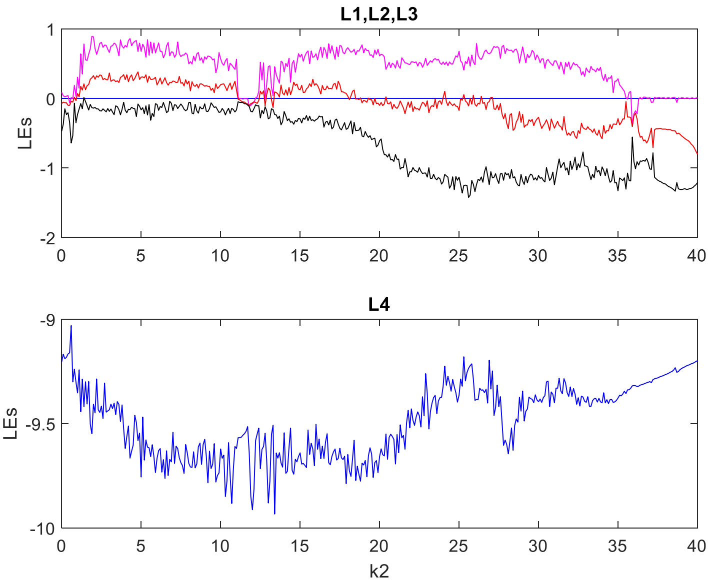

Figure 6:
Lyapunov exponents (with $q = 0.89$ fractional order) for the k_2 parameter of the Pang system, which varies in the range [0 40].

3.2. The Effect of k_2 Change on the System When $q = 0.88$

In this part, k_2 parameter of the system is changed for the fraction degree $q = 0.88$, and the Lyapunov exponents are calculated again. In the graph shown in Figure 7, when k_2 is in the range of *[0 4]*, only λ_{L1} is positive and the system is chaotic. However, when the k_2 value continues to increase and is brought to the *[5 10]* range, λ_{L2} also becomes positive, and thus two positive Lyapunov exponents are obtained in the system. Although System (3) appears chaotic at $q = 0.88$ according to the values in Table 1, it has also been revealed that a hyperchaotic structure can be formed when the k_2 value is changed for the same system. Thus, in the fractional degree system given in Eq. (3), when $q = 0.88$, $a = 36$, $b = 3$, $c = 20$, and $k_1 = 2$, when $k_2 = [5\ 10]\ U\ [17\ 20]$, a hyperchaotic structure is obtained.

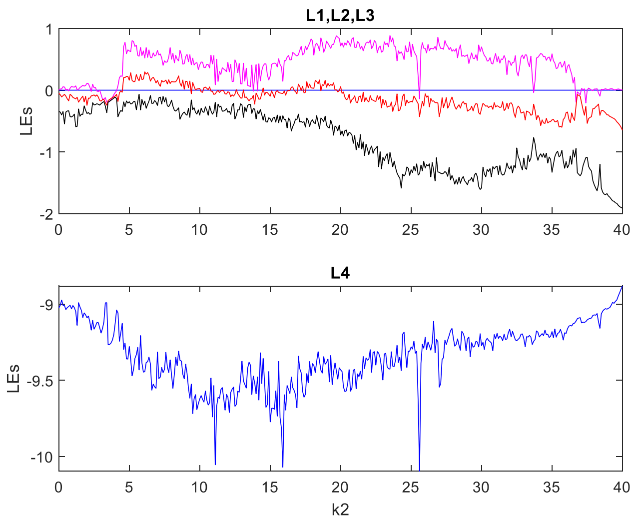

Figure 7:
Lyapunov exponents (with q = 0.88 fractional order) for the k_2 parameter of the Pang system, which varies in the range [0 40].

4. ADAPTIVE SYNCHRONIZATION OF FRACTIONAL ORDER HYPERCHAOTIC PANG SYSTEM

This section deals with the adaptive synchronization of the fractional order hyperchaotic Pang system with different levels of fractions. First, define the master and slave systems for synchronization given as Eq. (4) and (5), respectively:

$$\begin{aligned} D_t^{q_1}x_m &= a(y_m - x_m) \\ D_t^{q_2}y_m &= cy_m - x_m z_m + w_m \\ D_t^{q_3}z_m &= -bz_m + x_m y_m \\ D_t^{q_4}w_m &= -k_1 x_m - k_2 y_m \end{aligned} \tag{4}$$

and the slave system is:

$$\begin{aligned} D_t^{q_1}x_s &= a(y_s - x_s) + u_1 \\ D_t^{q_2}y_s &= cy_s - x_s z_s + w_s + u_2 \\ D_t^{q_3}z_s &= -bz_s + x_s y_s + u_3 \\ D_t^{q_4}w_s &= -k_1 x_s - k_2 y_s + u_4 \end{aligned} \tag{5}$$

where $u_i(t)$ $(i = 1, 2, 3, 4)$ represents the control function. The convenient choice of $u_i(t)$ function synchronizes slave system to the master system. Therefore, the error function between (4) and (5) should be calculated and minimized as given:

$$e_1 = x_s - x_m \qquad e_2 = y_s - y_m \qquad e_3 = z_s - z_m \qquad e_4 = w_s - w_m$$

The error dynamic system can be described as:

$$\begin{aligned} D_t^{q_1} e_1 &= a(e_2 - e_1) + u_1 \\ D_t^{q_2} e_2 &= c e_2 + x_m z_m - x_s z_s + e_4 + u_2 \\ D_t^{q_3} e_3 &= -b e_3 - x_m y_m + x_s y_s + u_3 \\ D_t^{q_4} e_4 &= -k_1 e_1 - k_2 e_2 + u_4 \end{aligned} \tag{6}$$

Following the control function $u_i(t)$ is defined:

$$\begin{aligned} u_1 &= V_1(t) \\ u_2 &= -x_m z_m + x_s z_s + V_2(t) \\ u_3 &= x_m z_m - x_s z_s + V_3(t) \\ u_4 &= V_4(t) \end{aligned} \tag{7}$$

where $V_i(t)$ is the control input.

Substituting Eq. (7) to Eq. (6) gives the error dynamics with control input:

$$\begin{aligned} D_t^{q_1} e_1 &= a(e_2 - e_1) + V_1(t) \\ D_t^{q_2} e_2 &= c e_2 + e_4 + V_2(t) \\ D_t^{q_3} e_3 &= -b e_3 + V_3(t) \\ D_t^{q_4} e_4 &= -k_1 e_1 - k_2 e_2 + V_4(t) \end{aligned} \tag{8}$$

According to Eq. (8), control inputs can be obtained as:

$$(V) = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} a-1 & -a & 0 & 0 \\ 0 & -1-c & 0 & -1 \\ 0 & 0 & b-1 & 0 \\ k-1 & k-1 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} \tag{9}$$

Finally control inputs are formed for the parameter values $(a, b, c, k_1, k_2) = (36, 3, 20, 2, 2)$ as given:

$$(V) = \begin{pmatrix} 35 & -35 & 0 & 0 \\ 0 & -21 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} \tag{10}$$

Calculated control function is used in the slave system to control its dynamics and hence synchronize to the master system.

5. SIMULATION RESULTS

For the numerical simulation, adaptive synchronization is observed in three separate cases, summarized in Table 2, according to the fraction degrees. System parameters are defined as $(a, b, c, k_1, k_2) = (36, 3, 20, 2, 2)$ for all cases.

Table 2. Summary of the parameters for different type of synchronization

Synchronization cases	Fraction degree (q)		Initial conditions (x_0, y_0, z_0, w_0)	
	Master	Slave	Master	Slave
Integer order	1	1	(1, 1, 1, 1)	(1.1, 1.1, 1.1, 1.1)
Fractional order	0.89	0.89	(1, 1, 1, 1)	(1.1, 1.1, 1.1, 1.1)
Integer vs. fractional order	1	0.89	(1, 1, 1, 1)	(1, 1, 1, 1)

5.1. Adaptive synchronization of integer order system

For the case of integer order synchronization, fraction degrees of both master and slave systems are assigned as q = 1. Initial conditions of the master system are $(x_{m0}, y_{m0}, z_{m0}, w_{m0}) = (1, 1, 1, 1)$ while $(x_{s0}, y_{s0}, z_{s0}, w_{s0}) = (1.1, 1.1, 1.1, 1.1)$ for the slave system. Synchronization results based on the time series of x variables for master and slave systems are illustrated in Figure 8. It is seen that the behavior of x_m and x_s is almost the same as time changes. Furthermore, Figure 9 also demonstrates the synchronization of two systems with phase-space representation on x-y planes.

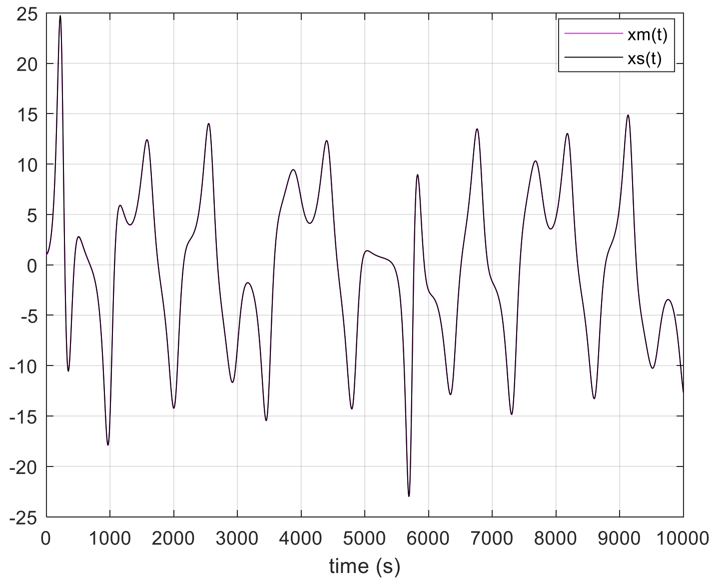

Figure 8:
Time series of x variables for master ($x_m(t)$) and slave ($x_s(t)$) systems when q = 1.

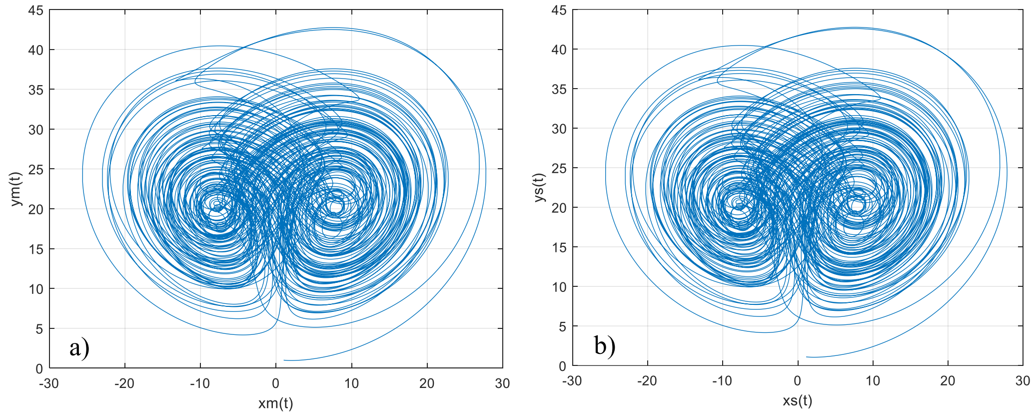

Figure 9:
Phase-space representation on x-y plane when q = 1: a) master system, b) slave system.

5.2. Adaptive synchronization of fractional order system

The case of fractional order synchronization is carried out when fraction degrees for the master and slave systems are equal to 0.89. Initial conditions of the master and slave systems are defined as *(1, 1, 1, 1)* and *(1.1, 1.1, 1.1, 1.1)*, respectively. Synchronization results are given in Figures 10 and 11 based on the time series analysis and phase-space representation, respectively.

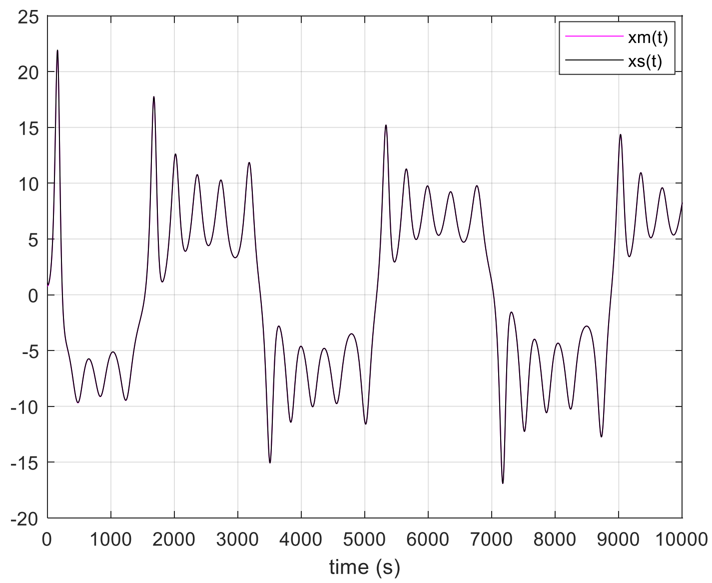

Figure 10:
Time series of x variables for master ($x_m(t)$) and slave ($x_s(t)$) systems when q = 0.89.

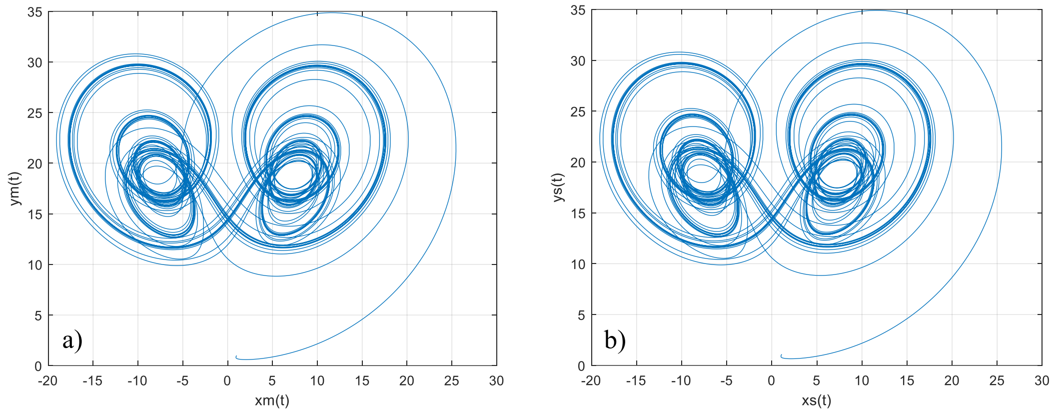

Figure 11:
Phase-space representation on x-y plane when q = 0.89: a) master system, b) slave system.

5.3. Adaptive synchronization of integer order vs. fractional order system

The last case for the synchronization is obtained for different numerical values of the fraction degree. Under the circumstances, the master system is considered an integer order system where the fractional degree equals 1. On the other hand, the slave system is a fractional order system with q = 0.89. The situation here is to synchronize a fractional order system to an integer order system. Initial conditions of these two systems are defined as $(x_{m0}, y_{m0}, z_{m0}, w_{m0}) = (x_{s0}, y_{s0}, z_{s0}, w_{s0}) = (1, 1, 1, 1)$. Simulation results including the time series analysis and the phase portraits are given in Figures 12 and 13, respectively.

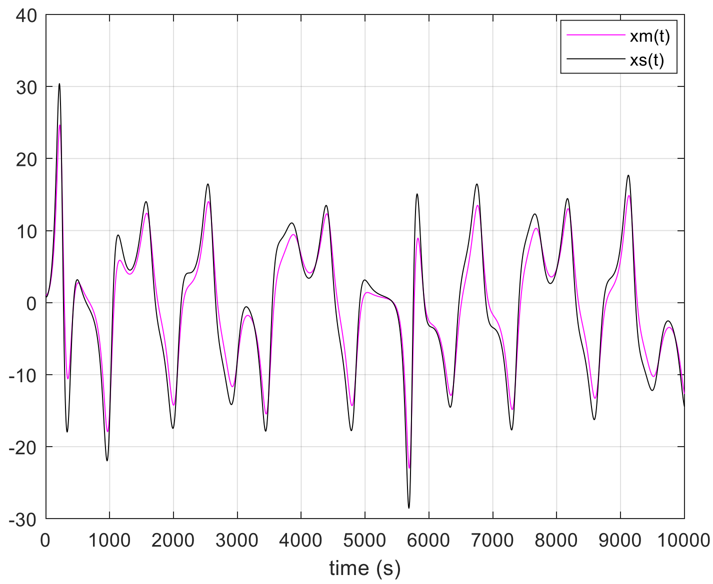

Figure 12:
Time series of x variables for master ($x_m(t)$-q = 1) and slave ($x_s(t)$-q = 0.89) systems.

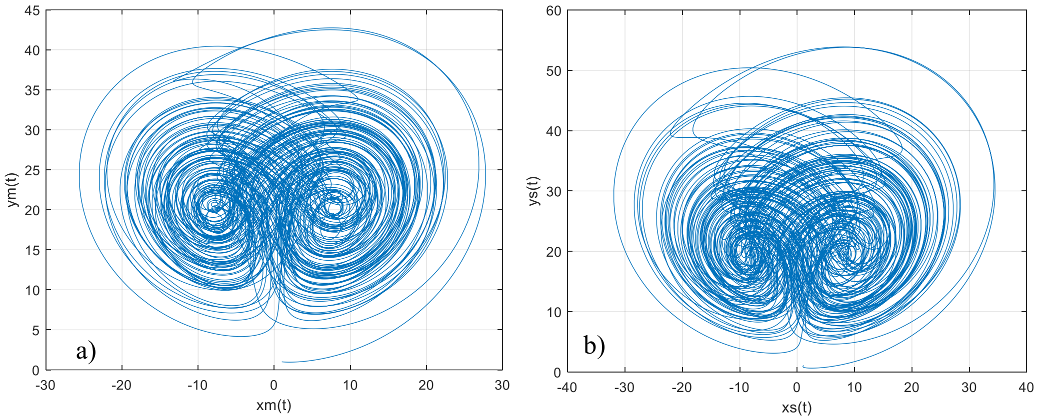

Figure 13:
Phase-space representation on x-y plane: a) master system (q = 1), b) slave system (q = 0.89).

6. CONCLUSION

In this study, fractional order analyses of the 4-dimensional hyperchaotic Pang system are performed based on the Caputo method. First of all, Lyapunov exponents of different fraction degrees are calculated and it is determined which dynamics the system has at which fraction degree. Then, for the fraction degree $q = 0.89$, where it exhibits hyperchaotic behavior, the k_2 system parameter is changed within a certain range and the Lyapunov exponents are calculated again. From here, it has been observed that even if the k_2 parameter of the system is changed, it remains in a hyperchaotic structure over a wide range. In addition, it has been revealed by the analysis that the Pang system has chaotic dynamics at fractional order $q = 0.88$. However, when the k_2 value is changed in the same range, hyperchaotic dynamics are observed in the system at the *[5 10]* values of k_2, although the fractional degree remains constant. The conclusion drawn from this is that the required system size for the fractional Pang system to exhibit hyperchaotic dynamics is at least *3.52*. In this study, the structure of dynamics determined by Lyapunov exponents is also confirmed by phase-space representation.

Adaptive synchronization of the hyperchaotic Pang system is also investigated after fractional order analyses. The point of motivation is to provide synchronization between an integer order and a fractional order system besides obtaining integer order synchronization and fractional order synchronization separately. Simulation results demonstrate that an integer order master system forces a fractional order slave system to behave as itself and synchronization for all three cases is achieved.

CONFLICT OF INTEREST

Authors approve that to the best of their knowledge, there is not any conflict of interest or common interest with an institution/organization or a person that may affect the review process of the paper.

AUTHOR CONTRIBUTION

Enis Günay determining the concept and design process of the research and research management, Gülnur Yılmaz data collection and analysis, and interpretation of results.

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