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Homogeneous and anisotropic universe models with bouncing deceleration parameter in $f(R,T)$ theory

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Bouncing deceleration parameter, Cloud of string with perfect fluid, Kantowski-Sachs, Bianchi I, Bianchi III

Abstract — In this study, we have investigated the cloud of string with perfect fluid (CSPF) matter distribution for the homogeneous and anisotropic (Bianchi I, III, and Kantowski-Sachs) universe models in the $f(R,T)$ theory. We have used the bouncing deceleration parameter, anisotropy parameter, and equation of state to obtain the exact solutions of field equations. The obtained physical and kinematic quantities were analyzed with the help of graphics. If the anisotropy parameter is zero, then our model becomes an isotropic model of the universe and the string tension vanishes in LRS Bianchi I metric. Finally, we discuss the need for further research.

Subject Classification (2020): 83C05, 83C15

1. Introduction

The expansion of the universe is one of the hot topics in cosmology. Today we know that our universe is expanding [1, 2]. According to research, there are two methods to solve this mysterious event. One of these methods is dark energy (DE) models, and the other one is alternative gravitation theories such as Brans-Dicke [3], $f(R)$ [4], and $f(R,T)$ [5] also other alternative gravitation theories. Since its significant contribution to explaining the expansion of the universe, the interest in alternative gravitational theories is increasing day by day. $f(R,T)$ theory [5] modifying matter and geometry has been studied by many scientists in recent years.

Aygün et al. [6, 7] have studied Marder and Friedmann-Robertson-Walker (FRW) universes in $f(R,T)$ theory. Aktaş and Aygün [8] have investigated magnetized strange quark matter (MSQM) solutions for the FRW universe model in the framework $f(R,T)$ gravity. They [9] then have researched MSQM solutions obtained for a Marder-type universe. Pawar et al. [10] have analyzed Kaluza-Klein string cosmological model (SCM) in $f(R,T)$ theory. The solutions of the SCMs in $f(R,T)$ gravity for Bianchi-type models have been studied by several authors, e.g., Kanakavalli and Ananda Rao [11], Sahoo [12], Rao and Papa Rao [13], Rani et al. [14], Mishra and Sahoo [15], and Samanta and Dhal [16], using different techniques. Nagpal et al. [17] have studied the strange quark matter (SQM) and magnetized

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quark matter (MQM) distributions in the presence of $f(R,T)$ gravity in the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric. Sahoo et al. [18, 19] have investigated the Bianchi type-I model with MSQM in $f(R,T)$ theory. Çağlar and Aygün [20] have researched string cloud and strange quark matter distributions for higher dimension FRW universe in Creation Field theory. Magnetized string matter distribution has been studied by Kömürcü and Aktaş [21] for the Marder universe within the framework of $f(R,T)$ theory. Morais Graça et al. [22] have studied the spherically symmetrical string cloud solution in d-dimensional space-time within the framework of $f(R)$ gravity theory. String cosmology was studied by Sharma et al. [23] for the spatially symmetrical Bianchi-IX universe considering the flat potential. Vijaya Santhi and Chinnappalanaidu [24] have investigated cosmological models of strange quark matter attached to the string cloud in the $f(R)$ theory. Sobhanbabu and Vijaya Santhi [25] have researched strange quark matter attached to string cloud matter distribution for five-dimensional Kaluza-Klein cosmology in $f(R)$ gravity theory.

This study aims to analyze the behavior of cloud of string with perfect fluid matter by LRS Bianchi I, Kantowski-Sachs, and Bianchi III metrics in the $f(R,T)$ theory, explaining the accelerated expansion of the universe, and to form a model of the universe. The present study is organized as follows: Section 2 provides some basic notions related to the $f(R,T)$ theory. Section 3 presents metric and field equations and three universe models. Section 4 interprets the results and some kinematical and physical properties of the models. The last section concludes the study by pointing out future studies.

2. Preliminaries

This section provides some basic notions related to the $f(R,T)$ theory. The action integral is given by [5]

$$S = \int \left(\frac{f(R,T) + 2\Lambda}{16\pi} + L_m \right) \sqrt{-g} d^4x \tag{2.1}$$

where $f(R,T)$ is a function of Ricci scalar and trace of the energy-momentum tensor, g denotes the determinant of the metric tensor, L_m shows the matter of Lagrangian, and Λ is the cosmological term [5]. The energy-momentum tensor is defined as follows [5]:

$$T_{ab} = g_{ab}L_m - \frac{2\partial L_m}{\partial g^{ab}}$$

The variation of (2.1) is as follows:

$$f_R(R,T)R_{ab} - \frac{1}{2}f(R,T)g_{ab} + (g_{ab}\Box - \nabla_\alpha\nabla_\beta)f_R(R,T) = 8\pi T_{ab} - f_T(R,T)T_{ab} - f_T(R,T)\Theta_{ab} + \Lambda g_{ab} \tag{2.2}$$

where $f_R(R,T)$ and $f_T(R,T)$ represent the derivative of the function $f(R,T)$ with respect to R and T, respectively. Moreover, ∇_α shows the covariant derivative and $\Box = \nabla_\alpha\nabla^\alpha$ [5]. Θ_{ab} is defined as follows [5]:

$$\Theta_{ab} = -2T_{ab} + g_{ab}L_m - 2g^{ik}\frac{\partial^2 L_m}{\partial g^{ab} g^{ik}}$$

Contracting (2.2), we obtain

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)T - f_T(R,T)\Theta + \Lambda g_{ab} \tag{2.3}$$

where $\Theta = g^{ab}\Theta_{ab}$ [5]. From (2.2) and (2.3), the field equations of $f(R,T)$ gravity are as follows [5]:

$$\begin{aligned} f_R(R,T)\left(R_{ab} - \tfrac{1}{3}Rg_{ab}\right) + \tfrac{1}{6}f(R,T)g_{ab} = 8\pi\left(T_{ab} - \tfrac{1}{3}Tg_{ab}\right) - f_T(R,T)\left(T_{ab} - \tfrac{Tg_{ab}}{3}\right) \\ + f_T(R,T)\left(\Theta_{ab} - \tfrac{\Theta g_{ab}}{3}\right) + \nabla_\alpha\nabla_\beta f_R(R,T) + \Lambda g_{ab} \end{aligned} \tag{2.4}$$

In this study, we consider $f(R,T) = R + 2\mu T$. If we take $f(R,T) = R + 2\mu T$ in (2.4), we obtain the field equations in $f(R,T)$ gravity with Λ as follows:

$$G_{ab} = [8\pi + 2\mu]T_{ab} + [\mu\rho - p\mu + \Lambda]g_{ab} \tag{2.5}$$

3. Metric and Field Equations

The general form of anisotropic and homogeneous LRS Bianchi I, III, and Kantowski-Sachs metric in spherical coordinates as follows:

$$ds^2 = -A^2(t)dr^2 - B^2(t)\left(d\theta^2 + K(\theta)^2 d\phi^2\right) + dt^2 \tag{3.1}$$

If we take $K(\theta) = \theta$, $K(\theta) = \sin(\theta)$, and $K(\theta) = \sinh(\theta)$, we obtain LRS Bianchi I, III, and Kantowski-Sachs space-times, respectively. In this study, we consider the source of gravitational CSPF. Then, the new energy-momentum tensor is provided by

$$T_{ab} = (\rho + p)u_a u_b - p g_{ab} - \lambda x_a x_b \tag{3.2}$$

where p is the isotropic pressure, ρ is the energy density, λ shows that string tension density, u_a denotes four-velocity vector, and x_a represents direction of the string. Moreover, $x_a x^a = -u_a u^a = -1$ and $x_a u^a = 0$. The particle density ρ_p is defined as follows:

$$\rho = \rho_p + \lambda \tag{3.3}$$

From (2.5) and (3.1)–(3.3), we obtain field equations in $f(R,T)$ gravity with Λ as follows:

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}}{B^2} - \frac{\delta}{B^2} = 8\lambda\pi - 8p\pi + 3\lambda\mu - 3p\mu + \rho\mu + \Lambda \tag{3.4}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} = -8p\pi + \lambda\mu - 3p\mu + \rho\mu + \Lambda \tag{3.5}$$

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{\delta}{B^2} = 8\rho\pi + 3\rho\mu - p\mu + \lambda\mu + \Lambda \tag{3.6}$$

where the dot represents derivation with respect to time. Besides, δ is defined as follows:

$$\delta = \frac{K''}{K} = \begin{cases} -1, & \text{Kantowski-Sachs universe} \\ 0, & \text{LRS Bianchi I universe} \\ 1, & \text{Bianchi III universe} \end{cases}$$

According to (3.1), the kinematical quantities, i.e., the scale factor a, the Hubble parameter H, the expansion parameter θ, and the shear scalar σ^2, are as follows:

$$a = AB^2 \tag{3.7}$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \tag{3.8}$$

$$\theta = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \tag{3.9}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{3.10}$$

(3.4)–(3.6) are a system of their equations with six unknown parameters (A, B, p, ρ, λ, and Λ). Three additional equations are required to obtain exact solutions of the system.

i. The anisotropy parameter is a parameter that indicates whether the universe is anisotropic or not. The anisotropy parameter is defined as $\frac{\sigma}{\theta}$. Since the metric given by (3.1) is anisotropic, we can choose the anisotropy parameter as a constant:

$$\frac{\sigma}{\theta}=\frac{\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)}{\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}}=\xi \tag{3.11}$$

where ξ is a constant and $0\leq\xi<1$. If (3.11) is solved, then we obtain

$$A=c_1B^n \tag{3.12}$$

Here, $n=\frac{\sqrt{3}-6\xi}{\sqrt{3}+3\xi}$ or $n=\frac{\sqrt{3}+6\xi}{\sqrt{3}-3\xi}$.

ii. We can use of state (EoS)

$$p=w\rho$$

iii. One of the important parameters that shows whether the universe is accelerating or not is the deceleration parameter. While the deceleration parameter was taken in a fixed form before, it is taken depending on the time in the last studies because it is observed that the acceleration of the universe changes with time. Therefore, in this study, we obtain a solution by taking the deceleration parameter in time-dependent form [26].

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1=-\frac{\alpha}{t^2}+\beta-1 \tag{3.13}$$

Here, α and β are constants.

From the solution of (3.13), the Hubble parameter is obtained as follows:

$$H=\frac{\dot{a}}{a}=\frac{t}{\beta t^2+c_2t+\alpha} \tag{3.14}$$

From (3.14), we get the scale factor as follows:

$$a=\int\frac{t}{\beta(t^2+\frac{c_2t}{\beta}+\frac{\alpha}{\beta})}dt$$

In order to solve this integral, it is possible to investigate three different cases, such as $c_2=2\sqrt{\alpha\beta}$, $c_2\neq 2\sqrt{\alpha\beta}$, and $c_2=0$.

i. If $c_2=2\sqrt{\alpha\beta}$, then the scale factor is

$$a=\left(t+\sqrt{\frac{\alpha}{\beta}}\right)^{\frac{1}{\beta}}e^{\left(\frac{\sqrt{\alpha}}{\beta(\sqrt{\beta}t+\sqrt{\alpha})}\right)} \tag{3.15}$$

ii. If $c_2\neq 2\sqrt{\alpha\beta}$, then the scale factor is obtained as follows:

$$a=\left(t^2+\frac{c_2}{\beta}+\frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}}\left(\frac{2\beta t+c_2+\sqrt{c_2^2-4\alpha\beta}}{2\beta t+c_2-\sqrt{c_2^2-4\alpha\beta}}\right)^{\frac{c_2}{2\beta\sqrt{c_2^2-4\alpha\beta}}} \tag{3.16}$$

iii. If $c_2=0$, then the scale factor is obtained as

$$a=\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}} \tag{3.17}$$

As can be seen from (3.15)–(3.17), $a=a_0\neq 0$, for $t=0$. However, for the point $t=0$, $\dot{a}=0$, and $\ddot{a}=\frac{a_0}{\alpha}=$ constant. This indicates that in all three cases the models do not have an initial singularity and start with a finite acceleration. Figure 1 shows that variation of the scale factor with respect to time.

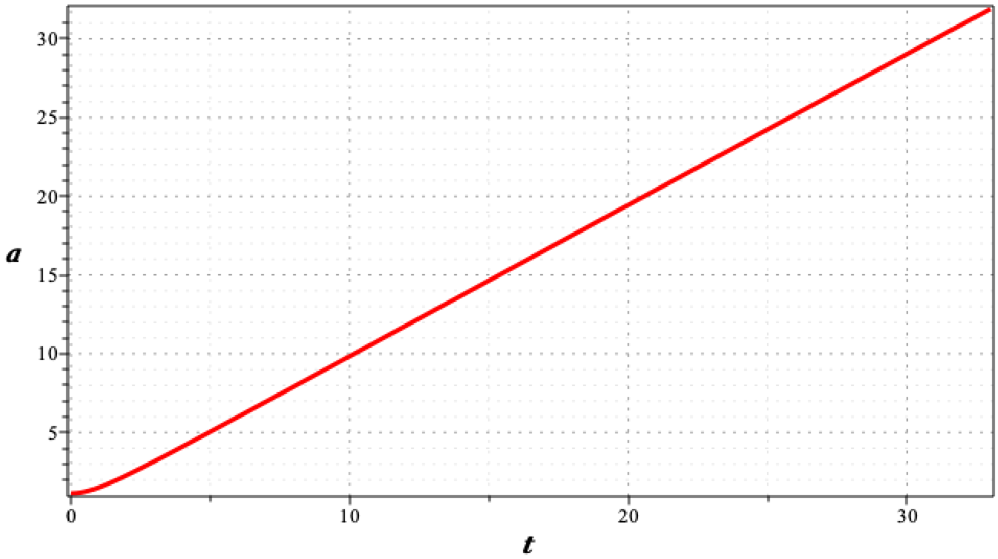

Figure 1. Variation of scale factor with time

In this study, we take $c_2 = 0$. In this case, the Hubble parameter is as follows:

$$H = \frac{t}{\beta t^2 + \alpha}$$

From (3.17), the metric potential A is obtained as

$$A = c_1 c_3^n (\beta t^2 + \alpha)^{\frac{3n}{2\beta(n+2)}} \tag{3.18}$$

From (3.7), (3.17), and (3.18), the metric potential B takes the following form:

$$B = c_3 (\beta t^2 + \alpha)^{\frac{3}{2\beta(n+2)}} \tag{3.19}$$

where $c_3 = \frac{1}{\left(c_1 \beta^{\frac{1}{2\beta}}\right)^{\frac{3}{n+2}}}$. Graphs of the metric potentials A and B are presented in Figures 2 and 3, respectively.

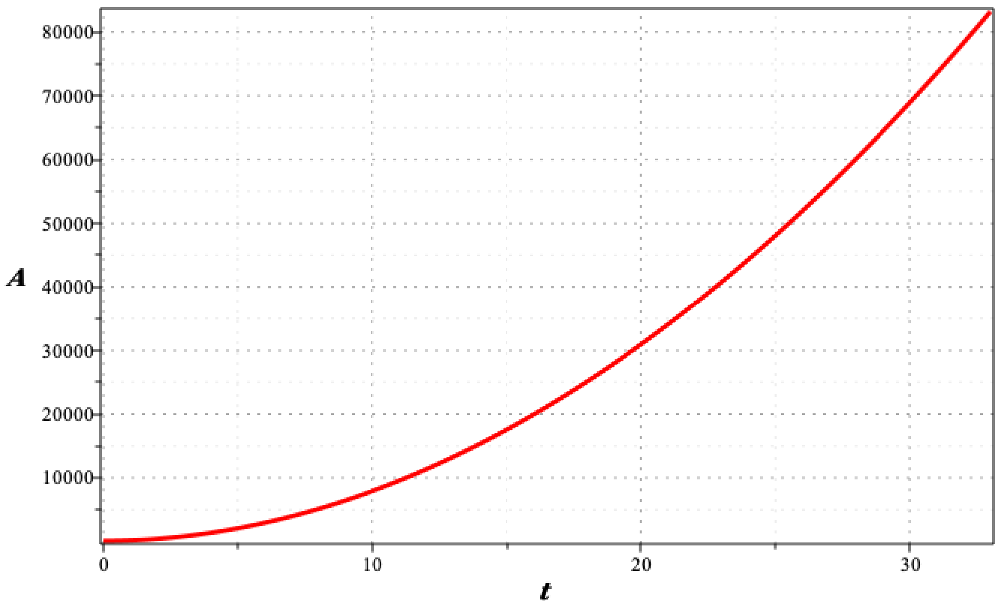

Figure 2. Variation of metric potential A with time

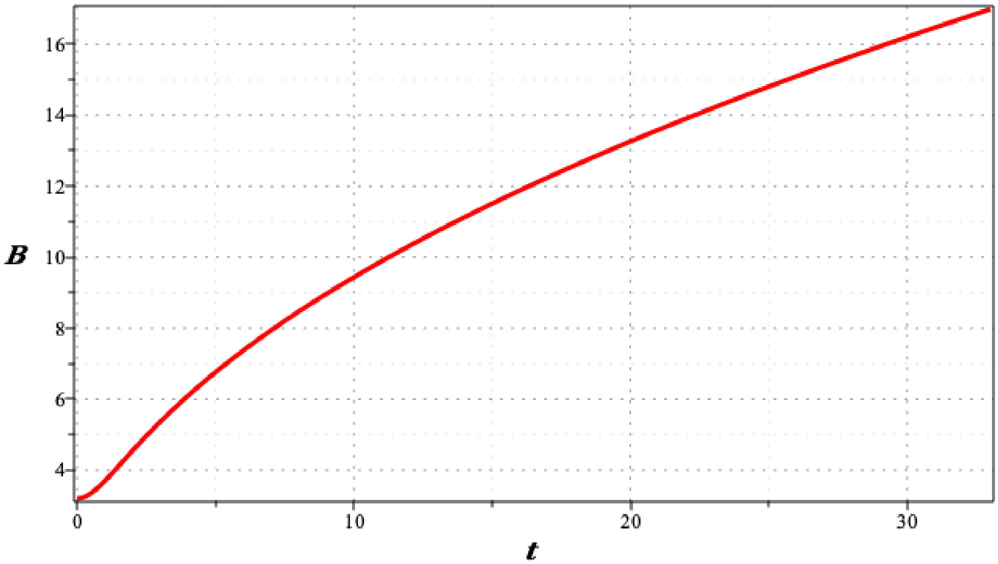

Figure 3. Variation of metric potential B with time

Furthermore, pressure, energy density, string tension, and particle density are obtained as follows:

$$p = \frac{w}{2(4\pi+\mu)(w+1)}\left(\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2} - \frac{\delta}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right) \tag{3.20}$$

$$\rho = \frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2} - \frac{\delta}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right) \tag{3.21}$$

$$\lambda = \frac{1}{2(4\pi+\mu)}\left(\frac{3(n-1)\left((\beta-3)\ t^2-\alpha\right)}{(n+2)(\beta t^2+\alpha)^2} - \frac{\delta}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right) \tag{3.22}$$

and

$$\rho_p = \frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{3t^2(1-n)\left(3nw(\beta-1)-6(w+1)\right)+3(n+2)\left(\alpha w(n-1)+2(\beta t^2+\alpha)\right)}{(n+2)^2(\beta t^2+\alpha)^2} + \frac{w\delta}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right) \tag{3.23}$$

The cosmological term is as follows:

$$\Lambda = \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4 \tag{3.24}$$

such that

$$\Lambda_1 = \frac{6n^2}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left((2\pi+\mu)\left(\alpha-(\beta-3)t^2\right)\right)$$

$$\Lambda_2 = \frac{3n}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu\left((w-5)(\beta t-\alpha)+3t^2(3w+1)\right)+12\pi(2wt^2-\beta t^2+t^2+\alpha)\right)$$

$$\Lambda_3 = \frac{6}{(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu(w-1)(\beta t^2+\alpha)+2\pi(3wt^2-2\beta t^2+t^2-4\alpha)\right)$$

and

$$\Lambda_4 = \frac{(\mu-\mu w-4\pi w)\delta}{(w+1)c_3^2(4\pi+\mu)(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}$$

We investigate the results obtained using the bouncing deceleration parameter for LRS Bianchi I, Kantowski-Sachs, and Bianchi III.

3.1. LRS Binachi I Universe Model

If $K(\theta)=\theta$, then $\delta=0$, for the LRS Bianchi I metric. In this case, the pressure

$$p^{BI}=\frac{w}{2(4\pi+\mu)(w+1)}\left(\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2}\right) \tag{3.25}$$

energy density

$$\rho^{BI}=\frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2}\right) \tag{3.26}$$

sting tension

$$\lambda^{BI}=\frac{3}{2(4\pi+\mu)}\left(\frac{(n-1)\left((\beta-3)\ t^2-\alpha\right)}{(n+2)(\beta t^2+\alpha)^2}\right) \tag{3.27}$$

and particle density

$${\rho_p}^{BI}=\frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{3t^2(1-n)\left(3nw(\beta-1)-6(w+1)\right)+3(n+2)\left(\alpha w(n-1)+2(\beta t^2+\alpha)\right)}{(n+2)^2(\beta t^2+\alpha)^2}\right)$$

Further, cosmological term is as follows:

$$\Lambda^{BI}=\Lambda_1^{BI}+\Lambda_2^{BI}+\Lambda_3^{BI} \tag{3.28}$$

Here,

$$\Lambda_1^{BI}=\frac{6n^2}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left((2\pi+\mu)\left(\alpha-(\beta-3)t^2\right)\right)$$

$$\Lambda_2^B I=\frac{3n}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu\left((w-5)(\beta t-\alpha)+3t^2(3w+1)\right)+12\pi(2wt^2-\beta t^2+t^2+\alpha)\right)$$

and

$$\Lambda_3^{BI}=\frac{6}{(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu(w-1)(\beta t^2+\alpha)+2\pi(3wt^2-2\beta t^2+t^2-4\alpha)\right)$$

3.2. Kantowski-Sachs Universe Model

If $K(\theta)=\sin(\theta)$, then solutions are obtained in the Kantowski-Sachs universe model for CSPF distribution. Thus, pressure

$$p^{KS}=\frac{w}{2(4\pi+\mu)(w+1)}\left(\frac{1}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}+\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2}\right)$$

energy density

$$\rho^{KS}=\frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{1}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}+\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2}\right)$$

string tension

$$\lambda^{KS}=\frac{1}{2(4\pi+\mu)}\left(\frac{1}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}+\frac{3(n-1)\left((\beta-3)\ t^2-\alpha\right)}{(n+2)(\beta t^2+\alpha)^2}\right)$$

particle density

$${\rho_p}^{KS}=\frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{-w}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}+\frac{3t^2(1-n)\left(3nw(\beta-1)-6(w+1)\right)+3(n+2)\left(\alpha w(n-1)+2(\beta t^2+\alpha)\right)}{(n+2)^2(\beta t^2+\alpha)^2}\right)$$

and cosmological term

$$\Lambda^{KS}=\Lambda_{1}{}_1^{KS}+\Lambda_{1}{}_2^{KS}+\Lambda_{1}{}_3^{KS}+\Lambda_{1}{}_4^{KS}$$

where

$$\Lambda_1^{KS}\frac{6n^2}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left((2\pi+\mu)\left(\alpha-(\beta-3)t^2\right)\right)$$

$$\Lambda_2^{KS}=\frac{3n}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu\left((w-5)(\beta t-\alpha)+3t^2(3w+1)\right)+12\pi(2wt^2-\beta t^2+t^2+\alpha)\right)$$

$$\Lambda_3^{KS}=\frac{6}{(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu(w-1)(\beta t^2+\alpha)+2\pi(3wt^2-2\beta t^2+t^2-4\alpha)\right)$$

and

$$\Lambda_4^{KS}=\frac{(\mu w+4\pi w-\mu)}{(w+1)c_3^2(4\pi+\mu)(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}$$

3.3. Bianchi III Universe Model

If $(\theta)=\sinh(\theta)$, then $\delta=1$, for the Bianchi III metric. In this case, the pressure and energy density of the matter are obtained as follows:

$$p^{BIII}=\frac{w}{2(4\pi+\mu)(w+1)}\left(\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2}-\frac{1}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right)$$

and

$$\rho^{BIII}=\frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{3n^2\left((\beta-3)t^2-\alpha\right)+9n\left((\beta+1)t^2-\alpha\right)+6(\beta t^2-\alpha)}{(n+2)^2(\beta t^2+\alpha)^2}-\frac{1}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right)$$

The string tension and particle density are as follows, respectively.

$$\lambda^{BIII}=\frac{1}{2(4\pi+\mu)}\left(\frac{3(n-1)\left((\beta-3)\ t^2-\alpha\right)}{(n+2)(\beta t^2+\alpha)^2}-\frac{1}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right)$$

and

$${\rho_p}^{BIII}=\frac{1}{2(4\pi+\mu)(w+1)}\left(\frac{3t^2(1-n)\left(3nw(\beta-1)-6(w+1)\right)+3(n+2)\left(\alpha w(n-1)+2(\beta t^2+\alpha)\right)}{(n+2)^2(\beta t^2+\alpha)^2}+\frac{w}{c_3^2(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}\right)$$

Cosmological term is

$$\Lambda^{BIII}=\Lambda_1^{BIII}+\Lambda_2^{BIII}+\Lambda_3^{BIII}+\Lambda_4^{BIII}$$

where

$$\Lambda_1^{BIII}=\frac{6n^2}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left((2\pi+\mu)\left(\alpha-(\beta-3)t^2\right)\right)$$

$$\Lambda_2^{BIII}=\frac{3n}{(n+2)^2(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu\left((w-5)(\beta t-\alpha)+3t^2(3w+1)\right)+12\pi(2wt^2-\beta t^2+t^2+\alpha)\right)$$

$$\Lambda_3^{BIII}=\frac{6}{(\beta t^2+\alpha)^2(w+1)(4\pi+\mu)}\left(\mu(w-1)(\beta t^2+\alpha)+2\pi(3wt^2-2\beta t^2+t^2-4\alpha)\right)$$

and

$$\Lambda_4^{BIII}=\frac{(\mu-\mu w-4\pi w)}{(w+1)c_3^2(4\pi+\mu)(\beta t^2+\alpha)^{\frac{3}{(n+2)\beta}}}$$

4. Results and Discussions

From (3.8)-(3.10) and (3.18)-(3.19), we obtain

$$\frac{\sigma}{\theta} = \frac{\sqrt{3}(n-1)}{3(n+2)} \tag{4.1}$$

$$\frac{\sigma}{H} = \frac{\sqrt{3}(n-1)}{n+2} \tag{4.2}$$

In (4.1) and (4.2), $\frac{\sigma}{\theta} = \frac{\sqrt{3}}{6} = 0.2827$ and $\frac{\sigma}{H} = \frac{2\sqrt{3}}{3} = 0.866$ is obtained, for $n = 4$. The present values of $\frac{\sigma}{\theta}$ and $\frac{\sigma}{H}$ are 0.1 and 0.3, respectively. The values we have obtained are considerably greater than these values. This indicates that our models belong to the earliest moments of the universe. In our solutions, ξ refers to the anisotropy parameter. If $\xi = 0$, our model turns into an isotropic universe model. μ is also an important parameter in our solutions. If we take $\mu = 0$ in our solutions, we obtain the General Relativity theory solutions of the cloud of string with perfect fluid for the LRS Bianchi I, Kantowski-Sachs, and Bianchi III universe models.

If $q > 0$ there is slowing expansion, if $q < 0$ there is accelerated expansion. The value that makes $q = 0$ is called transit time and is denoted by t_{tr}. The value t_{tr} for the deceleration parameter given by (3.13) is $t_{tr} = \frac{\sqrt{\alpha}}{\sqrt{\beta-1}}$. Since the value t_{tr} will be positive, it should be $\alpha > 0$ and $\beta > 1$. It should be $\alpha < t^2(\beta - 1)$ and $\beta > 1$ for a decelerating expansion ($q > 0$), and $\alpha > t^2(\beta - 1)$ and $\beta > 1$ for an accelerated expansion ($q < 0$).

Our model starts with the Big Bang at $t = 0$. The expansion in the model increases with time. It is obtained as $t_{tr} = \sqrt{\frac{\alpha}{\beta-1}}$. If the α and β values we use to draw the graphs are substituted in t_{tr}, we get $t_{tr} = 11.4018$. While the Hubble parameter increases in the time interval $(0, 1.1345)$, it decreases and approaches zero after $t = 1.1345$. While the scale factor takes the value of approximately 1.13 at$t = 0$, it approaches infinity as time increases (see Figure 1). As can be seen in Figures 2 and 3, the metric potentials A and B increase with time and go to infinity. However, when these graphs are examined, it is seen that the metric potential A approaches infinity faster than the metric potential B.

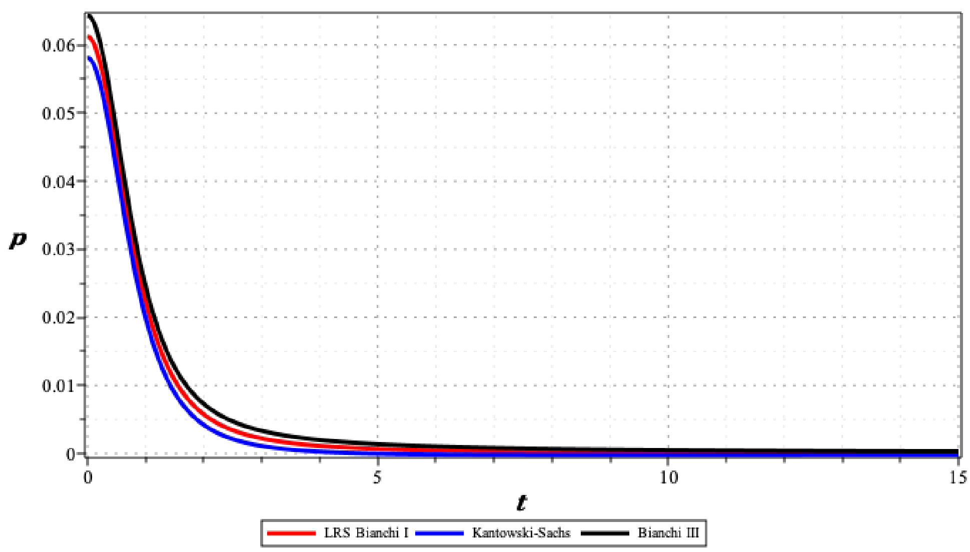

Figure 4. Variation of p with time

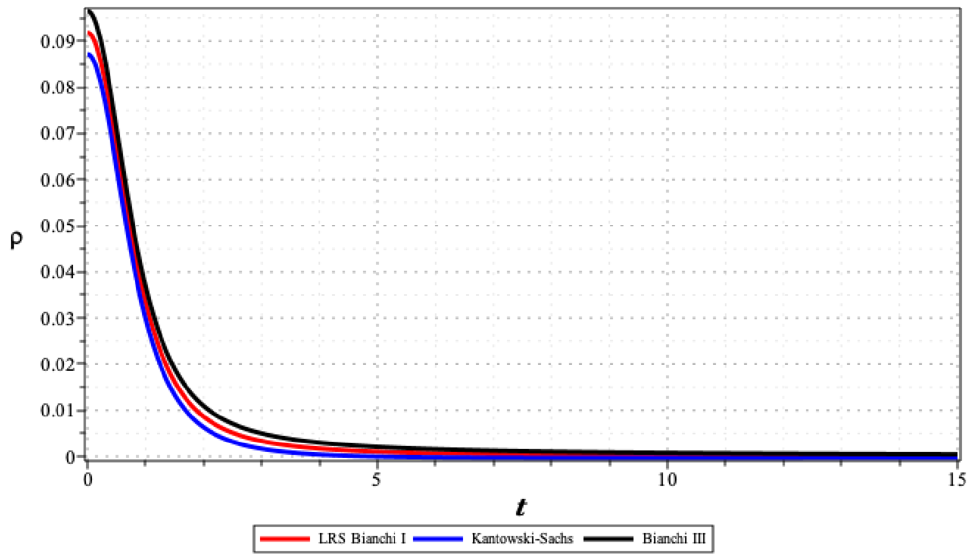

Figure 5. Variation of ρ with time

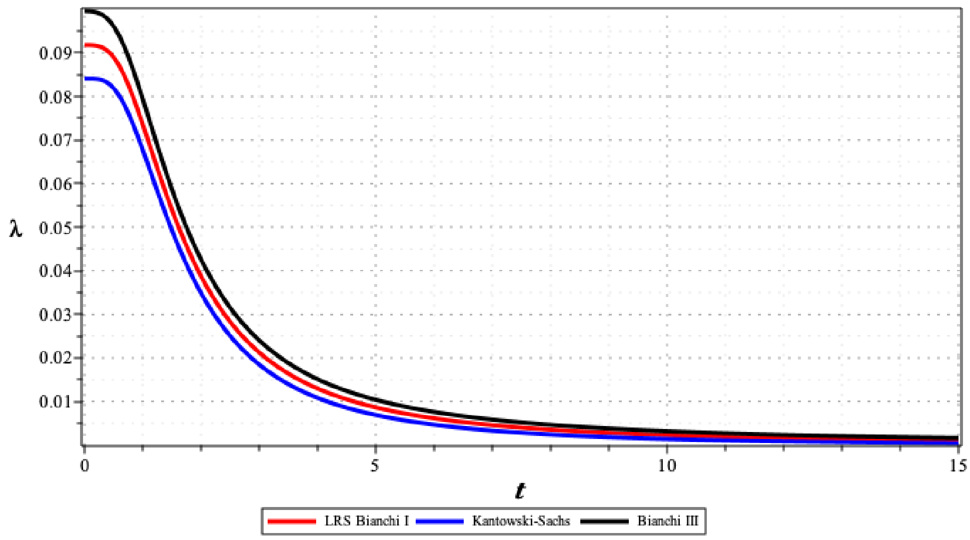

Figure 6. Variation of λ with time

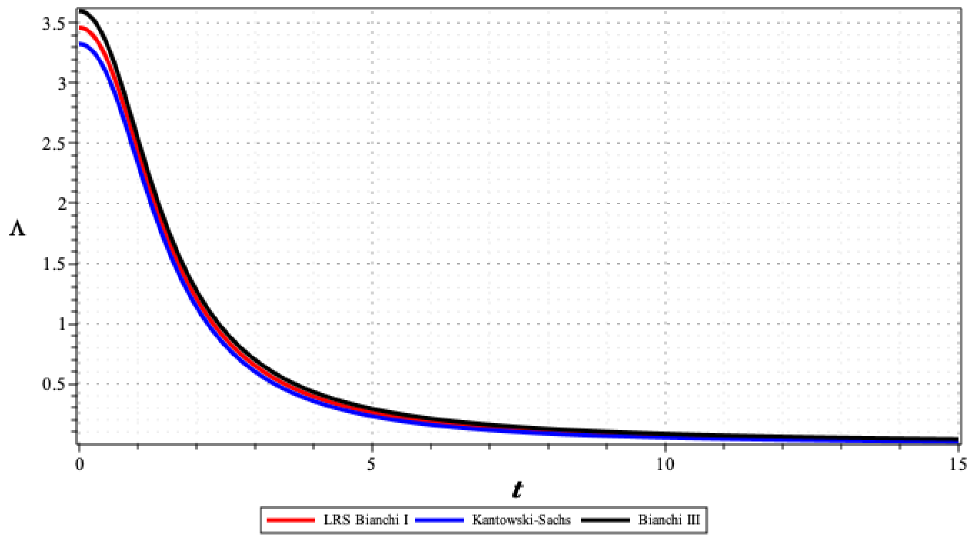

Figure 7. Variation of Λ with time

Figures 4-7 show the graphs of pressure, energy density, string tension, and cosmological term, respectively, for the solutions obtained from the considered universe models. In all LRS Bianchi I, Kantowski-Sachs, and Bianchi III universe models, total energy density, pressure, string tension, and cosmological term constant values are taken for $t \to 0$. On the other hand, it can be seen from Figures 4-7 that p, ρ, λ, and Λ decrease and approach zero in case of $t \to \infty$. When their graphs are examined, all three universe models show similar behavior. If (3.27) is examined, for $n = 1$ (i.e., $\xi = 0$) the string tension for the LRS Bianchi I universe model becomes zero.

As it is known, singular points are places where the theory of gravity is not valid. When the Hubble parameter is examined for the bouncing deceleration parameter, it is seen that it has a singularity of $t = \sqrt{-\frac{\alpha}{\beta}}$. Moreover, as can be seen from (3.20)-(3.24), p, ρ, ρ_p, and Λ have singularity in $t = \sqrt{-\frac{\alpha}{\beta}}$, $w = -1$, and $\mu = -4\pi$ values, while λ has singularity in $t = \sqrt{-\frac{\alpha}{\beta}}$ and $\mu = -4\pi$ values. For our solutions to be valid, $t \neq \sqrt{-\frac{\alpha}{\beta}}$, $w \neq -1$, and $\mu \neq -4\pi$.

5. Conclusion

In this study, universe models were studied using the bouncing deceleration parameter within the framework of $f(R,T)$ theory for the cloud of string with perfect fluid matter distribution in the LRS Bianchi I, Kantowski-Sachs and Bianchi III metrics. In order to obtain the exact solutions of the modified field equations in $f(R,T)$ theory, the bouncing deceleration parameter, the state equation and the anisotropy parameter are used. In future studies, it will be worthwhile to investigate the cloud of stirng with perfect fluid matter distribution within the framework of other alternative gravity theories such as $f(Q)$, $f(\tau)$, and $f(T,\tau)$ using different deceleration parameters of the space-time geometry.

Author Contributions

All the authors equally contributed to this work. This paper is derived from the first author's master's thesis supervised by the second author. They all read and approved the final version of the paper.

Conflicts of Interest

All the authors declare no conflict of interest.

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